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The Deductibility of the Aristotelian Modal Syllogism ◇E□I◇O-1 from the Perspective of Knowledge Reasoning

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ABSTRACT

Syllogistic reasoning has a significant impact on human society and human thinking. This paper firstly proves the validity of ◇E□I◇O-1, which has great significance to ensure the soundness of all subsequent proofs, then there is other 41 valid Aristotelian modal syllogisms derived from ◇E□I◇O-1 with the help of relevant rules, facts, and definitions. There are reducible relations between different Aristotelian modal syllogisms, so it has been given that consistent results for them. This paper studies their formalization and deductibility from the perspective of knowledge reasoning, and contributes to providing theoretical support for natural language information processing in computer science area.

Keywords: Aristotelian modal syllogisms, Deductibility, Quantifiers, Knowledge reasoning

INTRODUCTION

Syllogistic reasoning has a significant impact on human society and human thinking. There are different types of syllogisms, such as Aristotelian syllogisms [6], Aristotelian modal syllogisms [13], generalized syllogisms [4], etc. The topic of this paper is Aristotelian modal syllogisms. Some scholars studied Aristotelian modal syllogisms, such as [1, 3, 7, 8, 9, 10], and so on. But some of their results are inconsistent.

This paper aims to give consistent results for the syllogistic. The specific method firstly is to prove the validity of the syllogism ◇E□I◇O-1, and then regards it as a basic axiom to derive the other 41 valid Aristotelian modal syllogisms on the basis of classical logic [5], modal logic [2] and generalized quantifier theory [11].

PRELIMINARIES

In this paper, the letters *m*, *r* and *x* represent lexical variables. The sets composed of *m*, *r* and *x* are respectively *M*, *R* and *X*. The domain of lexical variables is denoted by *D*. Let *s*, *t*, *u* and *v* be propositional variables, and '$\vdash s$' means that *s* is provable. '$s =_{\text{def}} t$' means that *s* is defined by *t*. 'if and only if' is shortened as 'iff'. There are four Aristotelian quantifiers (i.e., *all*, *no*, *some*, *not all*), any of which is denoted by *Q*, its inner negation by $Q\neg$ and its outer negation by $\neg Q$.

Aristotelian syllogisms involve the following four types of propositions: 'all *m*s are *x*s', 'no *m*s are *x*s', 'some *m*s are *x*s', and 'not all *m*s are *x*s', which can be symbolized as *all* (*m*, *x*), *no* (*m*, *x*), *some* (*m*, *x*), and *not all* (*m*, *x*), and be shortened to Proposition *A*, *E*, *I*, *O*, respectively. Figures of syllogisms are defined as usual [12]. An Aristotelian modal syllogism is obtained only by

adding one to three non-overlapping possible modalities (◇) and/or necessary ones (□) to an Aristotelian syllogism.

The following example demonstrates how to formalize instances of Aristotelian modal syllogisms in natural language:

- Major premise: No English books are possibly on the desk.
- Minor premise: Some books are necessarily English books.
- Conclusion: Not all books are possibly on the desk.

m stands for all books in the domain, *x* for all things on the desk, and *r* for all English books. Then this example can be formalized as $\Diamond no\ (r, x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond not\ all\ (m, x))$, and abbreviated as ◇E□I◇O-1, that is a first figure syllogism. The others are similar.

According to modal logic and generalized quantifier theory, it is obtained that the following definitions, facts and rules:

Definition 1:

