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Optimum Sampling Plan on Quality Indices AOQL and MAPD

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ABSTRACT

This paper describes a selection procedure for an Optimum Sampling Plan, offering maximum consumer protection in terms of AOQL and MAPD. The greatest lower bound (glb) property of AOQL for a fixed MAPD is used to design the plan offering highest precision on outgoing quality for the lot. Tables for optimum sampling plans corresponding to specified MAPD and g l b of AOQL are listed along with AQL. Empirical relation to determine AOQL for given acceptance number and MAPD is determined. Also an approximated acceptance number function in terms of (MAPD, AOQL) is developed. Lower and Upper bounds of AOQL for some parametric sampling plans are listed.

Key words: Optimum Sampling Plan, MAPD, glb(AOQL), Empirical relations

INTRODUCTION

AOQL is the maximum average outgoing quality over all possible levels of incoming quality as per American National Standard, A2-1987. It is the worst average quality that the consumer will receive in the long run, when defectives are replaced by non-defectives and so the consumers prefer this quality index. Mandelson (1962) aspire an engineer's quality standard and established a relation MAPD =p*=c/n where c is the acceptance number and n is the sample size. Mayer (1959 &1967) has established the properties of inflection point and termed it as MAPD. Ramkumar &Suresh (1996) described the glb property of AOQ curves leading to optimum sampling plans. Soundararajan (1975 &1981) had established a sampling plan on AQL,AOQL and MAPD and proved that MAPD is the inflexion point of OC curve. Ramkumar (2013) mentioned more than 40 properties of MAPD as an incoming quality measure. The slope of OC curve is horizontal at the inflection point and a sharp decline in the probability of acceptance is expected for the proportion of defective beyond p*. So MAPD is a good and consumer protective quality index.

NEED OF THE SAMPLING PLAN

The glb of AOQL is derived here assuring maximum protection for the outgoing quality, where the OC curve is identified by MAPD in a SSP. Thus MAPD & glb (AOQL) will doubly protect the interest of the consumer. Also MAPD-AOQL indexed sampling plan is more balanced as MAPD protects incoming lot quality while AOQL take care of out going quality on an average. The sampling plan designed here can be considered in the floor shop situations, where stiff

competition is involved. But for destructive and costly testing, this sampling plan is less applicable, because the sampling plans with c=0 is inapplicable. The glb of AOQL provides a sharpened plan, safeguarding the outgoing quality occurring within stringent limit.

It was found that the ratio AOQL/MAPD is monotonic decreasing upto c<5 and it become monotonic increasing after c=5 (See Table:1 & Figure:1). Thus the ratio is minimum at c=5 and glb of AOQL is attained for a given MAPD. The glb of AOQL is defined as greatest lower bound of all outgoing quality over any sampling plan with a fixed c/n ratio. When MAPD is fixed this sampling plan provide highest outgoing quality protection. 'It is the limiting value of AOQL admissible in a MAPD fixed sampling plan. Such a sampling plan could not attain further improvement on outgoing quality and so the unique sampling plan (n,c) realized on MAPD and glb of AOQL is considered as an Optimum Plan'. The glb of AOQL is attained at 0.6336p* as the ratio of AOQL/MAPD=0.6336 is uniquely minimum for any combination of n and c and it is reached at c=5. Hence an optimum plan attains an outgoing quality protection of least proportion 0.6336p* (glb of AOQL) with c=5. It is clear that there exists a unique sampling plan only at glb of AOQL (at c=5) and for all other higher values of AOQL there exist two sampling plans ((c=1 & c>40), (c=2 & c=17 or 18), (c=3 & c=8 &9), (c=4 &c=6&7)).

SELECTION OF OPTIMUM PLAN

For any given MAPD (p*), the optimum plan can be determined as (n=5/p*, 5). Thus the sample size will be too large if MAPD is too small and it will be moderate if the MAPD is sizeable.

