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A Study on the Eigenmodes of the Ridge Waveguide

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ABSTRACT

The finite element method is carried out to investigate eigenmodes established in the ridge waveguide. Structure of the waveguide is divided into the mesh of triangular element. To obtain consistent eigenmodes, the edges and nodes of a triangular element are shared with neighboring elements. The eigen equation reveal large dimensionalmatrix so that Arnoldi methods is applied to compress it. Krylov-Schur iteration method is used to make it Schurform. The eigenpairs are obtained from the column matrix of similarity transformation matrix of an iterative process and consist of electric fields and potentials. The results are schematically represented with the electric field, 3-dimensional electric potential and electric equipotential contour. The eigenmodes represented by the electric field and potential are compared with each other. And the precise mode type is determined through the space distribution of those spectra.

Keyword: ridge waveguide, Arnoldi compression, Krylov-Schur iteration method, Schur matrix, eigenpair, similarity transform.

INTRODUCTION

As mentioned in the previous manuscript, FEM (Finite Element Method) is a reliable and robust numerical interpretation method for simulating integrated optical devices [1]. By using this method, the electric field and electric equipotential eigenpairs for micro-optical waveguide like CBCPW (Conductor Backed Coplanar Waveguide) etc., have been calculated and revealed in the schematic representations [2]. In FEM process, the matrix of eigen equation is too large to manipulate by personal computer. So, the sophisticate method like Arnoldialgorithm is applied to compress the large dimensional eigen equation into the smaller one [3]. Krylov-Schur iterative method is used to finds several major eigenpairs. This method stands out uniquely and has been used in research to reveal the unique characteristics of electromagnetic waves formed in photonic crystal waveguides. [4][5]. FEM is also a method that can be easily applied numerically to physical systems exhibiting complex structures with difficult boundary conditions.

In this study, FEM is carried out to investigate the eigenpairs of the dielectric single ridge waveguide. The waveguide is rectangular shape with rectangular ridge protruding from the dielectric substrate. The reason for using the ridge waveguide is to ensure that the wave propagating in a single mode couple with the structural channel of the waveguide to preserve and maintain its physical properties. It is well known that an eigenmode propagating through the waveguide depend on the geometrical factor like the width and height of the dielectric ridge. Those factors must be adjusted according to the wavelength and polarization of the propagating wave. It isnecessary to know information about the eigenmodes of the waveguide that can

cause resonance with the propagating wave. The eigenmode of the waveguide interferes with the traveling wave and causes resonance, so that a conserved signal can be transmitted to the circuit without energy leakage. To achieve this goal, it is important to know the eigenmodes that a waveguide can form in a given geometry and physical property. So, the purpose ofthis study is to reveal the properties of various eigenmodes that can be formed by ridge waveguides with the constant ratio of the width and height.

What is noteworthy in this manuscript is that the theory of FEM and the subsequent development of formulas have been mentioned in detail in previously published works, so they are omitted. This manuscript describes the structure and spectrum properties of the ridge waveguide, the result and discussion of the simulation, and conclusion in order.

THE STRUCTURE AND SPECTRUM PROPERTIES OF THE RIDGE WAVEGUIDE

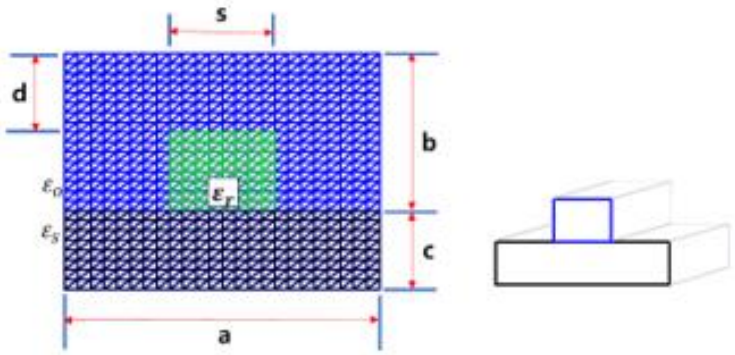


Figure 1: Schematic representation of the ridge waveguide.

