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SERVICES FOR SCIENCE AND EDUCATION

Quadratic Stability of Switched affine DAEs Via State-Dependent Switching law

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ABSTRACT

In this paper, we deal with quadratic stabilization for a particular class of impulse-free switched differential affine algebraic systems (switched affine DAEs) which consists of a finite set of affine subsystems with the same algebraic constraint, where both subsystem matrices and affine vectors in the vector fields are switched independently and no single subsystem has desired quadratic stability. We show that if there exist a convex combination of subsystem matrices such that the result Linear Time-Invariant (LTI) algebraic system is asymptotically stable, and another convex combination of affine vectors is zero, then we can design a state-dependent switching law such that the entire switched affine DAEs system is quadratically stable at the origin. But when the convex combination of affine vectors is not zero, we discuss the quadratic stabilization to a convergence set defined by the convex combination of subsystem matrices and the convex combination of affine vectors.

Keywords: switched affine DAEs system, quadratically stabilization, convex combination, state-dependent switching, convergence set.

INTRODUCTION

A switched differential algebraic affine system consists of a finite number of algebraic affine subsystems and a switching law that determines which subsystem is activated at each switching instant described as:

$$E_\sigma \dot{\mathrm{x}}(t) = A_\sigma x(t) + b_\sigma, \tag{1.1}$$

where the switching signal $\sigma: \mathbb{R} \to \{1,2, \dots, p\}$, $p \in \mathbb{N}$, is a right-continuous (with a locally finite set of discontinuities), piecewise constant function and for each parameter $i \in \{1,2, \dots, p\}$, E_i, A_i are a square matrix of $\mathcal{M}_n(\mathbb{R})$, and b_i a vector of $\mathbb{R}^n$. It is a more general class of differential

switched systems with continuous state evolution which has been studied extensively in the past few decades, [5, 6, 11, 21, 22, 24, 25, 26, 27].

In the literature, it is well known that all solutions of each subsystem evolve within a so-called consistency space, which is a linear subspace of $\mathbb{R}^n$ [2, 23]. In general, at a switching time $t \in \mathbb{R}$, there does not exist any continuous extension of the solution into the future, because the value $x(t-)$ does not need to be within the consistency space corresponding to the DAE after the switch. Therefore, it is necessary to allow for solutions with jumps. However, this leads to difficulties in evaluating the derivative of the solutions. To resolve this problem, we adopt the distributional framework introduced in [23] i.e. as solutions of the switched (1.1) distributions (generalized functions), in particular Dirac impulses, are considered. For this, we have to assume that the switching signal has only a locally finite set of switching times. Furthermore, to ensure existence and uniqueness of solutions, we have to assume that each matrix pair (E_i, A_i) is regular. We will make one more assumption which ensures that no impulse occur in the solutions of the switched (1.1). A consequence of these assumptions is that although a distributional solution framework is necessary as a theoretical basis for treating switched DAEs of the form (1.1), the only solutions that arise in this paper are piecewise-smooth functions with locally finitely many discontinuities.

During the last two decades, there has been increasing interest in stability analysis and controller design for switched systems; see the survey papers (Branicky 1998; DeCarlo [7], Branicky, Pettersson and Lennartson [18, 19]; Liberzon and Morse [1, 10, 13, 14, 15, 16]; Lin and Antsaklis, 2009), the books (Liberzon, 2003; Sun and Ge, 2005) and the references cited therein. As pointed out in Liberzon and Morse (1999), Liberzon (2003), Feron (1996), [16, 1, 8, 21] one basic but very important problem for switched systems is to design a stabilizing switching law (strategy) for the case where each single subsystem is not stable. There are several approaches dealing with this issue, especially for switched linear and affine systems. Minqing Xiao, Guisheng Zhai and Chi Huang(2019)[17] prove quadratic stabilization with convex combination approach; Pettersson and Lennartson (2002)[5] use Lyapunov-like functions to partition the entire state space into different regions, and proposes Linear Matrix Inequality (LMI) based state-dependent switching laws, such that the switched system is stabilized. In Feron (1996), Wicks, Peleties, and DeCarlo (1998), it is shown that the existence of a stable convex combination of subsystem matrices implies the existence of a state-dependent switching rule, which stabilises the switched system along with a quadratic Lyapunov function. It is also proved in Feron (1996) that when the number of subsystems is two, the existence of a stable convex combination of the subsystem matrices is both necessary and sufficient for quadratic stabilizability of the switched system by state-dependent switching.

