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ADDENDUM TO: A Study on the Eigen Properties of the Coaxial Waveguide of Cylindrical Form

Yeong Min Kim
Kyonggi University, Korea

ABSTRACT

The physical properties of the dielectric cladding wrapping the cylindrical waveguide were investigated by finite element method. These properties are described by transverse electric and magnetic field modes. To obtain these results, the eigen matrix equation basing on the Helmholtz vector equation must be solved. But the size of this equation is too large to derive the results using a personal computer. Therefore, the eigen equation is compressed using the Arnoldi algorithm and after that the results are derived using the Krylov-Schur iteration method. The similarity transformation matrix to make this equation into the Schure form contains the desired eigenmode pairs of electric field and electric potential. As results, these are schematically represented with the electric field, 3-dimensional electric potential and electric potential contour.

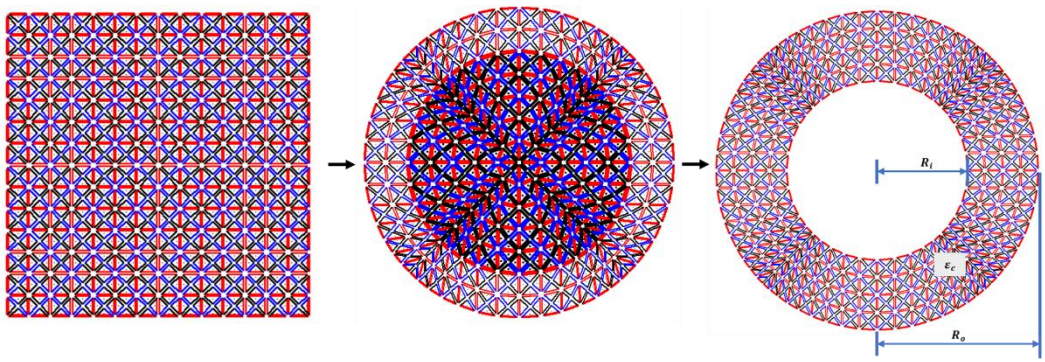
Keyword: dielectric cladding, similarity transformation, eigenmode, Helmholtz vector equation, Arnoldi algorithm, Krylov-Schur iteration method.

INTRODUCTION

In a previous study, it was reported the results of the eigen properties of a cylindrical coaxial waveguide using FEM [1]. In this manuscript, the results of FEM (Finite Element Method) calculation were revealed with the schematic representation of the TE (Transverse Electric) and TM (Transverse Magnetic)-modes. In each eigenmode, the electric field clearly showed the physical characteristics of the waveguide. However, small and complex peaks coexisting with the main peaks made understanding the characterization of the electric potential difficult. In reference [2] we have discussed the properties of hollow but similar shaped waveguides in the same way. Although the waveguide structure discussed there is hollow, the exterior cladding forms a thick body. This physical system is the main guiding channel of the electromagnetic wave rather than a cladding. There has been no discussion of the electric potential, so comprehensive comparison of eigen properties with those in this study is impossible. However, the electric field distribution is clear, and this could be used to understand the eigen characteristics of the result obtained in this study.

This study, as an extension of previous research, examines eigenmodes created in the dielectric cladding which may influence on the eigen characteristics of the coaxial waveguides. The thickness of the cladding is assumed to be sufficiently thick to form eigenmodes within it. The geometry of the dielectric cladding is a hollow cylindrical coaxial structure. The electrical dielectric constant of the cladding is imposed to be significantly higher compared to its value in free space, in order to increase the resolution of the spectrum. The eigenmode generated in the