Example:1 At a fixed MAPD

For a product at each level of inspection =MAPD =6% (fixed)
The optimum plan hold the parameters c=5, n=5/0.06=83.
The optimum plan (83,5) has glb of AOQL=0.6336×0.06 =0.03802. =3.8%
The producer's risk for the optimum plan is fixed at AQL=0.5226×0.06 =0.031356.=3.14%
Hence the optimum plan (83,5) will have AQL=3.14% and AOQL=3.80%.
The other parameters are LTPD= 10.53%, IQL =6.74% and MAAOQ =3.70%.

Example:2 At a fixed glb(AOQL)

A company producing electric switches claim that the product attains the glb of AOQL=4% (fixed)
Then MAPD is p*= 0.04/0.6336 =0.0631, so the parameters of optimum plan=5/0.0631=80 and c=5. The optimum plan (80, 5) has MAPD = 6.31% and AOQL =4%. AQL=3.30%
LTPD= 11.70% IQL=7.09

COMPARISON OF AOQ CURVES

The outgoing quality over different levels of proportion of defectives is observed in an AOQ curve. The peak of AOQ curve exhibits the utmost proportion of average defectives inhibited in the lot and it is called the AOQL of a single sampling plan. Figure:2 shows that the Curve:1 has a minimum peak compared to Curve:2,3 and 4. Fixing MAPD, one can identify a number of plans with less than or greater than 5 as acceptance number and the plan with c=5 has a greatest lower bound for AOQL (See Figure:2). Fixing MAPD=5%, take the AOQ curves od SSP (50, 5) curve-1,(40,4) curve-2, (100,10)-curve-3 and (200,20) –curve-4 , it is clear that AOQL for (50,5) is only .6336 while it is larger values for all other curves.

CONSTRUCTION OF THE PLAN

For a single sampling plan, assuming the Poisson model for the probability of acceptance for defectives.

$$AOQL = p_m P_a(p_m) = p_m * \sum_0^c \frac{e^{-np_m}(np_m)^r}{r!} \quad (1)$$

$$\frac{AOQL}{p^*} = \frac{nAOQL}{np^*} = \frac{np_m * \sum_0^c \frac{e^{-np_m}(np_m)^r}{r!}}{p^*} \quad (2)$$

$$\frac{AOQL}{p^*} = \frac{e^{-np_m}(np_m)^{(c+2)}}{c * c!} \quad \text{(Dodge1957)} \quad (3)$$

$np^* = c$ (Mandelson 1959) (4)

Thus AOQL/p* is a function of c and np_m where np_m satisfies the relations $1 + \frac{\mathbf{n\,p_m}}{1!} + \frac{(\mathbf{n\,p_m})^2}{2!} + \ldots\ldots\ldots \frac{(\mathbf{n\,p_m})^{\mathbf{c}}}{\mathbf{c}!} := \frac{(\mathbf{n\,p_m})^{(\mathbf{c}+2)}}{(\mathbf{c}+2)!}$, which is also a function of c. Dodge and Romig Table evaluates the value of np_m in terms of c for 1 to 40. Thus AOQL/MAPD is a function of c only.

TABLES AND GRAPHS

Table:1 is showing the ratio AOQL/MAPD for selected values of c. It is capable of determining the unique sampling plans for the ratio approximated to nearest fraction less than it up to 5 and more than it for c<5. Thus the unique sampling plan exist only for c=5 and for all other fractions other than 0.6336 there exists two sampling plans. So it is fixed according to the OC curve required. Table:2 provides the value of glb of AOQL & AQL and the parameters of optimum plan for selected values of MAPD. The glb of AOQL is obtained from the relation, glb of AOQL=0.6336.p*. The relation gives AQL where AQL = 0.5226.p*.since $p_1/p^* = 0.5226$ at c=5 for a single sampling plan. Similarly LTPD =1.8549 p^* and IQL =1.1240 p*

Table:3 gives the value of MAPD, AQL and the parameters of sampling plan for selected values the glb of AOQL. The relationship MAPD= glb of AOQL/0.6336 determines the MAPD, hence n is determined.