The structure of the ridge waveguide is schematically represented in figure 1. The relative permittivity of the rectangular ridge is assumed to be ε_r (=11.68). It is protruded from the dielectric substrate whose assumed relative permittivity to be ε_S (=3.9). The ridge is surround with the substrate and the air assuming relative permittivity to be ε_O (=1.00). The relation of the relative permittivity is $\varepsilon_O < \varepsilon_S < \varepsilon_r$ so that the electromagneticwave is expected to existed mostly in the region of the rectangular ridge or substrate by Snell's law. For FEM calculation, the space of the waveguide is divided into the mesh of elementary triangular. As mentioned in the previous manuscript, in order to derive a consistent physical meaning, the edge and vertices of triangular elementsmust be continuously connected. As can be seen from figure 1, except for the edge exists on the surface, the one that exists inside the waveguide is shared with two neighboring triangular elements. An edge present on the surface belongs to only one triangular element. The eigen electric field of a triangular element reveal at the barycentric point and it is determined by combination of the edge vectors of the triangular element. Except for the nodes on the surface, those in the inside of the waveguide are shared with six neighboring triangular elements. A node present on the surface of the waveguide is shared with three neighboring triangular elements. And the node existat the conner of the waveguide are involved only one elementary triangle. The eigen electric potential is describedas scalar quantities. Those values are used to construct the 3D representation and the electric equipotential contours. The

equipotential line is drawn by connecting the points obtained from interpolating the potential valuesof nodal points of the triangular element.

The cutoff wavelength λc of the eigenmode is depending on the geometrical factor of the rectangular ridge waveguide [6]. The ratio b/a from the figure 1, is an important factor in determining the wavelength of the cutoff eigenmode. There is precise discussion by FEM, to reveal the relation between $s/a$and $\lambda c/a$ for given the ratios of b/a. According to this discussion, wise selection of waveguide geometry is an important factor in maximizing propagation efficiency. The resonate interference between the propagating single-mode wave and the eigenmode established in the waveguide dominantly depend on the unique structure of the ridge channel. The methods of generating resonance are largely divided into two class. The first method is fixing the propagating single mode wave and varying the geometric structure to find the eigenmode matching to it. The second method is fixing the geometric structure of the ridge waveguide so that the eigenmode established in it is not varied. And it is to find a single-mode propagating wave that can resonate with a stable eigenmode.

In this study, FEM is carried out to investigate variety eigenmodes of the ridge waveguide for the purpose of assisting the latter method. As can be seen in figure 1., the ridge and the substrate ensure sufficient space to achieve variety eigenpairs. Each spectrum is complicated by mixing of unique pairs from ridges and substrates. In order to understand the physical properties of the ridge waveguide, each spectrum is represented by an electric field, 3Dpotential distribution, and equipotential contour. Each representation is compared and identified to determine the eigenmodes of the waveguide. For the precisely interpretation of spectra, it is necessary to identify the peak position of the electric field and potential. The symmetry of the peak is also an important factor in determining the eigenmodes. Except for the ground state, higher order eigenmodes exhibit more than one peak. If the material of the waveguide is uniformly distributed, the peak positions and shapes in most spectra are symmetric about the center. So, the eigenmodes for the spectrum are determined based on those criterions.

