ISSN: 2756-2638
Transactions on Engineering and Computing Sciences; Vol. 14 No. 04 (2026) (38-48)
https://doi.org/10.14738/tmlai.1404.11936

Laplace's Demon: The Deterministic versus the Probabilistic Nature of the Universe

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Abstract: *Essai philosophique sur les probabilités* (*A Philosophical Essay on Probability*) was a nontechnical exposition of the laws of chance written by Pierre-Simon Laplace in 1814 as an elaboration of a lecture delivered at the École Normale in 1795. In contrast to his monumental 1812 treatise, *Théorie analytique des probabilités* (*The Analytical Theory of Probability*), Laplace sought to introduce the concept of probability independently of the methods of calculus and to demonstrate that probabilistic reasoning permeates all aspects of human existence, including expectation, uncertainty, hope, and fear. He concluded the introduction of the essay with the observation: I hope that the reflections given in this essay may merit the attention of philosophers and direct it to a subject so worthy of their engaging minds. This philosophical program, however, gave rise to a profound conceptual tension. Specifically, Laplace was compelled to reconcile the apparent randomness inherent in probabilistic phenomena with his uncompromising commitment to universal determinism. This tension culminated in the celebrated thought experiment commonly referred to as *Laplace's Demon*: the hypothetical intelligence capable of knowing, at a given instant, all forces acting in nature together with the precise positions and motions of every particle in the universe. Under such conditions, Laplace argued, the future and the past would be entirely determined and therefore fully predictable. The coexistence of probabilistic analysis and strict determinism remains one of the most significant philosophical issues in the foundations of probability theory and the philosophy of science.

INTRODUCTION

Pierre-Simon de Laplace (1749 – 1827) was born in Beaumont-en-Auge, in Normandy, and attended a Benedictine school there in accordance with his father's wishes that he be ordained in the Roman Catholic Church. Fittingly, in 1765, he was sent to the University of Caen to study theology.

At the University, his aptitude for mathematics soon became evident, in particular, when his paper titled *Sur le Calcul integral aux differences infinitment petites et aux differences finies,* got accepted for publication in *Miscellanea Taurinensia*, a journal founded by Joseph Louis Lagrange (1736 – 1813) in his native city of Turin. In 1768, Laplace left the University of Caen and went to Paris to study mathematics under Jean le Rond d'Alembert (1717–1783) and shortly thereafter was appointed professor of mathematics in École Militaire de Paris. In the years that followed, he published several books and papers on mathematics, physics, and celestial mechanics, prompting him to be referred to as the *Newton of France*, and to be placed, rightfully, among the greatest scientists of all time.

As a mathematician, Laplace worked on the general theory of determinants, the theory of equations, and solutions of linear partial differential equations of the second order. He also gave a general proof of the Lagrange inversion theorem, which expresses the Taylor series expansion of the inverse of an analytic function. He formulated what is now known as the Laplace's equation, and introduced the Laplace transform which appears in many branches of mathematical physics. As well known, this is an integral transform that takes a function of a positive real variable t (usually time), $f(t)$, to a function of a complex variable s (usually frequency), $\mathcal{L}\{f\}$, by

$$\mathcal{L}\{f\} = \int_0^\infty f(t)e^{-st}dt$$

The Laplacian differential operator, which is defined for a function $f(x_1, x_2, \dots, x_n)$ as

$$\Delta f = \frac{\partial^2 f}{\partial x_1^2} + \frac{\partial^2 f}{\partial x_2^2} + \cdots + \frac{\partial^2 f}{\partial x_n^2}$$

widely used in mathematics, is also named after him. There is also a Laplace method for approximating integrals of the form

$$I = \int_a^b e^{Kf(x)}\,dx$$

where f is a twice differentiable function and K is a large number. Laplace showed that as $K \to \infty$,

$$I \approx \sqrt{\frac{2\pi}{K|f''(x_o)|}}\, e^{Kf(x_o)}$$

Here, x_0 denotes the point where f attains its global maximum.

One should note that Laplace's proofs are not always sufficiently rigorous when assessed by modern standards, but his conclusions remain essentially unassailable (Stigler 1986). His focus in physics and celestial mechanics included development of the theory of capillary action (the Young-Laplace Equation), and some work on the speed of sound. He also postulated the existence of black holes and the notion of gravitational collapse and completely determined the attraction of a spheroid on a particle outside using spherical harmonics.