cladding is predominantly determined by boundary conditions. To obtain the result of the eigenmode, the previously used FEM technique is used. The space in the dielectric cladding is decomposed into a triangular mesh element to facilitate FEM calculations. The eigen matrix equation is constructed based on the Helmholtz vector equation. The size of the matrix equation is proportional to the number of edges and nodes of the triangle elements. The greater the density of triangular elements, the more accurate the calculation results will be. However, because the capacity of personal computers is limited, it is difficult to handle the inverse matrix of the large size eigen equation. To overcome this contradiction, FEM uses the Arnoldi algorithm to compress the matrix equations into the smaller one as mentioned in the previous manuscript. Afterwards, the Krylov-Schur iteration method is used to find several prominent eigenmodes with the highest reliability. This method is used in this study to find the desired eigenmodes. Eigenmodes consist of electric fields and potential pairs. These constitute the column matrix of the similarity transformation matrix used in the iterative method. The mathematical derivation process for FEM has been mentioned in a previous manuscript [2][3]. Therefore, this process is not mentioned again in this manuscript. In other words, the theory of FEM is omitted in this manuscript. The description format of this manuscript deviates from the existing order and is described as follows. This manuscript describes the structure of waveguide for FEM calculation, the result and discussion of the simulation, and conclusion in order.



(a) (b) (c)

Figure 1: The schematic representation of the cladding

THE STRUCTURE OF WAVEGUIDE

In general, there are two important factors in determining the eigenmodes in a waveguide. The first is the geometric structure of the waveguide and the second is the dielectric properties of the cladding that makes it up. Firstly, the cladding wrapping the core of the waveguide used in this study form a cylindrical structure, as shown in Figure 1. The figure shows the triangular element mesh for the cladding developed from the previously discussed cylindrical coaxial waveguide. Figure 1, (a) shows a structure of the square mesh in a two-dimensional plane. It is convenient to create a triangular unit element from a square structure. The desired mesh structure is established by starting from this and gradually modifying it. Figure 1, (b) shows the cross section of a coaxial waveguide completed by transforming the square mesh into a circular shape. The edges and nodes of the inner core, indicated by thick lines, are removed leaving a final cladding area. In Figure 1, (c), the edges and nodes surrounding the inner and outer surfaces are boundary values that are eliminated when determining the TE mode in FEM calculation. If these are removed, only the tangential component of the electric field appears

and only the perpendicular segment of the electric potential is allowed [4]. The thickness of the cladding layer is an important factor in generating eigenmodes. If it is too thin, eigenmodes cannot be generated due to geometric factors. Conversely, if it is too thick, eigenmodes are generated, but in this case, the spectrum represents the characteristics of the waveguide, not the cladding. Therefore, selecting the layer thickness of the cladding requires a lot of trial and error. The spectrum shown in the next section is the result of struggle to find the most ideal thickness of the cladding. In this study, the thickness of the cladding layer is set to be $d = R_o - R_i = 0.5 - 0.4(arb.units)$.

Secondly, permittivity is an important parameter in determining the eigenmodes in a dielectric cladding. In most waveguides, the dielectric constant of the cladding is configured to have a lower value than that of the core. By doing so, electromagnetic waves are concentrated in the core area according to Snell's law, making it easy to generate eigenmodes. In previous studies, the dielectric constant of the core area of the coaxial waveguide was 10 or higher and that value of the cladding region was made to be 3 or lower. With those dielectric constant, the spatial distribution of electric field and electric potential was obtained as an eigenmode pair. There was no difficulty in accepting the spatial distribution of the electric field as an eigenmode. But there has been hard points to interpret the electric potential obtained together with them. Although the main peak of the electric potential was visible, the spatial distribution of the small and complex peaks made interpretation of the spectrum difficult. Therefore, it became necessary to separate the cladding from the coaxial waveguide and independently investigate its physical properties. The geometric structure is similar to that discussed in the previous study, but the eigenmodes are obtained by slightly changing the dielectric constant. The permittivity value of the cladding is varied gradually from 1 to 15 and the spectra are obtained at that point when it exhibits the most ideal eigenmode. In this study, the best resolution was confirmed when the value was 13, and based on this, a higher order spectra are obtained. The relative permittivity of 13 is too large for the cladding. However, since it is considered an independent waveguide and only the eigenmodes generated in this system are considered hypothetically. To facilitate FEM calculations, decomposing the space of cladding layer into triangular elements is the same as in the previous study. In addition, the process of deriving and solving the vector eigenmatrix equation from the Helmholtz vector equation was also mentioned in the previous manuscript.

