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SERVICES FOR SCIENCE AND EDUCATION

Knowledge Reasoning Based on the Generalized Modal Syllogism □AM◇M-1 with '*Most*'

Liheng Hao
School of Engineering and Materials Science,
Queen Mary University of London, London, United Kingdom

Chongzi Cao
School of Engineering and Materials Science,
Queen Mary University of London, London, United Kingdom

ABSTRACT

This paper firstly illustrates knowledge representation of generalized modal syllogisms based on the structure of generalized quantification propositions, and proves the validity of the generalized modal syllogism □AM◇M-1 on the basis of set theory and modal logic. And then the paper shows that the other 37 valid generalized modal syllogisms can be deduced from □AM◇M-1. Similarly, more valid generalized modal syllogisms can be derived from it. The reason why these syllogisms are reducible is that the necessary modality □ and possible one ◇ are able to define mutually, and that the two Aristotelian quantifiers *some* and *no* are symmetric.

Keywords: generalized modal syllogisms, knowledge reasoning, reducibility, validity.

INTRODUCTION

Syllogistic reasoning has become an increasingly important form of reasoning in daily natural language. The common types of syllogisms are as follows: Aristotelian syllogisms [3, 7] Aristotelian modal syllogisms [4, 5, 8, 9], generalized syllogisms [1, 6], and generalized modal syllogisms [2]. It's important to note that the following syllogisms in this paper refer to generalized modal syllogisms.

At present, there are some studies on the reducibility of generalized modal syllogisms. [10, 11] discussed the validity of other generalized modal syllogisms based on □AM◇I-1and E□M◇O-3, respectively. This paper studies the reducibility of generalized modal syllogisms based on the validity of generalized modal one □AM◇M-1. Therefore, this paper deduces the validity of other syllogisms by means of reduction on the basis of proving its validity.

KNOWLEDGE REPRESENTATION FOR GENERALIZED MODAL SLLOGISMS

In the following, let *c*, *g, x* be lexical variables, and the sets of the three variables be *C*, *G*, and *X,* respectively, and *D* be the domain of lexical variables. The cardinality of the set *C* is represented by $|C|$, and a generalized quantifier by *Q*. And let β, δ, η, and μ be well-formed formulas (abbreviated as wff). '$\vdash\beta$' says that β is provable. '$\beta=_{def}\delta$' shows that β can be defined by δ. The others are similar.

The generalized modal syllogisms discussed in this paper are composed of three of the following 24 propositions: (1) 'All *g*s are *xs*', 'No *g*s are *xs*', 'Some *g*s are *xs*', 'Not all *g*s are *xs*', 'Most *g*s are *xs*', 'Fewer than half of the *g*s are *xs*', 'At most half of the *g*s are *xs*', and 'At least half of the *g*s are *xs*'. The eight propositions can be respectively symbolized as *all(g, x)*, *no(g, x)*, *some(g, x)*, *not all(g, x)*, *most(g, x)*, *fewer than half of the(g, x)*, *at most half of the(g, x)*, *at least half of the(g, x)*, and abbreviated as the proposition *A*, *E*, *I*, *O*, *M*, *F*, *H*, and *S* respectively; (2) The necessary propositions corresponding to the above 8 propositions, that is, □*all(g, x)*, □*no(g, x)*, □*some(g, x)*, □*not all(g, x)*, □*most(g, x)*, ☑*fewer than half of the(g, x)*, □*at most half of the(g, x)*, □*at least half of the(g, x)*, and abbreviated as the proposition □*A*, □*E*, □*I*, □*O*, □*M*, □*F*, □*H*, and □*S* respectively; (3) The possible propositions corresponding to the above 8 propositions, that is, the proposition ◇*A*, ◇*E*, ◇*I*, ◇*O*, ◇*M*, ◇*F*, ◇*H*, and ◇*S* respectively.

