Transactions on Engineering and Computing Sciences - Vol. 11, No. 5
Publication Date: October 25, 2023
DOI:10.14738/tecs.115.15656.
Kim, Y. M. (2023). A Study on the Eigen Properties of the Coaxial Waveguide of Cylindrical Form. Transactions on Engineering and Computing Sciences, 11(5). 69-77.
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A Study on the Eigen Properties of the Coaxial Waveguide of Cylindrical Form

Yeong Min Kim
Kyonggi University, Korea

ABSTRACT

In this study, the eigen properties of the coaxial waveguide of cylindrical form is investigated by using finite element method. The eigenmatrix equation constructed from the Helmholtz vector equation is too large to derivethe results using a personal computer. Therefore, the eigen equations are compressed using the Arnoldi algorithm and after that the results are derived using the Krylov-Schur iteration method. The similarity transformation matrixused during this process contains the desired eigenmode pair. The eigenmodes are simultaneously included in thecolumn matrix components of the transform matrix. These are represented with the pairs of the electric field and electric potential. The eigenmodes have been divided into two classes: transverse magnetic modes and transverseelectric modes. As results, in order to more clearly reveal the characteristics of the eigenmodes, these results are shown in the figure.

Keyword: coaxial waveguide, similarity transformation, eigenmode, transverse magnetic mode, transverse electric mode, iteration method.

INTRODUCTION

In the previous paper, it was mentioned that the eigenmodes of a waveguide or the cavity are related to the properties of the constituent materials and geometric structure [1]. In this paper, the eigenmode formed in a cavity whose horizontal length of the rectangular is 1.5 times larger than that in reference [2] has been revealed. The eigenmodes were promoted similar to them of reference [2] as the order increased. In particular, it was confirmed that as the order of the eigenmode increases, complex characteristics appear in the feature of electric field. From these results, it was identified that various types of eigenmodes can be expected simply by changing the geometric structure. A rectangular cavity is obtained by a cross-section of infinitely extended waveguide. Therefore, the cavity exhibits the transverse cross-sectional characteristics of the waveguide. Meanwhile, there are results showing that when the composition of the waveguide changes, the characteristics of the eigenmode take on various forms. Photonic crystal waveguides are the most representative example of this result [3][4]. In these studies, various dielectrics are symmetrically inserted into the waveguide, and the eigenmodes of square or cylindrical photonic crystals are determined. The results obtained from these studies showed that the eigenmodes varied depending on the relative position and permittivity of the inserted dielectrics. Based on such studies, various waveguide characteristics can be expected to be able to obtained through changes in the geometrical structure and diversity of constituent materials.

In this study, the eigen properties of the coaxial waveguide is investigated by FEM (Finite Element Method). So far, many experiments and studies have been reported on coaxial waveguides. Among those, we have published the studies and results on the eigen properties of coaxial waveguides at reference [2][3]. In reference [2], the waveguide has a structure similar to that of this study, but the core region is an empty space without material. The waveguide in reference [3] is similar to that in this study, but the core and the cladding region wrapping it have different material properties. In this manuscript, the results of a study on those waveguides that similar to previous studies in terms of composite materials and geometry is revealed. This study is intended to reveal the unique characteristics of coaxial waveguides, such as in these references, and is therefore extending those studies with a review and supplement from them.

For the calculation of FEM, the space of waveguide cross section is divided into the triangular mash. Using the edge and node of the mesh element, the vector Helmholtz equation is reconstructed into the eigen matrix form. The size of the eigen matrix equation is proportional to the number of meshes made up with edges and nodes.

However, the accuracy of calculation increases as the number of triangular elements making up the mesh increases. To overcome this contradiction, FEM uses the Arnoldi algorithm to compress the matrix equations. Afterwards, the Krylov-Schur circulation method is used to find several eigenmodes with the highest reliability. This method is used in this study to find the desired eigenmodes and these results are revealed with the schematic representation. As stated in previous studies, this study omits the theory and formula derivation process for FEM. Therefore, the description format of this manuscript deviates from the existing order and is described as follows. This manuscript describes the structure of waveguide for FEM calculation, the result and discussion of the simulation, and conclusion in order.

