Transactions on Engineering and Computing Sciences - Vol. 11, No. 3
Publication Date: June 25, 2023
DOI:10.14738/tecs.113.14896.
Kim, Y. M. (2023). A Study on the Eigenmodes Constructed in the Conductor Backed Coplanar Waveguide. Transactions on Engineering and Computing Sciences, 11(3).108-121.

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A Study on the Eigenmodes Constructed in the Conductor Backed Coplanar Waveguide

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ABSTRACT

Finite element method (FEM) is applied to obtain eigenmodes constructed in the conductor backed coplanar waveguide. The eigenmodes for coplanar and microstrip waveguides are also calculated for understanding the eigen properties of this waveguide. The cross sections of the waveguides are divided into the simple triangular elements in the calculations. The edge vectors and nodes of an element are basic components for constructing the eigenmodes of the waveguides. The vector Helmholtz equation is made to the eigen matrices in the process of FEM. Those eigen matrices are diagonalized by the Krylov-Schur iteration method. The eigenmodes are obtained from the similar transform matrices diagonalizing the eigen matrices. As results, specific eigen spectra representing the properties of the waveguides have been revealed schematically and compared with each other.

Keywords: finite element method, eigenmode, conductor backed coplanar waveguide, Krylov-Schur iteration method

INTRODUCTION

The conductor backed coplanar waveguide (CBCPW) is a common variant of the coplanar waveguide (CPW). It is formed by attaching a conductor to the back of the CPW's dielectric substrate. Conventional CPW consists of a signal conducting track printed onto a dielectric substrate, together with a pair of ground conductors, one to either side of the track. The advantage of CPW is that a signal transmission track and a pair of ground conductors on the same plan can achieve an electric field of a more plausible transverse electromagnetic mode (TEM). The detailed information of the CPW helps to understand the characteristics of the CBCPW's eigenmodes. Especially it is known that when the distance between the top and rear conductor plates of the dielectric are increased, the two waveguides exhibit similar eigen properties [1][2].

The microstrip waveguide (MSW) is constructed with a single signal conducting track over a dielectric substrate which is backed by a conductive ground plane. In MSW, the electromagnetic fields are contained mainly within the dielectric substrate between the two conducting planes, with a higher field concentrating at the edges of the signal conducting track. MSW shows different properties from CBCPW in many respects. It is because CBCPW is constructed with the ground planes on the backed the dielectric substrate and either side of a signal conducting track. But, if spacing of CBCPW circuit's top grounds from the signal conducting track increased, those grounds become less impacted on the circuit and the eigenmode to be similar with MSW. In other word, MSW operate as like CBCPW by means of a dominant quasi-TEM propagation mode [3][4].

In this manuscript, the eigenmodes for above mentioned waveguides are calculated and reviewed by using the FEM [5][6]. The cross section of waveguide is divided into simple triangular elements. The vector Helmholtz equation governing FEM process is represented through the linearized simultaneous equations. Those are rebuilt into a matrix eigen equation of high-dimensional symmetric form. The square matrix is made to be a Schur matrix by similarity transform. The eigenmodes are represented by the column components of similarity transformation matrix. The Krylov-Schur iteration method is used to obtain the prominent several eigenmodes in calculation processes. The characteristics of eigenmodes are described by the electric field distributions and the contour lines of the electric equipotential. These are obtained by extrapolating the column matrices corresponding to the edges and nodes of triangular mesh elements. The electric fields are represented at the center of a mesh by combining field components of its edges. The electric equipotential is obtained from the extrapolation of node components of the column matrices. As results, the eigenmodes that characterize MSW, CPW and CBCPW are depicted in the simplified spectra. The characteristics of these eigenmodes are briefly mentioned and their association is examined by comparing them with each other.

THE WAVEGUIDES

The waveguides studied in this manuscript are schematically represented in Figure 1. All waveguides have the rectangular cross section of the same width, height and area. The height and width of the waveguide must be sufficient size for building the eigenmodes of the electromagnetic waves in it. Especially, the area of the waveguide is an important factor in determining the characteristics of the eigenmodes. This area must be sufficiently large to be considered that the wave emitted by the signal conducting track is disappeared from its edge. This is because it can cause unwanted higher order modes to propagate in the waveguide. The upper part of the dielectric is considered free space and the permittivity is assumed to be ε_0=1. FEM perform the calculation including this area in order to know how the electromagnetic waves emitted by the signal conduction track spread