Example 1:

- Major premise: All dogs are necessarily omnivores animals.
- Minor premise: Most of the pets in our community are dogs.
- Conclusion: Most of the pets in our community are possibly omnivores animals.

Let *g* be a variable representing pets in the domain, *c* be a variable representing dogs, and *x* be a variable representing omnivores animals. The above syllogism can be symbolized as □*all (c, x)*∧*most (g, c)*→◇*most (g, x)*, abbreviated as □AM◇M-1.

FORMAL SYSTEM OF GENERALIZED MODAL SYLLOSITIC

In order to analyze valid generalized modal syllogisms, primitive symbols, formation rules, related definitions and basic axioms, and deductive rules are as follows.

Primitive Symbols

1. lexical variables: *c*, *g*, *x*
2. quantifier: *all*, *most*
3. operators: ¬, →
4. modality: □
5. brackets: (,)

Formation Rules

1. If *Q* is a quantifier, *g* and *x* are lexical variables, then *Q (g, x)* is a wff.
2. If β and δ are wffs, then so are ¬β and β→δ.
3. Only the formulas obtained by (1) and (2) are wffs.

Basic Axioms

1. A1: If δ is a valid formula in first-order logic, then ⊢δ.
2. A2: ⊢□*all (c, x)*∧*most (g, c)*→◇*most (g, x)* (that is, the syllogism □AM◇M-1).

Relevant Definitions

- Definition 1 (truth value):
  1. *most (g, x)* is true when and only when $|G \cap X| > 0.5|G|$ is true in any real world.

2. □*all (g, x)* is true when and only when $G \subseteq X$ is true in any possible world.
3. ◇*most (g, x)* is true when and only when $|G \cap X| > 0.5|G|$ is true in at least one possible world.

- Definition 2 (biconditional): $(\beta \leftrightarrow \delta) =_{def} (\beta \rightarrow \delta) \wedge (\delta \rightarrow \beta)$.
- Definition 3 (conjunction): $(\beta \wedge \psi) =_{def} \neg(\beta \rightarrow \neg\delta)$.
- Definition 4 (possibility): $\Diamond\beta =_{def} \neg\Box\neg\beta$.
- Definition 5 (inner negation): $Q\neg(g, x) =_{def} Q(g, D-x)$.
- Definition 6 (outer negation): $(\neg Q)(g, x) =_{def}$ It is not that $Q(g, x)$.

Inference Rules

- Rule 1 (subsequent weakening): If $\vdash(\beta \wedge \delta \rightarrow \eta)$ and $\vdash(\eta \rightarrow \mu)$, then $\vdash(\beta \wedge \delta \rightarrow \mu)$.
- Rule 2 (anti-syllogism): If $\vdash(\beta \wedge \delta \rightarrow \eta)$, then $\vdash(\neg\eta \wedge \beta \rightarrow \neg\delta)$.
- Rule 3 (anti-syllogism): If $\vdash(\beta \wedge \delta \rightarrow \eta)$, then $\vdash(\neg\eta \wedge \delta \rightarrow \neg\beta)$.

