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Solution to n-Queens Problem: Heuristic Approach

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ABSTRACT

Finding non-backtracking solutions to n-queens problems, has been very challenging. Several researchers have attempting to find non-backtracking solutions to n-queens problem, although, some of them recorded huge successes in their researches, but none of them have attempting using the implications of even and odd n-queens, to find solutions to the n-queens problem. In this paper, an attempt was made to extract patterns created from even-numbered n-queens and odd-numbered n-queens placements on the chessboard. The research started from experimenting the placements of non-attacking queens for 4-queens, 16-queens, 32-queens and n-queens. After the experimenting with the placements of different number of queens on the chessboard, it was revealed that even-numbered and odd-numbered queens, have pattern of placement of queens. The results of this research work show how to start and finish placements of queens on the chessboard, depending on whether the n number of queens are either even or odd, using heuristic approach.

Keywords: n-queens, backtracking, heuristic and even/odd numbers.

INTRODUCTION

N-Queens is a classical combinatorial problem that was invented in the 19th century. This problem entails the challenge of attempting to place the queens on the chessboard, so that no queen attacks another (on their rows, columns and diagonals) [1],[3] and [4]. Some of the algorithms for solving n-queens problem are backtracking, Ant Colony Optimization, DNASticker, Genetic Algorithm, etc. In backtracking algorithm, number of queens are placed on the chessboard, one after another, if they are not attacking each other, on their row, columns and diagonals. If these queens attack each other, the initial queen is backtracked to its another non-attacking tile on the row, and a retrial of the process. When the two queens no longer attack each other, then there is an attempt to place a third queen on any of the tiles on the third row. Subsequently, more queens are added to the existing queens and the backtracking process is applied until n-queens are placed on the chessboard, without any of them attacking each other [7]. Even and odd numbers in puzzles can help derive the rule of thumb, which helps to find novel solutions to the puzzles [10] and [11]. Even and odd number can also help to find solution of an algorithms, using solution method of another algorithm [12]. In some cases, even and odd number of n-queens revealed that odd-numbered n-queens, executes faster than even-numbered n-queens [13]. In this paper, there will be an attempt to propose non-backtracking solution method of solving n-queens problem, by extracting even-numbered and odd-numbered sets of n-queens from the placement of queens in the chessboard.

LITERATURE REVIEW

The solution of n-queens problem starts by attempting to place four queens on a 4x4 chessboard so that no two queens can capture each other. That is, no two queens are allowed to be placed on the same row, the same column, or the same diagonal. In the general n-queens, a set of n-queens is to be placed on an *nxn* chessboard so that no two queens attack each other. In a 4-queens problem, any possible solution can be represented as the 4-tuple (q_1, q_2, q_3, q_4), where q_i is a column position on which the queen in the *i-th* row is placed [2]. One method for solving the n-queens problem, which systematically generates all possible solutions is known as backtracking search. Since the nature of backtracking search is exponential in time, backtracking search is not able to solve the large size n-queens problem [2].

[2] developed a probabilistic local search algorithm which is based on gradient based heuristic. This algorithm is capable of finding a solution for extremely large size n-queens problems. They gave the execution statistics for the algorithm with n up to 500,000.

[5] proposed an algorithm which has less computational complexity that predict a candidate solution, based on arithmetic progression. The experiments of the research were conducted on a single machine and expecting improved results from a distributed computing environment. The research has two limitations; the algorithm is not optimized and there is a need to reduce the number of false positive prediction.

[6] applied a variant of Cooperative particle Swarm Optimization (CPSO2) to solve n-queen problem in a more efficient way. Their approach showed better result when there are only two particles in the swarm. They suggested that further study of some parameters, such as learning factor, inertia weight and update constant could improve the result.

[7] proposed a solution to n-queen problem using conditional probability and Bayes theorem. They use the Bayes theorem to update the prior probability in order to arrive at the posterior probability on the basis of given evidence, that is, to find that present position is the safe position. This algorithm will increase the reliability of the solution obtained.