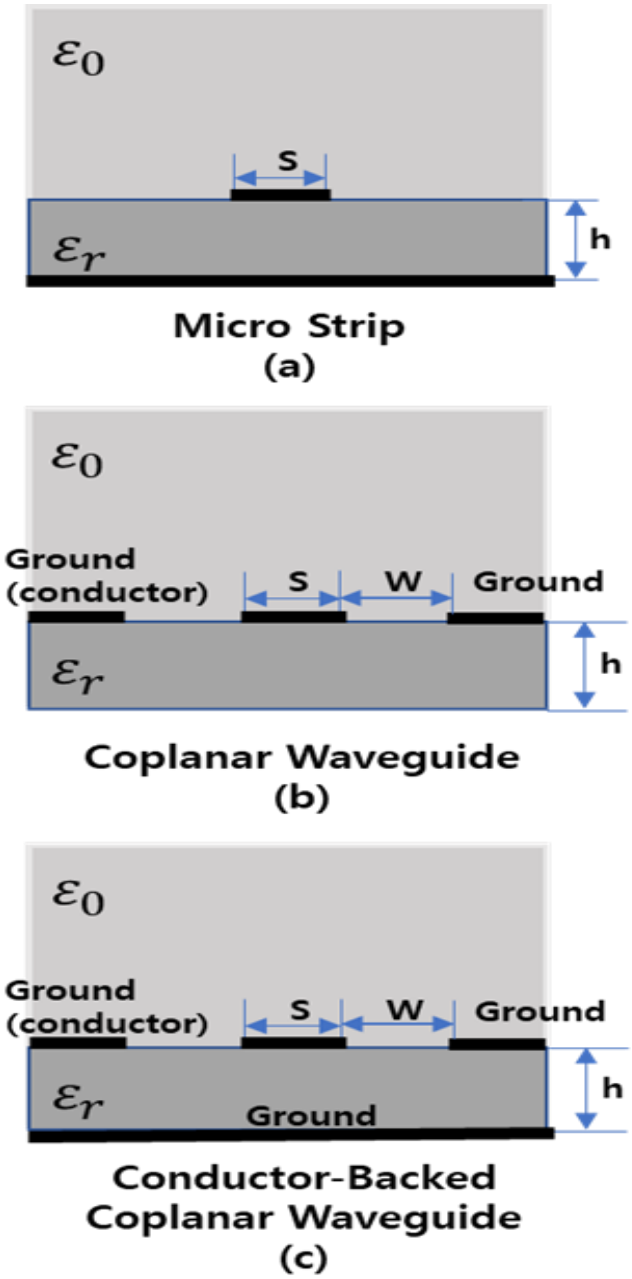


Figure 1 The Waveguides.

out into free space. The relative permittivity of the dielectric substrate has larger value $\varepsilon_0 < \varepsilon_r$ than the free space. According to Snell's law, electromagnetic waves are expected to be concentrated in areas with a large permittivity. The height of the dielectric substrate is 1/4 of its width. The ground conductor and signal conducting track onto the substrate are symmetrically arranged with respect to the vertical center line of waveguides. The width of the signal conducting track is 1/5 of that of substrate. In the case of CBCPW and CPW, the distance between the signal conducting track and a ground conductor on the same plan is 1/5 of the substrate width. In CBCPW, this distance has a profound effect on determining the nature of the eigenmode. It has been known that as this distance increases, the eigenmode takes on a shape

similar to MSW. In other words, as this distance increases, the influence of the ground conductors has less effect on the electromagnetic waves emitted from the signal conducting track [7][8]. In this study, in order to reveal more clearly the characteristics of the eigenmodes, the most prominent distance is chosen and fixed in all waveguides as shown in the Figure 1.

CBCPW is a common variant of CPW. The difference between those waveguides is whether the conductor is attached to the bottom of the dielectric substrate. The ground conductor attached on the bottom of CBCPW circuit provides additional mechanical stability compared to CPW circuit, with improved thermal management for higher power circuits and devices. As the conductor is attached to the bottom, the boundary conditions of CBCPW are different from CPW. The electric field is incident perpendicularly from the outer surface of the conductor and their tangential component is not permitted. These constraints affect the formation and characterization of eigenmodes, as mentioned in the result and discussion section of this manuscript. In practice, the microwave and millimeter wave integrated circuits (MMIC) will have a backside ground plane intentionally added for heat removal or inadvertently added by the package. Although the dielectric thickness "h" is the same, finding and comparing the eigenmodes of CBCPW and CPW provides an understanding of the characteristics of the eigenmode depending on the presence or absence of the attached back conductor. For FEM calculation, cross sectional area of the waveguide is uniformly divided into simple triangular elements. The important components of FEM calculation for triangular elements are the edges and nodes of these elements. The edges are for calculating the electromagnetic transverse field and the nodes are for obtaining the scalar potential. The direction of the edge is especially important to calculate the electromagnetic field distribution. The direction of the common edge of two adjacent triangular elements must be the same. As mentioned in the previous study, the reason for it is to adhere to the consistent direction of the electric field. [9]. This restriction constrains the mesh construction in several ways when decomposing space for FEM calculation. The spatial arrangement of the triangular elements must be carefully considered. To achieve this relation, when the edge vectors of one triangular element are set clockwise, those of another adjacent triangular elements must be arranged counterclockwise as like Figure 2. As shown in this figure, the length and the orientation of one edge is shared by two triangular elements, and a node is shared by six triangular elements.