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Result:1

Empirical Relation connecting AOQL, Sample size and MAPD

a) Finding AOQL for a given (n, p*) i.e. (for given n, c)

$$AOQL = \frac{\mathbf{a}\cdot(\mathbf{b})^{\frac{1}{\mathbf{c}}}\,\mathbf{c}^{(\mathbf{k}+1)}}{\mathbf{n}} \quad \text{=for c=1 to 40.}$$

where **a**=**0.45832955**, **b**=**1.8366067**, **k**=**0.12802105** from the best fitted polynomial with a

$$AOQL := MAPD\cdot\frac{a\cdot c^{k}}{b^{c}}$$

maximum error <0.0067

Also an empirical relation is possible on Dodge Romig Table as

where **a=0.225561,b=0.566595,k=0.007244,d=-7.45E-05** having a maximum error of

$$AOQL := \frac{a + b\cdot c + k\cdot c^{2} + d\cdot c^{3}}{n}$$

0.0062 for a minimum sample size 10

Example:3

For a sampling plan sample size is fixed at n= 30 and inflection point of OC curve at p*=0.08. Then AOQL determined from empirical relation is 0.064571.

b) Finding c for a given AOQL & MAPD

The Single Sampling Plan on product quality MAPD and outgoing quality AOQL can be derived from the empirical relation **x= a/(1+be^{-kc})** where x=AOQL/MAPD

$$\text{Then} \quad c := \frac{1}{k}\cdot\ln\left(\frac{x\cdot b}{a - x}\right)$$

where **a**=**0.631478**, **b**=**-0.78491**, **k**=**1.151094**; for c=1to 5 having error in the estimation of the ratio < 0.0005 where **a**=**0.896979**, **b**= **0.48153**, **k**=**0.025388**. for c=6 to 20, having error in the estimation in the ratio <. 0009.

Example:4

A lot size of 1000 packets claim stable product quality MAPD=0.082 &AOQL=0.058.

Then **x**=0.707317; From the empirical relation **c**=1.72938≈**2** and **n**=34.48 = **35**. Thus the sampling plan is **(35,2)** (Using the relation c= a/(1+be^{-kx}) with first set of parameters). The suitable sampling plan is (35,2) and it displays only less discrimination in the OC curve. (From figure:3)

Another suitable sampling plan with the same quality indices having high discrimination can also be determined from the empirical relation, as **c**=23.06007≈**23** and **n**= 396.5517=**397**. The sampling plan **(397,23)** also satisfies the same product requirements. (Using the relation c=

$a/(1+be^{-kx})$ with second set of parameters). Thus for the same characteristics of MAPD and AOQL, two distinct sampling plans are available. The first is useful when the customer is rather thirst about the quality and high sampling cost is acceptable; the second is preferred when quick decision are to be taken and the customer is not that particular about high quality.

Result:2
Relation between AOQL and p*
Among all plans AOQL/p* is minimum for c=5.
AOQL ≥ 0.6336 p* for all sampling plans i.e. **AOQL/p* ≥ 0.6336**

Range of AOQL
From the Table:1, AOQL/p* ≤ 0.84 for c=1,2,.....40 and
from result:2, AOQL/p* ≥ 0.6336 for c=1,2,.....40,
Therefore **0.6336p* ≤ AOQL ≤ 0.84p*** for c=1,2,3,4,5.
and **0.6336p* ≤ AOQL ≤ 0.744325p*** for c=5……..40

The minimum and maximum value of AOQL for different sampling plans can be obtained using Table:1. Thus the producer is capable of selecting the plan with specified AOQL along with some other specifications like fixing the sample size (Table:4).

For example, sample size is fixed at n=60 and AOQL =0.07432.From Table:4, 0.07413 ≤AOQL ≤ 0.0745267 for n=60,with c=7.

Sampling plan corresponding to the fixed sample size and AOQL need not always exist. For example, sample size is fixed at n=60 and AOQL =0.0746.

Since U & L of AOQL for (n=60,c=7) is (0.07413,0.0745267) and for (n=60,c=8) it is (0.0851867,0.08576), the value for AOQL=0.0746 has no solution with the sample size 60.