THE COAXIAL WAVEGUIDE AND FEM

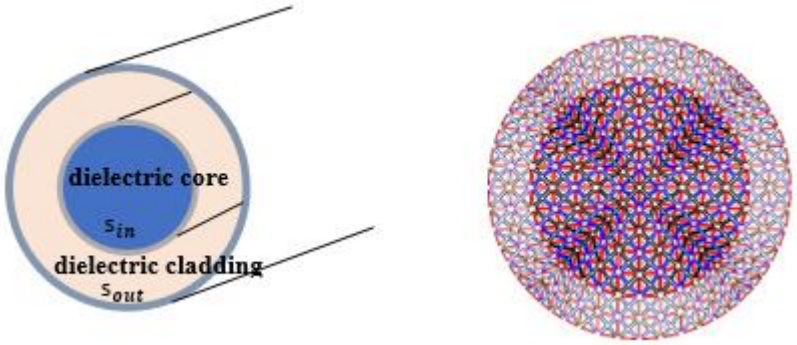


Figure 1: The coaxial waveguide and the mesh for FEM calculation

Figure 1 is the schematic representation of the transverse cross section of coaxial waveguide. A waveguide exhibits a homogeneous dielectric constant in the longitudinal direction, and its value changes in a stepwise manner in the transverse direction. The relative dielectric constants are assumed to be ε_{in} = 10 and ε_{out}=1 for the dielectric core and cladding respectively. Therefore, the generated eigenmodes are expected to be concentrated in the core

region with high dielectric constant according to Snell's law. The mesh is as first set as two-dimensional body centered square unit cell and then refined into a circularly symmetric structure. Each unit cell is composed of four triangular elements. In this study, cross sectional space is divided into triangular elements of 15 × 15 × 4 = 900. Here, the number 15 can be artificially converted to a another set externally to perform FEM calculations. Among the triangular elements of the mesh, the edges and nodes form the electric field and electric potential respectively.

In particular, the edges of the triangular elements that make up the mesh must form a consistent direction with their neighbors, as shown in the figure 2.

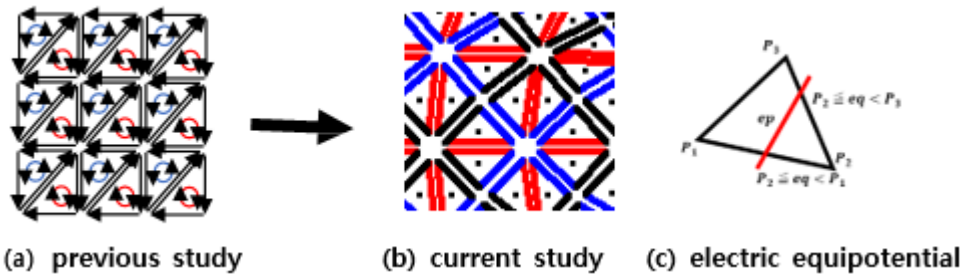


Figure 2: The mesh, edges and nodes of triangular elements.

Figure 2(a) is the relationship between edges used previously when performing FEM, and (b) is the correlation between edges used in the current study. In the case of Figure 2(b), the rotational direction is $\mathbf{1} \rightarrow \mathbf{2} \rightarrow \mathbf{3}$ depending on the color of each edge. The electric field vectors at each edge obtained in these directions are combined according to reference [1] and marked at the barycentric point of the triangular element. In Figure 2(b), the barycentric point of each triangular element is indicated by a black dot. Since figure (b) decomposes the space in more detail compared to (a), the accuracy of calculations is expected to be further improved. The edge and the node numbers are in this study is 1204 and 421 respectively. The node points of the triangular element describe the electric potential which is simultaneously obtained with the electric field. These values are also used when drawing equipotential contours by interpolation as shown in (c). The electric potential at a point inside the triangular element can be obtained by combining the shape function as reference [1].