[8] proposed a Genetic Algorithm (GA) to solve n-queen problem with 16 queens and its performance is analyzed in terms of quality of the solution achieved and time taken. The algorithm takes parameters such as high value of population size, cross over rate and mutation rate. The main challenge of this algorithm is that it takes more time for high value of these parameters.

[9] proposed a dynamic programming technique, based on the description of a simple $O(f(n)8^n)$, where $f(n)$ is a low-order polynomial, to solve n-queen problem. At the time their paper was published, it appears to be the first nontrivial upper bound for the problem.

METHODOLOGY

In this paper, there will be an attempt to place the n-queens on the chessboard by using heuristic approach. The set of queens placements is arranged in row order, therefore, formally, a queen and its placement can be expressed as $Q_i = P_{i,j}$. Q_i is the number and $P_{i,j}$ is

the location where the queen is placed. Where *i* is the placement row number, while *j* is the column number. It was observed in Table 1 that a complete placement of 4-queens on the chessboard in row order. After rigorous trial and error approach, it was observed that the best placement position for the starting queen is in the first row and second column. Formally, it can be written as $Q_1 = P_{1,2}$. Following this approach, we can write as Q_2, Q_3 and Q_4 as $Q_2 = P_{2,4}$, $Q_3 = P_{3,1}$ and $Q_4 = P_{4,3}$ respectively.

Table 1: 4-queens chessboard

	Q_1		
			Q_2
Q_3			
		Q_4	

Table 2 shows the solution placements of 4-queens problem viewed in a tabular form is:

Table 2: Solution Table of n-queens Problem

n-queens	Q_1	Q_2	Q_3	Q_4
4	$P_{1,2}$	$P_{2,4}$	$P_{3,1}$	$P_{4,3}$

Now, let apply this heuristic to solving 8-queens problem.

Table 3: 8-queens chessboard

				Q3			
	Q1						
			Q2				
					Q4		
							Q5
		Q6					
Q7							
						Q8	

Again, in Table 3, presents a complete placement of 8-queens on the chessboard and after rigorous trial and error approach, it was observed that the first placement position for Q_1, should start from the second row. Consequently, the placements can be formally written as $Q_1 = P_{2,2}$, $Q_2 = P_{3,4}$, $Q_3 = P_{4,6}$, $Q_4 = P_{5,8}$, $Q_5 = P_{1,5}$, $Q_6 = P_{6,3}$, $Q_7 = P_{7,1}$ and $Q_8 = P_{8,7}$.

Table 4 shows an updated solution placement of 4 and 8 queens problems viewed in a tabular form. The purpose of the updated tabular form is to see if there is a trend in the placement values. The results on the table shows that the placement of the first queen, when n is 4, starts from the first row and when n is 8, the placement starts from the second row. It is also observed that last placement of queen 4 and 8 are in row n(4 and 8 respectively) and the column n-1(3 and 7 respectively).

Table 4: Updated1 Solution Table of n-queens Problem

n-queens	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
4	$P_{1,2}$	$P_{2,4}$	$P_{3,1}$	$P_{4,3}$				
8	$P_{2,2}$	$P_{3,4}$	$P_{4,6}$	$P_{5,8}$	$P_{1,5}$	$P_{6,3}$	$P_{7,1}$	$P_{8,7}$

Repeating this process in Table 5, presents a complete placement of 16-queens on the chessboard and after rigorous trial and error approach, it was again observed that the first placement position for Q_1, starts from the first row, just like 4-queens placement.

Table 5: 16-queens chessboard

	Q1														
			Q2												
					Q3										
							Q4								
									Q5						
											Q6				
													Q7		
															Q8
Q9															
		Q10													
				Q11											
						Q12									
								Q13							
										Q14					
												Q15			
														Q16	

It is further observed that there is a symmetric arrangement partner of 16-queens and with the 4-queens placements.