The present paper aims to extend the approach of convex combination of subsystems, proposed and used in [17] to quadratic stabilization for a particular class of switched affine DAEs system (1.1), assuming that both subsystem matrices and affine vectors are switched by different switching signals. Since we assume that the affine subsystems do not have common equilibrium states, stabilization to a desired 'equilibrium' state is generally difficult and even not easy to formulate. Some authors have been working long in this area [3, 20, 17, 4], and have proposed the idea of practical stability/stabilization together with several efficient switching laws (Xu and Zhai, 2005; Xu, Zhai, and He, 2008, Xiao, 2019). We show that if for a convex combination of subsystems matrices and vectors, the result Linear Time-Invariant (LTI) algebraic system is

asymptotically stable and another convex combination of affine vectors is zero, then we can design a state-dependent switching law such that the state of the switched affine DAEs can converge to the origin . We also show that when the convex combination of affine vectors is not zero, a convex set can be established such that the state of the switched affine DAEs can converge to any point in the set.

PROBLEM FORMULATION

In this paper, let us consider the switched DAEs system composed of a finite set of time-invariant affine subsystems where E is the same non-invertible matrix for all subsystems described by:

$$E\dot{x}(t) = A_{\sigma_1}x(t) + b_{\sigma_2} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $\sigma_1 \colon \mathbb{R} \to I_{N_1} = \{1, \dots, N_1\}$ and $\sigma_2 \colon \mathbb{R} \to I_{N_2} = \{1, \dots, N_2\}$ are switching laws. Here E, A_i and b_j are constant matrices/vectors of appropriate dimensions.

Definition : The switched DAEs (2.1) is quadratically stabilizable via state-dependent switching if there exist switching laws (σ_1, σ_2) which vary explicitly with respect to the system state x, a Lyapunov function V and a positive scalar ε such that

$$\dot{V}(x) < -\varepsilon \|x\|^2$$

holds for all nonzero trajectories x of the switched affine DAEs system.

***Quadratic Stabilization Problem*:**

Under several assumptions, we formulate our stability problem as follows:

• Designing the state-dependent switching laws $\sigma_1(x)$ and $\sigma_2(x)$, such that the switched affine system (2.1) is quadratically stable.

• Establishing the convergence set of points when the switched affine DAEs system is not stabilizable at the origin.

It is noted from the above problem formulation and the discussion later that since we assume the asymptotic stability of a result LTI algebraic system based on a convex combi-nation A_λ and b_μ, we use a Lyapunov matrix for A_λ vigorously to obtain quadratic stability for the swicthed affine DAEs.

Assumption 1 : The swicthed (2.1) is impulse-free and there exist a convex combination A_λ and b_μ of the matrices A_i; $(i = 1, \dots, N_1)$ and the vectors b_j; $(j = 1, \dots, N_2)$ respectively such that the linear time-invariant algebraic system $E\dot{x} = A_\lambda x + b_\mu$ is asymptotically stable.

STATE-DEPENDENT SWITCHING LAW

1- Stabilization at the origin

Assumption 2: $b_\mu = 0_{\mathbb{R}^n}$.

Theorem 3.1. Under assumptions 1 and 2, the switched affine system (2.1) is quadratically stabilizable at the origin via state-dependent switching laws.

Proof.