The numbers of edges and nodes increase as the mesh number increased. The matrix eigen equation is composited with edges and nodes of the triangular mesh. Therefore, the size of the matrix eigen equation increase proportional to the number of the mesh. The size of the matrix eigen equation constructed by adding the number of edges and nodes is 1625 × 1625. In order to handle the matrix equation of this size, sophisticated algorithm must be used. In this study, the Arnoldi algorithm is applied to compress this equation so that it can be easily handled. As a result, the compressed matrix equation has the size of 20 × 20 square matrix. Afterwards, this square matrix is transformed into a Schur form using the Krylov-Schur iteration method [5]. In iteration process, the Schur matrix is established by similarity transformation matrices of 20 × 1625. The eigenmodes of the coaxial waveguide are described by column matrices of the similarity transforming matrix. Those column matrices contain the contribution from the edges

and nodes combing to represent the electric field and electric potential according to reference [1]. As a result, the electric field and electric potential obtained in this way are shown in the figures of schematic representation.

THE RESULTS AND DISCUSSION

The results of FEM calculation are summarized with the schematic representation of figures 4 and 5. They are divided into two classes: TM (Transverse Magnetic modes) and TE (Transverse Electric modes) modes. The two types proceed the same in FEM calculations, but one difference is the boundary conditions. In the case of TM mode, the calculation is performed by reconstructing the eigen equation with the surface components of the waveguide removed. Therefore, the tangential component of electric field only permitted without that of vertical one at the surface. The eigenmodes are represented with two component pairs of the electric field and electric potential pairs. Those eigen pairs are compensated for each other when determining the eigen properties. The eigenvalues are obtained simultaneously with the eigen pairs and appeared on the diagonal component of the Schur matrix. However, these values were omitted in this study. This is because the mesh is adjusted depending on where the electric field density is concentrated. When the mesh changes, the eigenvalues will show different values accordingly, resulting in a lack of consistency compared to other spectra.

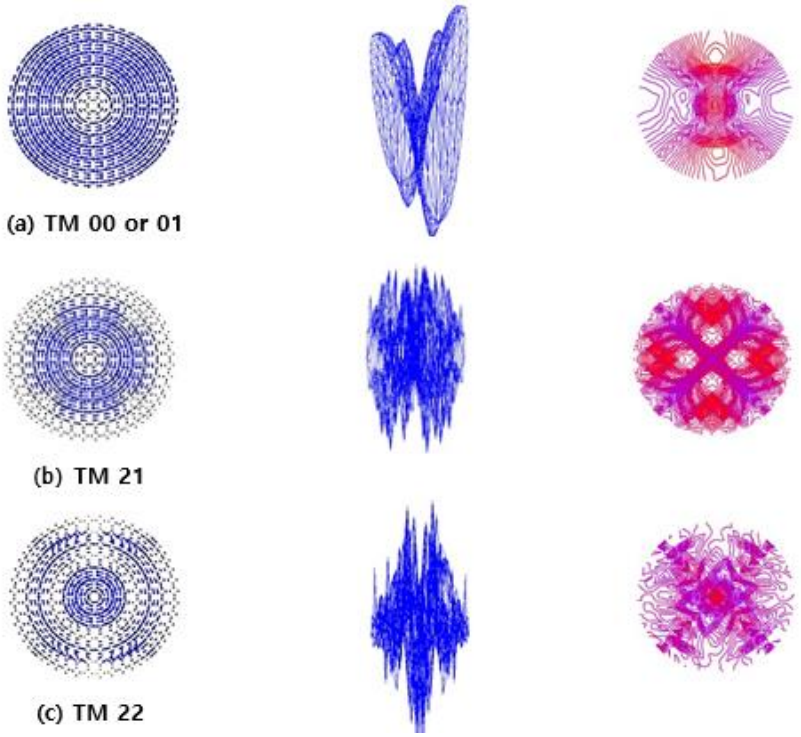
Figure 4 shows the eigenpairs of TM mode. The eigenmode of electric field well represents the intrinsic characteristics of the waveguide with high resolution. As can be seen in the figure, the electric field in TM mode only has a tangential component on the surface of waveguide and excludes the perpendicular one. Comparing to this, the resolution of electric potential is complicated. Therefore, the eigenmode is determined dominantly with the electric field and the electric potential only assistant it.