Result:3
MAPD is greater than AOQL (for c=1-40)
The result is direct from ranges of AOQL. The producer to fix the upper bound for AOQL can use this result. AOQL can be fixed at 65%MAPD or 70% MAPD etc.

Table:1 The ratio of AOQL/MAPD for the values of c

c	AOQL/p*	c	AOQL/p*	c	AOQL/p*	c	AOQL/p*
1	0.8400	11	0.657545	21	0.697952	31	0.725774
2	0.6855	12	0.662333	22	0.701227	32	0.728062
3	0.647333	13	0.666923	23	0.704348	33	0.730303
4	0.6360	14	0.671286	24	0.707333	34	0.732471
5	0.6336	15	0.6756	25	0.71024	35	0.7346
6	0.635333	16	0.679688	26	0.713077	36	0.736639
7	0.638857	17	0.683647	27	0.715778	37	0.738649
8	0.64325	18	0.687444	28	0.718393	38	0.740579
9	0.647889	19	0.691105	29	0.720931	39	0.742462
10	0.6528	20	0.6946	30	0.7234	40	0.744325

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Table:2 Some parametric values for specified MAPD

MAPD%	(n, c)	glb (AOQL)%	AQL%	MAPD%	(n, c)	glb (AOQL)%	AQL%
0.25	(2000, 5)	0.1584	0.1307	7.00	(72,5)	4.4352	3.6582
0.50	(1000, 5)	0.3168	0.2613	8.00	(63,5)	5.0688	4.1808
1.00	(500, 5)	0.6336	0.5226	9.00	(56,5)	5.7024	4.7034
2.00	(250, 5)	1.2672	1.0452	10.00	(50,5)	6.3360	5.2260
3.00	(167, 5)	1.9008	1.5678	12.50	(40,5)	7.9200	6.5325
4.00	(125, 5)	2.5344	2.0904	15.00	(34,5)	9.5040	7.8390
5.00	(100, 5)	3.1680	2.6130	17.00	(29,5)	11.0880	9.1455
6.00	(84, 5)	3.8016	3.1356	20.00	(25,5)	12.6720	10.4520

Table:3 Some characteristic values for specified glb of AOQL

glb (AOQL)	(n, c)	MAPD	AQL	glb (AOQL)	(n, c)	MAPD	AQL
0.01	(319,5)	0.0157	0.0082	0.07	(46,5)	0.1104	0.0576
0.02	(159,5)	0.0315	0.0165	0.08	(40,5)	0.1262	0.0659
0.03	(106,5)	0.0473	0.0247	0.09	(36,5)	0.1420	0.0742
0.04	(82,5)	0.0613	0.0320	0.10	(33,5)	0.1578	0.0824
0.05	(64,5)	0.0789	0.0412	0.11	(29,5)	0.1736	0.0907
0.06	(53,5)	0.0946	0.0494	0.12	(27,5)	0.1894	0.0990