The placements can be formally written as
$Q_1 = P_{1,2}$, $Q_2 = P_{2,4}$, $Q_3 = P_{3,6}$, $Q_4 = P_{4,8}$, $Q_5 = P_{5,10}$, $Q_6 = P_{6,12}$, $Q_7 = P_{7,14}$ and $Q_8 = P_{8,16}$,
$Q_9 = P_{9,1}$, $Q_{10} = P_{10,3}$, $Q_{11} = P_{11,5}$, $Q_{12} = P_{12,7}$, $Q_{13} = P_{13,9}$, $Q_{14} = P_{14,11}$, $Q_{15} = P_{15,13}$ and $Q_{16} = P_{16,15}$.

Table 6 shows a second update of solution placements of 4, 8 and 16 queens problems viewed in a tabular form. The purpose of the updated tabular form is to see if there is a trend in the placement values.

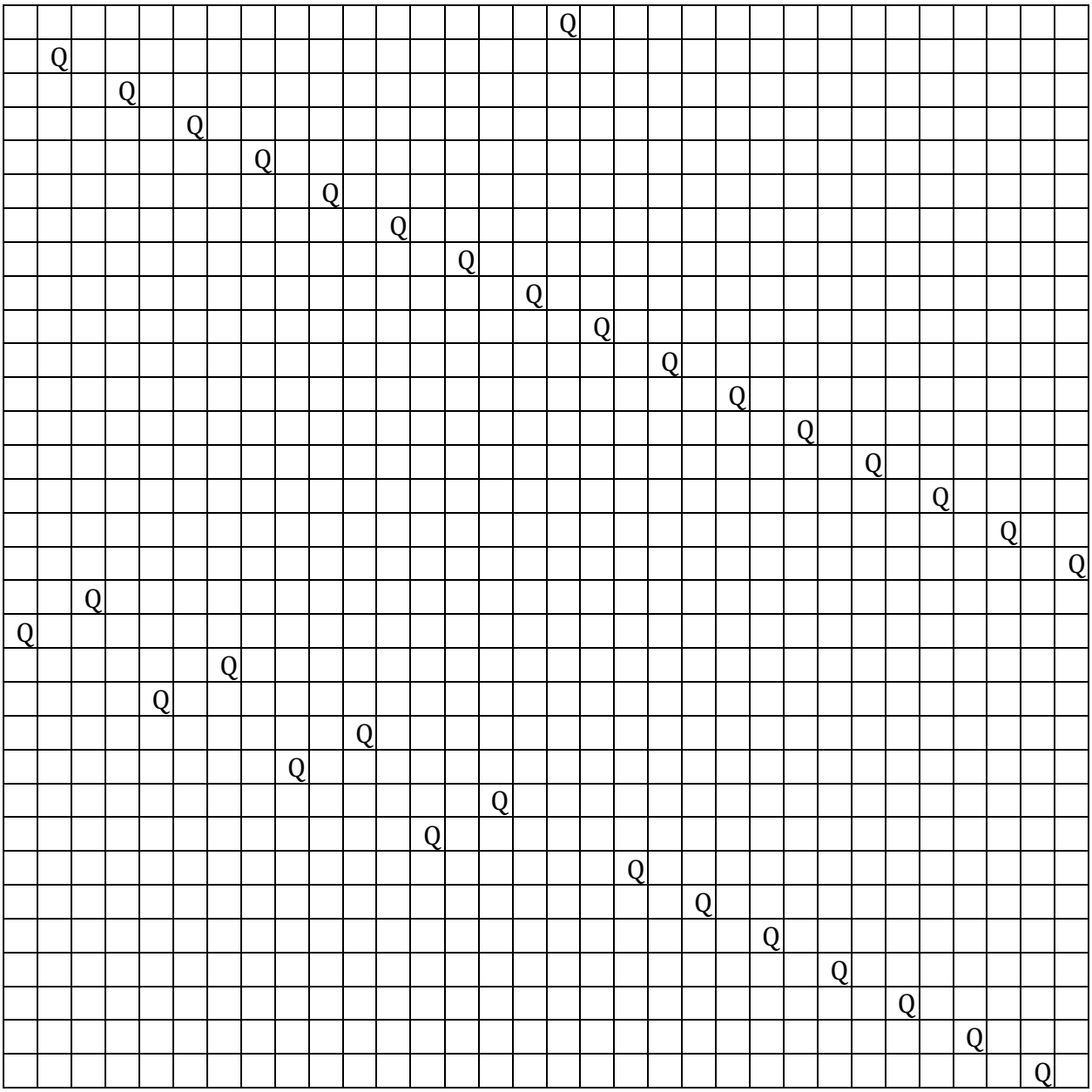
Table 6: Updated2 Solution Table of n-queens Problem

