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Analytical Modeling of the Binary Dynamic Circuit Motion

Anatolii Alpatov
The Institute of Technical Mechanics of the National Academy of Sciences of Ukraine and the State Space Agency of Ukraine, Dnipro, Ukraine

Victor Kravets
Department of Mechanics
Dnipro University of Technology, Dnipro, Ukraine

Volodymyr Kravets
Department of Mechanics
Dnipro State Agrarian and Economic University, Dnipro, Ukraine

Erik Lapkhanov
The Institute of Technical Mechanics of the National Academy of Sciences of Ukraine and the State Space Agency of Ukraine, Dnipro, Ukraine

ABSTRACT

The binary dynamic circuit, which can be a design scheme for a number of technical systems is considered in the paper. The dynamic circuit is characterized by the kinetic energy of the translational motion of two masses, the potential energy of these masses' elastic interaction and the dissipative function of energy dissipation during their motion. The free motion of a binary dynamic circuit is found according to a given initial phase state. A mathematical model of the binary dynamic circuit motion in the canonical form and the corresponding characteristic equation of the fourth degree are compiled. Analytical modeling of the binary dynamic circuit is carried out on the basis of the proposed particular solution of the complete algebraic equation of the fourth degree. A homogeneous dynamic circuit is considered and the reduced coefficients of elasticity and damping are introduced. The dependence of the reduced coefficients of elasticity and damping is established, which provides the required class of solutions to the characteristic equation. An ordered form of the analytical representation of a dynamic process is proposed in symmetric determinants, which is distinguished by the conservatism of notation with respect to the roots of the characteristic equation and phase coordinates.

Keywords: dynamic design; dynamic circuit; mathematical model; characteristic equation; analytical solution.

INTRODUCTION

The design scheme in the form of a dynamic circuit is widely used in the development and improvement of various types of vehicles, which include conveyor transport, multi-mass vibratory conveyors, pneumatic transport, railway transport, road transport, etc. [1-7].

Analytical modeling is an important stage in the dynamic design of vehicles, which precedes computational and full-scale experiments [8]. Analytical modeling is carried out on the basis of classical mathematical methods for solving systems of linear differential equations, operational calculus, residue theory, modal control theory, root hodograph method, root distribution method in the complex plane [9-13]. Dynamic processes in a single-mass system were analytically probed in the works [14-17]. It shows the possibility of analytical modeling of the binary dynamic circuit free motion which is described by the fourth-order canonical mathematical model [18].

FORMULATION OF THE PROBLEM

The design scheme of the considered mechanical system is shown in Fig. 1 and is defined as a binary dynamic circuit [19].

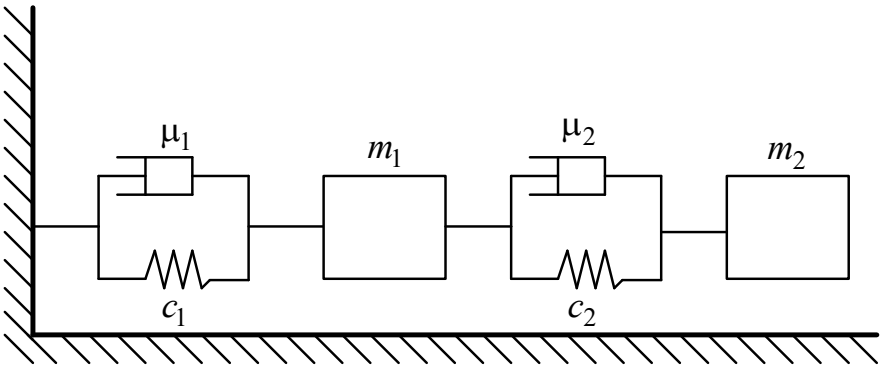


Fig. 1. Binary dynamic circuit

Here m_1, m_2 are the masses of the circuit elements;

c_1, c_2 are the elastic interaction coefficients;

μ_1, μ_2 are the damping coefficients.

The initial perturbation of the binary dynamic circuit is assumed to be given:

$q_1(0)$, $q_2(0)$ are the deviation of masses from natural configuration;

$\dot{q}_1(0)$, $\dot{q}_2(0)$ are the velocities of masses deviations.

It is required to select the class of technically realizable solutions that provide analytical modeling of the binary circuit dynamics.

