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Control of following Force in Gimbal Suspension by Programmed Transfer of Vehicle in the Class of Spiral-Screw Trajectories

Kravets V.V.[1], Kravets Vl.V.[2], Artemchuk V.V.[2]
[1] *National Technical University "Dnipro Politechnic", Dnipro, Ukraine.*
[2]*Dnipro National University of Railway Transport, Dnipro, Ukraine.*
prof.w.kravets@gmail.com; vladkravetsphd@gmail.com

ABSTRACT

The programmed transfer of the transport vehicle in space is carried out in the class of helical trajectories, using forcing (throttling) and deviation of the following driving force in the gimbal. The paper introduces the mathematical models of the transport vehicle kinetics in space in the terrestrial reference system and in the basis of the natural trihedral of the trajectory, using the quaternion form. The kinematics of the transport vehicle in the fixed and mobile reference systems, as well as the orientation of the natural trihedral in the inertial space, are represented by the hodograph of the program helix trajectory in vector and quaternion forms. The components of the controlling driving force in the basis of the natural trihedral are determined by the kinetostatics equations of the programmed transfer of the transport vehicle along a helical trajectory in the required speed mode. The authors proposed a structural scheme of the gimbal suspension, providing the required driving force components. The authors considered two possible sequences of rotations of the moving gimbal rings and demonstrated their equivalence. Laconic formulas are established for the control angles of rotation of the moving gimbal rings.

Keywords: gimbal system, hodograph, Rodrigues-Hamilton parameters, components of control forces, spiral-screw trajectory, natural trihedral, quaternion matrices.

1 Introduction

The central stage of the dynamic design of transport vehicles for various purposes is the formation of a control system that provides the objective function controlling for the objective constraints [1, 2]. The design of advanced algorithms for managing the spatial transfer and turn of the transport vehicle in the required speed mode according to the prescribed program path [3, 4] is possible based on the development of special mathematical methods for modelling dynamic processes [5, 6]. For example, programmed transfer is exhaustively described by a vector-hodograph in the class of helical trajectories [7, 8], programmed turn – by the Gibbs vector or quaternion according to Rodrigues-Hamilton parameters [9, 10]. The search for feasible structural schemes for implementing the control actions constitutes the logical continuation of the work "The Vehicle Controlling near the Screening Surface Using Thrust Vector Deflection of the Electric Motor with Gimbal Mounted Propeller" [11].

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2 Problem statement

The vehicle is considered as a material point of a given mass m, to which inertia forces, gravity, aerodynamic forces, following force (engine tractive effort) in the gimbal suspension are applied. The programmed transfer of the vehicle in space is carried out in the class of spiral-screw trajectories specified in the earth reference by time-variable radius vector $\bar{r}(t)$. It is necessary to determine the driving force variable in magnitude and direction, which provides the programmed transfer of the vehicle in space. The driving force is formed by forcing (throttling) of the following force magnitude and by the controlled rotation of the gimbal suspension. We determine a sequence of turns and the magnitude of the control turning angles of the gimbal suspension rings.

Kinetics of the vehicle in space. The mathematical model of vehicle kinetics is constructed in Newton's and Euler's form [12], respectively, in the rigid and moving frames of reference. The rigid (earth) reference $O\bar{i}\bar{j}\bar{k}$ is assumed to be inertial (Figure 1). The moving frame of reference is associated with the natural trihedral of the vehicle trajectory $M\bar{\tau}\bar{n}\bar{b}$ (Fig. 1)

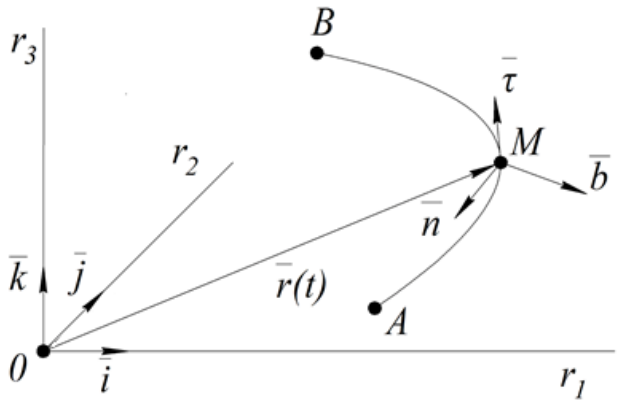

Fig.1 Reference frames and hodograph.