Figure 3: Comparison between the spectra of reference [2] and TM 12

What makes this study different from previous ones is that the dielectric cladding wrapping the core thickly. The cladding affect forming the eigenmode as shown the figure 3. The spectrum revealed in reference [2] shows similar eigenmodes although there is a difference in resolution compared to that in this study. As shown in this figure, the electric field eigenmode spreads out in concentric circles from the center. Figure 3(a) is result from the cylindrical waveguide without the dielectric core unlike that of (b). The eigenmode of figure (b) shows that the electric field is shielded by the cladding and do not extending further. However, the electric field is concentrated in the core, which has a high dielectric constant, similar to the geometric shape of a waveguide. Contrast to the electric field, the electric potential is not simple. Because three-dimensional expressions show complex patterns with various small peaks overlapping with the

main peak, it is difficult to determine the eigenmode with the electric potential alone. The electric field manifestly describe the eigenmode. But the electric potential must be deeply considered for confirming it. Including the electric field, they do not harmoniously describe a simple eigenmode. Therefore, it would be reasonable to analyze the eigenmode through the distribution of the electric field by excluding the small peak of the electric potential and considering only the main peak. Then, TM 12 can be assigned to the eigenmode of spectrum (b).



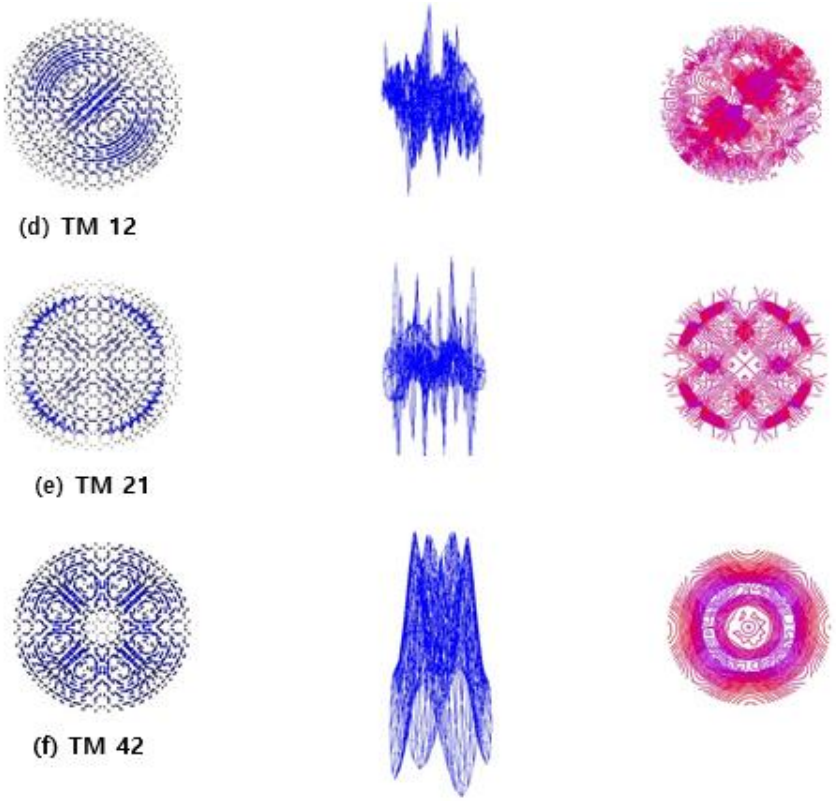


Figure 4: The schematic representation of TM mode

The eigenmode can be assigned for each spectrum referring to the electric field assisted with the electric potential. The decisive factor for this process is the electric field component distributed in the core area. The intensity of the electric field distributed in the cladding area is small and does not affect the intensity of the core. Therefore, as discussed in Figure 3, the potential distribution in the cladding region, which complicates the spectrum, is ignored. Based on these standards, the eigenmodes assigned to each spectrum are inserted in the figure. In the TE mode, like the TM mode, small potential peaks in the cladding region make it difficult to determine the eigenmode. Figure 5 shows spectra composed of low-order TE modes. In each spectrum, the contribution of the cladding region is ignored and an eigenmode is imposed according to the distribution of electric field and potential in the core region. One exception is the assignment of eigenmodes to the spectrum (a) of the lowest order. As shown in the figure 6 (a) and (b), this spectrum shows the shape of the electric field extending across the entire waveguide, including the cladding region. Changes in permittivity with the medium does not significantly affect the distribution of electric field in this mode. Even if the same eigenmode TE 10 is assigned, the spatial distribution of electric fields is not the same. As shown in (b), it is presumed that the narrow electric field distribution connected to the center and both ends of the waveguide is due to the influence of cladding, but the cause cannot be determined. Cladding appears to influence the spatial distribution of the electric field and the formation of the eigenmode.