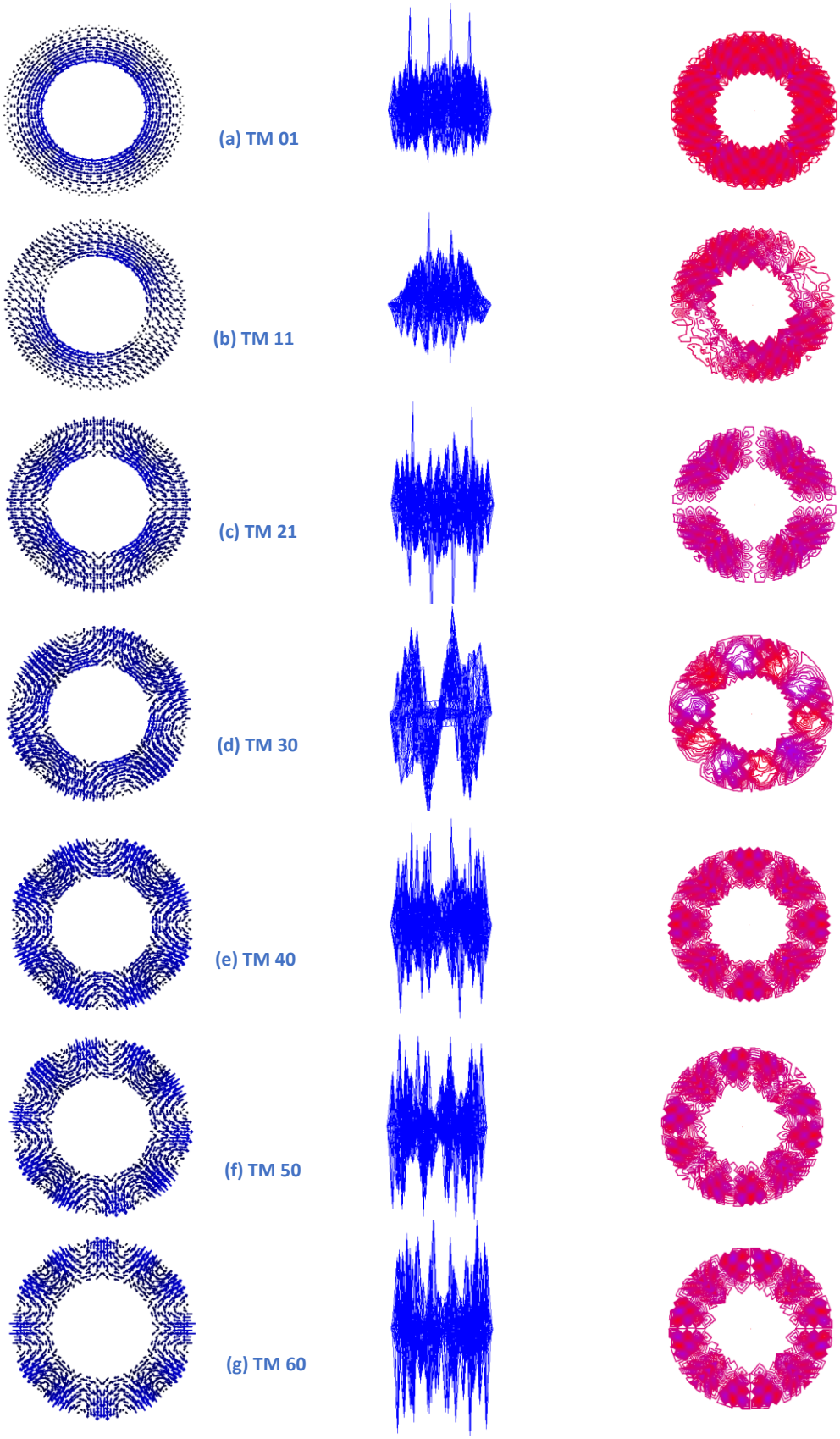


Figure 2: TM mode

THE RESULTS AND DISCUSSION

Figure 2 shows the schematic represntatopn of the TM eigenmodes formed in the cylindrical cladding. As shown in this figure, it can be identified that the eigenmode expressed as an electric field (in FEM calculations, the magnetic field is obtained through the same process as the electric field) is systematically developed as the order increases. Both the magnitude and direction of the electric field clearly explain the characteristics of the eigenmode. However, as found in previous studies on coaxial waveguides, small and complex components of the electric potential appear in this spectra too. Various attempts were made to exclude them and obtain the spectrum of the electric potential corresponding to the electric field, but these components always appeared. However, if each spectra for the electric field and electric potential are compared, it can be identified that although the peaks are complex, the electric potentials assist the eigenmodes determined by the electric field. Overall, it can be seen that the two spectra describe the same eigenmodes. Therefore, the eigenmodes assigned to each spectrum were inserted in Figure 2 to reflect the auxiliary characteristics of the potential according to the electric field.