n-queens	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}
4	$P_{1,2}$	$P_{2,4}$	$P_{3,1}$	$P_{4,3}$												
8	$P_{2,2}$	$P_{3,4}$	$P_{4,6}$	$P_{5,8}$	$P_{1,5}$	$P_{6,3}$	$P_{7,1}$	$P_{8,7}$								
16	$P_{1,2}$	$P_{2,4}$	$P_{3,6}$	$P_{4,8}$	$P_{5,10}$	$P_{6,12}$	$P_{7,14}$	$P_{8,16}$	$P_{9,1}$	$P_{10,3}$	$P_{11,5}$	$P_{12,7}$	$P_{13,9}$	$P_{14,11}$	$P_{15,13}$	$P_{16,15}$

Applying the same process again, for Table 7, we represent a complete placement of 32-queens on the chessboard and after rigorous trial and error approach, it was observed that the first placement position for Q_1, starts from the second row, just like 8-queens problem.

From the placement of queens in 32-queens and 8-queens on the chessboard, it is observed that there is a pattern on the arrangements of the queens. It is observed that the upper regions of the chessboard reveal almost symmetric arrangements of half of the number of queens, except a queen on the middle column of the chessboard. These queens (on 8 and 32 queens placements) on the middle column on the first row, is observed to be approximately on the middle of the columns.

Table 7: 32-queens chessboard



The placements 32-queens on the chessboard can be formally written as
$Q_1 = P_{2,2}$, $Q_2 = P_{3,4}$, $Q_3 = P_{3,6}$, $Q_4 = P_{4,8}$, $Q_5 = P_{5,10}$, $Q_6 = P_{6,12}$, $Q_7 = P_{7,14}$,$Q_8 = P_{8,16}$,
$Q_9 = P_{1,17}$, $Q_{10} = P_{10,18}$, $Q_{11} = P_{11,22}$, $Q_{12} = P_{12,24}$, $Q_{13} = P_{13,26}$, $Q_{14} = P_{14,28}$, $Q_{15} = P_{15,30}$, $Q_{16} = P_{16,32}$, $Q_{17} = P_{18,3}$, $Q_{18} = P_{17,3}$, $Q_{19} = P_{3,6}$, $Q_{20} = P_{4,8}$, $Q_{21} = P_{5,10}$, $Q_{22} = P_{6,12}$, $Q_{23} = P_{7,14}$,$Q_{24} = P_{8,16}$, $Q_{25} = P_{1,17}$, $Q_{26} = P_{22,18}$, $Q_{27} = P_{24,20}$, $Q_{28} = P_{26,22}$, $Q_{29} = P_{28,24}$, $Q_{30} = P_{30,26}$, $Q_{31} = P_{31,28}$ and $Q_{32} = P_{32,31}$

Table 8 shows a third update of solution placements of 4, 8,16 and n queens placements, viewed in a tabular form. The purpose of the updated tabular form is to see if there is a trend in the placement values. Let's give the definitions of the variables used and they are:
n: the number of queens

s: are numbers ranging from 1, 2, 3,…and are used to describe set of queens placements

located at the lower section of the chessboard.
t: are also numbers ranging from 1, 2, 3,…and are used to describe set of queens placements located at the second right hand side of chessboard, to help complete the placement of the queens.

There is an attempt to represent the n-queens number as an indices values, to see, if more information can be revealed from the placement process. There is also and attempt to convert the power of the indices to either even or an odd number.