In the basis $O\bar{i}\bar{j}\bar{k}$ we represent the Newton's motion equations as the sum of the following quaternion-columns:

$$F_0^r + G_0^r + Q_0^r + N_0^r = 0\,.$$

Here is the quaternion-column of inertial forces:

$$F_0^r = -m\left\|\begin{matrix}0\\ w_1\\ w_2\\ w_3\end{matrix}\right\|,$$

where w_1, w_2, w_3 are the components of acceleration in the rigid basis;

Quaternion-column of gravity:

$$G_0^r = mg \begin{Vmatrix} 0 \\ 0 \\ 0 \\ -1 \end{Vmatrix},$$

where g is the gravity acceleration.

Quaternion-column of aerodynamic forces:

$$Q_0^r = qS\, A \cdot {}^t A \cdot R_d \cdot {}^t R_d \begin{Vmatrix} 0 \\ c_{1d} \\ c_{2d} \\ c_{3d} \end{Vmatrix},$$

where c_{1d}, c_{2d}, c_{3d} are the aerodynamic coefficients of longitudinal, transverse, normal forces;

q is the velocity head;

S is the midsection area;

R_d, ${}^t R_d$ are the quaternion matrices of transformation of the vehicle aerodynamic axes to the natural trihedral basis;

A, ${}^t A$ are the quaternion matrices of transformation of the natural trihedral basis to the earth reference basis.

Quaternion-column of driving force:

$$N_0^r = A \cdot {}^t A \begin{Vmatrix} 0 \\ N_\tau \\ N_n \\ N_b \end{Vmatrix},$$

where N_τ, N_n, N_b are the tangential, normal and binormal components of the driving force in the natural trihedral basis.

In the basis $M\bar{\tau}\bar{n}\bar{b}$ of the equation of motion in the Eulerian form, we represent the sum of the following quaternion columns:

$$F_0 + G_0 + Q_0 + N_0 = 0.$$

Here is the quaternion-column of inertial forces:

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$$F_0 = -m \begin{Vmatrix} 0 \\ w_\tau \\ w_n \\ w_b \end{Vmatrix},$$

where w_τ, w_n, w_b are the acceleration components in the natural trihedral basis.

Quaternion-column of gravity:

$$G_0 = mgA^t \cdot {}^tA^t \begin{Vmatrix} 0 \\ 0 \\ 0 \\ -1 \end{Vmatrix},$$

where A^t, ${}^tA^t$ are the quaternion matrix of transformation of the earth reference basis to the natural trihedral basis.

Quaternion-column of aerodynamic forces:

$$Q_0 = qS\,A \cdot R_d \cdot {}^tR_d \begin{Vmatrix} 0 \\ c_{1d} \\ c_{2d} \\ c_{3d} \end{Vmatrix},$$

Quaternion-column of driving force:

$$N_0 = \begin{Vmatrix} 0 \\ N_\tau \\ N_n \\ N_b \end{Vmatrix}.$$

3 Programmed spiral-screw trajectory

The variety of program motions of vehicles in space is proposed to be formed in the spiral-screw line class using the following hodograph [3]:

$$\bar{r}(t) = \|\rho_0\rho_1\rho_2\rho_3\| \begin{Vmatrix} 1 \\ t \\ t^2 \\ t^3 \end{Vmatrix} \left(\bar{i}\cos\omega t + \bar{j}\sin\omega t\right) + \bar{k}\,\|h_0h_1h_2h_3\| \begin{Vmatrix} 1 \\ t \\ t^2 \\ t^3 \end{Vmatrix},$$

where ρ_i, h_i $(i = 0, 1, 2, 3)$ are the variable parameters determined by given boundary conditions;

t is time (independent parameter);