Table:4 The Lower & Upper limit of AOQL for parametric sampling plans

c	Limit	n=10	n=20	n=30	n=40	n=50	n=60	n=70	n=80	n=90
1	U	0.06856	0.03428	0.022853	0.01714	0.013712	0.011426	0.0097943	0.00857	0.007618
	L	0.084	0.042	0.028	0.021	0.0168	0.014	0.012	0.0105	0.009333
2	U	0.12948	0.06474	0.04316	0.03237	0.025896	0.02158	0.0184971	0.016185	0.014387
	L	0.1371	0.06855	0.0457	0.034275	0.02742	0.02285	0.0195857	0.0171375	0.015233
3	U	0.19083	0.09541	0.06361	0.047707	0.038166	0.031805	0.0272614	0.0238537	0.021203
	L	0.19419	0.09709	0.06473	0.048547	0.038838	0.032365	0.0277414	0.0242737	0.021577
4	U	0.25348	0.12674	0.084493	0.06337	0.050696	0.042246	0.0362114	0.031685	0.028164
	L	0.2544	0.1272	0.0848	0.0636	0.05088	0.0424	0.0363429	0.0318	0.028267
5	U	0	0	0	0	0	0	0	0	0
	L	0.3168	0.1584	0.1056	0.0792	0.06336	0.0528	0.0452571	0.0396	0.0352
6	U		0.19011	0.12674	0.095055	0.076044	0.06337	0.0543171	0.0475275	0.042247
	L		0.19059	0.12706	0.095295	0.076236	0.06353	0.0544543	0.0476475	0.042353
7	U		0.22239	0.14826	0.111195	0.088956	0.07413	0.06354	0.0555975	0.04942
	L		0.22358	0.149053	0.11179	0.089432	0.074526	0.06388	0.055895	0.049684
8	U		0.25556	0.170373	0.12778	0.102224	0.085186	0.0730171	0.06389	0.056791
	L		0.25728	0.17152	0.12864	0.102912	0.08576	0.0735086	0.06432	0.057173
9	U		0.28948	0.19299	0.144742	0.115794	0.096495	0.08271	0.0723712 5	0.06433
	L		0.29151	0.19434	0.145755	0.116604	0.09717	0.0832886	0.0728775	0.06478
10	U			0.215966	0.161975	0.12958	0.107983	0.0925571	0.0809875	0.071989
	L			0.2176	0.1632	0.13056	0.1088	0.0932571	0.0816	0.072533

Table:5 Slopes of AOQL/MAPD curve on unit intervals

Interval	1-2	2- 3	3- 4	4- 5	5- 6	6-7	7- 8	8- 9	9- 10
slope	-0.155	-0.036	-0.013	-0.002	0.0009	0.002	0.007	0.004	0.002

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Table:6 Difference table to determine minimum of the function

c	R=AOQL/P*	D1	D2	D3	D4	D5	D6	D7	D8	D9
1	0.84137	-0.1555								
2	0.68579	-0.0361	0.11941							
3	0.64963	-0.0135	0.02259	-0.09682						
4	0.63607	-0.0020	0.01149	-0.0111	0.08571					
5	0.63400	0.00089	0.00296	-0.00854	0.00256	-0.08316				
6	0.63490	0.00250	0.00160	-0.00136	0.00718	0.00461	0.08777			
7	0.63740	0.00702	0.00451	0.00291	0.00426\	-0.00291	-0.00753	-0.09531		
8	0.64442	0.00450	-0.00252	-0.00704	-0.00995	-0.01422	-0.01131	-0.00378	0.09152	
9	0.64892	-0.64892	-0.65343	-0.65091	-0.64387	-0.63392	-0.61969	-0.60838	-0.6046	-0.69613

Appendix :1

Result:4

1) AOQL/MAPD is minimum at c=5

a) Graphical

An approximate best fit curve with standard error in estimation less than 0.00197174 and correlation coefficient .999895271 (s=.0019, r=.99989) with actual curve is determined in figure 6 The curve of AOQL/p* for the values, 1 to 40 is found to be minimum at c=5.The curve decreases from c=1 to c=5 and then it increases for all values from c=5 to c=40.Thus AOQL/MAPD attains at least a local minimum in the neighborhood of c=5.

b) Analytical

From Table:5 It is found that slope is negative for c=1-5 and then it is relatively increasing positively for c=5 –40 Thus the slope is zero at c=5, while it is negative in the interval 4-5 and it is positive in the interval 5-6. Hence there is a local minimum at c=5 and no other turning points are found for the practical values of c Thus the curve is getting a minimum at c=5.

c) Numerical differentiation

Taking values from 1to 9, the minima is found to be at c=5, since $D^1=dy/dx=0$ at c=5 and $D^2=d^2y/dx^2>0$ at this point.