(1.1) $all\ (m, x)$ is true iff $M \subseteq X$ is true in any real world.
(1.2) $no\ (m, x)$ is true iff $M \cap X = \emptyset$ is true in any real world.
(1.3) $some\ (m, x)$ is true iff $M \cap X \neq \emptyset$ is true in any real world.
(1.4) $not\ all\ (m, x)$ is true iff $M \nsubseteq X$ is true in any real world.
(1.5) $\Box all\ (m, x)$ is true iff $M \subseteq X$ is true in any possible world.
(1.6) $\Box no\ (m, x)$ is true iff $M \cap X = \emptyset$ is true in any possible world.
(1.7) $\Box some\ (m, x)$ is true iff $M \cap X \neq \emptyset$ is true in any possible world.
(1.8) $\Box not\ all\ (m, x)$ is true iff $M \nsubseteq X$ is true in any possible world.
(1.9) $\Diamond all\ (m, x)$ is true iff $M \subseteq X$ is true in at least one possible world.
(1.10) $\Diamond no\ (m, x)$ is true iff $M \cap X = \emptyset$ is true in at least one possible world.
(1.11) $\Diamond some\ (m, x)$ is true iff $M \cap X \neq \emptyset$ is true in at least one possible world.
(1.12) $\Diamond not\ all\ (m, x)$ is true iff $M \nsubseteq X$ is true in at least one possible world.

Definition 2

$Q\neg(m, x) =_{\text{def}} Q\ (m, D-x)$.

Definition 3

$\neg Q\ (m, x) =_{\text{def}}$ It is not that $Q\ (m, x)$.

Fact 1 (outer negation):

(1.1) $\vdash \neg all\ (m, x) \rightarrow not\ all\ (m, x)$;
(1.2) $\vdash \neg no\ (m, x) \rightarrow some\ (m, x)$;
(1.3) $\vdash \neg some\ (m, x) \rightarrow no\ (m, x)$;
(1.4) $\vdash \neg not\ all\ (m, x) \rightarrow all\ (m, x)$.

Fact 2 (inner negation):

(2.1) $\vdash all\ (m, x) \rightarrow no\neg(m, x)$;

(2.2) $\vdash no\ (m, x) \rightarrow all \neg (m, x)$;
(2.3) $\vdash some\ (m, x) \rightarrow not\ all \neg (m, x)$;
(2.4) $\vdash not\ all\ (m, x) \rightarrow some \neg (m, x)$.

Fact 3 (symmetry):
(3.1) $\vdash some\ (m, x) \rightarrow some\ (x, m)$;
(3.2) $\vdash no\ (m, x) \rightarrow no\ (x, m)$.

Fact 4 (subalternation):
(4.1) $\vdash all\ (m, x) \rightarrow some\ (m, x)$;
(4.2) $\vdash no\ (m, x) \rightarrow not\ all\ (m, x)$;
(4.3) $\vdash \Box Q\ (m, x) \rightarrow Q\ (m, x)$;
(4.4) $\vdash \Box Q\ (m, x) \rightarrow \Diamond Q\ (m, x)$.

Fact 5 (dual):
(5.1) $\vdash \neg \Diamond Q\ (m, x) \rightarrow \Box \neg Q\ (m, x)$;
(5.2) $\vdash \neg \Box \mathrm{Q}\ (m, x) \rightarrow \Diamond \neg Q\ (m, x)$.

Rule 1 (subsequent weakening): If $\vdash (s \rightarrow (t \rightarrow u))$ and $\vdash (u \rightarrow v)$, then $\vdash (s \rightarrow (t \rightarrow v))$.

Rule 2 (anti-syllogism): If $\vdash (s \rightarrow (t \rightarrow u))$, then $\vdash (\neg u \rightarrow (s \rightarrow \neg t))$ or $\vdash (\neg u \rightarrow (t \rightarrow \neg s))$.

KNOWLEDGE REASONING BASED ON THE ARISTOTELIAN MODAL SYLLOGISM ◇E□I◇O-1

In the following, the validity of the Aristotelian modal syllogism ◇E□I◇O-1 is proved in Theorem 1. '(2.1) ◇E□I◇O-1→◇E□I◇O-3' in Theorem 2 characterizes the valid syllogism ◇E□I◇O-3 is derived from ◇E□I◇O-1. This indicates that the syllogism ◇E□I◇O-1 is deductive. Others are similar to this.

Theorem 1 (◇E□I◇O-1)
The Aristotelian modal syllogism $\Diamond no\ (r, x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond not\ all\ (m, x))$ is valid.