ω is the average turning rate, i.e. $\omega = \dfrac{\varphi_0}{t_0}$,

φ_0 is the full turning angle;

t_0 is the required time of transfer in space;

$\bar{i}$, $\bar{j}$, $\bar{k}$ is the basis of the earth-centred Cartesian reference system.

The method for constructing the hodograph of program motion in the considered class of spiral-screw trajectories consists in determining the introduced variable parameters by the required boundary conditions. Additional conditions include limitations on the technical feasibility of both the geometry of the trajectory shape and the mode of vehicle motion along this trajectory.

4 Kinematics of the vehicle in space

The given hodograph of the vehicle motion determines the acceleration components in the basis $O\bar{i}\bar{j}\bar{k}$:

$$w_1 = \ddot{r}_1; \quad w_2 = \ddot{r}_2; \quad w_3 = \ddot{r}_3;$$

in the basis $M\bar{\tau}\bar{n}\bar{b}$:

$$\bar{\tau}W_\tau = \frac{\left(\dot{\bar{r}}\cdot\ddot{\bar{r}}\right)}{\left(\dot{\bar{r}}\cdot\dot{\bar{r}}\right)}\dot{\bar{r}}; \; \bar{n}W_n = -\frac{\left[\bar{r}\times\left(\dot{\bar{r}}\times\ddot{\bar{r}}\right)\right]}{\left(\dot{\bar{r}}\cdot\dot{\bar{r}}\right)}; \; \bar{b}W_b = 0.$$

For the vehicle velocity component, respectively, we obtain in the basis $O\bar{i}\bar{j}\bar{k}$:

$$V_1 = \dot{r}_1; \; V_2 = \dot{r}_2; \; V_3 = \dot{r}_3;$$

in the basis $M\bar{\tau}\bar{n}\bar{b}$:

$$\bar{\tau}V_\tau = \dot{\bar{r}}; \; \bar{n}V_n = 0; \; \bar{b}V_b = 0.$$

The relationship between the acceleration and velocity components in the moving and rigid reference frames is established by the kinematic equations:

$$\begin{Vmatrix} 0 \\ W_1 \\ W_2 \\ W_3 \end{Vmatrix} = A\cdot{}^tA \begin{Vmatrix} 0 \\ W_\tau \\ W_n \\ W_b \end{Vmatrix}; \quad \begin{Vmatrix} 0 \\ W_\tau \\ W_n \\ W_b \end{Vmatrix} = A^t\cdot{}^tA^t \begin{Vmatrix} 0 \\ W_1 \\ W_2 \\ W_3 \end{Vmatrix}; \quad \begin{Vmatrix} 0 \\ V_1 \\ V_2 \\ V_3 \end{Vmatrix} = A\cdot{}^tA \begin{Vmatrix} 0 \\ V_\tau \\ V_n \\ V_b \end{Vmatrix}; \quad \begin{Vmatrix} 0 \\ V_\tau \\ V_n \\ V_b \end{Vmatrix} = A^t\cdot{}^tA^t \begin{Vmatrix} 0 \\ V_1 \\ V_2 \\ V_3 \end{Vmatrix}.$$

Here, the orientation of the natural trihedral in the inertial space for an arbitrary time moment of the vehicle motion along the programmed trajectory is determined in the Rodrigues-Hamilton parameters a_0, a_1, a_2, a_3 using the following matrix formula [10]:

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$$\begin{Vmatrix} a_0^2 \\ a_1^2 \\ a_2^2 \\ a_3^2 \end{Vmatrix} = \frac{1}{4\left|\dot{\bar{r}}_1\right| \cdot \left|\dot{\bar{r}}_1 \times \ddot{\bar{r}}_1\right|} \begin{Vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{Vmatrix} \begin{Vmatrix} \left|\dot{\bar{r}}_1\right| \cdot \left|\dot{\bar{r}}_1 \times \ddot{\bar{r}}_1\right| \\ \left|\dot{\bar{r}}_1 \times \ddot{\bar{r}}_1\right| \dot{r}_1 \\ \ddot{r}_2 \left(\dot{r}_1^2 + \dot{r}_3^2 \right) - \ddot{r}_2 \dot{r}_1 \dot{r}_2 - \ddot{r}_3 \dot{r}_2 \dot{r}_3 \\ \left|\dot{\bar{r}}_1\right| \begin{pmatrix} \ddot{r}_2 \dot{r}_1 - \ddot{r}_1 \dot{r}_2 \end{pmatrix} \end{Vmatrix}$$

where

$$\left|\dot{\bar{r}}\right|^2 = \dot{\bar{r}} \cdot \dot{\bar{r}}; \quad \left|\dot{\bar{r}} \times \ddot{\bar{r}}\right|^2 = \left(\dot{\bar{r}} \times \ddot{\bar{r}}\right) \cdot \left(\dot{\bar{r}} \times \ddot{\bar{r}}\right).$$

5 Kinetostatics of programmed motion of the vehicle

The force that drives the vehicle along the programmed spiral-screw trajectory in the required speed rate is directly found from the Euler's equations of motion in the following form:

$$N_0 = -F_0 - G_0 - Q_0$$

or in the expansion of equation:

$$\frac{1}{m} \begin{Vmatrix} 0 \\ N_\tau \\ N_n \\ N_b \end{Vmatrix} = \begin{Vmatrix} 0 \\ W_\tau \\ W_n \\ 0 \end{Vmatrix} + gA^t \cdot {}^tA^t \begin{Vmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Vmatrix} - \frac{qS}{m} R_d \cdot {}^tR_d \begin{Vmatrix} 0 \\ c_{1d} \\ c_{2d} \\ c_{3d} \end{Vmatrix},$$

where

$$W_\tau = \frac{\dot{\bar{r}} \cdot \ddot{\bar{r}}}{\left|\dot{\bar{r}}\right|}; \quad W_\tau = \frac{\left|\dot{\bar{r}} \times \ddot{\bar{r}}\right|}{\left|\dot{\bar{r}}\right|}.$$

6 Controlling the following force in the gimbal suspension

The controlled vehicle motion along the programmed trajectory is carried out by the following force $\overline{P}$ in the gimbal suspension according to the proposed constructive scheme (Fig.2) [11]:

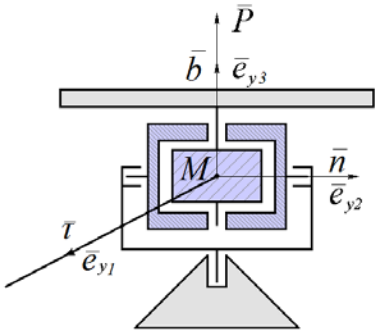

Fig. 2 Constructive scheme of the following force in gimbal suspension.

It is believed that in the initial position the basis $M\overline{e}_{y1}\overline{e}_{y2}\overline{e}_{y3}$, associated with the gimbal suspension, is combined with the natural trihedral basis $M\overline{\tau}\overline{n}\overline{b}$, associated with the vehicle according to Fig. 2. The magnitude of the following force is determined from the found components of the driving force in the form

$$P = \sqrt{N_\tau^2 + N_n^2 + N_b^2}$$

and varies (is forced, throttled) depending, for example, on the rotor speed around the axis: $\overline{e}_{y3} - \dot{\varphi}$:

$$P\left(\dot{\varphi}\right),$$

i.e. $\dot{\varphi}$ is a control parameter.