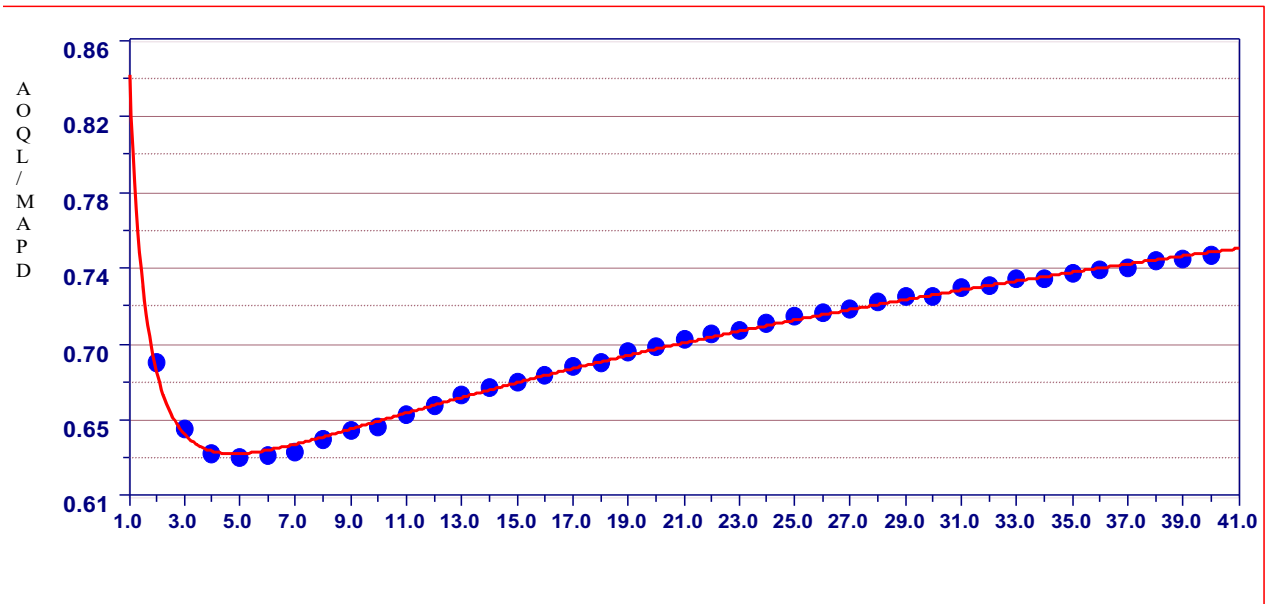
d) Empirical

An empirical function is suggested by the most suitable curve for the data from2-20.The fittest curve is suggested as

for **x**=AOQL/p* where **a=0.4403493**,**b=1.9927082**,**k=0.14087239**,are determined having SE (standard error) in the estimation, less than 0.00000239(approximated to 4 digits). The first

$$\mathbf{x} := \mathbf{a} \cdot \left[\mathbf{b}^{\left(\frac{1}{\mathbf{c}}\right)} \cdot \mathbf{c}^{\mathbf{k}} \right]$$

derivative is zero at c=5 and second derivative is positive at this point. Thus from the above methods it is found that AOQL/p* is minimum at c=5.



Acceptance number

Figure:1 Fittest curves representing the data

1.f(p)=(50,5) 2. g(p)=(40,4) 3. h(p)=(100,10) 4.j(p)=(200,2)