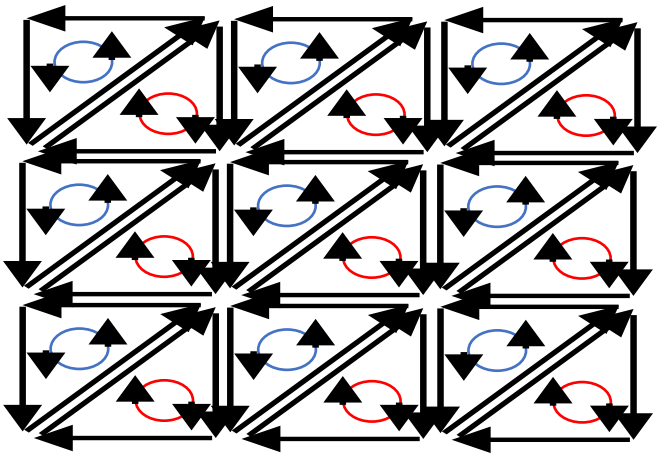


Figure 2 Mesh and directions of the edge vectors.

FEM THEORY

Generally, it is well known that the above-mentioned waveguides form quasi-TEM modes. Those waveguides include some TE and TM modes independently but the TEM eigenmodes are dominant comparing to them. This is because the conductors are partially attached to the surface of the substrates. The conducting surface is very important in determining the eigenmode. The Neumann and Dirichlet boundary condition are applied for TM and TE modes respectively. For the perfect conductor attached on the dielectric substrate, the electric field only incident on it perpendicularly. Tangential components of the electric field are not allowed along the conductor in the equilibrium state. To apply this fact to FEM, the edge vectors and nodes of the triangular elements each contacting the conductor boundary are excluded for TE mode. In FEM process, those components describe the electric field and electric potential respectively. The boundary conditions mentioned above cannot be applied to the rest of the parts where conductors are excluded. TM mode predominates in this area where conducting material is excluded. In a structure in which TE and TM characteristics are mixed, those modes are combined into quasi-TEM modes. In this manuscript, quasi-TEM eigen properties are considered for above mentioned waveguides. The theory base on the vector Helmholtz equation. FEM is applied to construct matrix eigen equations that are solvable through numerical analysis [10]. In FEM calculating process, TM is similarly described to that of TE.

These modes can be consistently described together by the function $\vec{F}$. For convenience of discussion, it is assumed that the cross section and the axis of the waveguides are the xy-plane and z-axis of the Cartesian coordinate, respectively. In this coordinate, the vector Helmholtz equation is given by

$$\vec{\nabla} \times (\bar{\bar{p}}\vec{\nabla} \times \vec{F})\text{-}k_o^2\bar{\bar{q}}\vec{F} = 0 \tag{1}$$

and the wavefunction

$$\vec{F} = (F_x\hat{x} + F_y\hat{y} + F_z\hat{z})e^{-j\beta z} \tag{2}$$

where β is the propagation constant and for electric field strength $\vec{F} = \vec{E}, \bar{\bar{p}} = \frac{1}{\bar{\bar{\mu}}_r}$($\mu_r$: relative permeability μ/μ_o) $\bar{\bar{q}} = \bar{\bar{\varepsilon_r}}$ (ε_r: relative permitivity $\varepsilon/\varepsilon_o$), and for magne-tic field strength $\vec{F} = \vec{H}$(magnetic field strength),$\vec{p} = 1/\bar{\bar{\varepsilon}}_r, \bar{\bar{q}} = \bar{\bar{\mu}}_r$ .In this study, it is assumed that the waveguides have the homogeneous dielectric substrates. So, the following relation is reflected in the tensor matrix of the equation

$$\begin{cases} \varepsilon_{ij} = \varepsilon, \mu_{ij} = \mu \ \text{when} \ i = j \\ \varepsilon_{ij} = 0, \mu_{ij} = \mu \ \text{when} \ i \neq j \end{cases} \tag{3}$$

