



Deterministic Path Planning Algorithms for Manipulator Chains and Mobile Manipulators (Path Planning of Manipulator Chains and Mobile Manipulators)

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Abstract: This paper proposes 2 schemes for calculating paths of manipulators that convey objects in a large space. The one is for a manipulator chain and to relay objects to distant goal positions finds attitudes of manipulator pairs that exchange objects. The other is for a mobile manipulator and calculates paths of the manipulator and the base vehicle simultaneously by regarding the vehicle with nonholonomic constraints as an end of the base arm. Both schemes exploit Backtrack free path planning algorithm (BFA), therefore they are deterministic, i.e. they determine existence of paths and calculate paths without fail if paths exist. Also, their computation volumes are predictable and increase only linearly with numbers of arms.

Keywords: Manipulators, Path planning, Deterministic algorithms, Linear computation volume, Nonholonomic constraints.

INTRODUCTION

A chain of manipulators distributed on a floor can convey objects to distant places, and when the floor has enough free space, a single mobile manipulator consists of a base vehicle and a manipulator fixed on the vehicle satisfies the same requirement. But to find their paths for accomplishing given tasks is difficult, and various available path planning algorithms are not practical enough.

Namely, attitudes of manipulator pairs by which they exchange objects must be determined in the former case, and in the latter case, paths of a manipulator and a base vehicle with nonholonomic constraints must be coordinated. But many available algorithms are non-deterministic ones that exploit heuristics extensively, and numbers of try and errors required for them increase enormously.

To cope with these difficulties, this paper proposes path planning schemes that apply Backtrack free path planning algorithm (*BFA*). A scheme for a manipulator chain determines attitudes of manipulator pairs for exchanging objects without backtracks, and a scheme for a mobile manipulator regards a vehicle as an end of a base arm to calculate paths of the arms and the vehicle simultaneously.

Because *BFA* is deterministic, proposed schemes determine existence of paths and calculate paths without fail if they exist. Besides, their computation volumes are predictable, i.e. they calculate paths of a manipulator chain and a mobile manipulator with computation volumes of $O(HN)$ and $O(N)$ respectively, where N and H are the number of arms in each manipulator and the number of manipulators in the chain.

RELATED WORKS

Flood algorithm-based schemes are deterministic and solve path planning problems completely, but they are practical only for manipulators with small numbers of arms, because their computation volumes increase exponentially with the number of arms.

Therefore, many currently available algorithms such as probabilistic roadmap methods (PRM, [3, 8] and potential field algorithms [1, 4] extensively exploit heuristics that include numbers of try and errors, *i.e.* they are non-deterministic [2, 11]. As a result, although they efficiently calculate paths in many cases even if manipulators have many arms, they are not practical enough in complicated environments, *e.g.* ones with scattered obstacles or narrow passages. Besides, it is difficult to predict computation volumes, especially they cannot determine the existence of paths. Modern heuristics-based algorithms such as evolutionary algorithms [7], swarm intelligence algorithms [6], and neural networks [9] also have the same drawbacks.

These drawbacks become more serious when they are applied to a manipulator chain or a mobile manipulator. Because paths of multiple manipulators or those of a manipulator and a vase vehicle must be coordinated, numbers of try and errors increase enormously.

BACKTRACK FREE PATH PLANNING ALGORITHM (BFA)

BFA is constructed based on Proposition 1, where manipulator M is a sequence of arms which are connected by joints, and contiguous numbers from 1 to N (N is the number of arms of M) are assigned to arms so that the base arm becomes the 1st or the N -th arm. Here, arms move in a 2 or 3-dimensional workspace, and positions in the space are approximated by points in a 2 or 3-dimensional grid. By defining following terms, Proposition 1 is stated as below.

Arm $M(n)$: The n -th arm of M is denoted by $M(n)$.

B-end and T-end: A base (B)-end of $M(n)$ is $M(n)$'s end which is connected to $M(n-1)$ and denoted by $M(n, B)$. A top (T)-end is another end, and denoted by $M(n, T)$.

E_n : E_n is the space where $M(n, T)$ moves ($M(1, B)$ moves in E_0).