THE RESULT AND DISCUSSION OF THE SIMULATION

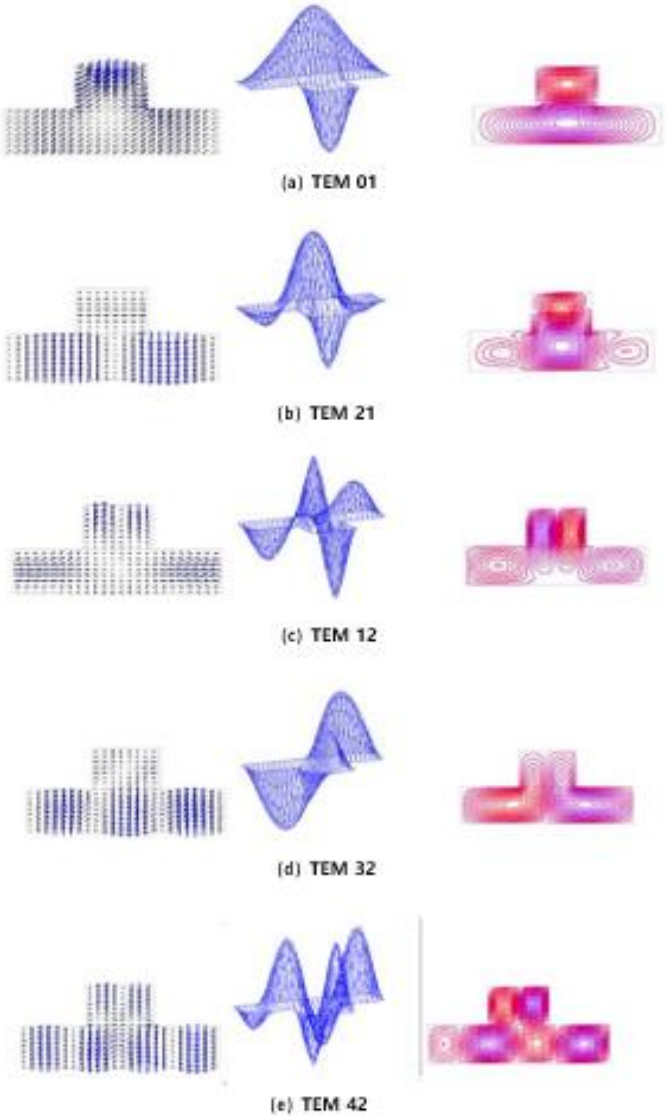


Figure 2: The spectra of the ridge waveguide

The spectra of the ridge waveguide are schematically represented in the figure 2. As can be seen in this representation, each spectrum is complex because it contains a mixture of contributions from different regions ofthe geometric space. However, each spectrum exhibits its own physical properties due to the contribution of waves adapt to waveguide. The simplest spectrum is shown in figure 2(a). The electric field is concentrated in the upperregion of ridge. The electric field spreads from the center of the peak toward the substrate. The potential forms a left-right symmetric distribution about the center of the waveguide. The electric potential distributed on the ridgeand the substrate are continuously connected based on their boundary. the separate line in the y-mode is established by the boundary between the ridge and substrate. Therefore, it is reasonable to assign the eigenmode TEM 01 to this spectrum. This eigenmode is the lowest order spectrum which a ridge waveguide can form. When the signal as a single mode coming from the outside resonates with this mode, it will show the most useful characteristics. When

resonance occurs, there is little energy leakage of the signal and the characteristics of it can be preserved over a long range and wide regime. The concentration of the electric field and the peak position of equipotential coincide with each other, giving reliability in determining the eigenmode. The 3D representation of the electric potential emphasizes the correspondence between the spectrum and the eigenmodes assigned to it.

From the figure 2(b, c), it is obviously identified that the eigenmodes are oriented in directions perpendicular to each other. The electric field in the region of ridge represented in figure 2(b) is in the horizontal direction. And the electric field in the region of substrate is the vertical direction. However, the electric field in the region of ridge represented in figure 2(c) is in the vertical direction. And the electric field in the area of substrate is arranged in the horizontal direction.

These spectra are different but exhibit similar characteristics. Referring to the expression of spectrum (b, c), those state can be considered that the electric fields of the ridge portion and the substrate region are separated from each other. The direction of the electric field in the ridge and substrate are perpendicular each other. Therefore, the boundary between the ridge portion and the substrate region is clear. It is certainly identified that each region forms independent eigenmodes. From figure 2(b, c), it is identified that the eigenmodes of TEM 01 and TEM 10 are established in the region of ridge respectively.

However, it is identified that in the region of substrate the eigenmodes of TEM 20 and TEM 02 can be independently imposed for (b) and (c) respectively. There is more to be said about the eigenmode TEM 02 of substrate of figure 2(c). There are two separation lines in the y-mode, one for the interface between the regions of ridge and substrate, and the other is between the substrate and the conductor backed flat plane. Referring those discussion, the eigenmode of the spectrum of figure 2(b) can be defined to be TEM 21. In the similar way, by combining those discussion, the eigenmode of the spectrum of figure 2(c) is imposed to be TEM 12. Figure 2(e) is the spectrum representing the another eigenmode manifestly. The eigenmode can be identified through the spectrum of the electric field of (e). For the region of ridge, it is certain that there are two peaks of the electric field. And there are four peaks of the electric field in the substrate. The directions of those electric field all orient to the y-axis. For more conclusive evidence, it is referred to the spectrum describing the electric potential. As in figure 2(c) for the electric potential, the separation line of x-mode extends from the center of substrate to the region of ridge in the vertical direction. It means that the electric potential formed on the substrate and that of the ridge are continuously connected. It can be said that the electric potential formed in the vicinity of the ridge combine with that of one in the substrate area to establish more comprehensive one. In other word, they are not two independent peaks of the electric potential, but a modified inclusive one. Hence, the electric potential of the ridge is combined with that of the substrate, and as a whole the eigenmode of figure 2(e) can be determined to be TEM 42.