Proof
'◇E□I◇O-1' is the abbreviation of the syllogism $\Diamond no\ (r, x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond not\ all\ (m, x))$. Assumed that $\Diamond no\ (r, x)$ and $\Box some\ (m, r)$ are true, then $R \cap X = \emptyset$ is true in at least one possible world by Definition (1.10), and $M \cap R \neq \emptyset$ is true in any possible world by Definition (1.7). Thus, it is obvious that $M \nsubseteq X$ is true in at least one possible world by Definition (1.12). This can also be proved by reductio ad absurdum. Supposed that $M \nsubseteq X$ is not true. That is to say, $M \subseteq X$ is true. And it has been proven that $R \cap X = \emptyset$ is true. Then, it can be obtained that $M \cap R = \emptyset$, which contradicts with $M \cap R \neq \emptyset$. So, $M \subseteq X$ is not true, which means that $M \nsubseteq X$ is true. Therefore, it follows that $\Diamond no\ (r, x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond not\ all\ (m, x))$ is valid, exactly as desired.

Theorem 2

There are at least the following 41 valid Aristotelian modal syllogisms deduced from the syllogism ◇E□I◇O-1.

(2.1) ◇E□I◇O-1→◇E□I◇O-3
(2.2) ◇E□I◇O-1→◇E□I◇O-3→◇E□I◇O-4
(2.3) ◇E□I◇O-1→◇E□I◇O-3→◇E□I◇O-4→◇E□I◇O-2
(2.4) ◇E□I◇O-1→□A□I□I-3
(2.5) ◇E□I◇O-1→□A□I□I-3→□I□A□I-3
(2.6) ◇E□I◇O-1→□A□I□I-3→□I□A□I-3→□I□A□I-4
(2.7) ◇E□I◇O-1→□A□I□I-3→□I□A□I-3→□I□A□I-4→□A□I□I-1
(2.8) ◇E□I◇O-1→□A□I□I-3→□A□II-3
(2.9) ◇E□I◇O-1→□A□I□I-3→□A□II-3→□I□AI-3
(2.10) ◇E□I◇O-1→□A□I□I-3→□A□II-3→□I□AI-3→□A□II-1
(2.11) ◇E□I◇O-1→□A□I□I-3→□A□II-3→□I□AI-3→□A□II-1→□I□AI-4
(2.12) ◇E□I◇O-1→□A□I□I-3→□A□I◇I-3
(2.13) ◇E□I◇O-1→□A□I□I-3→□A□I◇I-3→□A□I◇I-1
(2.14) ◇E□I◇O-1→□A□I□I-3→□A□I◇I-3→□A□I◇I-1→□I□A◇I-4
(2.15) ◇E□I◇O-1→□A□I□I-3→□A□I◇I-3→□A□I◇I-1→□I□A◇I-4→□I□A◇I-3
(2.16) ◇E□I◇O-1→◇E□A◇E-2
(2.17) ◇E□I◇O-1→◇E□A◇E-2→◇E□A◇O-2
(2.18) ◇E□I◇O-1→◇E□A◇E-2→□A◇E◇E-2
(2.19) ◇E□I◇O-1→◇E□A◇E-2→□A◇E◇E-2→□A◇E◇O-2
(2.20) ◇E□I◇O-1→◇E□A◇E-2→◇E□A◇E-1
(2.21) ◇E□I◇O-1→◇E□A◇E-2→◇E□A◇E-1→◇E□A◇O-1
(2.22) ◇E□I◇O-1→◇A□I◇I-1
(2.23) ◇E□I◇O-1→◇A□I◇I-1→◇A□I◇I-3
(2.24) ◇E□I◇O-1→◇A□I◇I-1→◇A□I◇I-3→□I◇A◇I-3
(2.25) ◇E□I◇O-1→◇A□I◇I-1→◇A□I◇I-3→□I◇A◇I-3→□I◇A◇I-4
(2.26) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3
(2.27) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□I□O-4
(2.28) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□I□O-4→□E□I□O-2
(2.29) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□I□O-4→□E□I□E-2→□E□I□O-1
(2.30) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□IO-3
(2.31) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□IO-3→□E□IO-4
(2.32) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□IO-3→□E□IO-4→□E□IO-2
(2.33) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□IO-3→□E□IO-4→□E□IO-2→□E□IO-1
(2.34) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□I◇O-3
(2.35) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□I◇O-3→□E□I◇O-4
(2.36) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□I◇O-3→□E□I◇O-4→□E□I◇O-2
(2.37) ◇E□I◇O-1→◇A□I◇I-1→□E□I□O-3→□E□I◇O-3→□E□I◇O-1
(2.38) ◇E□I◇O-1→◇A□I◇I-1→◇A□E◇E-2
(2.39) ◇E□I◇O-1→◇A□I◇I-1→◇A□E◇E-2→◇A□E◇E-4
(2.40) ◇E□I◇O-1→◇A□I◇I-1→◇A□E◇E-2→◇A□E◇E-4→□E◇A◇E-1