Mathematical model

The motion of the binary dynamic circuit is described by a system of differential equations in the next form:

$$\begin{aligned} m_1\ddot{q}_1 &= -(\mu_1+\mu_2)\dot{q}_1 - (c_1+c_2)q_1 + \mu_1\dot{q}_2 + c_2q_2, \\ m_2\ddot{q}_2 &= \mu_2\dot{q}_1 + c_2q_1 - \mu_2\dot{q}_2 - c_2q_2, \end{aligned} \qquad (1)$$

where $q_1(t)$, $q_2(t)$ are the generalized coordinates.

Assuming that

$$\dot{q}_1(t) = x_1(t), \quad q_1(t) = x_2(t),$$
$$\dot{q}_2(t) = x_3(t), \quad q_2(t) = x_3(t), \qquad (2)$$

the equations of motion are reduced to the canonical form [13]:

$$\dot{x}_1 = -\frac{(\mu_1 + \mu_2)}{m_1} x_1 - \frac{(c_1 + c_2)}{m_1} x_2 + \frac{\mu_1}{m_1} x_3 + \frac{c_2}{m_1} x_4,$$
$$\dot{x}_2 = x_1,$$
$$\dot{x}_3 = \frac{\mu_2}{m_2} x_1 + \frac{c_2}{m_2} x_2 - \frac{\mu_2}{m_2} x_3 - \frac{c_2}{m_2} x_4,$$
$$\dot{x}_4 = x_3. \qquad (3)$$

The characteristic determinant corresponding to this system of differential equations (3) has the form:

$$\begin{vmatrix} -\frac{\mu_1 + \mu_2}{m_1} - \lambda & -\frac{c_1 + c_2}{m_1} & \frac{\mu_2}{m_1} & \frac{c_2}{m_1} \\ 1 & 0 - \lambda & 0 & 0 \\ \frac{\mu_2}{m_2} & \frac{c_2}{m_2} & -\frac{\mu_2}{m_2} - \lambda & -\frac{c_2}{m_2} \\ 0 & 0 & 1 & 0 - \lambda \end{vmatrix}. \qquad (4)$$

The characteristic equation reduces to a complete algebraic equation of the fourth degree, as:

$$a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0, \qquad (5)$$

where the coefficients of the algebraic equation are represented by symmetric determinants in the ordered form [18]:

$$a_0 = \begin{vmatrix} -\frac{\mu_1 + \mu_2}{m_1} & -\frac{c_1 + c_2}{m_1} & \frac{\mu_2}{m_1} & \frac{c_2}{m_1} \\ 1 & 0 & 0 & 0 \\ \frac{\mu_2}{m_2} & \frac{c_2}{m_2} & -\frac{\mu_2}{m_2} & -\frac{c_2}{m_2} \\ 0 & 0 & 1 & 0 \end{vmatrix};$$

$$a_1 = \begin{vmatrix} -\frac{\mu_1+\mu_2}{m_1} & -\frac{c_1+c_2}{m_1} & \frac{\mu_2}{m_1} & 0 \\ 1 & 0 & 0 & 0 \\ \frac{\mu_2}{m_2} & \frac{c_2}{m_2} & -\frac{\mu_2}{m_2} & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} + \begin{vmatrix} -\frac{\mu_1+\mu_2}{m_1} & -\frac{c_1+c_2}{m_1} & 0 & \frac{c_2}{m_1} \\ 1 & 0 & 0 & 0 \\ \frac{\mu_2}{m_2} & \frac{c_2}{m_2} & -1 & -\frac{c_2}{m_2} \\ 0 & 0 & 0 & 0 \end{vmatrix} +$$

$$+ \begin{vmatrix} -\frac{\mu_1+\mu_2}{m_1} & 0 & \frac{\mu_2}{m_1} & \frac{c_2}{m_1} \\ 1 & -1 & 0 & 0 \\ \frac{\mu_2}{m_2} & 0 & -\frac{\mu_2}{m_2} & -\frac{c_2}{m_2} \\ 0 & 0 & 1 & 0 \end{vmatrix} + \begin{vmatrix} -1 & -\frac{c_1+c_2}{m_1} & \frac{\mu_2}{m_1} & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{c_2}{m_2} & -\frac{\mu_2}{m_2} & -\frac{c_2}{m_2} \\ 0 & 0 & 1 & 0 \end{vmatrix};$$

$$a_2 = \begin{vmatrix} -\frac{\mu_1+\mu_2}{m_1} & -\frac{c_1+c_2}{m_1} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{\mu_2}{m_2} & \frac{c_2}{m_2} & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} + \begin{vmatrix} -\frac{\mu_1+\mu_2}{m_1} & 0 & \frac{\mu_2}{m_1} & 0 \\ 1 & -1 & 0 & 0 \\ \frac{\mu_2}{m_2} & 0 & -\frac{\mu_2}{m_2} & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} +$$