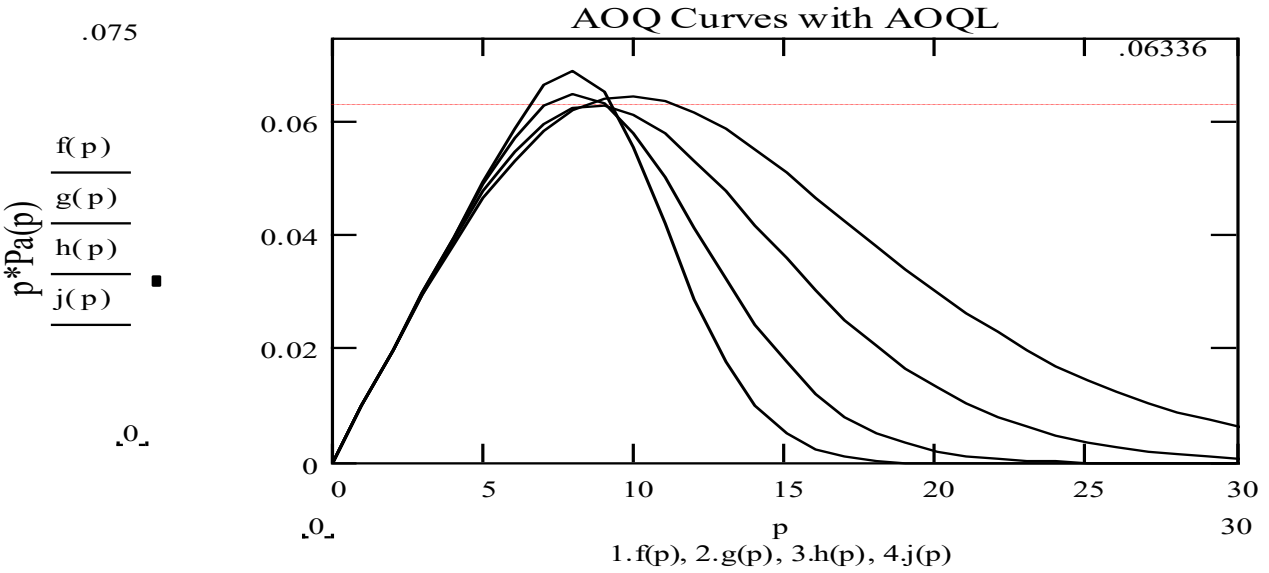


Figure:2 AOQ curve showing AOQL for fixed MAPD=10%

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AOQL =5.8%, MAPD =8.2% , 1.f(p) = (n=35, c=2), 2.g(p) = (n=397, c=23)

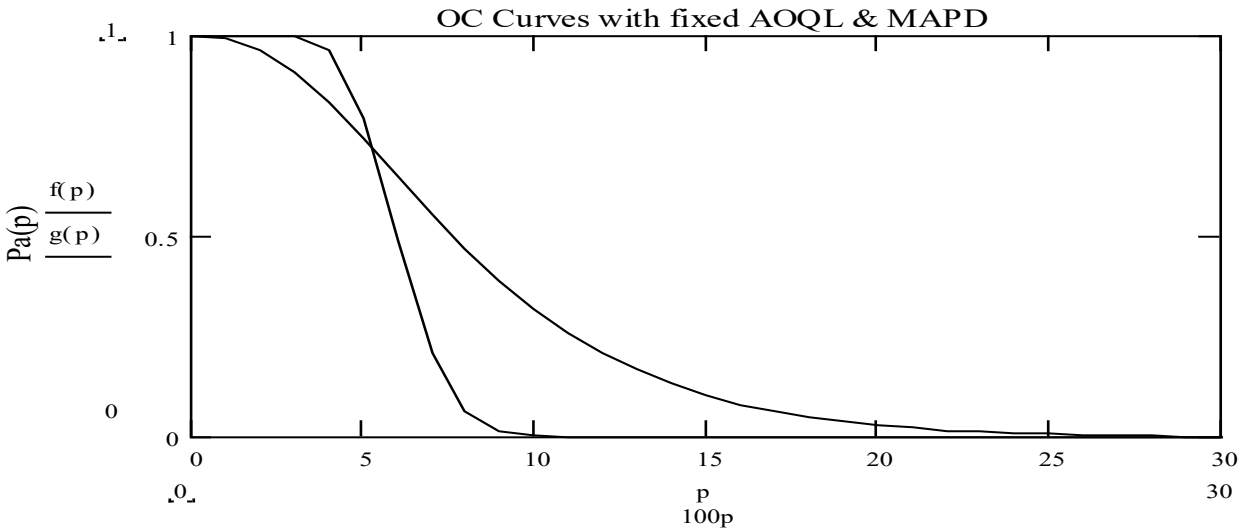


Figure:3 OC curve with same characteristics (AOQL, MAPD)

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