For the convenience of the description, the following calculating process focus on the electric field. By separating the transverse vector fields and the z-components scalar potential, equation (1) for electric field can be described as

$$\vec{\nabla}_t \times (\vec{\nabla}_t \times \vec{E}_t) + \frac{1}{\mu_r}(\beta^2 \vec{\nabla}_t E_Z + \beta^2 \vec{E}_t) = k_o^2 \varepsilon_r \vec{E}_t \tag{4}$$

$$-\frac{1}{\mu_r}[\vec{\nabla}_t \cdot (\vec{\nabla}_t E_Z + \beta \vec{E}_t] = k_o^2 \varepsilon_r \vec{E}_z \tag{5}$$

Where,

$$\vec{E}_t = E_x \hat{x} + E_y \hat{y} \tag{6}$$

When the vector Helmholtz equation is divided into two equations, one equation is described by the tangential edge vectors and other by the node components of the triangular element. Then the transverse electric field is approximately given by the tangential edge vectors of a triangular element as follows

$$\vec{E}_t = \sum_{m=1}^{3} e_{tm} \vec{W}_{tm} \tag{7}$$

where $\vec{W}_{tm}$ is the edge m vector of the triangular element. Its components are given by the simplex coordinate as like

$$\vec{W}_{t1} = L_{t1}(\alpha_3 \vec{\nabla}_t \alpha_2 - \alpha_2 \vec{\nabla}_t \alpha_3 \tag{8}$$

$$\vec{W}_{t2} = L_{t2}(\alpha_1 \vec{\nabla}_t \alpha_3 - \alpha_3 \vec{\nabla}_t \alpha_1 \tag{9}$$

$$\vec{W}_{t3} = L_{t3}(\alpha_2 \vec{\nabla}_t \alpha_1 - \alpha_1 \vec{\nabla}_t \alpha_2 \tag{10}$$

where L_{ti} is a length of the edge connecting the nodal points j and k. The scalar potential of z-component equation is described by the nodal based first order Lagrangian interpolation functions as

$$E_z = \sum_{i=1}^{3} e_{zi} \alpha_i \tag{11}$$

where α_i is the simplex coordinate of node i. The simplex coordinates are given with nodal points of the triangular element of Figure 3.

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}^{-1} \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} \tag{12}$$

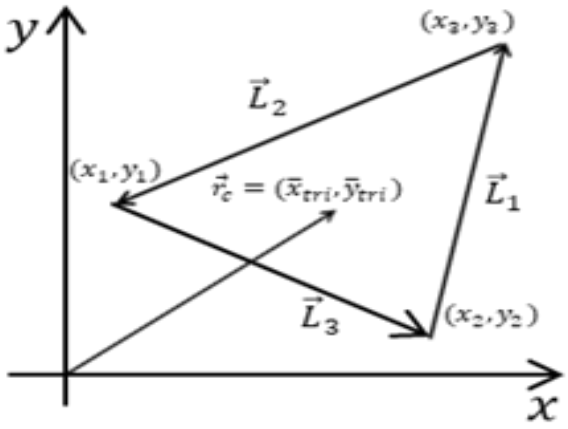


Figure 3 The simplex coordinate.

From Figure 3, the relationship between tangential edge vectors and nodes of a triangular element can be identified.

The vector Helmholtz equation for the triangular element can be made into the matrix eigen equation

$$\begin{pmatrix} S_{el(tt)} & S_{el(tz)} \\ S_{el(zt)} & S_{el(zz)} \end{pmatrix} \begin{pmatrix} e_t \\ e_z \end{pmatrix} = \begin{bmatrix} T_{el(tt)} & 0 \\ 0 & T_{el\,(zz)} \end{bmatrix} \begin{pmatrix} e_t \\ e_z \end{pmatrix} \tag{13}$$

where each component of the square matric are given as

$$S_{el(tt)} = \frac{1}{\mu_r} \iint_{\nabla} (\vec{\nabla}_t \times \vec{W}_{tm}) \cdot (\vec{\nabla}_t \times \vec{W}_{tn}) ds - \frac{\beta^2}{\mu_r} \iint_{\nabla} (\vec{W}_{tm} \times \vec{W}_{tn}) ds \tag{14}$$

$$S_{el(tz)} = \frac{\beta^2}{\mu_r} \iint_{\nabla} \vec{W}_{tm} \cdot \vec{\nabla}_t \alpha_j) \, ds \tag{15}$$