$$+ \begin{vmatrix} -1 & -\frac{c_1+c_2}{m_1} & \frac{\mu_2}{m_1} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{c_2}{m_2} & -\frac{\mu_2}{m_2} & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} + \begin{vmatrix} -\frac{\mu_1+\mu_2}{m_1} & 0 & 0 & \frac{c_2}{m_1} \\ 1 & -1 & 0 & 0 \\ \frac{\mu_2}{m_2} & 0 & -1 & -\frac{c_2}{m_2} \\ 0 & 0 & 0 & 0 \end{vmatrix} +$$

$$+ \begin{vmatrix} -1 & -\frac{c_1+c_2}{m_1} & 0 & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{c_2}{m_2} & -1 & -\frac{c_2}{m_2} \\ 0 & 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} -1 & 0 & \frac{\mu_2}{m_1} & \frac{c_2}{m_1} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{\mu_2}{m_2} & -\frac{c_2}{m_2} \\ 0 & 0 & 1 & 0 \end{vmatrix};$$

$$a_3 = \begin{vmatrix} -\frac{\mu_1+\mu_2}{m_1} & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ \frac{\mu_2}{m_2} & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} + \begin{vmatrix} -1 & -\frac{c_1+c_2}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{c_2}{m_2} & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} +$$

$$+ \begin{vmatrix} -1 & 0 & \frac{\mu_2}{m_1} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{\mu_2}{m_2} & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} + \begin{vmatrix} -1 & 0 & 0 & \frac{c_2}{m_1} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -\frac{c_2}{m_2} \\ 0 & 0 & 0 & 0 \end{vmatrix};$$

$$a_4 = \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}.$$

Evaluating these determinants, it can be obtained laconic formulas for the coefficients of the characteristic equation of the binary dynamic circuit:

$$a_0 = \frac{c_1 c_2}{m_1 m_2};$$

$$a_1 = \frac{1}{m_1 m_2}\left[c_1\mu_2 + c_2\mu_1\right];$$

$$a_2 = \frac{1}{m_1 m_2}\left[m_2 c_1 + (m_1+m_2)c_2 + \mu_1\mu_2\right];$$

$$a_3 = \frac{1}{m_1 m_2}\left[m_2\mu_1 + (m_1+m_2)\mu_2\right];$$

$$a_4 = 1.$$

For the homogeneous dynamic circuit, (where $m_1 = m_2 = m$, $c_1 = c_2 = c$, $\mu_1 = \mu_2 = \mu$) these formulas for the coefficients of the characteristic equation are simplified:

$$a_0 = \frac{c^2}{m^2}; \quad a_1 = \frac{2c\mu}{m^2}; \quad a_2 = \frac{3mc+\mu^2}{m^2}; \quad a_3 = \frac{3\mu}{m}; \quad a_4 = 1.$$

Introducing the elasticity and damping coefficients reduced to mass:

$$\overline{c} = \frac{c}{m}, \quad \overline{\mu} = \frac{\mu}{m}, \tag{6}$$

it will be obtained:

$$a_0 = \bar{c}^2; \quad a_1 = 2 \cdot \bar{c} \cdot \bar{\mu}; \quad a_2 = 3\bar{c} + \bar{\mu}^2; \quad a_3 = 3\bar{\mu}; \quad a_4 = 1. \qquad (7)$$

It has been established [20] and tested [18, 21] that the solution of the complete algebraic equation of the fourth degree takes the foreseeable form:

$$\lambda_{1,2,3,4} = -\frac{a_3}{4} \pm \frac{1}{\sqrt{2}} \sqrt{\left[\frac{3}{2}\left(\frac{a_3}{2}\right)^2 - a_2\right] \pm \sqrt{\left[\left(\frac{a_3}{2}\right)^2 - a_2\right]^2 - 4a_0}}, \qquad (8)$$

if it's coefficients satisfy the condition:

$$a_1 + \frac{a_3}{2}\left[\left(\frac{a_3}{2}\right)^2 - a_2\right] = 0. \qquad (9)$$

It is easy to show that for a homogeneous, binary dynamic circuit, this condition is reduced to the following dependence of the given coefficients of elasticity and damping:

$$\bar{c} = \frac{3}{4}\bar{\mu}^2. \qquad (10)$$

Providing the required ratio of parameters in the dynamic design process of a homogeneous binary circuit, it is possible to obtain an analytical solution of the characteristic equation in the form:

$$\lambda_{1,2,3,4} = -\frac{3}{4}\bar{\mu} \pm \frac{1}{\sqrt{2}} \sqrt{\frac{1}{2^3}\bar{\mu}^2 \pm \sqrt{-\frac{5}{4}\bar{\mu}^4}}, \qquad (11)$$

or

$$\lambda_{1,2,3,4} = \frac{1}{4}\bar{\mu}\left[-3 \pm \sqrt{1 \pm 4\sqrt{5}i}\right]. \qquad (12)$$