Since $E\dot{x} = A_\lambda x + b_\mu$ is asymptotically stable, there exists a positive definite function V described by $V(x) = (Ex)^\top P(Ex)$ where $P > 0$ satisfying

$$A_\lambda^\top PE + E^\top PA_\lambda < 0. \tag{3.1}$$

Since this is a strict matrix inequality, there always exists a positive scalar ε such that

$$A_\lambda^\top PE + E^\top PA_\lambda + \varepsilon P < 0. \tag{3.2}$$

We use the matrix P to define the switching law

$$(\sigma_1(x), \sigma_2(x)) = \arg\min_{i\in I_{N_1}, j\in I_{N_2}} \{x^\top (A_i^\top PE + E^\top PA_i)x + 2x^\top E^\top Pb_j\}. \tag{3.3}$$

In order to implement the above switching law clearly, we need to take care of the case in where there are several indexes of subsystems obtained from (3.3). If these indexes include the present subsystem, that is,

$$(\sigma_1(x(t^-)), \sigma_2(x(t^-))) \in \arg\min_{i\in I_{N_1}, j\in I_{N_2}} \{x(t)^\top (A_i^\top PE + E^\top PA_i)x(t) + 2x(t)^\top E^\top Pb_j\}.$$

we choose to stay at the same subsystem without switching. Otherwise, we choose any subsystem or the least activated subsystem index generated by (3.3).

Under the switching law (3.3), the inequality

$$x^\top (A_{\sigma_1}^\top PE + E^\top PA_{\sigma_1})x + 2x^\top E^\top Pb_{\sigma_2} \le x^\top (A_i^\top PE + E^\top PA_i)x + 2x^\top E^\top Pb_j \tag{3.4}$$

holds for all $i = 1, \dots, N_1$ and $j = 1, \dots, N_2$. Multiplying both sides of the above inequality by nonnegative scalars λ_i; μ_j, and then adding all the inequalities results in

$$\begin{aligned}\sum_{i=1}^{N_1}\sum_{j=1}^{N_2} \lambda_i \quad & \mu_j x^\top (A_{\sigma_1}^\top PE + E^\top PA_{\sigma_1})x + 2x^\top E^\top Pb_{\sigma_2} \\ \leq \quad & \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} \lambda_i\, \mu_j x^\top (A_i^\top PE + E^\top PA_i)x + 2x^\top E^\top Pb_j \\ = \quad & \sum_{i=1}^{N_1} \lambda_i\, x^\top (A_i^\top PE + E^\top PA_i)x + 2x^\top E^\top Pb_\mu) \\ = \quad & x^\top (A_\lambda^\top PE + E^\top PA_\lambda)x + 2x^\top E^\top Pb_\mu \\ = \quad & x^\top (A_\lambda^\top PE + E^\top PA_\lambda)x\end{aligned}$$

Then, from (3.2), we obtain

$$x^\top (A_{\sigma_1}^\top PE + E^\top PA_{\sigma_1})x + 2x^\top E^\top Pb_{\sigma_2} < -\varepsilon x^\top Px \tag{3.5}$$

for any nonzero x

Next, the derivative of the Lyapunov function candidate $V(x) = (Ex)^\top P(Ex)$ along the trajectories of the switched affine system (2.1) is computed and evaluated as:

$$\begin{aligned}\dot{V}(x) \quad = \quad & \frac{d}{dt}((Ex)^\top P(Ex)) \\ = \quad & (E\dot{x})^\top P(Ex) + (Ex)^\top P(E\dot{x}) \\ = \quad & (A_{\sigma_1}x + b_{\sigma_2})^\top P(Ex) + (Ex)^\top P(A_{\sigma_1}x + b_{\sigma_2}) \\ = \quad & x^\top (A_{\sigma_1}^\top PE + E^\top PA_{\sigma_1})x + 2x^\top E^\top Pb_{\sigma_2} \\ < \quad & -\varepsilon \parallel P \parallel \parallel x \parallel^2 \\ < \quad & -\tilde{\varepsilon} \parallel x \parallel^2\end{aligned}$$

for any nonzero x. This implies that the switched affine system (2.1) is quadratically stable at the origin