$$S_{el(zt)} = \frac{\beta^2}{\mu_r} \iint_{\nabla} \vec{\nabla}_t \alpha_j \cdot \vec{W}_{tm}) \, ds \tag{16}$$

$$S_{el(zz)} = \frac{\beta^2}{\mu_r} \iint_{\nabla} \vec{\nabla} \alpha_i \cdot \vec{\nabla} \alpha_j) \, ds \tag{17}$$

$$T_{el(tt)} = \varepsilon_r \iint_{\nabla} \vec{W}_{tm} \cdot \vec{W}_{tn}) \, ds \tag{18}$$

$$T_{el(zz)} = \varepsilon_r \beta^2 \iint_{\nabla} \alpha_i \cdot \alpha_j \, ds \tag{19}$$

The global eigen equation is resulted by combining these element matrices for total meshes.

$$[S]\{e_{tot}\} = k_o^2 [T]\{e_{glob}\} \tag{20}$$

As results, the eigenmodes are obtained by similarity transforming the matrix eigen equation into the Schur matrix. The column vectors of the similarity transforming matrix are the eigenmodes and the diagonal components of the Schur matrix are eigenvalues.

In general, the dimensionality of the matrix eigen equation is very large. The personal computer cannot perform the calculation especially for the inverse matrix of large dimensionality in ordinary manner. Therefore, a sophisticated method such as the Krylov-Schur iteration method is applied to overcome this problem [11]. It is well known that this method gives several prominent eigenmodes for the waveguides robustly. Previously, this iteration method has applied to the varied optical fiber and revealed their eigen properties of propagating waves [12]. To apply this iteration method, the global eigen equation is firstly converted to the shift invert form as like

$$\frac{1}{k_0^2-\sigma}\{e_{glob}\} = \frac{T}{[S]-\sigma[T]}\{e_{glob}\} = [M]\{e_{glob}\} \tag{21}$$

The Krylov-Schur iteration method is applied to the matrix [M]. By doing so, this strategy may be more efficiently implemented in finding specific eigenmodes at σ value.

RESULT AND DISCUSSION

The eigenmodes of above mention waveguides are obtained by applying the Krylov-Schur iteration method to the matrix [$\boldsymbol{M}$] of the equation (21). Each column matrix of $\{e_{glob}\}$ is composed of the eigenmodes of electric field and electric potential simultaneously. The electric field vectors and the electric potential components are described by the edges and the nodes of the triangular elements respectively. Electric field is expressed at the center of a triangular element using equation (7). This electric field is established by combining edge components of $\{e_{glob}\}$ of a triangular mesh. The electric potential contour is obtained by combining and interpolating node components of $\{e_{glob}\}$ of the triangular element described by equation (11). Those results are expressed as the eigenpairs of each waveguide which describes the electric field and electric potential.