Here, according to the second Moivre formula [22], it can be written:

$$\sqrt{1 + 4\sqrt{5}i} = \sqrt{\left|1 + 4\sqrt{5}i\right|} \cdot$$

$$\cdot \left[\cos\left(\frac{\arg(1 + 4\sqrt{5}i) + 360^o \cdot k}{2}\right) + i \cdot \sin\left(\frac{\arg(1 + 4\sqrt{5}i) + 360^o \cdot k}{2}\right)\right]$$

$$(k = 0,1), \text{ where}$$

$$\left|1 + 4\sqrt{5}i\right| = 9, \quad \arg(1 + 4\sqrt{5}i) = \varphi, \quad \arctan(\varphi) = 4\sqrt{5}.$$

To simplify calculations, it is used the approximate equality $\varphi \approx 90^o$.

Then, if $k=0$ it will be obtained: $3\frac{\sqrt{2}}{2}(1+i)$;

if $k=1$ it will be obtained: $3\frac{\sqrt{2}}{2}(-1-i)$.

Likewise for $\sqrt{1-4\sqrt{5}i}$, $\left|1-4\sqrt{5}i\right|=9$,

$\arg\left(1-4\sqrt{5}i\right)\approx 270^{\circ}$ and correspondingly

if $k=0$ it will be obtained: $3\frac{\sqrt{2}}{2}(-1+i)$;

if $k=1$ it will be obtained: $3\frac{\sqrt{2}}{2}(1-i)$.

Thus, the approximate solution of the characteristic equation is illustrated by the following qualitatively ordered notation in the follow form:

$$\lambda_{1,2}\approx\frac{\overline{c}}{\overline{\mu}}\left[\left(-1+\frac{\sqrt{2}}{2}\right)\pm i\frac{\sqrt{2}}{2}\right];$$
$$\lambda_{3,4}\approx\frac{\overline{c}}{\overline{\mu}}\left[\left(-1-\frac{\sqrt{2}}{2}\right)\pm i\frac{\sqrt{2}}{2}\right]. \quad (13)$$

The distribution of these roots in the complex plane is shown in Fig. 2 and corresponds to the result given in [18].

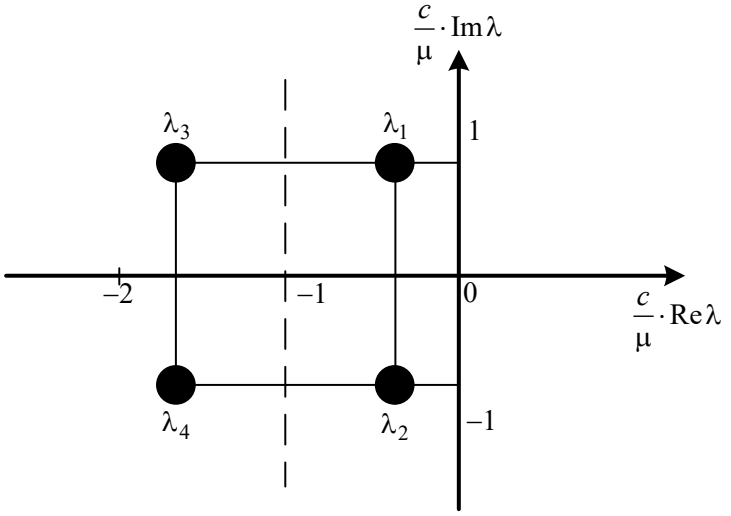


Fig. 2. Distribution of the characteristic equation roots of a homogeneous binary dynamic circuit

Verification of results (Error estimation of the characteristic equation approximate solution)

The estimation of the error in calculating the roots of the characteristic equation is carried out using the Vieta's formulas [22]:

$$\lambda_1+\lambda_2+\lambda_3+\lambda_4=-a_3;$$
$$\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4=a_2;$$
$$\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4=-a_1;$$
$$\lambda_1\lambda_2\lambda_3\lambda_4=a_0. \quad . \qquad (14)$$