(2.41) ◇E□I◇O-1→◇A□I◇I-1→◇A□E◇E-2→◇A□E◇E-4→□E◇A◇E-1→□E◇A◇E-2

Proof

[1] $\vdash \Diamond no\ (r, x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond not\ all\ (m, x))$ (i.e., ◇E□I◇O-1, Theorem 1)
[2] $\vdash \Diamond no\ (r, x) \rightarrow (\Box some\ (r, m) \rightarrow \Diamond not\ all\ (m, x))$ (i.e., ◇E□I◇O-3, by [1] and Fact (3.1))
[3] $\vdash \Diamond no\ (x, r) \rightarrow (\Box some\ (r, m) \rightarrow \Diamond not\ all\ (m, x))$ (i.e., ◇E□I◇O-4, by [2] and Fact (3.2))
[4] $\vdash \Diamond no\ (x, r) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond not\ all\ (m, x))$ (i.e., ◇E□I◇O-2, by [3] and Fact (3.1))
[5] $\vdash \neg\Diamond not\ all\ (m, x) \rightarrow (\Box some\ (m, r) \rightarrow \neg\Diamond no\ (r, x))$ (by [1] and Rule 2)
[6] $\vdash \Box\neg not\ all\ (m, x) \rightarrow (\Box some\ (m, r) \rightarrow \Box\neg no\ (r, x))$ (by [5], Fact (5.1) and Fact (5.2))
[7] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (m, r) \rightarrow \Box some\ (r, x))$ (i.e., □A□I□I-3, by [6], Fact (1.2) and (1.4))
[8] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (m, r) \rightarrow \Box some\ (x, r))$ (i.e., □I□A□I-3, by [7] and Fact (3.1))
[9] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (r, m) \rightarrow \Box some\ (x, r))$ (i.e., □I□A□I-4, by [8] and Fact (3.1))
[10] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (r, m) \rightarrow \Box some\ (r, x))$ (i.e., □A□I□I-1, by [9] and Fact (3.1))
[11] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (m, r) \rightarrow some\ (r, x))$ (i.e., □A□II-3, by [7] and Fact (4.3))
[12] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (m, r) \rightarrow some\ (x, r))$ (i.e., □I□AI-3, by [11] and Fact (3.1))
[13] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (r, m) \rightarrow some\ (r, x))$ (i.e., □A□II-1, by [12] and Fact (3.1))
[14] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (r, m) \rightarrow some\ (x, r))$ (i.e., □I□AI-4, by [13] and Fact (3.1))
[15] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond some\ (r, x))$ (i.e., □A□I◇I-3, by [7] and Fact (4.4))
[16] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (r, m) \rightarrow \Diamond some\ (r, x))$ (i.e., □A□I◇I-1, by [15] and Fact (3.1))
[17] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (r, m) \rightarrow \Diamond some\ (x, r))$ (i.e., □I□A◇I-4, by [16] and Fact (3.1))
