Transactions on Engineering and Computing Sciences - Vol. 11, No. 4
Publication Date: August 25, 2023
DOI:10.14738/tecs.114.15383.
Kim, Y. M. (2023). A Study on the Eigenmodes Constructed in the Double Ridge Waveguide. Transactions on Engineering and Computing Sciences, 11(4). 175-183.

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A Study on the Eigenmodes Constructed in the Double Ridge Waveguide

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ABSTRACT

In this study, the eigenmodes of a double ridge waveguide has been investigated by finite element method. The eigenmodes are schematically represented by the electric field, 3D electric potential and equipotential contour. The electric field together with the electric potential compensate each other in understanding the eigenmodes. The eigenmodes are divided into two classes: transverse electric modes and transverse magnetic modes. They are determined by the intensity of electric field and the peak position of electric potential. The several prominent eigenmodes from the lower order are placed in the spectra. They are more complicated by boundary conditions than single-ridged waveguides. By symmetrically arranging the two ridges up and down, the eigenmodes show various shapes in the double ridge waveguide. Since the waveguide forms a bilaterally symmetrical structure with respect to the center line, the eigenmodes exhibit such a symmetrical shape.

Keyword: double wave guide, eigenmode, transverse electric mode, transverse magnetic mode, ridge, symmetry.

INTRODUCTION

Previously, FEM (Finite Element Method) has been carried out to investigate the eigenmodes constructed in the CBCW (Conductor Backed Coplanar Waveguide) [1]. The eigen modes resulted from the FEM were schematically represented with electric field and electric potential pairs. In this result, it has been identified that the electric field together with the electric potential compensate each other in understanding the eigenmodes. If the resolution of the electric field is better than the potential, the eigenmode was determined through this. In addition, when the resolution of the potential is better than that of the electric field, the eigenmode was confirmed through the potential. Using this process, the effect of the ridges included in the waveguide on the generation of eigenmodes was also explored [2]. It was understood that the position of ridge was important in the waveguide and that the eigenmodes were complex even if the ratio between ridge and the waveguides are not so large. Ridges were placed symmetrically with the waveguides in the bottom plane. Geometries such as the width, height and shape of the ridges affected eigenmode formation. From this study, it was confirmed that the single ridge waveguide created eigenmode of low order involving the ground state even if the geometrical factors were varied.

In this study, the eigenmodes formed in double ridge waveguides is discussed using FEM. Two ridges of dielectric material protrude from the bottom and top flat of the waveguide. They are

arranged symmetrically left and right with respect to the center line. The space of the waveguide propagating electromagnetic waves is delimited by a mesh of triangular elements. Generally, the eigen-equation is expressed as high-dimensional matrix equations proportional to the number of the triangular element. Since the same trend is also found in this study, the eigenequation of matrix form is too large to manipulate by the personal computer. So, the sophisticate method like Arnoldi algorithm is applied to compress the large dimensional eigen-equation into the smaller one [3]. From the resulted eigen-equation, Krylov-Schur iterative method is applied to finds several prominent eigenpairs. This method stands out uniquely and is used in this study to reveal the prominent characteristics of electromagnetic waves formed in waveguides [4][5]. As a result, the eigenpairs are schematically represented by the electric field and 3D (dimensional) potential and electric equipotential contour. The eigenpair is composited of the electric field and potential as described in the single ridge waveguide. What is noteworthy in this manuscript is that the theory of FEM and the subsequent development of formulas have been mentioned in detail in previously published works, so they are omitted. This manuscript describes the structure and spectrum properties of the ridge waveguide, the result and discussion of the simulation, and conclusion in order.