Substituting the approximate values of the roots into these formulas, it can be found:

$$\lambda_1+\lambda_2+\lambda_3+\lambda_4=-3\bar{\mu};$$
$$\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4=6\frac{\bar{c}^2}{\bar{\mu}^2}=\frac{9}{2}\bar{c};$$
$$\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4=-4\frac{\bar{c}^3}{\bar{\mu}^3}=-\frac{3^3}{4^2}\bar{\mu}^3;$$
$$\lambda_1\lambda_2\lambda_3\lambda_4=\frac{9}{8}\bar{c}^2. \quad .$$

Comparing the obtained approximate results with the exact values of the coefficients of the characteristic equation:

$$a_3=3\bar{\mu};\quad a_2=3\bar{c}+\bar{\mu}^2=\frac{13}{3}\bar{c};\quad a_1=2\bar{c}\bar{\mu}=\frac{3}{2}\bar{\mu}^3;\quad a_0=\bar{c}^2,$$

let's establish the degree of the admitted error:

$$-3\bar{\mu}=-3\bar{\mu} \quad \text{that is} \quad 1=1;$$
$$\frac{9}{2}\bar{c}\approx\frac{13}{3}\bar{c} \quad \text{that is} \quad 4,5\approx4,34;$$
$$-\frac{3^3}{4^2}\bar{\mu}^3\approx-\frac{3}{2}\bar{\mu}^3 \quad \text{that is} \quad 1,68\approx1,5;$$
$$\frac{9}{8}\bar{c}^2\approx\bar{c}^2 \quad \text{that is} \quad 1,13\approx1. \quad .$$

Analytical solution of the canonical system of differential equations

Using the theory of residues [9, 10], it is possible to obtain the following ordered notation of the desired analytical solution in the form of symmetric determinants [18]:

$$x_i(t)=\sum_{k=1}^{4}\frac{\Delta_i(\lambda_k)}{\prod\limits_{\substack{s=1\\k\neq s}}^{4}(\lambda_k-\lambda_s)}e^{\lambda_k\cdot t},(i=1,2,3,4) \quad , \qquad (15)$$

where $\Delta_i(\lambda_k)$ are the symmetric determinants, which are formed by the number of the phase coordinate by replacing the column corresponding to this number in the characteristic determinant with the column vector of the state of the dynamic circuit at the initial moment of time. The free motion of masses of a homogeneous binary dynamic circuit is analytically modeled by their generalized coordinates $q_1(t)$, $q_2(t)$ in the following expanded notation:

$$q_1(t)=\sum_{k=1}^{4}\frac{\begin{vmatrix} -2\bar{\mu}-\lambda_k & -\dot{q}_1(0) & \bar{\mu} & \bar{c} \\ 1 & -q_1(0) & 0 & 0 \\ \bar{\mu} & -\dot{q}_2(0) & -\bar{\mu}-\lambda_k & -\bar{c} \\ 0 & -q_2(0) & 1 & 0-\lambda_k \end{vmatrix}}{\prod_{\substack{s=1 \\ k\neq s}}^{4}(\lambda_k-\lambda_s)}e^{\lambda_k\cdot t};$$

$$q_2(t)=\sum_{k=1}^{4}\frac{\begin{vmatrix} -2\bar{\mu}-\lambda_k & -2\bar{c} & \bar{\mu} & -\dot{q}_1(0) \\ 1 & 0-\lambda_k & 0 & -q_1(0) \\ \bar{\mu} & \bar{c} & -\bar{\mu}-\lambda_k & -\dot{q}_2(0) \\ 0 & 0 & 1 & -q_2(0) \end{vmatrix}}{\prod_{\substack{s=1 \\ k\neq s}}^{4}(\lambda_k-\lambda_s)}e^{\lambda_k\cdot t}. \quad (15)$$

The simplicity of the formation of generalized coordinates shown here ensures the efficiency of the analysis of dynamic processes, both analytically and numerically. In the traditional notation, such formulas are devoid of structural order, cumbersome and boundless.

CONCLUSIONS

Analytical modeling of the free motion of a homogeneous binary dynamic circuit is carried out according to the ordered and conservative form of solving the canonical system of fourth-order differential equations. The ordering with respect to the index of generalized coordinates is provided by symmetric determinants. Conservativeness is due to the indexing of the roots of the characteristic equation of the fourth degree. The coefficients of this characteristic equation are presented in the form of symmetric determinants. The roots of this characteristic equation are in the class of the proposed particular analytical solution. In the space of inertial, elastic and damping parameters of a homogeneous binary dynamic circuit, it has been obtained the roots of the characteristic equation, their distribution in the complex plane and the corresponding functional dependence of these parameters.

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