The required components of the driving force N_τ, N_n, N_b are ensured by a controlled turn of the following force P in the movable gimbal rings relative to the vehicle.

The construction scheme shown in Fig. 2 admits two sequences of turns of the gimbal rings:

a. the first turn of the outer ring in the positive direction around the axis $\overline{e}_{y3}\left(\overline{b}\right)$ by a yaw angle ψ;
b. the second turn of the inner ring in the positive direction around the axis $\overline{e}_{y2}$ на by a pitch angle θ.
c. the first turn of the inner ring in the positive direction around the axis $\overline{e}_{y2}\left(\overline{n}\right)$ by a pitch angle θ;

the second turn of the outer ring in the positive direction around the axis $\overline{b}$ by a yaw angle ψ.

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Here, the turning angles of the outer and inner gimbal rings ψ and θ are the required control parameters for ensuring the programmed transfer of the vehicle. The mentioned two possible sequences of turns of the gimbal rings are shown in Fig. 3 a, b.

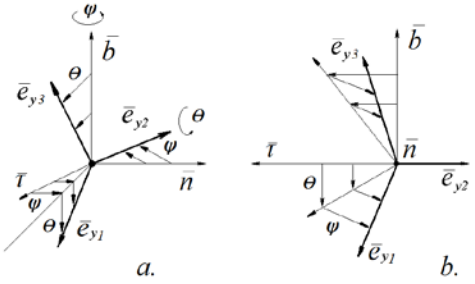

Figure 3. Sequence of turns of gimbal rings.

The dependence of the driving force components on the magnitude of the following force directed along the axis $\overline{e}_{y3}$, with full turn of the gimbal suspension to the angles ψ and θ, is determined by the formula:

$$\begin{Vmatrix} 0 \\ N_\tau \\ N_n \\ N_b \end{Vmatrix} = R \cdot {}^tR \begin{Vmatrix} 0 \\ 0 \\ 0 \\ P \end{Vmatrix},$$

where R, ${}^tR$ are the quaternion matrices of transformation of the gimbal suspension basis $M\overline{e}_{y1}\overline{e}_{y2}\overline{e}_{y3}$ to the natural trihedral basis $M\overline{\tau}\overline{n}\overline{b}$, associated with the vehicle. The resulting quaternion matrices R and ${}^tR$, corresponding to the full turn of the gimbal suspension, are determined by the quaternion matrices B, ${}^tB$ and C, ${}^tC$, constituting the sequence of two independent turns of the gimbal rings, in the form [13]:

$$R = B \cdot C, \quad {}^tR = {}^tB \cdot {}^tC$$

or

$$R \cdot {}^tR = B \cdot {}^tB \cdot C \cdot {}^tC.$$

The sequence of two independent turns of the gimbal rings, shown in Fig. 3a, is characterized by the following Rodrigues-Hamilton parameters:

$$1.\ b_0 = \cos\frac{\psi}{2},\ b_1 = 0,\ b_2 = 0,\ b_3 = \sin\frac{\psi}{2};$$

$$2.\ c_0 = \cos\frac{\theta}{2},\ c_1 = 0,\ c_2 = \sin\frac{\theta}{2},\ c_3 = 0$$

and the corresponding quaternion matrices:

$$1.\ B_a = \begin{Vmatrix} b_0 & 0 & 0 & b_3 \\ 0 & b_0 & -b_3 & 0 \\ 0 & b_3 & b_0 & 0 \\ -b_3 & 0 & 0 & b_0 \end{Vmatrix},\ {}^tB_a = \begin{Vmatrix} b_0 & 0 & 0 & -b_3 \\ 0 & b_0 & -b_3 & 0 \\ 0 & b_3 & b_0 & 0 \\ b_3 & 0 & 0 & b_0 \end{Vmatrix};$$

$$2.\ C_a = \begin{Vmatrix} c_0 & 0 & c_2 & 0 \\ 0 & c_0 & 0 & c_2 \\ -c_2 & 0 & c_0 & 0 \\ 0 & -c_2 & 0 & c_0 \end{Vmatrix},\ {}^tC_a = \begin{Vmatrix} c_0 & 0 & -c_2 & 0 \\ 0 & c_0 & 0 & c_2 \\ c_2 & 0 & c_0 & 0 \\ 0 & -c_2 & 0 & c_0 \end{Vmatrix}.$$