THE STRUCTURE AND SPECTRUM PROPERTIES

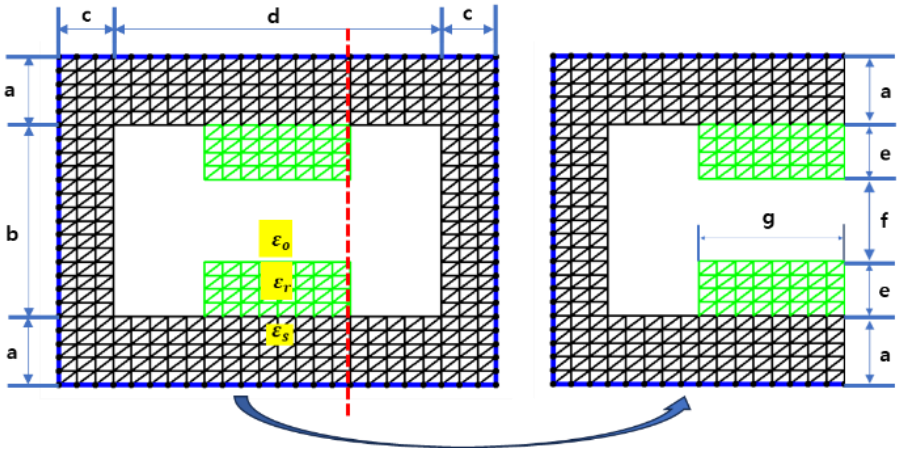


Figure 1: Schematic representation of the double ridge waveguide.

Schematically, the double ridge waveguide is represented in the figure 1. Two ridges of dielectric material are protruded from the top and bottom of the substrate. The geometry, such as the width, height and shape of the ridge, along with its relative permittivity, has a great influence on the formation of the eigenmodes. The horizontal width of the ridge represents a ratio of 8 : 25 = g : ($2c$ + d) based on that of the waveguide. The vertical height of the ridge represents a ratio of 4 : 25 = e : (2a + b) based on that of the waveguide. The shape of the ridge forms a right-angled rectangle with a slightly longer in width than the waveguide. The geometry of the waveguide does not vary during the computational process of FEM. The substrate as can be seen in the figure 1, is symmetrically arranged around the entire waveguide. The relativistic permittivity is ε_r(= 10) and ε_s(= 15) for the ridge and substrate respectively. The space enclosed by the substrate and the ridge is considered vacuum for convenience whose permittivity is assumed ε_o(= 1). Surface of the waveguide shown in blue greatly affects the formation of the eigenmode by setting the boundary conditions. If the surface is formed of the same material as the dielectric of the substrate, FEM calculations reveal the eigenmode of a TE

(Transverse Electric) mode. When the surface is formed with the conductor which permit only perpendicular electric field, the result of FEM calculations appears in the eigenmode of a TM (Transverse Magnetic) mode. The boundary conditions of TM and TE modes are made by excluding or retaining the matrix component of the surface among the eigen-equation established during the process of FEM calculation. The eigenpair of TM mode is obtained by excluding the surface element of eigenequation. Eigen pairs of TE modes are obtained by retaining the surface elements of the eigen-equations. The dimension of the matrix equation for the TM mode is smaller than that of TE mode. Therefore, FEM calculations allow TM mode to handle with a smaller scale capacity of personal computer than TE mode.

FEM is carried out to calculate the eigenmodes for the double ridge waveguide. The mathematical theory is not mentioned here because it has been discussed in detail in the previous manuscript. Only qualitative theories will be discussed in this manuscript. The eigenmode referred to in this manuscript consists of a pair of electric field and electric potential. These are simultaneously obtained through the calculation of the FEM in relation to the vector Helmholtz equation. For the calculation of FEM, the entire space where electromagnetic waves are formed is reconstructed as a triangular element mesh. In order to consistently describe the electric field represented by vector components, the edges of the triangular elements must be arranged regularly. As can be seen from figure 1, except for the edge exists on the surface, the one that exists inside the waveguide is shared with two neighboring triangular elements. An edge present on the surface belongs to only one triangular element. The eigen electric field of a triangular element reveal at the barycentric point and it is determined by combination of the edge vectors of the triangular element. Regardless of the location, the edges belonging to the neighbor triangles must always point in the same direction. The eigen electrical potential is determined by the nodes of the triangular element. Except for the nodes on the surface, those in the inside of the waveguide are shared with six neighboring triangular elements. A node present on the surface of the waveguide is shared with three neighboring triangular elements. And the node exist at the conner of the waveguide are involved only one elementary triangle. The eigen electric potential is described as scalar quantities at the nodes. Those values are used to construct the 3D representation and the electric equipotential contours. The equipotential line is drawn by connecting the points obtained from interpolating the potential values of nodal points of the triangular element.