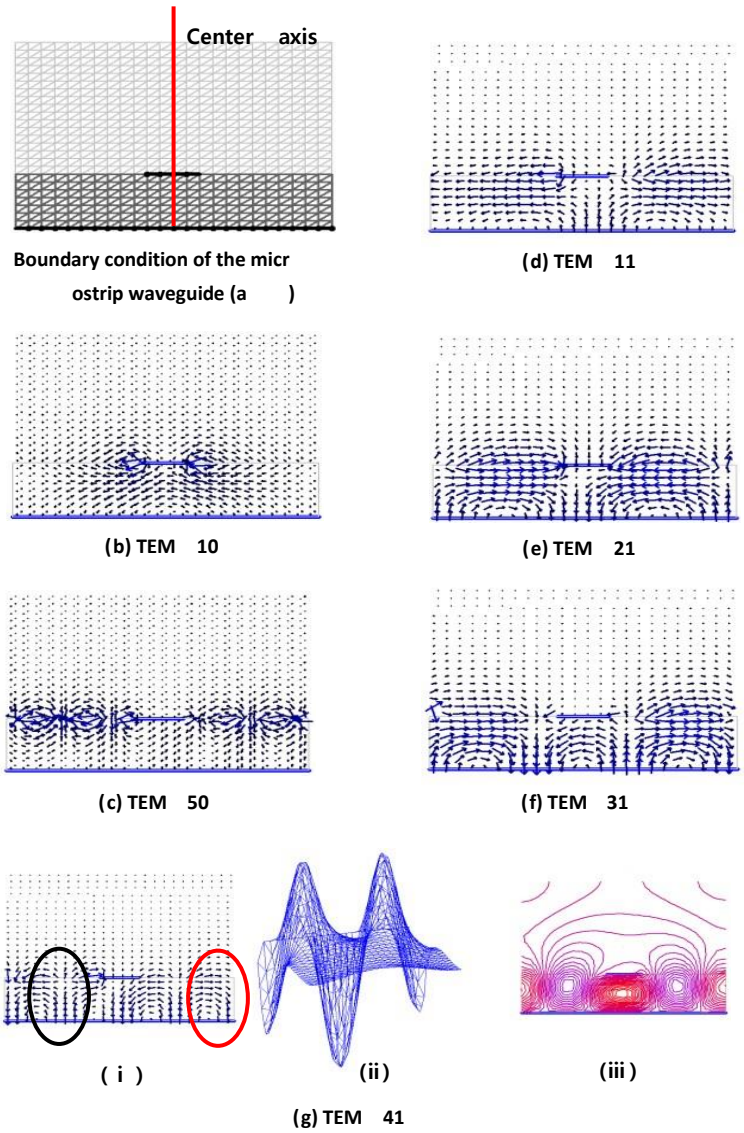


Figure 4 The spectra of the microstrip waveguide.

Figure 4. represent the eigenpairs of MSW. Figure 4. (a) describe the cross section of the waveguide dividing by triangular elements. The thick black lines represent the conductors and the boundary condition is applied to it in the process of FEM calculation. The edges and nodes of a triangular element are arranged as Figure 3. The edges vectors of the triangular elements are consistently arranged as Figure 2. to adhere to the coherent electric field. Figure 4. (b) and (c) are one series of eigenmodes formed near the interface between the substrate and the free space. Figure 4. (d)~(g) are the eigenmodes formed in the dielectric substrate and belong to another series. Figure 4. (b)~(f) and (g-i) represent eigenmodes of electric field formed in MSW. As can be seen from those spectra, the electric field are vertically incident on the conductors. The electric field is not confined to the dielectric substrate, but propagates weakly even in the vacuum. Figure 4. (g-ii) is the electric potential of TEM 41 eigenmode described in three-dimensional space. Figure 4. (g-iii) is the contour of equipotential for TEM 41 eigenmode projected on the xy plane of MSW. The mode types are determined based on the eigenmode

spectra describing the electric field and the electric potential. It is difficult to determine the mode type by looking at the contour of the equipotential line alone. The mode types are determined by comparing them with the three-dimensional representation of the electric potential and the distribution of the electric field. To determine the mode type of an eigenmode more clearly, the electric field strengths distributed in the black circle are compared with those in the red circle, as in Figure 4. (g-i). The electric field strength distributed in the black circle is twice as great as the one in the red circle. By summing the electric field strengths distributed at the edge of the substrate and comparing them with other intermediate electric field strengths, it is possible to know that they are parts of constituent to construct eigenmodes in succession. This indicates that the spectrum shown in Figure 4. (g) is TEM 41. The mode types of other spectra are determined in the similar way.