[18] $\vdash \Box all\ (m, x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond some\ (x, r))$ (i.e., □I□A◇I-3, by [17] and Fact (3.1))
[19] $\vdash \neg\Diamond not\ all\ (m, x) \rightarrow (\Diamond no\ (r, x) \rightarrow \neg\Box some\ (m, r))$ (by [1] and Rule 2)
[20] $\vdash \Box\neg not\ all\ (m, x)) \rightarrow (\Diamond no\ (r, x) \rightarrow \Diamond\neg some\ (m, r))$ (by [19], Fact (5.1) and (5.2))
[21] $\vdash \Box all\ (m, x) \rightarrow (\Diamond no\ (r, x) \rightarrow \Diamond no\ (m, r))$ (i.e., ◇E□A◇E-2, by [20], Fact (1.3) and (1.4))
[22] $\vdash \Box all\ (m, x) \rightarrow (\Diamond no\ (r, x) \rightarrow \Diamond not\ all\ (m, r))$ (i.e., ◇E□A◇O-2, by [21] and Fact (4.2))
[23] $\vdash \Box all\ (m, x) \rightarrow (\Diamond no\ (r, x) \rightarrow \Diamond no\ (r, m))$ (i.e., □A◇E◇E-2, by [21] and Fact (3.2))
[24] $\vdash \Box all\ (m, x) \rightarrow (\Diamond no\ (r, x) \rightarrow \Diamond not\ all\ (r, m))$ (i.e., □A◇E◇O-2, by [23] and Fact (4.2))
[25] $\vdash \Box all\ (m, x) \rightarrow (\Diamond no\ (x, r) \rightarrow \Diamond no\ (m, r))$ (i.e., ◇E□A◇E-1, by [21] and Fact (3.2))
[26] $\vdash \Box all\ (m, x) \rightarrow (\Diamond no\ (x, r) \rightarrow \Diamond not\ all\ (m, r))$ (i.e., ◇E□A◇O-1, by [25] and Fact (4.2))
[27] $\vdash \Diamond all\neg(r, x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond some\neg(m, x))$ (by [1], Fact (2.2) and (2.4))
[28] $\vdash \Diamond all\ (r, D-x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond some\ (m, D-x))$ (i.e., ◇A□I◇I-1, by [27] and Definition 2)
[29] $\vdash \Diamond all\ (r, D-x) \rightarrow (\Box some\ (r, m) \rightarrow \Diamond some\ (m, D-x))$ (i.e., ◇A□I◇I-3, by [28] and Fact (3.1))
[30] $\vdash \Diamond all\ (r, D-x) \rightarrow (\Box some\ (r, m) \rightarrow \Diamond some\ (D-x, m))$ (i.e., □I◇A◇I-3, by [29] and Fact (3.1))
[31] $\vdash \Diamond all\ (r, D-x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond some\ (D-x, m))$ (i.e., □I◇A◇I-4, by [30] and Fact (3.1))
[32] $\vdash \neg\Diamond some\ (m, D-x) \rightarrow (\Box some\ (m, r) \rightarrow \neg\Diamond all\ (r, D-x))$ (by [28] and Rule 2)
[33] $\vdash \Box\neg some\ (m, D-x) \rightarrow (\Box some\ (m, r) \rightarrow \Box\neg all\ (r, D-x))$ (by [32], Fact (5.1) and (5.2))
[34] $\vdash \Box no\ (m, D-x) \rightarrow (\Box some\ (m, r) \rightarrow \Box not\ all\ (r, D-x))$ (i.e., □E□I□O-3, by [33], Fact (1.1) and (1.3))