Whence it follows:

$$1.\ B_a \cdot {}^tB_a = \begin{Vmatrix} b_0^2 + b_3^2 & 0 & 0 & -b_0b_3 + b_3b_0 \\ 0 & b_0^2 - b_3^2 & -b_0b_3 - b_3b_0 & 0 \\ 0 & b_3b_0 + b_0b_3 & -b_3^2 + b_0^2 & 0 \\ -b_3b_0 + b_0b_3 & 0 & 0 & b_3^2 + b_0^2 \end{Vmatrix},$$

$$2.\ C_a \cdot {}^tC_a = \begin{Vmatrix} c_0^2 + c_2^2 & 0 & -c_0c_2 + c_2c_0 & 0 \\ 0 & c_0^2 - c_2^2 & 0 & c_0c_2 + c_2c_0 \\ -c_0c_2 + c_2c_0 & 0 & c_2^2 + c_0^2 & 0 \\ 0 & -c_2c_0 + c_0c_3 & 0 & -c_2^2 + c_0^2 \end{Vmatrix}$$

and, taking into account

$$b_0^2 + b_3^2 = \cos^2 \psi/2 + \sin^2 \psi/2 = 1,$$

$$b_0^2 - b_3^2 = \cos^2 \psi/2 - \sin^2 \psi/2 = \cos\psi,$$

$$2b_0b_3 = 2\cos \psi/2 \cdot \sin \psi/2 = \sin\psi,$$

$$c_0^2 + c_2^2 = \cos^2 \theta/2 + \sin^2 \psi/2 = 1,$$

$$c_0^2 - c_2^2 = \cos^2 \theta/2 - \sin^2 \theta/2 = \cos\theta,$$

$$2c_0c_2 = 2\cos \theta/2 \cdot \sin \theta/2 = \sin\theta,$$

we obtain:

$$1.\ B_a \cdot {}^tB_a = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi & 0 \\ 0 & \sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix},$$

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$$2.\ C_a \cdot {}^tC_a = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{Vmatrix}.$$

Thus, the resultant transformation matrix corresponding to the considered turn sequence (Fig. 3a) is found in the form:

$$R_a \cdot {}^tR_a = B_a \cdot {}^tB_a \cdot C_a \cdot {}^tC_a$$

or in the expansion of equation:

$$R_a \cdot {}^tR_a = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi \cdot \cos\theta & -\sin\psi & \cos\psi \cdot \sin\theta \\ 0 & \sin\psi \cdot \cos\theta & \cos\psi & \sin\psi \cdot \sin\theta \\ 0 & -\sin\theta & 0 & \cos\theta \end{Vmatrix}.$$

The sequence of two independent turns of the gimbals rings, shown in Fig. 3b, is characterized by the following Rodrigues-Hamilton parameters:

$$1.\ b_0 = \cos{}^{\theta}\!/_{2},\ b_1 = 0,\ b_2 = \sin{}^{\theta}\!/_{2},\ b_3 = 0;$$

$$2.\ c_0 = \cos{}^{\psi}\!/_{2},\ c_1 = -\sin\theta \cdot \sin{}^{\psi}\!/_{2},\ c_2 = 0,\ c_3 = \cos\theta \cdot \sin{}^{\psi}\!/_{2}$$

and the corresponding quaternion matrices:

$$1.\ B_b = \begin{Vmatrix} b_0 & 0 & b_2 & 0 \\ 0 & b_0 & 0 & b_2 \\ -b_2 & 0 & b_0 & 0 \\ 0 & -b_2 & 0 & b_0 \end{Vmatrix};\quad {}^tB_b = \begin{Vmatrix} b_0 & 0 & -b_2 & 0 \\ 0 & b_0 & 0 & b_2 \\ b_2 & 0 & b_0 & 0 \\ 0 & -b_2 & 0 & b_0 \end{Vmatrix};$$

$$2.\ C_b = \begin{Vmatrix} c_0 & c_1 & 0 & c_3 \\ -c_1 & c_0 & -c_3 & 0 \\ 0 & c_3 & c_0 & -c_1 \\ -c_3 & 0 & c_1 & c_0 \end{Vmatrix};\ {}^tC_b = \begin{Vmatrix} c_0 & -c_1 & 0 & -c_3 \\ c_1 & c_0 & -c_3 & 0 \\ 0 & c_3 & c_0 & -c_1 \\ c_3 & 0 & c_1 & c_0 \end{Vmatrix}.$$

Whence it follows:

$$1.\ B_{\dot{b}} \cdot {}^tB_b = \begin{Vmatrix} b_0^2 + b_2^2 & 0 & 0 & 0 \\ 0 & b_0^2 - b_2^2 & 0 & 2b_0b_2 \\ 0 & 0 & b_0^2 + b_2^2 & 0 \\ 0 & -2b_0b_2 & 0 & b_0^2 - b_2^2 \end{Vmatrix},$$

$$2.\ \underset{\dot{b}}{C}\cdot {}^{t}C_{b}=\begin{Vmatrix} c_0^2+c_1^2+c_3^2 & 0 & 0 & 0 \\ 0 & c_0^2+c_1^2-c_3^2 & -2c_0c_3 & 2c_1c_3 \\ 0 & 2c_0c_3 & c_0^2-c_1^2-c_3^2 & -2c_0c_1 \\ 0 & 2c_1c_3 & 2c_0c_1 & c_0^2-c_1^2+c_3^2 \end{Vmatrix}$$

and, taking into account

$$b_0^2+b_2^2=\cos^2\theta/2+\sin^2\theta/2=1,$$

$$b_0^2-b_2^2=\cos^2\theta/2-\sin^2\theta/2=\cos\theta,$$

$$2b_0^2-b_2^2=2\cos^2\theta/2\cdot\sin^2\theta/2=\sin\theta,$$

$$c_0^2+c_1^2+c_3^2=1,$$

$$c_0^2+c_1^2-c_3^2=\cos\psi\cdot\cos^2\theta+\sin^2\theta,$$

$$c_0^2-c_1^2-c_3^2=\cos\psi,$$

$$c_0^2-c_1^2+c_3^2=\cos\psi\cdot\sin^2\theta+\cos^2\theta,$$

$$2c_0c_3=\sin\psi\cdot\cos\theta,$$

$$2c_0c_1=-\sin\psi\cdot\sin\theta,$$

$$2c_1c_3=(\cos\psi-1)\sin\theta\cdot\cos\theta,$$

we obtain:

$$1.\ \underset{\dot{b}}{B}\cdot {}^{t}B_{b}=\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{Vmatrix},$$

$$2.\ \underset{\dot{b}}{C}\cdot {}^{t}C_{b}=\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi\cdot\cos^2\theta+\sin^2\theta & -\cos\theta\cdot\sin\psi & (\cos\psi-1)\sin\theta\cdot\cos\theta \\ 0 & \cos\theta\cdot\sin\psi & \cos\psi & \sin\theta\cdot\sin\psi \\ 0 & (\cos\psi-1)\sin\theta\cdot\cos\theta & -\sin\theta\cdot\sin\psi & \cos\psi\cdot\sin^2\theta+\cos^2\theta \end{Vmatrix}.$$