In 1773, he took on one of the most outstanding unsolved problems of celestial mechanics: Jupiter's orbit appeared to be continually shrinking, while Saturn's was continually expanding. That is why Newton believed that the planetary system would need divine intervention from time to time. Laplace showed that this phenomenon was of a periodic nature and could be expected to right itself every 929 years (Wilson 1985). Indeed, this problem had also another unforeseen consequence: to obtain the solution, Laplace had to develop the ***least squares method***.

In 1784 he published *Théorie du Mouvement et de la figure elliptique des planètes*. His monumental five-volume *Mécanique Céleste* was published between 1799 and 1825. These firmly placed celestial mechanics on a sound mathematical basis.

It is also interesting to note that he was quite an astute person with a knack for malleability which allowed him to blend in perfectly with the changing political climates of the time. Indeed, due to this “flexibility” not only did he manage to subsist through the French Revolution, the Napoleonic Era, and the Restoration, but he became a count of the Empire in 1806 and was named a marquis in 1817. See Gillespie (1997), Hahn (2005), or Rousse Ball (1908), for more information.

LAPLACE’S WORK ON PROBABILITY AND STATISTICS

Without a doubt, one of Laplace’s most well-known results in probability is the De Moivre-Laplace Theorem that approximates a binomial distribution with a normal distribution. This is, of course, a special case of the central limit theorem. In particular, the theorem showed that the probability mass function of the random number of successes observed in a series of n independent Bernoulli trials, each having probability p of success with $0 < p < 1$, converged to the probability density function of the normal distribution with expected value np and variance $np(1-p)$ as n got large. In other words, for large n, the probability of x successes is given as

$$\binom{n}{x} p^x (1-p)^{n-x} \approx \frac{1}{\sqrt{2\pi np(1-p)}} e^{-\frac{(x-np)^2}{2np(1-p)}}$$

This, of course, eventually led to the establishment of the central limit theorem. In fact, according to Henk.

> The central limit theorem has an interesting history. The first version of this theorem was postulated by the French-born mathematician Abraham De Moivre who, in a remarkable article published in 1733, used the normal distribution to approximate the distribution of the number of heads resulting from many tosses of a fair coin. This finding was far ahead of its time, and was nearly forgotten until the famous French mathematician Pierre-Simon Laplace rescued it from obscurity in his monumental work Théorie analytique des probabilités, which was published in 1812. Laplace expanded De Moivre's finding by approximating the binomial distribution with the normal distribution (Henk 2004, p. 169).

Of course, as is well known,

> Nowadays, the central limit theorem is considered to be the unofficial sovereign of probability theory (Henk 2004, p.169)

It is equally important to note that the Bayesian interpretation of probability was developed mainly by Laplace. Indeed, in probability one needs to answer the question “Does probability measure the real, physical tendency of something to occur or is it a measure of how strongly one believes it will occur?” In the Bayesian interpretation pioneered by Laplace, instead of being the frequency of some phenomenon, assigned probabilities represent states of knowledge or belief. (Stigler 1986, pp. 97 – 98).

Laplace published his Mémoire sur la probabilité des causes par les événements in 1774. But most of the abovementioned results were given in his magnum opus, Théorie analytique des probabilités (1812). The first half of this treatise was concerned with probabilistic methods and problems, the second half with statistical methods and applications. See Stigler (1986).

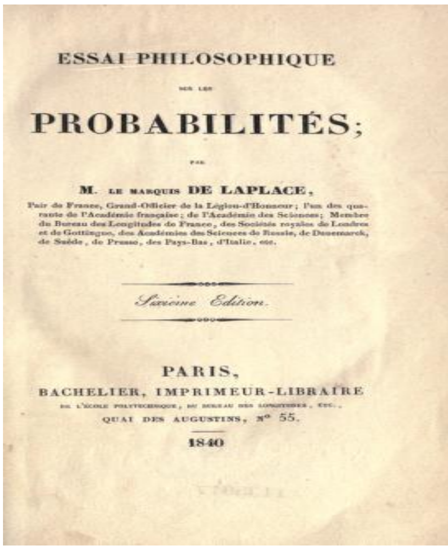
ESSAI PHILOSOPHIQUE SUR LES PROBABILITÉS AND THE TEN PRINCIPLES OF THE CALCULUS OF PROBABILITY

Essai philosophique sur les probabilités (1814) begins with Pierre-Simon Laplace clarifying that

> This philosophical essay is the development of a lecture on probabilities which I delivered in 1795 to the École Normale where I had been called, by a decree of the National Convention, as a professor of mathematics with Lagrange. I have recently published a work on the same topic entitled 'The analytical theory of probabilities'. I present here, without the aid of analysis, the principles and general results of this theory, applying them to the most important questions of life which are indeed, for the most part, only problems of probability (Laplace, 1).