Table 8: Updated3 Solution Table of n-queens Problem

n-queen	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}
4 (2^2)	$P_{1,2}$	$P_{2,4}$	$P_{3,1}$	$P_{4,3}$												
8 (2^3)	$P_{2,2}$	$P_{3,4}$	$P_{4,6}$	$P_{5,8}$	$P_{1,5}$	$P_{6,3}$	$P_{7,1}$	$P_{8,7}$								
16 (2^4)	$P_{1,2}$	$P_{2,4}$	$P_{3,6}$	$P_{4,8}$	$P_{5,10}$	$P_{6,12}$	$P_{7,14}$	$P_{8,16}$	$P_{9,1}$	$P_{10,3}$	$P_{11,5}$	$P_{12,7}$	$P_{13,9}$	$P_{14,11}$	$P_{15,13}$	$P_{16,15}$
32 (2^5)	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}
	$P_{2,2}$	$P_{3,4}$	$P_{4,6}$	$P_{5,8}$	$P_{6,10}$	$P_{7,12}$	$P_{8,14}$	$P_{9,16}$	$P_{1,17}$	$P_{10,18}$	$P_{11,20}$	$P_{12,22}$	$P_{13,24}$	$P_{14,26}$	$P_{15,28}$	$P_{16,30}$
32 (2^5)	Q_{17}	Q_{18}	Q_{19}	Q_{10}	Q_{21}	Q_{22}	Q_{23}	Q_{24}	Q_{25}	Q_{26}	Q_{27}	Q_{28}	Q_{29}	Q_{30}	Q_{31}	Q_{32}
	$P_{17,32}$	$P_{18,3}$	$P_{19,1}$	$P_{20,7}$	$P_{21,5}$	$P_{22,11}$	$P_{23,9}$	$P_{24,15}$	$P_{25,13}$	$P_{26,19}$	$P_{27,21}$	$P_{28,23}$	$P_{29,25}$	$P_{30,27}$	$P_{31,29}$	$P_{32,31}$
…	***…***	***…***	***…***	***…***	***…***	***…***	***…***	***…***	***…***	***…***	***…***	***…***	***…***	***…***	***…***	***…***
n (2^{even})	Q_1	Q_2	Q_3	…	$Q_{n/2}$	$Q_{n/2+1}$	$Q_{n/2+2}$	$Q_{n/2+3}$	…	$Q_{n/2+s}$	$Q_{n/2+s+1}$	$Q_{n/2+s+2}$	$Q_{n/2+s+3}$	***…***	Q_{n-1}	Q_n
	$P_{1,2}$	$P_{2,4}$	$P_{3,6}$	…	$P_{n/2,k}$	$P_{n/2+1,1}$	$P_{n/2+2,3}$	$P_{n/2+3,5}$	…	$P_{n/2+s,t}$	$P_{n/2+s+1,t+2}$	$P_{n/2+s+2,t+4}$	$P_{n/2+s+3,t+6}$	***…***	$P_{n-1,n-2}$	$P_{n,n-1}$
n (2^{odd})	Q_1	Q_2	Q_3	…	$Q_{n/2}$	$Q_{n/2+1}$	$Q_{n/2+2}$	$Q_{n/2+3}$	…	$Q_{n/2+s}$	$Q_{n/2+s+1}$	$Q_{n/2+s+2}$	$Q_{k/2+s+3}$	***…***	Q_{n-1}	Q_n
	$P_{2,2}$	$P_{3,4}$	$P_{4,6}$	…	$P_{n/2,n-2}$	$P_{n/2+1,n}$	$P_{n/2+2,3}$	$P_{n/2+3,1}$	…	$P_{n/2+s,t}$	$P_{n/2+s+1,t+2}$	$P_{n/2+s+2,t+4}$	$P_{n/2+s+3,t+6}$	***…***	$P_{n-1,n-2}$	$P_{n,n-1}$

From the results shown in Tables 8, it is observed that power of the indices values, represented as either even or odd numbers, has further reveal a significant pattern that will help infer how to generalize the solution of placements of n-queens on the chessboard. So, lets attempt to separate the n-even numbered queens placements from their odd-numbered queen placement and then, observed what was further revealed. We start with the extraction of n-even numbered queens pattern from Table 8 to get the placement pattern in Table 9. In Table 9, it can be seen that the set of queens are arranged as a set of queens in symmetric to another. Looking further at the arrangement, the two ranges of queens are like two parallel lines in elementary geometry.