Example

Consider the switched affine DAEs system (2.1) with different equilibrium points where the matrices and vectors are given by:

$$E = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix};\ A_1 = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix};\ A_2 = \begin{pmatrix}3 & 0\\ 0 & -4\end{pmatrix};\ b_1 = \begin{pmatrix}-2\\ 1\end{pmatrix};\ b_2 = \begin{pmatrix}8\\ -4\end{pmatrix}$$

We can see that A_1 and A_2 are not Hurwitz, but there exist a convex combination of A_i and b_j respectively for which we have:

$$A_\lambda = \frac{31}{40}A_1 + \frac{9}{40}A_2 = \begin{pmatrix} \frac{-1}{10} & 0 \\ 0 & \frac{-1}{8} \end{pmatrix}$$

and

$$b_\mu = \frac{4}{5}b_1 + \frac{1}{5}b_2 = 0_{\mathbb{R}^n}$$

Since for non-zero s, $dét(sE - A_\lambda) \neq 0$; there exist:

$$S_\lambda = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \; ; \; T_\lambda = \begin{pmatrix} 1 & 0 \\ 0 & -8 \end{pmatrix}$$

satisfying Quasi-Weierstrass form with $J_\lambda = \frac{-1}{10}$. So the LTI system $E\dot{x} = A_\lambda x + b_\mu$ is asymptotically stable. Moreover we have:

$$\Pi_1 = \Pi_2 = \Pi_\lambda = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \; ; \; C_{(E,A_1)} = C_{(E,A_2)} = C_{(E,A_\lambda)} = span\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\}$$

The positive definite matrix P is given by:

$$P = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$$

Starting from the initial state $x(0) = (10,0)$ and using the switching law (3.3), we obtain the state trajectories of the switched affine system which converge to zero.

2- Extension to convergence set

Assumption 3: $b_\mu \neq 0_{\mathbb{R}^n}$.

Theorem 3.2. Under assumption 1 and 3, the states of the switched affine system (2.1) converge to any $x_e \in \mathcal{S}_e$ defined as follows:

$$\mathcal{S}_e = \{x_e = -(\sum_{i=1}^{N_1} \lambda_i A_i)^{-1}(\sum_{j=1}^{N_2} \mu_j b_j) : \; \lambda_i \geq 0, \; \sum_{i=1}^{N_1} \lambda_i = 1 \; ; \; \mu_j \geq 0, \; \sum_{j=1}^{N_2} \mu_j = 1\}. \qquad (3.6)$$

under the state-dependent switching law

$$(\sigma_1(x, x_e), \sigma_2(x, x_e))$$

$$= \arg \min_{i \in I_{N_1}, j \in I_{N_2}} \{(x - x_e)^\top (A_i^\top PE + E^\top PA_i)(x - x_e) + 2(x - x_e)^\top E^\top P(A_i x_e + b_j)\}. \quad (3.7)$$

Proof. Let us set $z = x - x_e$. The switched affine DAEs system will be rewritten as:

$$E\dot{z} = A_{\sigma_1} z + (A_{\sigma_1} x_e + b_{\sigma_2}) \quad (3.8)$$

With the switching law (3.7), we obtain:

$$z^\top \left(A_{\sigma_1}^\top PE + E^\top PA_{\sigma_1}\right) z + 2z^\top E^\top P\left(A_{\sigma_1} x_e + b_{\sigma_2}\right)$$
$$\leq z^\top (A_i^\top PE + E^\top PA_i) z + 2z^\top E^\top P(A_i x_e + b_j) \quad (3.9)$$

for all $i = 1, \ldots, N_1$ and $j = 1, \ldots, N_2$. In the same way of the proof of the previous theorem 3.1, we obtain:

Set W = $z^\top (A_{\sigma_1}^\top PE + E^\top PA_{\sigma_1}) z + 2z^\top E^\top P(A_{\sigma_1} x_e + b_{\sigma_2})$. Then we have:

$$\begin{aligned} W &\leq \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \lambda_i \mu_j z^\top (A_i^\top PE + E^\top PA_i) z + 2z^\top E^\top P(A_i x_e + b_j) \\ &= \sum_{i=1}^{N_1} \lambda_i z^\top (A_i^\top PE + E^\top PA_i) z + 2z^\top E^\top P(A_i x_e + b_\mu) \\ &= z^\top (A_\lambda^\top PE + E^\top PA_\lambda) z + 2z^\top E^\top P(A_\lambda x_e + b_\mu) \\ &= z^\top (A_\lambda^\top PE + E^\top PA_\lambda) z \end{aligned}$$

where $x_e = -A_\lambda^{-1} b_\mu$ was used to obtain the final equation. Then, from (3.2), we obtain:

$$z^\top (A_{\sigma_1}^\top PE + E^\top PA_{\sigma_1}) z + 2z^\top E^\top P(A_{\sigma_1} x_e + b_{\sigma_2}) < -\varepsilon z^\top Pz \quad (3.10)$$

for any nonzero z. Next, we derivate the Lyapunov function $V(z) = (Ez)^\top P(Ez)$ along the trajectories of the switched affine system (2.1) is computed and evaluated as:

$$\begin{aligned}
\dot{V}(z) &= \frac{d}{dt}((Ez)^{\top}P(Ez)) \\
&= (E\dot{z})^{\top}P(Ez) + (Ez)^{\top}P(E\dot{z}) \\
&= (A_{\sigma_1}z + (A_{\sigma_1}x_e + b_{\sigma_2}))^{\top}P(Ez) + (Ez)^{\top}P(A_{\sigma_1}z + (A_{\sigma_1}x_e + b_{\sigma_2})) \\
&= z^{\top}(A_{\sigma_1}^{\top}PE + E^{\top}PA_{\sigma_1})z + 2z^{\top}E^{\top}P(A_{\sigma_1}x_e + b_{\sigma_2}) \\
&< -\varepsilon \parallel P \parallel \parallel z \parallel^2 \\
&< -\tilde{\varepsilon} \parallel z \parallel^2
\end{aligned}$$

for any nonzero z. This implies that the switched affine system (3.8) is quadratically stable at the origin and thus the state of the switched affine system (2.1) converges to x_e.

Remark : In the definition of the set S_e, the first combination dominates the stability of the system, and the second one helps to adjust the desired equilibrium point. And for a fixed λ_i satisfying assumption 1, the set S_e is convex; which is beneficial in real computation.

CONCLUSION

In this paper, we have extended the concept of convex combination of subsystems for switched ordinary affine systems to quadratic stabilization of a particular class of switched affine DAEs systems. Different from the literature, we have formulated a new type of switched affine DAEs systems, where both subsystem matrices and affine vectors are switched by different switching laws. Assuming that no single subsystem has desired quadratic stability, we have shown under impulse-freeness hypothesis, that if the swicthed there exist a convex combination A_λ and b_μ of the matrices $A_i, (i = 1, \dots, N_1)$ and the vectors $b_j; (j = 1, \dots, N_2)$ respectively such that the linear time-invariant algebraic system $E\dot{x} = A_\lambda x + b_\mu$ is asymptotically stable and another convex combination of affine vectors is zero, then we can design state-dependent switching laws such that the switched affine DAEs system is quadratically stable. We also prove that when the convex combination of affine vectors is not zero, we discuss the quadratic stabilization to a convergence set. In our future work, we will extend the present approach (combination of subsystem matrices and affine vectors) to more general class of switched affine DAEs systems where E is not the same for all subsystems.

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