After thus committing himself to a nontechnical exposition, Laplace proceeds to formulate the “theory of chance” as follows:

> … reducing all the events of the same kind to a certain number of cases equally possible, that is to say, to such as we may be equally undecided about in regard to their existence, and in determining the number of cases favorable to the event whose probability is sought. The ratio of this number to that of all the cases possible is the measure of this probability, which is thus simply a fraction whose numerator is the number of favorable cases and whose denominator is the number of all cases possible. (Laplace, 2)



ESSAI PHILOSOPHIQUE

SUR LES

PROBABILITÉS;

PAR

M. LE MARQUIS DE LAPLACE,

Pair de France, Grand-Officier de la Légion-d'Honneur; l'un des quarante de l'Académie française; de l'Académie des Sciences; Membre du Bureau des Longitudes de France, des Sociétés royales de Londres et de Gottingue, des Académies des Sciences de Russie, de Danemarck, de Suède, de Prusse, des Pays-Bas, d'Italie, etc.

Sixième Édition.

PARIS,

BACHELIER, IMPRIMEUR-LIBRAIRE

DE L'ÉCOLE POLYTECHNIQUE, DU BUREAU DES LONGITUDES, ETC.,

QUAI DES AUGUSTINS, N° 55.

1840

He then proceeds to examine the general principles of this theory in terms of ten general principles.

The First Principle. The first principle is the very definition of probability itself as the ratio of the favorable cases to that of all the cases possible.

The Second Principle. The first principle supposes that the various cases are equally possible. Else, we must first determine the respective probabilities of each event. Then, the probability is the sum of the probabilities of each favorable case.

Here Laplace gives the example of tossing an “ideal” coin twice and determining the probability of getting at least one head. He lists the four equally likely cases $\{HH, HT, TH, TT\}$ and concludes that this probability is

$$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$$

The Third Principle. This principle, which, Laplace calls “One of the most important points in the theory of probabilities and that which lends to most to illusions…” (Laplace, 5), states that if the events are independent of one another, the probability of their combined existence is the product of their respective probabilities.

Laplace’s simple example is as follows: probability of getting a one in a single roll of a die is $\frac{1}{6}$. So, probability of getting two ones in rolling two dice at the same time is $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$. Thus, Laplace concludes, probability that a simple event with probability p will occur consecutively m times is p^m, which makes such an occurrence extremely improbable. Consequently,

> … in the moral sciences, where each inference is deduced from that which precedes it only in a probable manner, however probable these deductions may be, the chance of error increases with their number, and ultimately surpasses the chance of truth… (Laplace, 5)

The Fourth Principle. “When two events depend upon each other, the probability of the compound event is the product of the probability of the first event and the probability that, this event having occurred, the second will occur” (Laplace, 6).

In modern notation,

$$P(A\cap B)=P(A)P(B|A)$$

Of course, if A, B are independent events, the fourth principle implies the third one.

Although Laplace claims that we see within this principle the influence of the past events upon the probability of the future events, this influence is, in fact, more distinctly detectable in his fifth principle:

The Fifth Principle. If we calculate *a priori* the probability of the occurred event and the probability of an event composed of that one and a second one which is expected, the second probability divided by the first will be the probability to the event expected, drawn from the observed event (Laplace, 6).

In modern notation,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The fifth principle, of course, follows from the fourth principle. The formula, clearly shows that if A is an event in the future and B an event in the past, the occurrence of B has some influence on the probability of the occurrence of A.

The Sixth Principle. The probability of any one of many events occurring is the ratio where the numerator is the probability of any of these events resulting from a cause and the denominator being the sum of all the individual events' probabilities.

The simplest way to express the sixth principle is this: If $A_1, A_2, \ldots, A_n, B$ are events in a sample space S with $A_1 \cup A_2 \cup \ldots \cup A_n = S$ and $A_i \cap A_j = \emptyset$ for all $i \neq j$, then

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n)}$$

Thus, the sixth principle is simply the Bayes' Theorem. Here is Laplace's example to illustrate this principle:

There are three urns. Urn 1 contains two white balls, urn 2 contains one white and one black, and urn 3 contains two black balls. We pick an urn and start drawing with replacement. If the first two balls are white, what is the probability that the third ball is white?