As mentioned in the introduction, the eigen-equation of the double waveguide is very large proportional to the number of the triangular elements. In FEM process, the sophisticate method such like Arnoldi algorithm and Krylov-Schur iterative method has been applied to find several significant eigenpairs. We also have carried out this method to reveal the prominent characteristics of electromagnetic waves formed in photonic crystal waveguides etc., [6][7]. Therefore, with these algorithms, it would be expected to obtain satisfactory results of the eigenpairs for the double ridge waveguide.

THE RESULT AND DISCUSSION

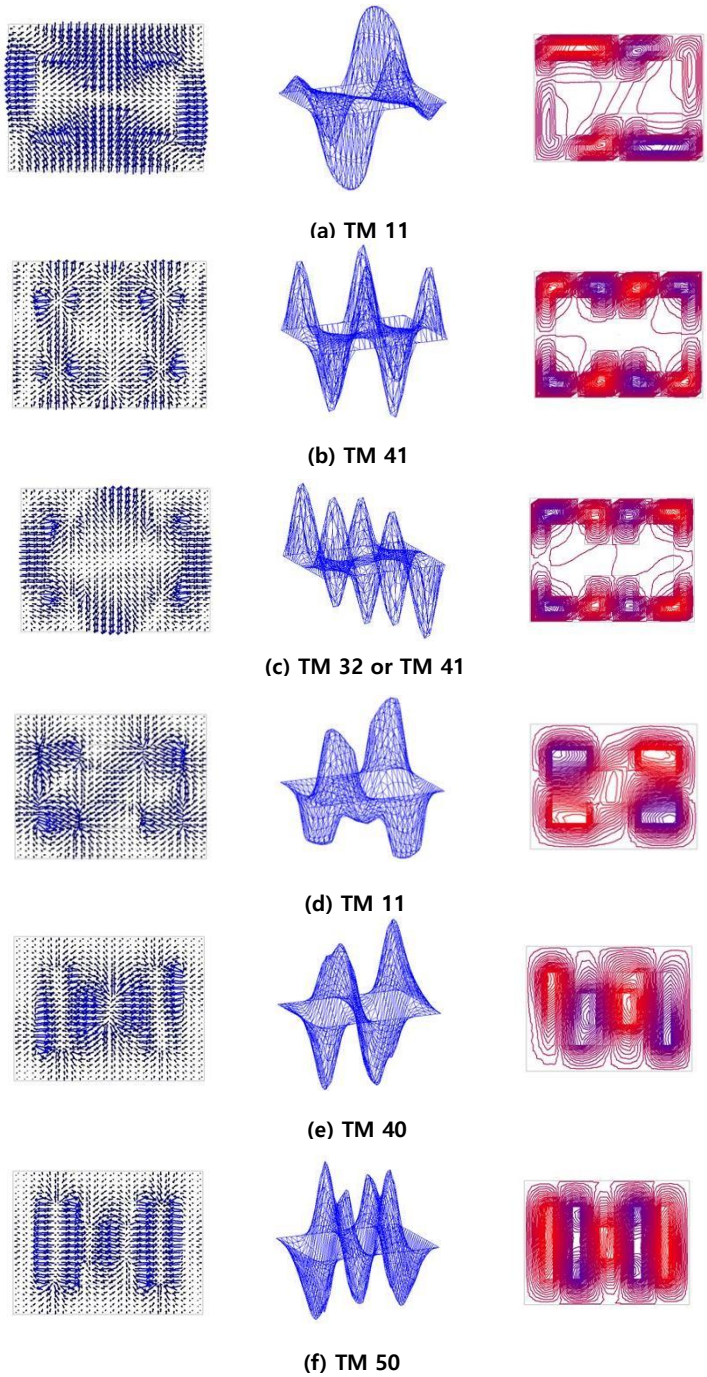


Figure 2: Schematic representation of TM eigenmodes

As can be seen in the figure 2, the spectra representing the eigenmodes of the double ridge waveguide are not simple as those of the single ridge waveguide. Two ridges protruding upward and downward combine with the substrate to form a complex eigenmode. In the process of FEM, The TM mode is obtained by excluding edge and nodal components present on the surface of the waveguide from the matrix eigen-equation. Thus, the electric field is formed perpendicular to the surface and the electric potential is established parallel to it. Figure 2