The most confusing spectrum which cannot be easily determined the eigen mode is figure 2(d). The distribution of the electric field in regions of the ridge and substrate is not continuously matched. In general, as the order of eigen mode increased, the number of peaks distributed on the substrate is greater than the number of peaks appearing on the ridge. This is because the space

where the electric field can be established is larger in the substrate than in the ridge. Considering the spatial distribution of the electric field, the ridges show even-numbered peaks and the substrate shows odd-numbered peaks. The eigen modes appearing in the region of ridge are estimated byTEM 20. It cannot be claimed that electric fields are continuously distributed between the ridge and substrate. However, the eigen mode appearing in the region of substrate is considered to be TEM 30. The spatial distributionof the electric potential does not consistently match with that of the electric field, as can be seen in figure 2(d). The electric potential generated in the ridge portion and the substrate are continuously connected. The eigen modes appearing in the region of ridge are estimated by TEM 20. The eigen modes appearing in the region of substrate are considered to be TEM 30. The positions and shapes are symmetrically distributed about centerline. And the electric potential generated in the ridge portion and the substrate are continuously connected. Considering the ridge and substrate as independent regions, these are typical eigenmodes of a waveguide of the rectangular structure. However, they are not self consistently implemented to each other.

When an eigenmode is expressed using an electric field, it can be described as TEM 32. When an eigenmode is marked using the electric potential,it can be represented by TEM 22. These expressions do not match with each other for the same waveguide. The eigenmodes represented by electric fields and potentials are so pronounced that it is impossible to say what wentwrong. Presumably, it is only estimating whether there is a peak hidden in the spectrum representing the electric potential. It would be needed a bit more study for the spectrum to resolve uncertainties. Figure 3, is a schematic representation another group of spectra for the ridge waveguide. They are different from that of the spectra figure 2.