[35] $\vdash \Box no\ (D\text{–}x, m) \rightarrow (\Box some\ (m, r) \rightarrow \Box not\ all\ (r, D\text{–}x))$ (i.e., □E□I□O-4, by [34] and Fact (3.2))
[36] $\vdash \Box no\ (D\text{–}x, m) \rightarrow (\Box some\ (r, m) \rightarrow \Box not\ all\ (r, D\text{–}x))$ (i.e., □E□I□O-2, by [35] and Fact (3.1))
[37] $\vdash \Box no\ (m, D\text{–}x) \rightarrow (\Box some\ (r, m) \rightarrow \Box not\ all\ (r, D\text{–}x))$ (i.e., □E□I□O-1, by [36] and Fact (3.2))
[38] $\vdash \Box no\ (m, D\text{–}x) \rightarrow (\Box some\ (m, r) \rightarrow not\ all\ (r, D\text{–}x))$ (i.e., □E□IO-3, by [34] and Fact (4.3))
[39] $\vdash \Box no\ (D\text{–}x, m) \rightarrow (\Box some\ (m, r) \rightarrow not\ all\ (r, D\text{–}x))$ (i.e., □E□IO-4, by [38] and Fact (3.2))
[40] $\vdash \Box no\ (D\text{–}x, m) \rightarrow (\Box some\ (r, m) \rightarrow not\ all\ (r, D\text{–}x))$ (i.e., □E□IO-2, by [39] and Fact (3.1))
[41] $\vdash \Box no\ (m, D\text{–}x) \rightarrow (\Box some\ (r, m) \rightarrow not\ all\ (r, D\text{–}x))$ (i.e., □E□IO-1, by [40] and Fact (3.2))
[42] $\vdash \Box no\ (m, D\text{–}x) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond not\ all\ (r, D\text{–}x))$ (i.e., □E□I◇O-3, by [34] and Fact (4.4))
[43] $\vdash \Box no\ (D\text{–}x, m) \rightarrow (\Box some\ (m, r) \rightarrow \Diamond not\ all\ (r, D\text{–}x))$ (i.e., □E□I◇O-4, by [42] and Fact (3.2))
[44] $\vdash \Box no\ (D\text{–}x, m) \rightarrow (\Box some\ (r, m) \rightarrow \Diamond not\ all\ (r, D\text{–}x))$ (i.e., □E□I◇O-2, by [43] and Fact (3.1))
[45] $\vdash \Box no\ (m, D\text{–}x) \rightarrow (\Box some\ (r, m) \rightarrow \Diamond not\ all\ (r, D\text{–}x))$ (i.e., □E□I◇O-1, by [42] and Fact (3.1))
[46] $\vdash \neg\Diamond some\ (m, D\text{–}x) \rightarrow (\Diamond all\ (r, D\text{–}x) \rightarrow \neg\Box some\ (m, r))$ (by [28] and Rule 2)
[47] $\vdash \Box\neg some\ (m, D\text{–}x) \rightarrow (\Diamond all\ (r, D\text{–}x) \rightarrow \Diamond\neg some\ (m, r))$ (by [46], Fact (5.1) and (5.2))
[48] $\vdash \Box no\ (m, D\text{–}x) \rightarrow (\Diamond all\ (r, D\text{–}x) \rightarrow \Diamond no\ (m, r))$ (i.e., ◇A□E◇E-2, by [47] and Fact (1.3))
[49] $\vdash \Box no\ (D\text{–}x, m) \rightarrow (\Diamond all\ (r, D\text{–}x) \rightarrow \Diamond no\ (m, r))$ (i.e., ◇A□E◇E-4, by [48] and Fact (3.2))
[50] $\vdash \Box no\ (D\text{–}x, m) \rightarrow (\Diamond all\ (r, D\text{–}x) \rightarrow \Diamond no\ (r, m))$ (i.e., □E◇A◇E-1, by [49] and Fact (3.2))
[51] $\vdash \Box no\ (m, D\text{–}x) \rightarrow (\Diamond all\ (r, D\text{–}x) \rightarrow \Diamond no\ (r, m))$ (i.e., □E◇A◇E-2, by [50] and Fact (3.2))

So far, it has been proven that there are 41 valid syllogisms deduced from ◇E□I◇O-1 by making full use of the deductibility of the Aristotelian modal syllogism ◇E□I◇O-1.

CONCLUSION AND FUTURE WORK

On the basis of classical logic, modal logic and generalized quantifier theory, this paper firstly proves the validity of ◇E□I◇O-1, which has great significance to ensure the soundness of all subsequent proofs, then there are the other 41 valid Aristotelian modal syllogisms derived from ◇E□I◇O-1 with the help of the above definitions, facts, and rules. It should be illustrated that there is a reducible relation between different Aristotelian modal syllogisms, so it has been given that consistent results for them. This paper studies their formalization and deductibility from the perspective of knowledge reasoning, and contributes to providing theoretical support for natural language information processing in computer science area.

How many Aristotelian modal syllogisms are needed at least to derive all the remaining valid Aristotelian modal ones? This question will be left for studying in the future.

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