illustrates these characteristics very well. Figure 2(a) represent most simple eigenmode MT 11. The electric field exists throughout the waveguide with the oblique direction as the boundary. Four peaks formed by the electric potential symmetrically exist on the dielectric substrate and ridge. When comparing the distribution of electric potential and electric field, there are some discrepancies, but there is no great difficulty in defining the eigenmode as TM 11. The peak difference in potential intensity is interpreted as being due to the different spatial and dielectric constants of the ridge and the substrate. Figure 2(b) is another eigenmode different from that of TM 11. The electric potential dominantly distributes in the region of substrate. The electric field is mainly distributed in the space enclosed by the ridge and the substrate. And the electric field is concentrated strongly in a place with a small radius of curvature such like the conner of ridge and substrate. The darker the blue color in the spectrum, it means that the stronger the intensity of electricity and electric potential. The electric field is created by the spatial gradient of the electric potential. As shown in Figure 2(a, b, c), the electric potential is highly developed on the substrate, and its value is rare in the vacuum region inside the waveguide.

In a vacuum region, an electric field is developed by the electric potential of the substrate surrounding it. There is four x-mode vertical lines and one y-mode horizontal line in the spectrum of electric field eigenmode. Therefore, it can be said that the spectrum of figure 2(b) where the electric field is formed in the space surrounded by the ridge and substrate reveal the eigenmode TM 41. Figure 2(c) shows another spectrum in which the eigenmodes are difficult to determine. Reflecting on the line of electric equipotential contour of figure 2(b), the eigenmode seems to be TM 41. However, comparing to the electric field of figure 2(b), it cannot be concluded that this spectrum represents the eigenmode as TM 41. The spectrum of the electric field eigenmodes has two distinct x-mode lines near the vertical substrate and a faint ymode horizontal line within them. In the vertical substrate, the strong electric field directed to the horizontal direction is established. It can be interpreted through the color representing the strength of the electric potential. Those colors mean that two strong peaks of the electric potential are formed on the left and right substrates.

From those features, it can be concluded that such a strong electric field can be formed in the substrate when these pairs with extreme electric potential interact with each other. Therefore, the eigenmode TM 32 can be also assigned to figure 2(c). The spectra of figure 2(d, e, f) are different from the spectra of figure 2(a, b, c). The electric potential is distributed in the vacuum region enclosed by the ridge and substrate. Eigenmodes can be assigned to each spectrum without difficulty. Firstly, the eigenmode TM 11 can be assigned to the spectrum shown in figure 2(d). One x-mode vertical line and one y-mode horizontal line manifestly appear in the spectrum of electric potential. Although the potential peak is not sharp, it is due to the geometrical characteristics of the double waveguide. The electric field is distinctly distributed in the vacuum region surrounded by ridges and substrates. The eigenmodes imparted through the electric field correspond to those TM 11 determined using the electric potential. The eigenmodes for the spectrum of figure 2(e) is easily determined using either electric field or the electric potential. The electric potential manifestly reveals four x-mode vertical lines. There is no y-mode horizontal line in the spectrum. Thus, the eigenmode TM 40 can be assigned to the spectrum. Some components of electric potential are included in the center of the three-dimensionally expression. However, this is considered to be part of the eigenmodes and does not seem to be the subject of an independent discussion. The spectrum representing the electric

field is more distinct for assigning TM 40 as an eigenmode. The vacuum region enclosed by the ridge and substrate is filled with four vertically aligned electric field components. The electric field is distributed symmetrically to the left and right with respect to the center of the waveguide. The electric field component does not exist beyond the ridge or substrate. They are distributed only in the vacuum region surrounded by them. Figure 2(f) shows a spectrum with similar characteristics as (e). The electric field exists only in the region surrounded by the ridge and the substrate.