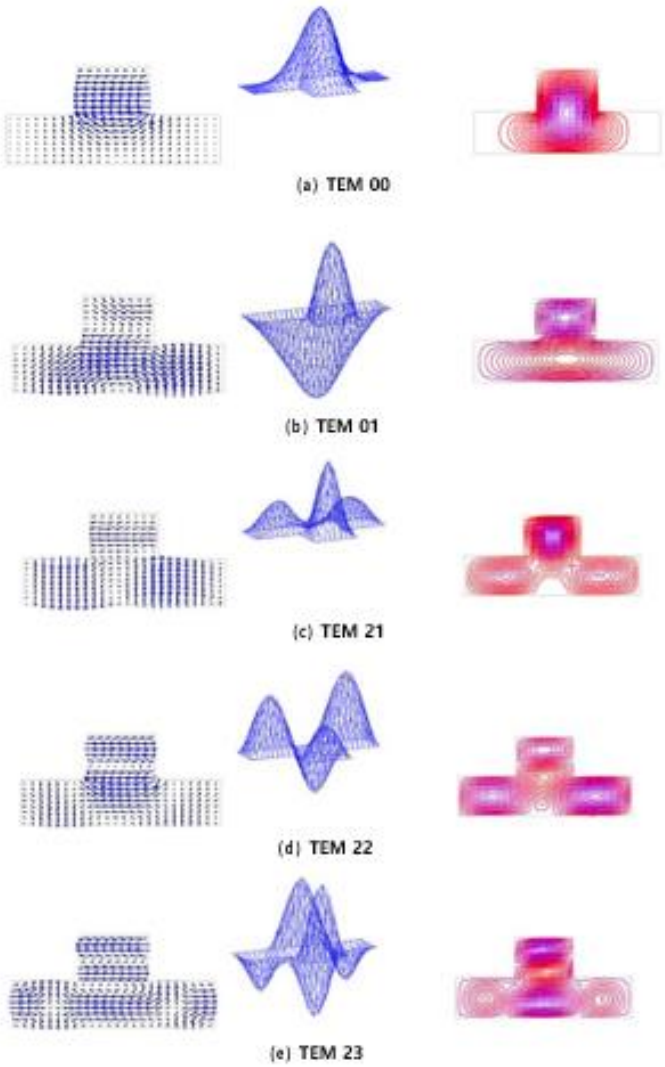


Figure 3: The schematic representation of the spectra of ridge waveguide

It can be seen from this figure that the electric field is predominantly oriented in the horizontal direction, which is different from the spectrum shown in figure 2. The most prominent spectrum is figure 3(a) which represent the lowest order eigenmode. The feature of this eigenmode different from that of figure 2(a) whose electric field orientvertical direction. It would be nice to find the eigenvalue and compare the energy values of the two eigenmodes with each other. However, this process could not be performed because the size of the waveguide was varied to obtain the best resolution of the eigenmodes.

Only with eigenmodes of the electric field cannot determine the lowest state between them. But, the eigenmodes described by the electric equipotential tell us the lowest state. Comparing with the electric equipotential contours of the spectra figure 2(a) and figure 3(a), it is manifest that figure 3(a) is lower than that of the figure 2(a). Since there is only one peak in the electric equipotential contour, it is reasonable to assign the eigenmode TEM 00 to this spectrum. Therefore, determination of accurate eigenmode for the spectrum is possible if the electric field and potential are used together in spectral decomposition and compared with each other. Fig.

3(b) shows the eigenmode one order higher than that of TEM 00. The electric fieldgenerated in the substrate and transferred to the ridge is separated at the interface between those regions by a y- mode transverse line. Referring to the 3D representation of the electric potential, it can be seen that the peaks of the ridge and substrate are separated by the interface region. This separation can also be confirmed through the electric equipotential contour. Looking closely at the electric field representation, an x-mode component is observed at the bottom of the substrate. This deformed portion does not appear to be involved in mode formation when compared to the electric equipotential contour. Therefore, it seems reasonable to assign the eigenmode of figure 3(b) to TEM 01. Another interesting spectrum is shown in the figure 3(c). The electric fields in the region of ridge and substrate array perpendicular each other. The electric field is therefore discontinuous at the interface.The electric field in the region of ridge is distributed uniformly (-x) direction. Two electric field peaks are distributed in the (+y) direction in the region of substrate. Information about the equipotential lines along with the spatial distribution of the electric field plays an important role in determining the eigenmodes of the spectrum. There is one peak in the region of ridge. There are two peaks of the electric equipotential in the region of substrate.The spectra representing the electric field and electric equipotential lines is clear, and the location of the peak is prominent. Based on the above description of the electric field and potential, it is reasonable to determine the eigenmodes in figure 3(c) by TEM 21.

Figure 3(d) are another spectrum characterizing the ridge waveguide. As shown in figure 3(d), the electric field and equipotential are evidently identified. Two separate lines of the y-mode appear from the region of ridge to the substrate. Two separate lines of the x-mode are expanded from the edges ofleft to right of the substrate. Those separation lines are evidently identified in the electric field and equipotential spectra. There is nothing wrong with this description. Basing on this description, the eigenmode TEM 22 can be imposed on the spectrum shown in figure 3(d). Figure 3(e) is a complex, but representative spectra of higher order showing the characteristics of the ridge waveguide. As shown in this figure, the electric field and equipotential can evidently identified. Three separate lines of the y-mode appear from the region of ridge to the substrate. Twoseparate lines of the x-mode are expanded from the edges of left to right of the substrate. Those separation lines are evidently identified in the electric field and equipotential spectra. There is nothing wrong with this descriptionas in the case of figure 3(d). Basing on this description, the eigenmode TEM 23 can be denoted on the spectrum shown in figure 3(e).

CONCLUSION

FEM has been carried out to investigate the eigen properties of the ridge waveguide. The electric field and potential were complementary to each other and used to determine the exact eigenmode. The eigenmodes generated in the space composed of the ridge and the substrate exhibited complex aspects due to the contribution of these spaces. The eigenmodes were divided into two groups in which the electric field was longitudinal and transverse. The eigenmodes were independently analyzed for the regions of ridge and substrate, and based on this,the results for the entire waveguide were obtained. The eigenmodes obtained in this way were schematically represented as electric fields, 3D potentials, and equipotential contour.

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