Relevant Facts

- **Fact 1** (inner negation):
  (1.1) $\vdash$all(g, x)$\leftrightarrow$no$\neg$(g, x);
  (1.2) $\vdash$no(g, x)$\leftrightarrow$all$\neg$(g, x);
  (1.3) $\vdash$some(g, x)$\leftrightarrow$not all$\neg$(g, x);
  (1.4) $\vdash$not all(g, x)$\leftrightarrow$some$\neg$(g, x);
  (1.5) $\vdash$most(g, x)$\leftrightarrow$fewer than half of the$\neg$(g, x);
  (1.6) $\vdash$fewer than half of the(g, x)$\leftrightarrow$most$\neg$(g, x);
  (1.7) $\vdash$at least half of the(g, x)$\leftrightarrow$at most half of the$\neg$ (g, x);
  (1.8) $\vdash$at most half of the(g, x)$\leftrightarrow$at least half of the$\neg$ (g, x).
- **Fact 2** (outer negation):
  (2.1) $\vdash\neg$*all(g, x)*$\leftrightarrow$*not all(g, x)*;
  (2.2) $\vdash\neg$*not all(g, x)*$\leftrightarrow$*all(g, x)*;
  (2.3) $\vdash\neg$*no((g, x)*$\leftrightarrow$*some(g, x)*;
  (2.4) $\vdash\neg$*some(g, x)*$\leftrightarrow$*no(g, x)*;
  (2.5) $\vdash\neg$*most(g, x)*$\leftrightarrow$*at most half of the(g, x)*;
  (2.6) $\vdash\neg$*at most half of the(g, x)*$\leftrightarrow$*most(g, x)*.
  (2.7) $\vdash\neg$*fewer than half of the(g, x)*$\leftrightarrow$*at least half of the(g, x)*;
  (2.8) $\vdash\neg$*at least half of the(g, x)*$\leftrightarrow$*fewer than half of the(g, x)*;
- **Fact 3** (symmetry):
  (3.1) $\vdash$*some(g, x)*$\leftrightarrow$*some(x, g)*;
  (3.2) $\vdash$*no(g, x)*$\leftrightarrow$*no(x, g)*.
- **Fact 4** (dual): (4.1) $\vdash\neg\Box$*Q(g, x)*$\leftrightarrow\Diamond\neg$*Q(g, x)*; (4.2) $\vdash\neg\Diamond$*Q(g, x)*$\leftrightarrow\Box\neg$*Q(g, x)*.
- **Fact 5** (a necessary proposition implies an assertoric one): $\vdash\Box$*Q(g, x)*$\rightarrow$*Q(g, x).*
- **Fact 6** (a necessary proposition implies a possible one): $\vdash\Box$*Q(g, x)*$\rightarrow\Diamond$*Q(g, x).*
- **Fact 7** (an assertoric proposition implies a possible one): $\vdash$*Q(g, x)*$\rightarrow\Diamond$*Q(g, x).*
- **Fact 8** (subalternations):
  (8.1) $\vdash$*all(g, x)*$\rightarrow$*some(g, x)*;
  (8.2) $\vdash$*no(g, x)*$\rightarrow$*not all(g, x)*;

(8.3) $\vdash$*all(g, x)*$\rightarrow$*most(g, x)*;
(8.4) $\vdash$*most(g, x)*$\rightarrow$*some(g, x)*;
(8.5) $\vdash$*at least half of the(g, x)*$\rightarrow$*some(g, x)*;
(8.6) $\vdash$*all(g, x)*$\rightarrow$*at least half of the(g, x)*;
(8.7) $\vdash$*at most half of the(g, x)*$\rightarrow$*not all(g, x)*;
(8.8) $\vdash$*fewer than half of the(g, x)*$\rightarrow$*not all(g, x).*

The above Fact 5 to Fact 8 are basic knowledge in modal logic or generalized quantifier theory. Then their proofs are omitted.

REDUCIBLE RELATIONS BETWEEN VALID GENERALIZED MODAL SYLLOGISMS

The following Theorem 1 shows that $\Box$AM$\Diamond$M-1 is valid. '$\Box$AM$\Diamond$M-1$\rightarrow\Box$AM$\Diamond$I-1' in Theorem 2 says that the validity of syllogism $\Box$AM$\Diamond$I-1 can be deduced from that of the syllogism $\Box$AM$\Diamond$M-1. In other words, there is reducible relations between these two generalized modal syllogisms. The others are similar.

Theorem 1 ($\Box$AM$\Diamond$M-1): The generalized modal syllogism $\Box$*all (c, x)*$\wedge$*most (g, c)*$\rightarrow$ $\Diamond$*most (g, x)* is valid.