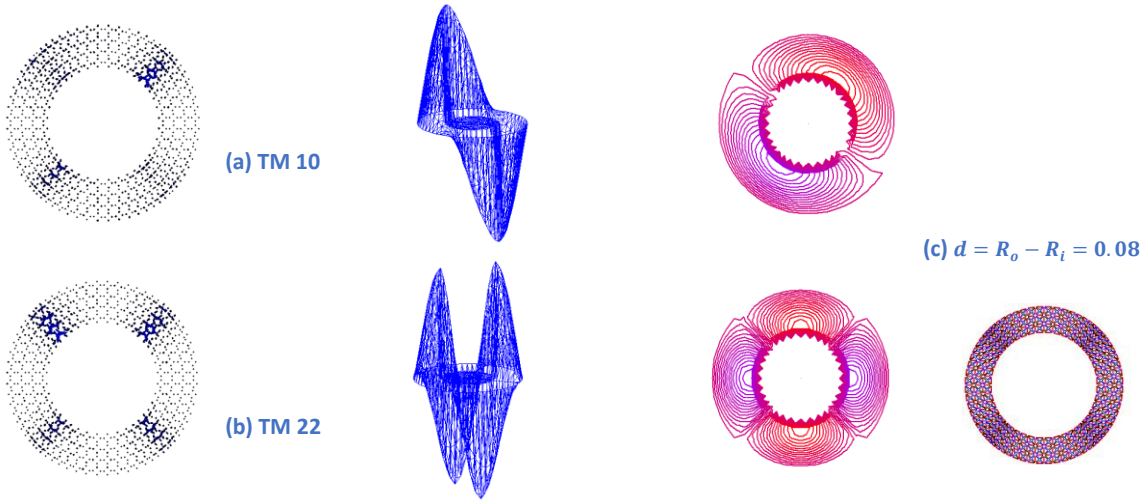


Figure 3: TM modes

Figure 3 shows the eigenmodes obtained from a waveguide with a cladding layer thickness of $d = R_o - R_i = 0.08$. Compared to Figure 2, the difference in thickness is only 0.02. The spectra for the cladding shown here reveal an eigenmode for the electric potential further enhanced. However, this cladding system has disadvantages that cannot be overlooked. As the order of the spectrum increases, small and complex potential components shown in Figure 2 also appear, reducing the resolution of the eigenmode. As mentioned at the beginning, it is confirmed that the thickness of the cladding layer has a significant influence on the generation of eigenmodes. The small and complex components mentioned in the previous manuscript rarely appear in the eigenmodes of a homogeneous waveguide. They usually appear near the surface or the interface between the two different dielectrics of the waveguide as can be seen in the Figure 3, (a) and (b). Therefore, it is presumed that their appearance confirmed in this study is probably due to the influence of the inner and outer surfaces of the cladding layer.