Let A be the event "the third ball is white," and B be the event "first two balls are white".

$$P(Urn\ 1|B) = \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times 0} = \frac{4}{5}$$

$$P(Urn\ 2|B) = \frac{\frac{1}{3} \times \frac{1}{4}}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times 0} = \frac{1}{5}$$

$$P(Urn\ 1|B) = \frac{\frac{1}{3} \times 0}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times 0} = 0$$

$$P(A) = 1 \times \frac{4}{5} + \frac{1}{2} \times \frac{1}{5} + 0 \times 0 = \frac{9}{10}$$

The Seventh Principle. The probability of a future event occurring is the sum of the products of the probability of each cause of the events. This is, of course, what we now refer to as the *law of total probability*:

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

As an example, suppose there are two urns, where the first one contains two white balls and the second one contains one white and one black ball. Let A be the event "the ball is white" and let B be the event urn 1 is selected. Then,

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c) = 1 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$$

One well-known formula derivable from this principle is ***the rule of succession*** (Gahramani, p. 125). Suppose that some trial has only two possible outcomes, labeled success and failure, with probability of success $p, 0 < p < 1$. Suppose this is repeated n times and we get a success each time. Then the probability of success at the next trial is

$$\frac{n+1}{n+2}$$

As an example, Laplace gives the ***sunrise problem***: Assume the earth is $5{,}000$ years old. Each year has 365.2426 days. Thus, the sun has risen for $1{,}826{,}213$ consecutive times. Hence the probability that the sun will rise the next day is

$$\frac{1{,}826{,}214}{1{,}826{,}215} = 0.9999994524$$

The rule of succession, and by extension, any result derived from it, thus in particular the sunrise problem, has been very contentious and has constantly been the object of ridicule. However, Laplace was quite mindful of the illogicality of the result:

> But this number is far greater for him who, seeing in the totality of phenomena the principle regulating the days and seasons, realizes that nothing at the present moment can arrest the course of it (Laplace, 11)

The next two principles are more philosophical in nature. According to Laplace, "The probability of events serves to determine the hope or the fear of persons interested in their existence" (Laplace, 11). Laplace defines *hope* as "the advantage of that one who expects a certain benefit in suppositions which are only probable" (Laplace, 11). He formulates this in his eighth principle:

The Eighth Principle. When the advantage depends on several events it is obtained by taking the sum of the products of the probability of each event by the benefit attached to its occurrence.

Here Laplace writes

> The probability of events serves to determine the hope or the fear of persons interested in their existence. The word *hope* has various acceptations; it expresses generally the advantage of that one who expects

a certain benefit in suppositions which are only probable…. We will call this advantage *mathematical hope* (Laplace, 11).

Laplace's example is as follows:

> Let us suppose that at the play of heads and tails Paul receives two francs if he throws heads at the first throw and five francs if he throws it at the second (Laplace, 11).

He then computes Paul's "advantage" as

$$\frac{1}{2} \times 2 + \frac{1}{4} \times 5 = 2\frac{1}{4}$$

Of course, this is simply the expected value associated with the game.

The Ninth Principle. In a series of probable events of which the ones that produce a benefit and the others a loss, we shall have the advantage which results from it by making a sum of the products of the probability of each favorable event by the benefit it procures, and subtracting from this sum that of the products of the probability of each unfavorable event by the loss which is attached to it. If the second sum is greater than the first, the benefit becomes a loss, and hope is changed to fear.

Thus, if the expected value is positive Laplace refers to it as *hope* or *advantage*, and if it is negative, he refers to it as *fear*.

Laplace goes on to write

> Consequently, we ought always in the conduct of life to make the product of the benefit hoped for, by its probability, at least equal to the similar product relative to the loss. But it is necessary, in order to attain this, to appreciate exactly the advantages, the losses, and their respective probabilities. For this a great accuracy of mind, a delicate judgement, and a great experience in affairs is necessary... (Laplace, 12)

In fact, he uses this principle to deal with the St. Petersburg Paradox.

Finally, Laplace invokes the concept of *mathematical utility*:

> Indeed, it is apparent that one franc has much greater value for him who possesses only a hundred than for a millionaire. We ought then to distinguish in the hoped-for benefit its absolute from its relative value.

To this end, he gives the tenth principle, which "… [was] proposed by Daniel Bernoulli…" (Laplace, 12):

The Tenth Principle. The relative value of an infinitely small sum is equal to its absolute value divided by the total benefit of the person interested.

Of course, this presupposes that everyone has a certain benefit whose value can never be estimated as zero. Laplace interprets this principle as follows:

> Let us designate by unity the part of the fortune of an individual, independent of his expectations. If we determine the different values that this fortune may have by virtue of these expectations and their probabilities, the product of these values raised respectively to the powers indicated by their probabilities will be the physical fortune which would procure for the individual the same moral advantage which he receives from the part of his fortune taken as unity and from his expectations; by subtracting unity from the product, the difference will be the increase of the physical fortune due to expectations: we will call this increase *moral hope* (Laplace, 13).