Again, the set of odd numbered arrangement of the queen is extracted from Table 8 and the placement is shown in Table 10. Unlike the pattern of queens created by even-numbered queens patten in Table 9, that revealed two sets of queens, arranged as one symmetric to the other. But in Table 10, the set of queens are irregularly arranged on the chessboard. To be able to derive meaning from the placement, the chessboard was demarcated into eight zones of which three zones are empty. The set of zones are A, B, C, D, E, F, G and H, of which C, F and G are empty. The placement of the queens starts from zone A, on the second row, in a slanty order, until it reaches the *n/2* column. The continuation of the placement of the queens starts from the row *1* and *n/2+1* column. No pattern of placement of queens was observed in zone C. And in zone D, is the continuation of the set of queen placement from zone B that reaches *n/2+1* and row *n* column. Apart from $P_{1, n/2+1}$ in zone B, the set of queens from zone A to C are arranged in a slanty order, which are *n/2* number of queens. Zone E comprises of the set of symmetric slantly arranged *n/4* number of queens.

Table 9: 2^{even}-queens chessboard

$Q_{n/2+1}$	Q_1	$Q_{n/2+2}$	Q_2		Q_3		$Q_{n/2-2}$	Q_{n-1}	$Q_{n/2-1}$	Q_n	$Q_{n/2}$
	$P_{1,2}$										
			$P_{2,4}$								
					$P_{3,6}$						...
						...					
							$P_{n/2-4,\ n-4}$				
									$P_{n/2-2,\ n-2}$		
											$P_{n/2,\ n}$
$P_{n/2+1,1}$				...							
		$P_{n/2+2,3}$		...							
				...							
								$P_{n,\ n-1}$			
				...						$P_{n,\ n-1}$	

Zone H is comprising of the a single slanty n/4-1 set of queens, which extends to n row and n-1 column.

Table 10: 2^{odd}-queens chessboard

$Q_{n/2+2}$	Q_1	$Q_{n/2+1}$	Q_2				$Q_{n/2+1}$	Q_{n-1}	$Q_{n/2-1}$	Q_n	$Q_{n/2}$
							$P_{1, n/2+1}$				
	$P_{2,2}$										
			$P_{3,4}$								
					…						…
	Zone A				…	$P_{n/4+1, n/2}$	…		Zone B		
							…				
	Zone C								$P_{n/2, n-2}$		
							Zone D				$P_{n/2+1, n}$
		$P_{n/2+2, 3}$		…			Zone F				
$P_{n/2+3, 1}$				…		Zone E		…			
Zone G				…				$P_{n-1,n-3}$			
							Zone H			$P_{n,n-1}$	

DISCUSSIONS

Placement of queens on the chessboard has always been challenging and this paper attempts to find how even and odd numbers can help to arrange the set of queens, without backtracking. This experiment starts from attempting to place 4-queens, 16-queens, 32-queens, and a set generalized n-queens for both even and odd numbered queens. In an n-queens problem, when *n* is even, the placement of the queen starts from row 1, column 2 and contrarily, when n is odd, the placement starts from row 2 column 2. In that way, the starting place for placement of queens on the chessboard is significant to this novel non-backtracking technique of solving n-queens problem. Although, there are more than one solutions of n-queens problems, but the focus of this research is on the solution to n-queens problems, when the *n* is either even or odd. When the *n* number of queens is even, then, it is expected that the set of placements of the queens is two slanty symmetric arrangement. When the number of n of the queens is odd, the placement pattern is now irregular and there would be a need to demarcate the contents of the chessboard into eight zones (A, B, C, D, E, F, G and H). There are set of queens that belong to these zones, which can help to accurately arrange the queens on the chessboard without backtracking.