Attitude of an arm: An attitude of $M(n)$ is ordered pair (P, Q) of points at which $M(n, B)$ and $M(n, T)$ are located.

Attitude of manipulator M : An attitude of M is tuple $(P_0, P_1, \dots, P_N)$, where P_n is a point in E_n which is occupied by $M(n, T)$ ($M(1, B)$ occupies $P_0 \in E_0$).

Feasible attitude set (FAS): Collision free attitude (P, Q) of $M(N)$ is called feasible. For $n < N$, collision free attitude (P, Q) of $M(n)$ is called feasible if at least one feasible attitude (Q, V) of $M(n+1)$ exists. A feasible attitude set (FAS) of $M(n)$ at P is a set of points that are occupied by the T-end of $M(n)$'s feasible attitudes of which B-end positions are P and denoted by $A(P, n)$.

Connecting point pair: Point pair (X, Y) included in $A(P, n)$ and $A(Q, n)$ respectively is called a connecting point pair of $A(P, n)$ and $A(Q, n)$, when P is adjacent to Q , and X is equal or adjacent to Y .

n-connectivity: Adjacent points P and Q in E_N are N -connective, when they are not occupied by obstacles. For $n < N$, adjacent points P and Q in E_n are n -connective, when FAS pair $\{A(P, n+1), A(Q, n+1)\}$ has at least one connecting point pair (X, Y) and $\{X, Y\}$ are $(n+1)$ -connective or $X = Y$.

n-reachable set $R(S; n)$: $R(S; n)$ is a set of points in E_n which are reachable from S by tracing n -connective points.

[Proposition 1]: Under assumptions that FAS $A(P, n)$ is a connected set in terms of n -connectivity for each P and n and collisions among arms themselves can be ignored, paths of manipulator M from initial attitude $\underline{S} = (S_0, S_1, \dots, S_N)$ to goal attitude $\underline{G} = (G_0, G_1, \dots, G_N)$ exist if and only if $R(S_0; 0)$ includes G_0 , and $G_n \in A(G_{n-1}, n)$ and $S_n \in A(S_{n-1}, n)$ hold for each n (≥ 1). Here, conditions $G_n \in A(G_{n-1}, n)$ ($m \leq n \leq N$) or $S_n \in A(S_{n-1}, n)$ ($k \leq n \leq N$) respectively are not necessary when $(G_m, \dots, G_N)$ in the goal attitude or $(S_k, \dots, S_N)$ in the initial attitude are not specified [10].

About assumptions, disconnected $A(P, n)$ can be made connected by generating copies of P in E_n and regarding different connected components of $A(P, n)$ as FASs at different copies. Also, collisions among arms can be removed by local adjustments of arm positions when the workspace is 3-dimensional, because finite volumes of free spaces exist around collision free points.

Based on the proposition, BFA behaves as in Figs. 1 and 2. Firstly, from $n = N$ to 0, connectivity determination (CD) part calculates FASs at points in E_n and determines the n -connectivity of adjacent points. After that from $n = 0$ to N , path generation (PG) part constructs L_n as a locus of $M(n, T)$ (L_0 is a locus of $M(1, B)$ to configure a path from $\underline{S}$ to $\underline{G}$). Here, Fig. 2 assumes that $S_n \in A(S_{n-1}, n)$ ($1 \leq n \leq N$) is satisfied, but when $\{G_m, \dots, G_N\}$ or $\{S_k, \dots, S_N\}$ are not specified, G_n ($m \leq n \leq N$) or S_n ($k \leq n \leq N$) is selected arbitrarily from points in $A(G_{n-1}, n)$ or $A(S_{n-1}, n)$.