Figure 4 is the schematic representation of the TE eigenmode formed in the cladding layer. As with the TM mode, the spectrum consistently shows that eigenmodes are enhanced as the order increases. As mentioned at the beginning, to obtain the TE mode, the edges and nodes of the cladding surface are removed from the FEM calculation by boundary conditions. At the surface of the cladding, for the TE mode only the tangential component of electric field is permitted and the perpendicular element is excluded. The spectra in Figure 4 represent eigenmodes that well reflect this principle. Equipotential lines organized in a two-dimensional plane are formed perpendicular to the surface because their spatial gradient forms the distribution of the electric field. As in the case of TM, small and complex potential components also appear. This phenomenon is recognized as being due to interference of the surface of the cladding material, as in the case of TM mode. Nevertheless, the overall spatial distribution of the electric potential, together with the electric field, contributes to imposing the eigenmodes for each spectrum. In other words, the eigenmodes for each spectrum can be easily imposed by analyzing the electric field and potential. The labels of eigenmode for the spectrum are attached right next to the electric field in Figure 4. Figure 5 is the TE mode spectrum obtained in a system with the same cladding layer thickness as that in Figure 3. As shown in Figure 5, the reliability of the spatial distribution of the electric field is low, but small and complex components in the electric potential are excluded. These spectra describe lower order eigenmodes. The equipotential contour expressed in a 2-dimensional plane and the peaks expressed in a 3-dimensional electric potential sufficiently describe the eigenmodes by themselves. The difference between the two spectra is that in Figure 4, unlike in Figure 5, the eigenmodes are consistently enhanced as the order increases. The eigenmodes shown in Figure 5 change irregularly and lack consistency as the order increases. The physical properties of the cladding revealed in Figure 4 are more reliable than those shown in Figure 5. Therefore, it can be assumed that the characteristics of the eigenmodes generated in the core of the previously discussed cylindrical coaxial waveguide will be greatly influenced by those in Figures 2 and 4. If so, it is believed that the desired shape of eigenmode can be obtained from the core by ideally controlling the geometric structure and dielectric properties of the cladding. Research on this is a goal we will continue to pursue in the future.