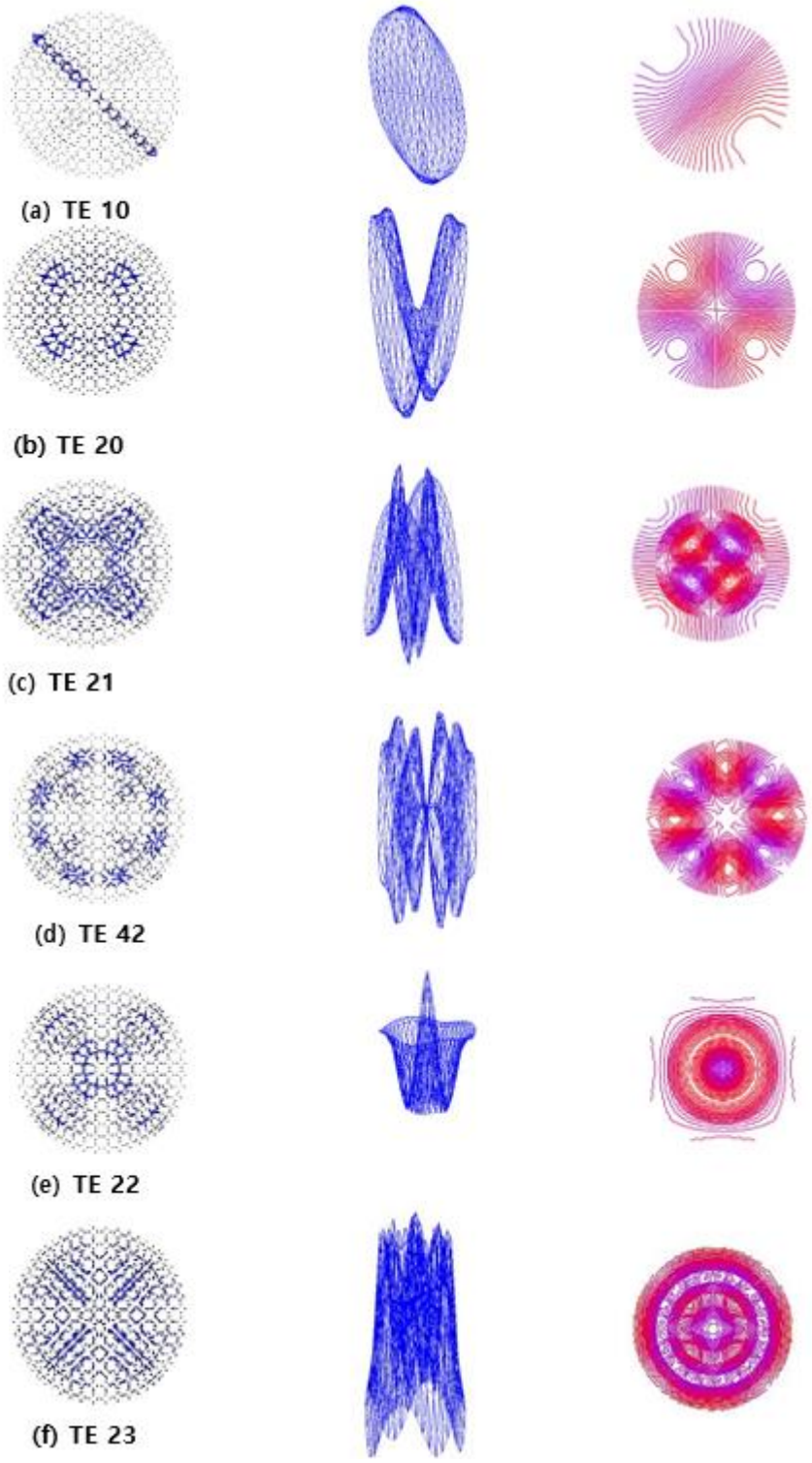


Figure 5: The schematic representation of TE modes



Figure 6: Comparison between the spectra of reference [2] and TE 10

The eigenmodes described by spectrum (b), (c) and (d) reflect the characteristics of the coaxial waveguide. The electric field is distributed intensively in the core area and is not so strong that its presence cannot be confirmed in the cladding area. The distribution of electrical equipotential also exhibits the same characteristics. The main peaks are distributed symmetrically around the horizontal and vertical axes. Despite the presence of small peaks, there is no difficulty in assigning an eigenmode because the shape of the electric potential is clear. Considering the electric field and potential distributed in the core excluding the cladding, eigenmodes of TE 20, TE 21 and TE 42 respectively can be assigned to these spectra. Figure 5(e) shows another spectrum which reveal eigenmode of TE 22.

To determine the component of radial eigenmode, the core region must be investigated precisely. The radial eigenmode in the core of electric potential shows the strong peak composited with four components. Since these peaks show strong characteristics in a small area, the equipotential line must be analyzed in detail to determine the eigenmode in the radial direction. This spectrum is combined with the electric field to be able to determining for the 2-radial component of eigenmode. The three-dimensional representation of the electric potential is combined with the electric field distribution to determine the azimuthal component of the eigenmode. Summarizing the above discussion, it is reasonable to determine that the eigenmode of spectrum (e) is TE 22. In Figure 5(f), the main peak and minor components are combined to complicate the spectrum. As mentioned earlier, the contribution of small components is excluded in the eigenmode assignment process. Then, the eigenmode component in the radial direction is determined by the electric field. The electric potential is used to determine the azimuth component. And, the eigenmode of this spectrum can be assigned to be TE 23.

CONCLUSION

The eigenmode pairs characterizing the coaxial waveguide have been investigated using FEM. The resulting spectra have been divided into TM-mode and TE-mode, and these were shown in schematic representation. The dielectric cladding has a great influence on the eigenmodes formed in the waveguide and complicates the spectrum. The eigenmodes determined by the electric field and potential have been revealed in the figure along with their respective spectra. The characteristics of the coaxial waveguide have been discussed by comparing several eigenmodes with those mentioned in reference [2]. From these discussions, it would be learned about the possibility of finely controlling the eigenmode of the core region by changing the geometric shape or physical properties of the dielectric cladding.

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