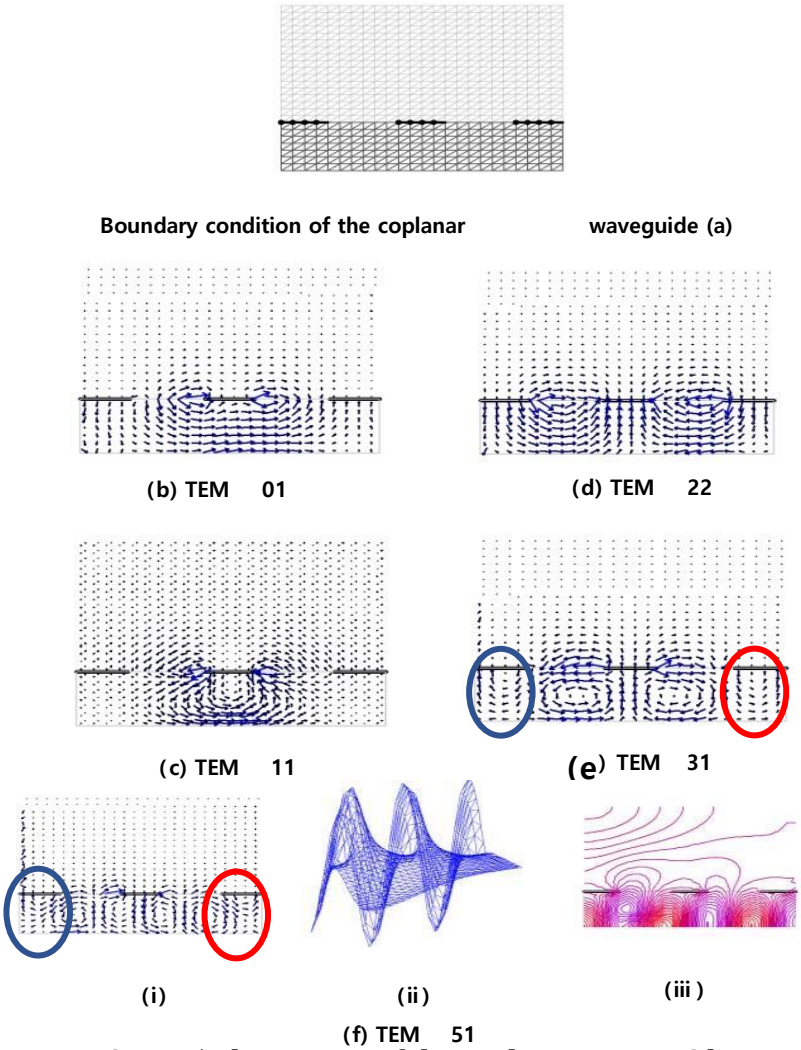


Figure 5 The spectra of the coplanar waveguide.

Figure 5. shows the spectra of CPW. The basic form for determining the properties of the spectra is mainly constructed on the substrate, as in MSW. The mode type is determined mainly through

the electric field seen in the spectra. The electric potential spectra are assistants to the electric fields in determining the mode type. There is a difference between MSW and CPW. As shown in Figures 4. and 5., TEM 31 can be clearly identified in MSW, but care must be taken to confirm this in CPW. The TEM 01 eigenmode seen in CPW is similar to TEM 10 in MSW, but it cannot be said that they are the same. It is inferred that the cause lies in the presence or absence of a ground conductor. The conductor attached to the bottom of dielectric substrate has a profound influence on the formation of the eigenmode. Unlike in dielectrics, electric fields are incident vertically on conductors. This restriction is an important factor in determining the characteristics of the eigenmode. The distribution of the electric field and electric potential formed in a space where conductor, dielectric and vacuum are coexisted is very complex. In particular, the electric potential distribution is too complex to prove the existence of an eigenmode, as shown in Figure 5. (f). When estimating the eigenmodes using the electric field described in Figure 5. (f-i), it can be concluded with TEM 51. Figure 5. (f-ii, iii) is a three-dimensional representation of the electrical potential obtained from the same program. The two expressions seem to describe different eigenmodes. But, when the electric field elements shown in the red and blue circles are combined, they can be considered as a component of the eigenmode TEM 31 and TEM 51 for Figure 5. (e) and (f) respectively. The electric potential is used as auxiliary data in determining the mode type. So, the mode type is determined by the electric field distribution assisted with the electric potential.