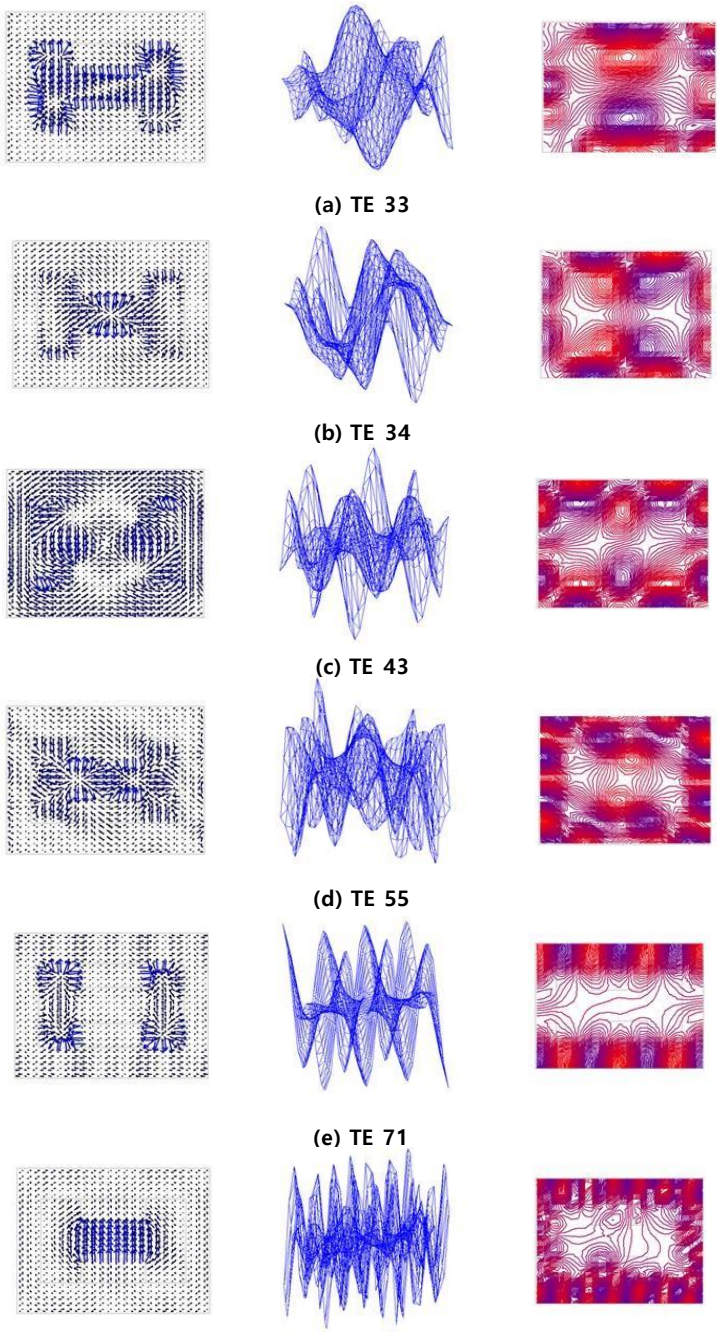


Figure 3: Schematic representation of TE eigenmodes

The electric potential is distributed symmetrically from side to side with respect to the center line of the waveguide. The spatial distribution of equipotential lines shows five peaks, which can be recognized in a three-dimensional representation. In addition, the spectrum representing the electric field shows a cluster of five electric field lines in the longitudinal direction. As in spectrum (e), the electric field is distributed symmetrically in the vacuum region with respect to the center of the waveguide. The electric potential is also distributed symmetrically from side to side with respect to the center line of the waveguide as in spectrum (e). Therefore, the eigenmodes for this spectrum assign as TM 50 without any problems.