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Make each adjacent grid point pair  $(P, Q)$  in  $E_N$   $N$ -connective if they do not collide with
obstacles;
for  $(n = N; n > 0; n--)$  {
  Calculate FAS  $A(P, n)$  for each point  $P$  in  $E_{n-1}$ ;
  if  $(A(P, n)$  is not a connected set) {
    Generate copies  $\{P_1, \dots, P_k\}$  of  $P$  ( $P_1 = P$ ) corresponding to connected components
     $\{A_1, \dots, A_k\}$  of  $A(P, n)$ , and assign  $A_k$  to  $P_k$  as FAS  $A(P_k, n)$ ;
  }
  Make each adjacent point pair  $\{P, Q\}$  in  $E_{n-1}$   $(n-1)$ -connective, if  $A(P, n)$  and  $A(Q, n)$ 
  have connecting point pair  $(X, Y)$ , and  $\{X, Y\}$  are  $n$ -connective or  $X = Y$ ;
}

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Figure 1: Connectivity determination (CD) part

In Fig. 2, the flood algorithm calculates 0-reachable set $R(S_0; 0)$ and distance $d(P, S_0)$ from S_0 to P for each P in E_0 . Then, locus L_0 of $M(1, B)$ is constructed by tracing 0-connective points from G_0 to S_0 by using the distance. L_n ($n \geq 1$) is constructed in the same way by tracing adjacent points in set Z from G_n to S_n , but Z is the union of FASs of $M(n)$ at points on L_{n-1} , and distance $d_Z(X, S_n)$ is calculated by tracing points in Z from S_n to X .

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Construct 0-reachable set  $R(S_0; 0)$  and calculate distance  $d(P, S_0)$  from  $S_0$  to  $P$  for each  $P$ 
in  $E_0$ ;
if  $(G_0 \notin R(S_0; 0) \cup G_{n+1} \notin A(G_n, n))$  for some  $n$  {Quit PG-part; } /* no path exists */
Construct sequence  $L_0 = \{G_0 = P_{(0, 1)}, \dots, P_{(0, k)}, \dots, S_0 = P_{(0, \kappa_0)}\}$  of 0-connective points in  $E_0$ 
as a locus of  $M(1, B)$  so that  $d(P_{(0, k)}, S_0)$  decreases with  $k$ ;
for  $(n = 1; n \leq N; n++)\{$ 
    Calculate  $Z = A(P_{(n-1, 0)}, n) \cup \dots \cup A(P_{(n-1, \kappa_{n-1})}, n)$  and distance  $d_Z(X, S_n)$  for each  $X$  in  $Z$ ;
    Construct sequence  $L_n = \{G_n = P_{(n, 0)}, \dots, P_{(n, k)}, \dots, S_n = P_{(n, \kappa_n)}\}$  of adjacent points in  $Z$  as a
    locus of  $M(n, T)$  so that  $d_Z(P_{(n, k)}, S_n)$  decreases with  $k$ ;
}
By using  $\{L_0, \dots, L_N\}$ , construct a sequence of attitudes that bring  $M$  from  $\underline{S}$  to  $\underline{G}$ .

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Figure 2: Path generation (PG) part

An important thing is that L_n is constructed without fail when L_{n-1} exists, because each $A(P_{(n-1, k)}, n)$ is a connected set and $A(P_{(n-1, k)}, n)$ and $A(P_{(n-1, k+1)}, n)$ on L_{n-1} have a connecting point pair. Therefore, BFA is deterministic, its computation time is predictable, and as a result, it is efficient even in complicated environments.

In fact, it calculates paths within few seconds even when non-deterministic algorithms require many hours, and its computation time changes only slightly with changes in obstacle locations [5]. Provided that N is the number of arms, and $\{D, U\}$ are maximum numbers of grid points in each E_n and FAS, computation volumes of the CD and PG-parts are $o(N \cdot D \cdot U)$ and $o(N \cdot D)$ respectively. Here, different from the CD-part, the PG-part is executed every time when initial or goal attitudes are changed, but $o(N \cdot D)$ is much less than $o(N \cdot D \cdot U)$.