CONCLUSION

Finding a non-backtracking method of n-queens problem has been very challenging and this paper has attempted to solve the problem, by using the even and odd number of the n-

queens. The experiment started by rigorously finding patterns of arrangement of queens on the chessboard, which later revealed that these patterns are order by either even or odd n number of queens. The even pattern shows that there are two slanty symmetric set of queens, where they are parallel to each other. Contrarily, the odd pattern shows that the queens placement on the chessboard are irregular and hence, the contents of the chessboard was demarcated into eight zones. Each zone has a particular unique set of queens that determines what number of queens are expected in each zone and their locations on the chessboard. In future research, there will be an attempt to derive an algorithm for this experiment and implement it in an ideal programming language.

Reference

[1] K. Vishal and C. Sarvesh, "Review on n-Queen Optimization using Tuned Hybrid ", *International Journal of Engineering Sciences and Research Technology*, vol. 6, no. 2, pp. 62-68, 2017.

[2] S. Rok and G. Jun, "A Polynomial Time Algorithm for the N-Queens Problem", *SIGART Bulletin*,vol. 1, no. 3, pp. 7-11, 1990.

[3] L. Marcin, "The N queens problem – new variants of the Wirth algorithm", *Annales UMCS Informatica AI*, vol.13, no. 1, pp. 53-61, 2013.

[4] R. E. Matthias, "A Group-based Search for Solutions of n-Queens Problem", *Discrete mathematics*, vol. 307, pp. 2535-2551, 2007.

[5] V. P. Lijo and T. J. Jasmin, "Solving N-Queen Problem by Prediction", *International Journal of Computer Science and Information Technologies*, vol. 6, no. 4, pp. 3844-3848, 2015.

[6] H. S. Fazlul, R. Mostafijur and H. Baker and Asrafullah, "N-Queen Problem Revisited – A Variant of Cooperative Particle Swarm Optimization-based Solution", *International Journal of Computer Science Trends and Technology*, vol. 6, no. 4, pp. 57-62, 2018.

[7] Madhusudan and R. Rachana, "Probability-based Solution to N-Queen Problem", *International Journal of Engineering Science and Innovation Technology*, vol. 3, no. 3, pp. 426-436, 2014.

[8] R. K. Singh, V. K. Panchal and B. K. Singh B. K, "Solving N-Queen Problem Using Genetic Algorithm and Analyzing the Effort of GA Parameters on its Performance", *International Journal of Research and Analytical and Reviews*, vol. 6, no. 2, pp. 680-683, 2019.

[9] R. Igor and Z. Ramin, A dynamic programming solution to n-queens problem, Information Processing Letter vol. 41 pp. 253-256, 1992.

[10] F. O. Ikpotokin, S. C. Chiemeke, and E. O. Osaghae, Alternative Approach to the Optimal Solution of Tower of Hanoi, Journal of Institute of Mathematics and Computer Sciences, 15 (2): 229-244, 2004.

[11] F. O. Ikpotokin, S. C. Chiemeke, and E. O. Osaghae, Generalized Recursive Optimal Solution for the Multi-Peg Towers of Hanoi, Journal of Institute of Mathematics and Computer Sciences, 15 (2): 297-309, 2004.

[12] S. C. Chiemeke, and E. O. Osaghae, Complete Binary Tree Implementation of Tower of Hanoi, Journal of Scientific and Industrial Studies, 4 (1): 59-65, 2006

[13] S. C. Chiemeke, and E. O. Osaghae, "The Turning Points in The Solution of n-Queens Problem using Backtracking Method", *Journal of Nigerian Association of Mathematical Physics*, vol. 2 vol. 10 pp. 107-114, 2006.

[14] E. O. Osaghae, A. A. G. Akanmu and A. E. Akhahowa A. E. (2007). Solution to A NON-STANDARD INITIAL STATE of The Tower of Hanoi Problem, International *Journal of Physical Sciences*, vol. 2 vol. 1, pp. 53-61, 2017.