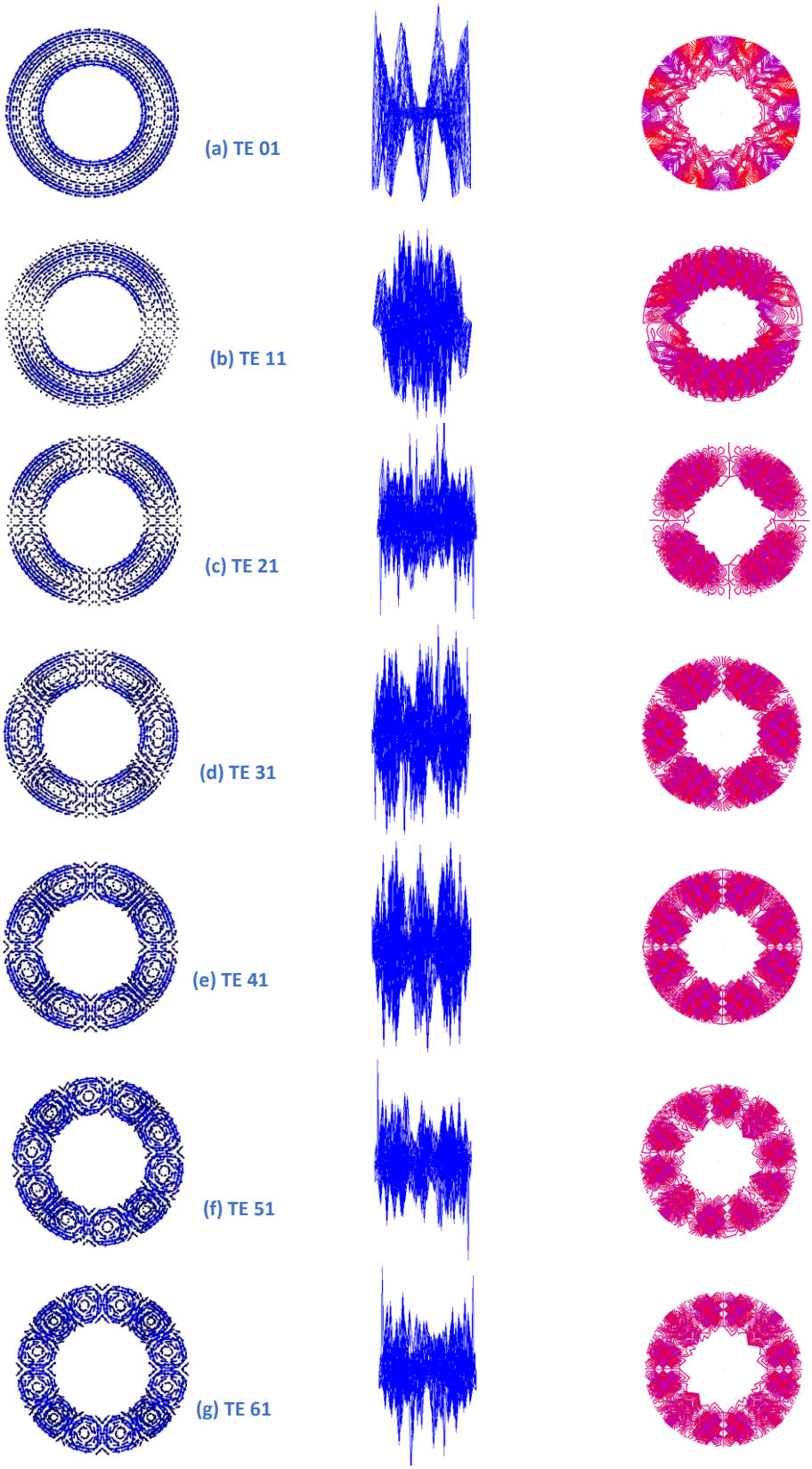


Figure 4: TE mode

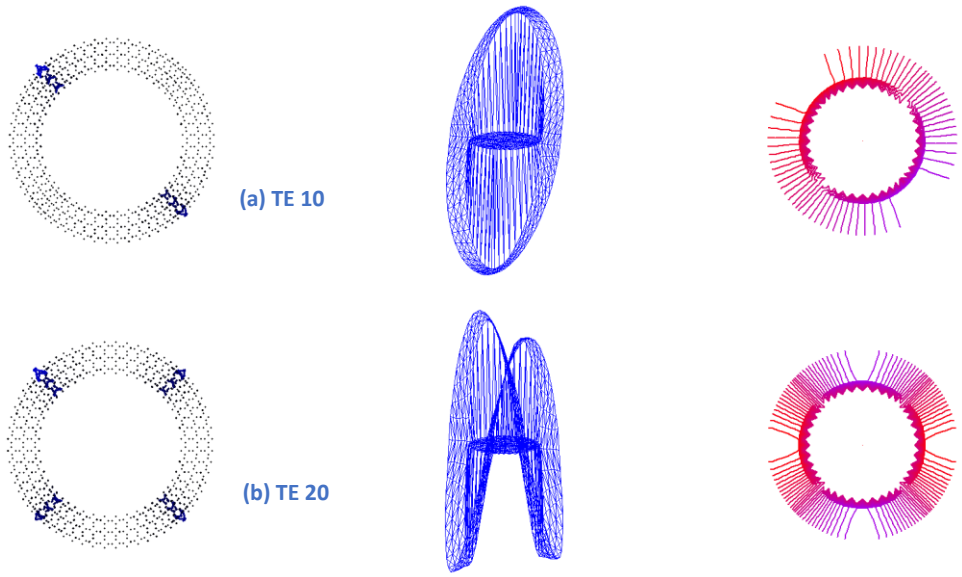


Figure 5: TE mode

CONCLUSION

The eigenmodes representing the physical characteristics of the cylindrical cladding have been obtained through FEM. Among the TM and TE eigenmodes dominated by the surface of the cladding, the most influential eigenmodes have been chosen and revealed in the schematical representation of Figure 2 and 4. It has been confirmed that these eigenmodes are consistently enhanced as the spectrum order increased. Therefore, it could be understood that these eigenmodes represent the physical properties of the dielectric cladding. It would be expected that the desired eigenmodes would be obtained in the coaxial waveguide core by ideally controlling the geometry and dielectric properties of the cladding.

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