SCHEMES FOR MANIPULATOR CHAINS AND MOBILE MANIPULATORS

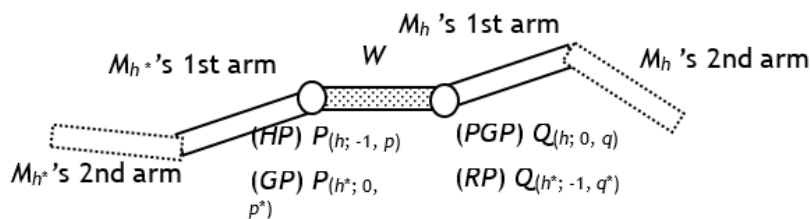
A Scheme for Manipulator Chains

This section proposes a path planning scheme for a chain of H -manipulators $\{M_h; 1 \leq h \leq H\}$. Here, each M_h has N_h -arms which are numbered so that the base arm becomes the N_h -th one, and T -end position of the base arm of each manipulator is fixed. Object W has a cylindrical shape, and its attitude is represented by its end positions pair $\{P, Q\}$ or $\{Q, P\}$. But when M_h grasps W at Q , M_h and W configure extended manipulator $\underline{M}_h$, and W is regarded as $\underline{M}_h$'s 0-th arm of which attitude is ordered pair (P, Q) .

Notations used in this section are in Table 1, and Fig.3. shows additional terms defined regarding manipulators that exchange object W . Namely, when M_{h^*} receives W from $\underline{M}_h$ as in Fig. 3, M_h is called a predecessor of M_{h^*} , and $Q_{(h^*; -1, q^*)}$, $P_{(h^*; 0, p^*)}$, $P_{(h; -1, p)}$ and $Q_{(h; 0, q)}$ respectively in attitudes $(Q_{(h^*; -1, q^*)}$, $P_{(h^*; 0, p^*)})$ and $(P_{(h; -1, p)}$, $Q_{(h; 0, q)})$ of $\underline{M}_{h^*}(0)$ and $\underline{M}_h(0)$ are called a receiving position (RP), grasping position (GP), handing position (HP) and predecessors' grasping position (PGP). About $E_n(h)$ is Table1, the same notation is used for both M_h and $\underline{M}_h$, because $\{P, Q\}$ are n -connective for $\underline{M}_h$ if and only if they are n -connective for M_h , i.e. n -connectivity of points in $E_n(h)$ ($n \leq 1$) is determined independently from points in $E_{n-1}(h)$.

Table 1: Notations.

$M_h(n)$ and $\underline{M}_h(n)$	n -th arm of M_h and $\underline{M}_h$.
$M_h(n, B)$ and $M_h(n, T)$	B -end and T -end of $M_h(n)$.
$\underline{M}_h(n, B)$ and $\underline{M}_h(n, T)$	B -end and T -end of $\underline{M}_h(n)$.
$E_n(h)$	workspace where $M_h(n, T)$ or $\underline{M}_h(n, T)$ moves ($\underline{M}_h(0, B)$ moves in $E_{-1}(h)$).
$P_{(h; n, k)}$	k -th copy of a point in $E_n(h)$ at position P .
$A_h(P_{(h; n-1, k)}, n)$	FAS of $M_h(n)$ or $\underline{M}_h(n)$ at $P_{(h; n-1, k)}$.
$(P_1, \dots, P_{m-1}, *)$	attitude of a manipulator which is locating $\{1\text{st}, \dots, m\text{-th}\}$ arms' B -ends at $\{P_1, \dots, P_{m-1}\}$.
$(P_0, \dots, P_{m-1}, *)$	attitude of an extended manipulator which is locating $\{0\text{-th}, \dots, m\text{-th}\}$ arms' B -ends at $\{P_0, \dots, P_{m-1}\}$.
$R_h(P_{(h; 0, k)})$	0 -reachable set of M_h from $P_{(h; 0, k)}$.
$\underline{R}_h(Q_{(h; -1, j)})$	(-1) -reachable set of $\underline{M}_h$ from $Q_{(h; -1, j)}$, which consists of points reachable from $Q_{(h; -1, j)}$ by tracing (-1) -connective points.
$I_n(h) \in E_n(h)$	initial position of $M_h(n, T)$ ($I_{-1}(h)$ and $I_0(h)$ are initial positions of $M_h(0, B)$ and $M_h(1, B)$).
$\{S^*, S\}$ and $\{G^*, G\}$	initial and goal attitudes of object W .

