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Algorithm for the Set of Similarity Criteria Formation for a Physical Process in the Class of Homogeneous Functions

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ABSTRACT

The efficiency of application of linear programming methods to problems of the theory of similarity and dimensions is shown. A general algorithm for formation of the set of similarity criteria for a physical process in the class of homogeneous functions is proposed. The set of systems of linear algebraic equations is created using the combinatorial method and chain diagrams. Basic and free variables and their corresponding variants of dimensionless sets of independent arguments, which are taken as the main similarity criteria, are distinguished. The set of derived similarity criteria is found using the basic criteria and the Cayley table.

Keywords: modeling, similarity criterion, homogeneous function, power complex, combinatorial method, chain diagrams.

INTRODUCTION

The similarity theory establishes the conditions for the equivalence of a natural and model physical phenomenon [1-3]. These conditions are expressed in similarity criteria [4-6]. Similarity criteria in solid mechanics, in the mechanics of bodies of variable mass, relativistic mechanics, mechanics of continuous media (aero, hydro and gas dynamics, electrodynamics, thermodynamics, elasticity, etc.), as well as in quantum physics, plasma physics, etc. are determined depending on accepted mathematical model of a physical phenomenon. In

problems of aerohydrodynamics, the following similarity criteria are known and widely used [7]:

- Reynolds number;
- Mach number;
- Froude number;
- Strouhal number,

in heat transfer problems (thermodynamics) [8]:

- Prandtl number;
- Nusselt number;
- Peclet number;
- Stanton number;
- Fourier number;
- Biot number,

in problems of rigid body dynamics [9]:

- Newton number,

in problems of elastic deformations of a rigid body [10]:

- Poisson's ratio,

Numerous similarity criteria correspond to various physical problems and their mathematical models. Mathematical models are constructed in various state spaces (Euclidean, vector, phase, Lorentz, Riemann, metric, Hilbert, etc.) using various transformation operators (differential, integral, Hamilton, Laplace, D'Alembert, Lagrange, Euler-Lagrange, etc.) to various representations of the sought functions by series (power series, Maclaurin, Taylor, Fourier, Laurent, Neumann, Fourier-Bessel, etc.) [11].

It was found that mathematical models constructed in Euclidean vector space using a homogeneous differential operator of transformations have the required solutions in the class of homogeneous functions represented by power complexes [5]. Then the construction of criteria for the similarity of a physical phenomenon is reduced to replacing the original independent arguments of the problem with arguments of a dimensionless form, using the dimensional analysis methods. In the general case, this problem has many solutions and, accordingly, many similarity criteria determined here by the combinatorial method.

Setting up a problem

Two physical processes (full-scale and model) are assumed to be similar if the numerical values of the model process can be used to obtain the numerical values of the full-scale process [4]. This goal is served by similarity criteria – dimensionless variables compiled from dimensional arguments that determine the physical process. Dimensional arguments $x_i \ (i = 1,2,...n)$ are expressed in terms of the accepted basic values of measurement $\mathrm{e}_j \ (j = 1,2,...k)$, depending on the system of units used. For the physical process under consideration, a system of units is established and the corresponding basic dimensional quantities of measurement in the amount of k. The number of independent dimensional arguments is established that determine the considered physical process in the form of a functional dependence:

$$y = f(x_1, x_2, ..., x_n).$$

It is assumed that functional dependence belongs to the class of homogeneous functions of its arguments [11].

It is required to find a set of dimensionless similarity criteria arguments describing a physical process in an equivalent way, which is invariant to the used system of units.

THE METHOD OF SOLUTION

By definition, a homogeneous function of degree α has the following property [11]:

$$f(\lambda x_1, \lambda x_2, ..., \lambda x_n) = \lambda^{\alpha} f(x_1, x_2, ..., x_n), \quad (1)$$

where $\lambda > 0$.