The mathematics is set up so that the relative value of a given risk or effort is the absolute value divided by the total worth of the individual involved. For example, if a destitute person wins the lottery it means a lot more *relatively speaking* than if a wealthy person won it. It would mean more because the gain would be so large in proportion to his existing “net worth.”

LAPLACE’S DEMON

What is rather perplexing is the fact that Laplace was a staunch and unfaltering determinist. Indeed, in this book he remarked that

> We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes (Laplace, 4).

Indeed, this “intellect” depicted in the last sentence above ("Une intelligence... Rien ne serait incertain pour elle, et l'avenir comme le passé, serait présent à ses yeux.") has come to be known as **Laplace’s demon**.

In other words, if the exact position and momentum of every particle in the universe were known—hypothetically by an intelligence such as Laplace’s demon—then, according to the laws of classical mechanics, both the past and future states of the universe could, in principle, be determined with complete precision. It is important to note, however, that Laplace himself never employed the term “demon”; subsequent commentators later introduced this designation.

Several significant objections have been raised against the notion of Laplace’s demon. First, the concept presupposes the universal validity of classical determinism and the reversibility of physical processes. Yet many thermodynamic processes are intrinsically irreversible. If thermodynamic quantities are regarded as fundamentally physical, then the irreversible increase of entropy prevents the exact reconstruction of prior states from present conditions, thereby undermining the possibility of such a demon (Ulanowicz, 1986).

Second, Laplacian determinism is fundamentally incompatible with the Copenhagen interpretation of quantum mechanics, according to which physical systems are intrinsically probabilistic rather than strictly deterministic. In quantum theory, the outcomes of measurements cannot, even in principle, be predicted with absolute certainty; instead, only probabilistic distributions can be specified.

A related objection arises from Bohr's principle of complementarity, which asserts that certain pairs of physical observables cannot be simultaneously measured with arbitrary precision. The canonical example is position and momentum, whose simultaneous determination is constrained by the structure of quantum mechanics (Bohr, 1996). Consequently, the complete information required by Laplace's demon is, in principle, unattainable.

A further challenge to strict determinism is provided by chaos theory, which demonstrates that even deterministic systems may exhibit behavior that is effectively unpredictable. Such systems display sensitive dependence on initial conditions, meaning that arbitrarily small differences in starting states can lead to dramatically divergent outcomes over time. Thus, although the governing equations are deterministic, long-term prediction becomes practically impossible.

A classical illustration of deterministic chaos is provided by the logistic map, widely used in mathematical biology to model population dynamics. The logistic map is defined by the quadratic recurrence relation

$$x_{n+1} = \gamma x_n(1 - x_n)$$

where x_0 denotes the initial population value, and the sequence $x_1, x_2, x_3, \ldots$represents successive stages in the temporal evolution of the system. The parameter γ, known as the growth rate, satisfies $0 \leq \gamma \leq 4$.

For $\gamma < 3$, the sequence converges rapidly to a stable equilibrium value. When $3 < \gamma < 3.57$, the system undergoes a sequence of period-doubling bifurcations: the population oscillates between two values, then four, then eight, and so forth. As γapproaches approximately 3.57, these bifurcations occur with increasing frequency. Finally, for $\gamma > 3.57$, the system enters a chaotic regime characterized by irregular and nonrepeating fluctuations. Although the system remains fully deterministic, its long-term behavior becomes effectively unpredictable.

The logistic map therefore illustrates a profound limitation of classical determinism: even when the underlying laws are entirely deterministic, infinitesimal uncertainties in initial conditions may render accurate long-term prediction impossible.

Thus, the question remains: is the universe fundamentally deterministic or probabilistic? Our position is that both descriptions are valid, depending on the scale under consideration. At the quantum level, namely at the subatomic scale, physical phenomena are governed by probabilistic laws rather than strict determinism. In quantum mechanics, particles are described by a wave function that represents a superposition of possible states, and only the probability of observing a particular outcome can be determined prior to measurement.

By contrast, at the macroscopic level – and that is the level Laplace was considering - the laws of physics are largely deterministic. According to both Newtonian mechanics and the theory of relativity, if the exact present state of a system and the forces acting upon it are known, its future evolution can, in principle, be predicted with precision. Consequently, while indeterminacy characterizes the quantum domain, determinism remains an effective and highly accurate description of physical reality at larger scales.

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