Proof: $\Box$AM$\Diamond$M-1 is the abbreviation of the syllogism $\Box$*all (c, x)*$\wedge$*most (g, c)*$\rightarrow\Diamond$*most (g, x)*. Suppose that $\Box$*all (c, x)* and *most (g, c)* are true. Then in virtue of Definition (1.2), $\Box$*all (c, x)* is true just in case $C\subseteq X$ is true in any possible world. In the same way, *most (g, c)* is true just in case $|G\cap C|>0.5|G|$ is true in line with Definition (1.1). Hence, it is easy to see that $C\subseteq X$ and $|G\cap C|>0.5|G|$ are true. It follows that $|G\cap X|>0.5|G|$ in at least one possible world. Then, $\Diamond$*most (g, x)* is true on the basis of the Definition (1.3). This proves that the first figure syllogism $\Box$*all (c, x)*$\wedge$*most (g, c)*$\rightarrow\Diamond$*most (g, x)* is valid, just as required.

Theorem 2: The following 37 valid generalized modal syllogisms can be deduced from $\Box$AM$\Diamond$M-1:

(1) $\Box$AM$\Diamond$M-1$\rightarrow\Box$AM$\Diamond$I-1
(2) $\Box$AM$\Diamond$M-1$\rightarrow\Box$AM$\Diamond$I-1$\rightarrow$M$\Box$A$\Diamond$I-4
(3) $\Box$AM$\Diamond$M-1$\rightarrow\Box$A$\Box$HH-2
(4) $\Box$AM$\Diamond$M-1$\rightarrow\Box$A$\Box$HH-2$\rightarrow\Box$A$\Box$HO-2
(5) $\Box$AM$\Diamond$M-1$\rightarrow\Box$A$\Box$HH-2$\rightarrow\Box$A$\Box$H$\Diamond$H-2
(6) $\Box$AM$\Diamond$M-1$\rightarrow\Box$A$\Box$HH-2$\rightarrow\Box$A$\Box$HO-2$\rightarrow\Box$A$\Box$H$\Diamond$O-2
(7) $\Box$AM$\Diamond$M-1$\rightarrow\Box$AM$\Diamond$I-1$\rightarrow\Box$A$\Box$EH-2
(8) $\Box$AM$\Diamond$M-1$\rightarrow\Box$AM$\Diamond$I-1$\rightarrow\Box$A$\Box$EH-2$\rightarrow\Box$A$\Box$EH-4
(9) $\Box$AM$\Diamond$M-1$\rightarrow\Box$AM$\Diamond$I-1$\rightarrow\Box$A$\Box$EH-2$\rightarrow\Box$A$\Box$E$\Diamond$H-2
(10) $\Box$AM$\Diamond$M-1$\rightarrow\Box$AM$\Diamond$I-1$\rightarrow\Box$A$\Box$EH-2$\rightarrow\Box$A$\Box$EH-4$\rightarrow\Box$A$\Box$E$\Diamond$H-4
(11) $\Box$AM$\Diamond$M-1$\rightarrow\Box$A$\Box$HH-2$\rightarrow\Box$HM$\Diamond$O-3
(12) $\Box$AM$\Diamond$M-1$\rightarrow\Box$A$\Box$HH-2$\rightarrow\Box$A$\Box$HO-2$\rightarrow\Box$HA$\Diamond$O-3
(13) $\Box$AM$\Diamond$M-1$\rightarrow\Box$A$\Box$HH-2$\rightarrow\Box$A$\Box$H$\Diamond$H-2$\rightarrow\Box$H$\Box$M$\Diamond$O-3
(14) $\Box$AM$\Diamond$M-1$\rightarrow\Box$A$\Box$HH-2$\rightarrow\Box$A$\Box$HO-2$\rightarrow\Box$A$\Box$H$\Diamond$O-2$\rightarrow\Box$H$\Box$A$\Diamond$O-3
(15) $\Box$AM$\Diamond$M-1$\rightarrow\Box$AM$\Diamond$I-1$\rightarrow\Box$A$\Box$EH-2$\rightarrow\Box$EM$\Diamond$O-3