In the case under consideration (Fig. 3b), the resultant transformation matrix is found similarly:

$$R_b\cdot {}^{t}R_b=B_b\cdot {}^{t}B_b\cdot C_b\cdot {}^{t}C_b.$$

Having executed the necessary operations, we establish:

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$$R_{b} \cdot {}^{t}R_{b} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta \cdot \cos\psi & -\sin\psi & \sin\theta \cdot \cos\psi \\ 0 & \cos\theta \cdot \sin\psi & \cos\psi & \sin\theta \cdot \sin\psi \\ 0 & -\sin\theta & 0 & \cos\theta \end{Vmatrix},$$

i.e.

$$R_{a} \cdot {}^{t}R_{a} = R_{b} \cdot {}^{t}R_{b}.$$

Thus, the considered two different sequences of turns of the gimbal rings (Fig. 3a, b) are equivalent.

Consequently, we find:

$$N_{\tau} = P\cos\psi\sin\theta,$$

$$N_{n} = P\sin\psi\sin\theta,$$

$$N_{b} = P\cos\theta$$

or

$$\frac{N_{\tau}}{P} = \cos\psi \cdot \sin\theta,$$

$$\frac{N_{n}}{P} = \sin\psi \cdot \sin\theta,$$

$$\frac{N_{b}}{P} = \cos\theta,$$

i.e.

$$\left(\frac{N_{\tau}}{P}\right)^{2} + \left(\frac{N_{n}}{P}\right)^{2} + \left(\frac{N_{b}}{P}\right)^{2} = 1,$$

$$\cos^{2}\psi \cdot \sin^{2}\theta + \sin^{2}\psi \cdot \sin^{2}\theta + \cos^{2}\theta = 1.$$

Whence we also find:

$$tg\psi = \frac{N_{n}}{N_{\tau}},$$

$$\frac{N_{n}}{N_{\tau}} = \cos\psi tg\theta,$$

$$\frac{N_{n}}{N_{b}} = \sin\psi tg\theta$$

or

$$tg\theta\left(\cos\psi+\sin\psi\right)=\frac{N_\tau+N_n}{N_b}.$$

As

$$\cos\psi+\sin\psi=\frac{1+tg\psi}{\sqrt{1+tg^2\psi}},$$

we obtain

$$\cos\psi+\sin\psi=\frac{N_\tau+N_n}{\sqrt{N_\tau^2+N_n^2}}.$$

It follows that:

$$tg\theta=\frac{\sqrt{N_\tau^2+N_n^2}}{N_b}.$$

Thus, the control turning angles of the following force for the rings of the gimbal suspension considered are determined by the components of the equivalent driving force in the natural trihedral basis according to the given spiral-screw programmed vehicle transfer trajectory:

$$tg\psi=\frac{N_n}{N_\tau};\quad tg\theta=\frac{\sqrt{N_\tau^2+N_n^2}}{N_b}.$$

For small turning angles of yaw motion - ψ and pitching motion θ the simple approximate formulas are valid:

$$\psi\approx\frac{N_n}{N_\tau};\quad \theta\approx\frac{\sqrt{N_\tau^2+N_n^2}}{N_b}.$$

7 Summary

The paper proposed a structural scheme of the following force (linkage) in the gimbal suspension to provide programmed controlled transfer of the transport vehicle in the class of helical trajectories. The control is implemented by changing the modulus and direction of the following force vector in the gimbal suspension according to the established algorithms. Algorithms for turning the gimbal rings and the magnitude of the following force are compiled by the components of the software driving force in the basis of the natural trihedral associated with the transport vehicle. The driving force components are found from the Euler-Lagrange kinetostatics equations according to the hodograph of the programmed transfer of the transport vehicle in the class of helical trajectories. The Euler-Lagrange kinetics equations and the complex vector kinematics formulas describing the transfer and turn of the transport vehicle in space are represented by quaternion matrices adapted to computer technologies.

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