According to Euler's theorem [12], a homogeneous function of degree α satisfies the equality:

$$\sum_{i=1}^{n} x_i \frac{\partial f(x_1, x_2, ...x_n)}{\partial x_i} = \alpha \cdot f(x_1, x_2, ...x_n). \quad (2)$$

In particular, for $n = 1$, it will be obtained:

$$x \frac{df(x)}{dx} = \alpha \cdot f(x). \quad (3)$$

Separating variables

$$\frac{df(x)}{f(x)} = \alpha \cdot \frac{dx}{x}$$

and integrating, it will be found:

$$\ln|f(x)| = \alpha \ln|x| + \ln|C|,$$

further, using potentiation, it will be obtained:

$$\ln|f(x)| = \ln|Cx^{\alpha}|.$$

Hence:

$$f(x) = Cx^{\alpha}.$$

In the general case, a homogeneous function can be represented by a power complex of its arguments [5] in the next form:

$$y = f(x_1, x_2, ..., x_n) = C \cdot x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot ... \cdot x_n^{\alpha_n}. \quad (4)$$

Passing to dimensions, the identity is established:

$$[y]=[x_1]^{\alpha_1}\cdot[x_2]^{\alpha_2}\cdot\ldots\cdot[x_n]^{\alpha_n}. \quad (5)$$

Here, due to the adopted system of measurement units, there is a well-known power-law representation of the dimensions of desired function and its arguments:

$$\begin{aligned}
&[y]=e_1^{\alpha_1}\cdot e_2^{\alpha_2}\cdot\ldots\cdot e_k^{\alpha_k},\\
&[x_1]=e_1^{\beta_{11}}\cdot e_2^{\beta_{12}}\cdot\ldots\cdot e_k^{\beta_{k1}},\\
&[x_2]=e_1^{\beta_{12}}\cdot e_2^{\beta_{22}}\cdot\ldots\cdot e_k^{\beta_{k2}},\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&[x_n]=e_1^{\beta_{1n}}\cdot e_2^{\beta_{2n}}\cdot\ldots\cdot e_k^{\beta_{kn}},
\end{aligned} \quad (6)$$

where α_j, β_{ij} ($j=1,2,...,k$; $i=1,2,...,n$) are specified dimensions (integers or fractional real numbers). Whence follows:

$$\left(e_1^{\beta_{11}}\cdot e_2^{\beta_{12}}\cdot\ldots\cdot e_k^{\beta_{k1}}\right)^{\alpha_1}\cdot\left(e_1^{\beta_{11}}\cdot e_2^{\beta_{12}}\cdot\ldots\cdot e_k^{\beta_{k1}}\right)^{\alpha_2}\cdot\ldots\cdot\left(e_1^{\beta_{1n}}\cdot e_2^{\beta_{2n}}\cdot\ldots\cdot e_k^{\beta_{kn}}\right)^{\alpha_n}=e_1^{\alpha_1}\cdot e_2^{\alpha_2}\cdot\ldots\cdot e_k^{\alpha_k},$$

or

$$e_1^{\beta_{11}\alpha_1}\cdot e_2^{\beta_{12}\alpha_1}\cdot\ldots\cdot e_k^{\beta_{k1}\alpha_1}\cdot e_1^{\beta_{11}\alpha_2}\cdot e_2^{\beta_{12}\alpha_2}\cdot\ldots\cdot e_k^{\beta_{k1}\alpha_2}\cdot\ldots\cdot e_1^{\beta_{1n}\alpha_n}\cdot e_2^{\beta_{2n}\alpha_n}\cdot\ldots\cdot e_k^{\beta_{kn}\alpha_n}=e_1^{\alpha_1}\cdot e_2^{\alpha_2}\cdot\ldots\cdot e_k^{\alpha_k},$$

and further

$$e_1^{\beta_{11}\alpha_1+\beta_{12}\alpha_2+\ldots+\beta_{1n}\alpha_n}\cdot e_2^{\beta_{21}\alpha_1+\beta_{22}\alpha_2+\ldots+\beta_{2n}\alpha_n}\cdot\ldots\cdot e_k^{\beta_{k1}\alpha_1+\beta_{k2}\alpha_2+\ldots+\beta_{kn}\alpha_n}=e_1^{\alpha_1}\cdot e_2^{\alpha_2}\cdot\ldots\cdot e_k^{\alpha_k},$$

which is equivalent:

$$\begin{aligned}
&\beta_{11}\alpha_1+\beta_{12}\alpha_2+\ldots+\beta_{1n}\alpha_n=\alpha_1,\\
&\beta_{21}\alpha_1+\beta_{22}\alpha_2+\ldots+\beta_{2n}\alpha_n=\alpha_2,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\beta_{k1}\alpha_1+\beta_{k2}\alpha_2+\ldots+\beta_{kn}\alpha_n=\alpha_n.
\end{aligned} \quad (7)$$

Thus, a system of k linear algebraic equations is established that contains n unknown powers α_i $(i=1,2,...,n)$ of dimensional arguments.