(16) □AM◇M-1→□AM◇I-1→□A□EH-2→□EM◇O-3→□EM◇O-4
(17) □AM◇M-1→□AM◇I-1→□A□EH-2→□A□E◇H-2→□E□M◇O-3
(18) □AM◇M-1→□AM◇I-1→□A□EH-2→□A□E◇H-2→□E□M◇O-3→□E□M◇O-4
(19) □AM◇M-1→□A□HH-2→□A□HO-2→□HA◇O-3→□AA◇M-1
(20) □AM◇M-1→□A□HH-2→□A□H◇H-2→□H□M◇O-3→□A□M◇M-1
(21) □AM◇M-1→□A□HH-2→□A□H◇H-2→□H□M◇O-3→□A□M◇M-1→□A□M◇I-1□
(22) □AM◇M-1→□A□HH-2→□A□H◇H-2→□H□M◇O-3→□A□M◇M-1→□A□M◇I-1→□M□A◇I-4
(23) □AM◇M-1→□A□HH-2→□A□HO-2→□A□H◇O-2→□H□A◇O-3→□A□A◇M-1
(24) □AM◇M-1→□AM◇I-1→□A□EH-2→□A□E◇H-2→□A□E◇O-2
(25) □AM◇M-1→□AM◇I-1→□A□EH-2→□A□EH-4→□A□E◇H-4→□A□E◇O-4
(26) □AM◇M-1→□EM◇F-1
(27) □AM◇M-1→□EM◇F-1→□EM◇F-2
(28) □AM◇M-1→□EM◇F-1→□EM◇O-1
(29) □AM◇M-1→□EM◇F-1→□EM◇O-1→□EM◇O-2
(30) □AM◇M-1→□A□HH-2→□E□SH-2
(31) □AM◇M-1→□A□HH-2→□E□SH-2→□E□SH-1
(32) □AM◇M-1→□A□HH-2→□E□SH-2→□E□SO-2
(33) □AM◇M-1→□A□HH-2→□E□SH-2→□E□SO-2→□E□SO-1
(34) □AM◇M-1→□A□HH-2→□E□SH-2→□E□S◇H-2
(35) □AM◇M-1→□A□HH-2→□E□SH-2→□E□S◇H-2→□E□S◇H-1
(36) □AM◇M-1→□A□HH-2→□E□SH-2→□E□SO-2→□E□S◇O-2
(37) □AM◇M-1→□A□HH-2→□E□SH-2→□E□SO-2→□E□S◇O-2→□E□S◇O-1

Proof:

[1] ⊢□*all (c, x)*∧*most (g, c)*→◇*most (g, x)* (i.e. □AM◇M-1)
[2] ⊢□*all(c, x)*∧*most(g, c)*→◇*some(g, x)* (i.e. □AM◇I-1, by [1] and Fact (8.4))
[3] ⊢□*all(c, x)*∧*most(g, c)*→◇*some(x, g)* (i.e. M□A◇I-4, by [2] and Fact (3.1))
[4] ⊢¬◇*most(g, x)*∧□*all(c, x)*→¬*most(g, c)* (by [1] and Rule 2)
[5] ⊢□¬*most(g, x)*∧□*all(c, x)*→*at most half of the(g, c)* (by [4], Fact (2.5) and Fact (4.2))
[6] ⊢□*at most half of the(g, x)*∧□*all(c, x)*→*at most half of the(g, c)* (i.e. □A□HH-2, by [5] and Fact (2.5))
[7] ⊢□*at most half of the(g, x)*∧□*all(c, x)*→*not all(g, c)*(i.e. □A□HO-2, by [6] and Fact (8.7))
[8] ⊢□*at most half of the(g, x)*∧□*all(c, x)*→◇*at most half of the(g, c)*(i.e. □A□H◇H-2, by [6] and Fact 7)
[9] ⊢□*at most half of the(g, x)*∧□*all(c, x)*→◇*not all(g, c)*(i.e. □A□H◇O-2, by [7] and Fact 7)
[10] ⊢¬◇*some(g, x)*∧□*all(c, x)*→¬*most(g, c)* (by [2] and Rule 2)
[11] ⊢□¬*some(g, x)*∧□*all(c, x)*→*at most half of the(g, c)* (by [10], Fact (2.5) and Fact (4.2))
[12] ⊢□*no(g, x)*∧□*all(c, x)*→*at most half of the(g, c)*(i.e. □A□EH-2, by [11] and Fact (2.4))
[13] ⊢□*no(x, g)*∧□*all(c, x)*→*at most half of the(g, c)* (i.e. □A□EH-4, by [12] and Fact (3.2))
[14] ⊢□*no(g, x)*∧□*all(c, x)*→◇*at most half of the(g, c)*(i.e. □A□E◇H-2, by [12] and Fact 7)
[15] ⊢□*no(x, g)*∧□*all(c, x)*→◇*at most half of the(g, c)*(i.e. □A□E◇H-4, by [13] and Fact 7)
[16] ⊢¬*at most half of the(g, c)*∧□*at most half of the(g, x)*→¬□*all(c, x)*(by [6] and Rule 2)