Figure 3 shows the TE mode spectra for the double waveguide. In the process of FEM calculation, the TE mode can be obtained from the boundary condition that the surface of the waveguide is made of a material such as a

dielectric inside. The matrix eigen-equations of electric field and potential have no canceling components. Unlike in TM mode calculations, there are no constraints on electric fields and potentials in the process of FEM. Reflecting this non-constraint, the result is complex and finding the eigenmodes is very difficult. Only higherorder eigenmodes exist, and it is difficult to find lower-order eigenmodes including the ground state. The spectra representing good resolution and reliable eigenmodes will be selected and discussed in the following description. The spectra of figure 3(a) represent the eigenmode of the lowest order for the TE mode in this study. As can be seen in these spectra, the electric potential developed distinctively in the ridge and substrate. However, its value is not so great in the vacuum region surrounded by the ridge and the substrate. Instead, the electric field is distinctly developed in that area. As mentioned in the description of the TM mode, the electric field appears proportionally to the spatial gradient of electric potential. In this context, when figure 3(a) is interpreted, the spatial distribution of electric potential and electric field is interpreted consistently. It can be confirmed from these spectra that the electric field develops in the space between them rather than at the peak point of the electric potential. Through this discussion, the eigenmode TE 33 can be assigned to figure 3(a).

Figure 3(b) shows another spectrum, although the electrical potential is complex. It is very difficult to determine eigenmodes through complex electrical potentials alone. By combining this with the spatial distribution of the electric field, it can be imposed eigenmodes that sufficiently characterize the spectra. The electric field is concentrated on the boundaries between the ridge and the vacuum, and between the substrate and the vacuum, respectively. Electric potentials are strongly developed in the ridges and substrates, but electric fields are faint in these regions. In a ridge or substrate, the effect of interacting with an opposing component is greater than the electric potential next to it. It is reasonable to confine the factors contributing to the formation of eigenmodes to the components in the region of the ridge and substrate. Under the premise that the interaction between the components facing each other across the vacuum is more dominant than the side component, the eigenmode in Fig. 3(b) can be determined as TE 34.

Figure 3(c) can also be explained in the same context. Electric potentials are highly developed in the ridges and substrates, but are sparse in the vacuum region they surround. Oppositely, the electric field is sparse in the ridge and substrate, but highly developed in the vacuum they

surround. One thing to note is that some component of the electric field appears between the substrate and the surface boundary of the waveguide. This can be understood by looking at the electric potential created between the substrate and the surface. There are weak components mediating between the extreme electric potentials. Thus, an electric field is created between neighboring extreme potentials. As in the case of 3(b), considering the electric potential on the ridges and substrate that generate electric fields in the internal vacuum, the eigenmode in figure 3(c) can be set to TE 43. Figure 3(d) is complex, but the eigenmodes can be set according to the above-mentioned discussion. As shown in 3(b, c), even in the spectra of figure 3(d), the electric potential is highly developed on the ridge and substrate and sparse in the inner vacuum region. The electric field is highly developed in the vacuum region enclosed by the ridge and the substrate and sparse in outside this region. The eigenmode for the spectrum 3(d) is mainly determined by the electric potential distributed in the regions of ridge and substrate and as a result it can be imposed to be TE 55.

Figure 3(e) is another different spectrum distinguished from figure 3(a, b, c, d). There is only one y-mode horizontal line across vacuum region. There are seven x-mode vertical lines across the top and bottom. These electrical potentials are distinct and play an important role in determining the eigenmodes. The eigenmode for it can be assigned to be TE 71 considering only the electric potential. The electric field is only established vertically in the inner vacuum region excluding between the region of the double ridge. This spectrum is contrast to figure 3(f). Figure 3(f) shows that the electric field only exists between the ridges. When the two spectra for the electric field are combined, it results in the form of Figure 3(a). But it cannot be said that these two spectra compensate each other. Because the electric potential does not reveal these features. This is because only the electric field shows this appearance externally, but the electric potential shows completely different characteristics. If the eigenmode of spectrum 3(f) is given through the electric potential, the result may be TE 95. And, if the eigenmode of spectrum 3(f) is given through the electric field, the result may be TE 10.

CONCLUSION

FEM has been carried out to investigate the eigen properties for the double ridge waveguide. The eigen pairs were divided into TM and TE modes composited with the electric fields and electric potentials. The structural imposed boundary conditions made complexity in results obtained through FEM calculations. Results were schematically represented as electric field, 3D potential and electric equipotential contour. The TE mode obtained a complex mode in which the ground state and low-dimensional results were excluded. However, eigenmodes were assigned to each spectrum including TM mode. And, they revealed the eigen-properties of the double ridge waveguide.

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