Analysis of this system of algebraic equations shows that in the general case, when $k<n$, the problem has a set of solutions determined by the choice of basic and free variables [13]. The number of basic variables is determined by the number of independent equations k that make up the system, the rest of the variables in the number $(n-k)$ are assumed to be free. The number of options for choosing basic and free variables is determined by the number of combinations:

$$N=C_n^k. \quad (8)$$

Combinations of basic and free variables $(m=1,2,...N)$ can be found using chain diagrams [14]. For each combination, k basic variables are expressed in terms of $(n-k)$ free variables. Then, excluding the basic variables in the original power complex, it is possible to represent the desired function depending on the free variables. By grouping dimensional arguments in the

desired function under the degree of the corresponding free variable, a set $(n-k)$ of new arguments $\pi_j^m\ (j=1,2,...,n-k)$ is set for each of the variants $(m=1,2,...,N)$. Due to the fact that free variables can take any arbitrary values, the complexes of initial dimensional arguments compiled under their sign are necessarily dimensionless, invariant to systems of units of measurement and are taken as the main similarity criteria π_j^m, that is $\pi_j^m = \text{idem}$ [5].

The set of basic similarity criteria are found by the number of dimensional arguments that determine the physical process, and the number of accepted basic units of measurement according to the formula:

$$(n-k)C_n^k. \qquad (9)$$

It can be noted that combinations of basic similarity criteria through rational operations (arithmetic multiplication, division, etc.) are dimensionless and refer to derived similarity criteria. It is efficient to construct derived similarity criteria using Cayley [15] tables and the corresponding rational operations.

In the special case, when $k=n$, the system of linearly independent algebraic equations has a unique solution, free variables are excluded from consideration and, therefore, it turns out to be impossible to compose similarity criteria, which is confirmed by the formula: $(n-n)C_n^n=0$. The absence of similarity criteria means that the model and natural physical processes are identical. The case when $k>n$ indicates the incorrectness of the problem and the absence of a solution.

CONCLUSIONS

The algorithm for constructing a set of similarity criteria for a physical process in the class of homogeneous functions is reduced to the sequential execution of the following operations:

1. The study of the physical process and the establishment of independent arguments x_i $(i=1,2,...n)$ that determine the desired function $y=f(x_1,x_2,...,x_n)$;
2. The selection of the system of measurement units $\mathrm{e}_j\ (j=1,2,...k)$ of independent arguments and the desired function;
3. Searching the required function in the class of homogeneous;
4. Represent the required function as a power-law complex of independent arguments with the corresponding unknown homogeneity indices $\alpha_i\ (i=1,2,...,n)$;
5. Express the dimension of the desired function $[y]$ and independent arguments $[x_i]$ in the accepted system of units of measurement e_j;
6. Represent the dimension of the power complex in the accepted system of units of measurement;
7. Transform in an equivalent way the identity equality of the dimension of the required function and the power complex to a system of k algebraic equations with respect to n unknown homogeneity exponents;

8. Assuming that k <n, using chain diagrams it is formed a variety C_n^k of selection of k basic variables and $(n-k)$ free variables from n unknown indicators of homogeneity;
9. Find the solution for each variant of system of linear algebraic equations for basic variables with respect to free variables;
10. Exclude the basic variables in the original power complex, respectively, for each of the variants;
11. Group independent arguments in a power complex under the sign of the degree of free variables for each of the variants;
12. Obtain dimensionless sets of independent arguments for each of the variants π_j^m, taken as the main criteria for the similarity of the full-scale and simulated physical process;
13. Find the derived similarity criteria for the main ones using the Cayley tables in accordance with the selected rational arithmetic operation.

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