[17] $\vdash most(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \Diamond \neg all(c, x)$ (by [16], Fact (2.6) and Fact (4.1))
[18] $\vdash most(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \Diamond not\ all(c, x)$ (i.e. $\Box$HM$\Diamond$O-3, by [17] and Fact (2.1))
[19] $\vdash \neg not\ all(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \neg \Box all(c, x)$ (by [7] and Rule 2)
[20] $\vdash all(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \Diamond \neg all(c, x)$ (by [19], Fact (2.2) and Fact (4.1))
[21] $\vdash all(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \Diamond not\ all(c, x)$(i.e. $\Box$HA$\Diamond$O-3, by [20] and Fact (2.1))
[22] $\vdash \neg \Diamond at\ most\ half\ of\ the(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \neg \Box all(c, x)$(by [8] and Rule 2)
[23] $\vdash \Box \neg at\ most\ half\ of\ the(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \Diamond \neg all(c, x)$(by [22], Fact (4.1) and Fact (4.2))
[24] $\vdash \Box most(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \Diamond not\ all(c, x)$(i.e. $\Box$H$\Box$M$\Diamond$O-3, by [23], Fact (2.1) and Fact (2.6))
[25] $\vdash \neg \Diamond not\ all(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \neg \Box all(c, x)$ (by [9] and Rule 2)
[26] $\vdash \Box \neg not\ all(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \Diamond \neg all(c, x)$ (by [25], Fact (4.1) and Fact (4.2))
[27] $\vdash \Box all(g, c) \wedge \Box at\ most\ half\ of\ the(g, x) \rightarrow \Diamond not\ all(c, x)$(i.e. $\Box$H$\Box$A$\Diamond$O-3, by [26], Fact (2.1) and Fact (2.2))
[28] $\vdash \neg at\ most\ half\ of\ the(g, c) \wedge \Box no(g, x) \rightarrow \neg \Box all(c, x)$ (by [12] and Rule 2)
[29] $\vdash most(g, c) \wedge \Box no(g, x) \rightarrow \Diamond \neg all(c, x)$ (by [28], Fact (2.6) and Fact (4.1))
[30] $\vdash most(g, c) \wedge \Box no(g, x) \rightarrow \Diamond not\ all(c, x)$ (i.e. $\Box$EM$\Diamond$O-3, by [29] and Fact (2.1))
[31] $\vdash most(g, c) \wedge \Box no(x, g) \rightarrow \Diamond not\ all(c, x)$ (i.e. $\Box$EM$\Diamond$O-4, by [30] and Fact (3.2))
[32] $\vdash \neg \Diamond at\ most\ half\ of\ the(g, c) \wedge \Box no(g, x) \rightarrow \neg \Box all(c, x)$ (by [14] and Rule 2)
[33] $\vdash \Box \neg at\ most\ half\ of\ the(g, c) \wedge \Box no(g, x) \rightarrow \Diamond \neg all(c, x)$ (by [32], Fact (4.1) and Fact (4.2))
[34] $\vdash \Box most(g, c) \wedge \Box no(g, x) \rightarrow \Diamond not\ all(c, x)$(i.e. $\Box$E$\Box$M$\Diamond$O-3, by [33], Fact (2.1) and Fact (2.6))
[35] $\vdash \Box most(g, c) \wedge \Box no(x, g) \rightarrow \Diamond not\ all(c, x)$ (i.e. $\Box$E$\Box$M$\Diamond$O-4, by [34] and Fact (3.2))
[36] $\vdash \neg \Diamond not\ all(c, x) \wedge all(g, c) \rightarrow \neg \Box at\ most\ half\ of\ the(g, x)$ (by [21] and Rule 2)
[37] $\vdash \Box \neg not\ all(c, x) \wedge all(g, c) \rightarrow \Diamond \neg at\ most\ half\ of\ the(g, x)$(by [36], Fact (4.1) and Fact (4.2))
[38] $\vdash \Box all(c, x) \wedge all(g, c) \rightarrow \Diamond most(g, x)$ (i.e. $\Box$AA$\Diamond$M-1, by [37], Fact (2.2) and Fact (2.6))
[39] $\vdash \neg \Diamond not\ all(c, x) \wedge \Box most(g, c) \rightarrow \neg \Box at\ most\ half\ of\ the(g, x)$ (by [24] and Rule 2)
[40] $\vdash \Box \neg not\ all(c, x) \wedge \Box most(g, c) \rightarrow \Diamond \neg at\ most\ half\ of\ the(g, x)$(by [39], Fact (4.1) and Fact (4.2))
[41] $\vdash \Box all(c, x) \wedge \Box most(g, c) \rightarrow \Diamond most(g, x)$ (i.e. $\Box$A$\Box$M$\Diamond$M-1, by [40], Fact (2.2) and Fact (2.6))
[42] $\vdash \Box all(c, x) \wedge \Box most(g, c) \rightarrow \Diamond some(g, x)$ (i.e. $\Box$A$\Box$M$\Diamond$I-1, by [41] and Fact (8.4))
[43] $\vdash \Box all(c, x) \wedge \Box most(g, c) \rightarrow \Diamond some(x, g)$ (i.e. $\Box$M$\Box$A$\Diamond$I-4, by [42] and Fact (3.1))
[44] $\vdash \neg \Diamond not\ all(c, x) \wedge \Box all(g, c) \rightarrow \neg \Box at\ most\ half\ of\ the(g, x)$ (by [27] and Rule 2)
[45] $\vdash \Box \neg not\ all(c, x) \wedge \Box all(g, c) \rightarrow \Diamond \neg at\ most\ half\ of\ the(g, x)$ (by [44], Fact (4.1) and Fact (4.2))
[46] $\vdash \Box all(c, x) \wedge \Box all(g, c) \rightarrow \Diamond most(g, x)$(i.e. $\Box$A$\Box$A$\Diamond$M-1, by [45], Fact (2.2) and Fact (2.6))
[47] $\vdash \Diamond at\ most\ half\ of\ the(g, c) \rightarrow not\ all(g, c)$ (by Fact (8.7))
[48] $\vdash \Box no(g, x) \wedge \Box all(c, x) \rightarrow \Diamond not\ all(g, c)$ (i.e. $\Box$A$\Box$E$\Diamond$O-2, by [14] and [47])
[49] $\vdash \Box no(x, g) \wedge \Box all(c, x) \rightarrow \Diamond not\ all(g, c)$ (i.e. $\Box$A$\Box$E$\Diamond$O-4, by [15] and Fact (8.7))
[50] $\vdash \Box no \neg (c, x) \wedge most(g, c) \rightarrow \Diamond fewer\ than\ half\ of\ the \neg (g, x)$ (by [1], Fact (1.1) and Fact (1.5))
[51] $\vdash \Box no\ (c, D–x) \wedge most\ (g, c) \rightarrow \Diamond fewer\ than\ half\ of\ the\ (g, D–x)$ (i.e. $\Box$EM$\Diamond$F-1, by [50] and Definition 5)