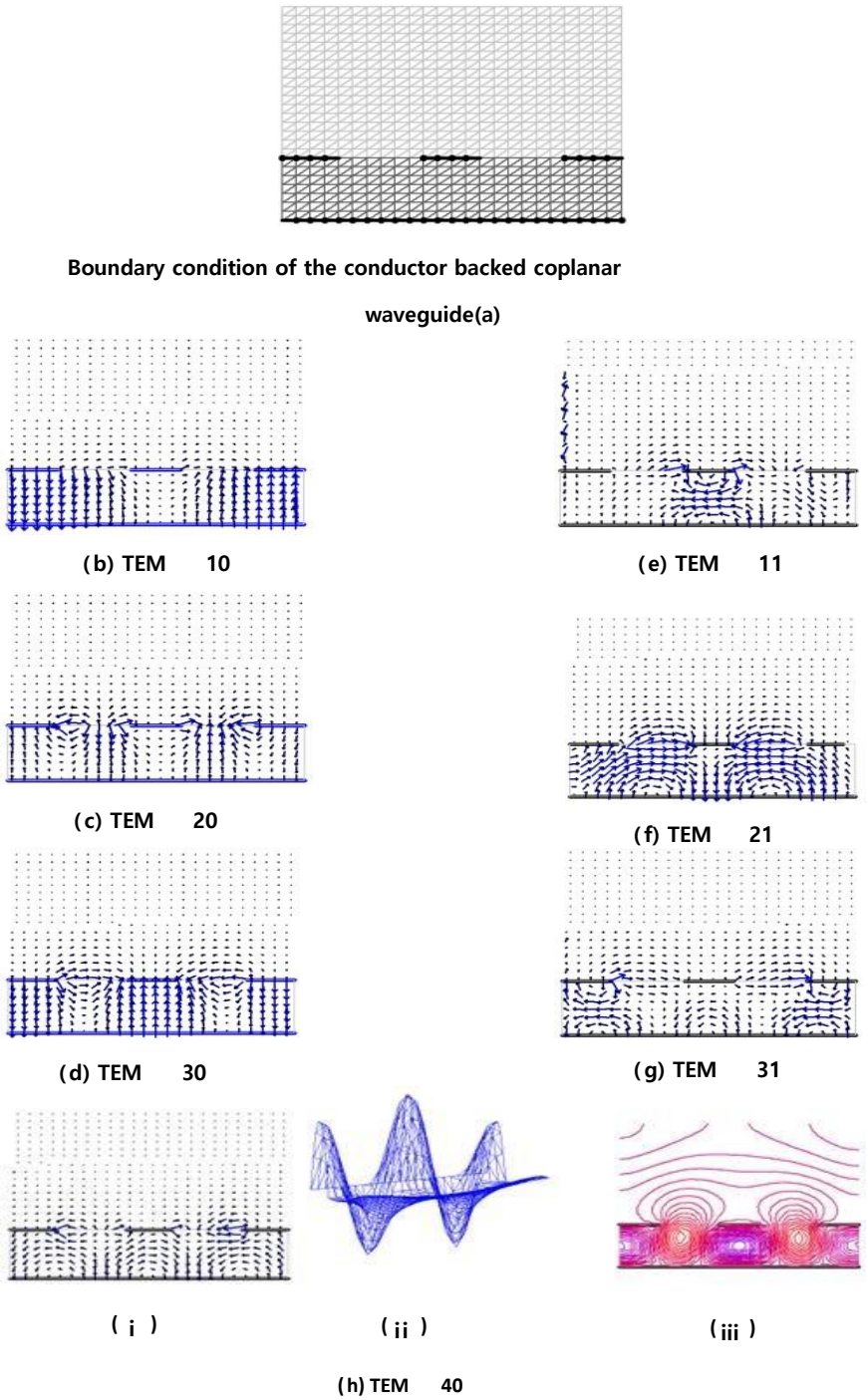


Figure 6 The spectra of the conductor backed coplanar waveguide.

Figure 6. is schematic representation of CBCPW eigenmodes. Unlike previous waveguides, the subdivided components of y-direction are shown here. As can be seen in the figure, the spectra are largely divided into two series. The (b) to (d) spectra show electric field lines mainly directed in the y direction. However, the (e) to (g) spectra also show a horizontally oriented electric field. The conductors attached to the top and bottom of the substrate have a profound

influence on the formation of eigenmodes. As mentioned above, the limitation that the electric field generated around the conductor is mostly incident vertically is an important factor in determining the characteristics of the eigenmode. Therefore, in this study, it is inferred that the characteristics of the spectra as shown in Figure 6. are due to the conductor plates attached to the substrate. The eigenmodes are symmetrical with respect to the center line which is represented as the vertical red line in the Figure 4. (a). This is because the conductors attached to the substrate in the waveguide are symmetrically arranged around the center line. In the case of MSW, the change in electric field is noticeable around the upper conductor region including partly the substrate. From those Figures 4., 5., 6., it can be identified that MSW and CBCPW show wider applicability because it exhibits more diverse eigenmodes under the same conditions.

One peculiar point is that the electric field distributions shown in Figures 4(e), 5(d) and 6(f) exhibit properties of the eigenmode TEM 21 similarly. These eigenmode are usually created on a dielectric substrate, including the region between the upper conductors of the substrate. From this, it can be seen that the three waveguides considered in this study exhibit similar functionality in some cases. As usually it is well known that, CBCPW shows the characteristics of MSW as the width 'W' of the Figure 1. between the upper conductor plates increases. And, as the height 'h' of the dielectric substrate increases, CBCPW exhibits the waveguide properties of CPW. It can be identified from those spectra that the waveguides share the same eigenmode as like TEM 21 under specified conditions [13]. From these results, it can be asserted that CBCPW comprehensively meets the requirements from the microwave applying field.

RESULT AND DISCUSSION

The eigenmodes of MSW, CPW and CBCPW are calculated using FEM and investigated the properties of them. The pairs of eigenmodes are obtained from the columns of the similarity transforming matrix for the eigen matrix of the Helmholtz equation. It is certified that the eigenmode for the electric field is influenced by the conductors attached to the substrate. The characteristics of each waveguide are mainly determined from the eigenmodes of the electric field distribution assisted by the electric potential. The distribution of electric fields shown from Figure 4. (e), Figure 5. (d) and Figure 6. (f) reveal similar eigenmode characteristics. From these results, it can be identified that CBCPW comprehensively characterizes the microwave waveguides.

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