[52] ⊢□*no (D–x, c)*∧*most (g, c)*→◇*fewer than half of the (g, D–x)* (i.e. □EM◇F-2, by [51] and Fact (3.2))
[53] ⊢□*no (c, D–x)*∧*most (g, c)*→◇*not all (g, D–x)* (i.e. □EM◇O-1, by [51] and Fact (8.8))
[54] ⊢□*no (D–x, c)*∧*most (g, c)*→◇*not all (g, D–x)* (i.e. □EM◇O-2, by [53] and Fact (3.2))
[55] ⊢□*at least half of the*¬ *(g, x)*∧□*no*¬*(c, x)*→*at most half of the (g, c)* (by [6], Fact (1.1) and Fact (1.8))
[56] ⊢□*at least half of the (g, D–x)*∧□*no (c, D–x)*→*at most half of the (g, c)* (i.e. □E□SH-2, by [55] and Definition 5)
[57] ⊢□*at least half of the (g, D–x)*∧□*no (D–x, c)* →*at most half of the (g, c)* (i.e. □E□SH-1, by [56] and Fact (3.2))
[58] ⊢□*at least half of the (g, D–x)*∧□*no (c, D–x)*→*not all (g, c)* (i.e. □E□SO-2, by [56] and Fact (8.7))
[59] ⊢□*at least half of the (g, D–x)*∧□*no (D–x, c)* →*not all (g, c)* (i.e. □E□SO-1, by [58] and Fact (3.2))
[60] ⊢□*at least half of the (g, D–x)*∧□*no (c, D–x)*→◇*at most half of the (g, c)* (i.e. □E□S◇H-2, by [56] and Fact 7)
[61] ⊢□*at least half of the (g, D–x)*∧□*no (D–x, c)* →◇*at most half of the (g, c)* (i.e. □E□S◇H-1, by [60] and Fact (3.2))
[62] ⊢□*at least half of the (g, D–x)*∧□*no (c, D–x)*→◇*not all (g, c)* (i.e. □E□S◇O-2, by [58] and Fact 7)
[63] ⊢□*at least half of the (g, D–x)*∧□*no (D–x, c)* →◇*not all (g, c)* (i.e. □E□S◇O-1, by [62] and Fact (3.2))

Up until now, the above 37 valid generalized modal syllogisms have been inferred from the syllogism □AM◇M-1 on the basis of 63 reasoning steps. This shows that there are reducible relationships between/among these 38 valid syllogisms.

CONCLUSION

This paper firstly illustrates knowledge representations of generalized modal syllogisms based on the structure of generalized quantification propositions, and proves the validity of the generalized modal syllogism □AM◇M-1 on the basis of set theory and modal logic. And then the paper shows that the other 37 valid generalized modal syllogisms can be deduced from □AM◇M-1. Similarly, more valid generalized modal syllogisms can be derived from □AM◇M-1. The reason why these syllogisms are reducible is that the necessary modality □ and possible one ◇ are able to define mutually, and that the two Aristotelian quantifiers *some* and *no* are symmetric. This paper not only provides a reducible approach for studying different generalized modal syllogisms in methodology, but also provides theoretical basis for knowledge representation and reasoning in related applications of artificial intelligence.

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