**Figure 3: Manipulators exchanging object W**

The scheme consists of *reachable set calculation (RC)* and *path generation (PG)* parts shown in Fig. 4 and 5, where to simplify discussions, it is assumed that each M_h grasps W in attitudes $\{S^*, S\}$ and $\{G^*, G\}$ at S and G , and attitudes of arms to grasp W except the 0 -th arm are not specified.

Reachable Set Calculation Part

RC -part calculates joint reachable set $J(h)$ for each h , where $J(h)$ consists of points to which $\underline{M}_h$ can bring $\underline{M}_h(0, B)$ by receiving W from other manipulators if possible.

Firstly, it determines n -connectivity of adjacent points in $E_n(h)$ for each h and n (≥ 1) and calculates 0 -reachable set $R_h(I_0(h))$. Then, it initializes $J(h)$, *i.e.* if $\underline{M}_h$ can bring $\underline{M}_h(0)$ to attitude $(S^*(h; -1, s^*(h)), S(h; 0, s(h)))$ (in other words, if M_h can grasp W in initial attitude $\{S^*, S\}$), it adds $S^*(h; -1, s^*(h))$ to $J(h)$ and attaches $S(h; 0, s(h))$ to $S^*(h; -1, s^*(h))$ as a GP .

After that, each $J(h)$ in set $\Theta = \{J(1), \dots, J(H)\}$ is expanded from each $P_{(h; -1, p)} \in J(h)$. In detail, $\underline{R}_h(P_{(h; -1, p)})$ is calculated as an additional connected component of $J(h)$, if $\underline{R}_h(P_{(h; -1, p)}) \subsetneq J(h)$ (if $d^*(P_{(h; -1, p)})$ is not defined yet). At the same time, sub-distance $d^*(Q_{(h; -1, q)})$ from $P_{(h; -1, p)}$ to $Q_{(h; -1, q)}$ is calculated for each $Q_{(h; -1, q)} \in \underline{R}_h(P_{(h; -1, p)})$. Here, $d^*(Q_{(h; -1, q)})$ is calculated by tracing (-1) -reachable points in $\underline{R}_h(P_{(h; -1, p)})$ from $P_{(h; -1, p)}$, therefore $d^*(Q_{(h; -1, q)}) = d(Q_{(h; -1, q)}, P_{(h; -1, p)})$ if $Q_{(h; -1, q)} \in \underline{R}_h(P_{(h; -1, p)})$. Also, $P_{(h; -1, p)}$ is regarded as a start point (SP)

of $J(h)$, and as in Proposition 2, each connected component of $J(h)$ has a single SP and $d^*(X) = 0$ holds only when X is an SP .

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for ( $h = 1; h \leq H; h++$ ) {
  Determine  $n$ -connectivity of adjacent points in  $E_n(h)$  for each  $n$ ;
  Construct  $R_h(I_0(h))$  and calculate distance  $d(P, I_0(h))$  for each  $P$  in  $E_0(h)$ ;
  Initialize joint reachable set  $J(h)$  by adding  $S_{(h; -1, s^*(h))}$  accompanied by  $S_{(h; 0, s(h))}$  as a  $GP$  to
   $J(h)$  if  $S_{(h; 0, s(h))}$  exists and  $S_{(h; 0, s(h))} \in \{R_h(I_0(h)) \cap A_h(S_{(h; -1, s^*(h)), 0)\}$  holds;
}
 $\Theta = \{J(1), \dots, J(H)\}$ ;
while ( $\Theta \neq \emptyset$ ) {
   $J(h) =$  1st element of  $\Theta$ ;
  while ( $(P_{(h; -1, p)} \in J(h)$  exists but sub-distance  $d^*(P_{(h; -1, p)})$  is not defined) {
    Calculate  $\underline{R}_h(P_{(h; -1, p)})$  and distance  $d(Q_{(h; -1, q)}, P_{(h; -1, p)})$  as sub-distance  $d^*(Q_{(h; -1, q)})$  for each
     $Q_{(h; -1, q)}$  in  $\underline{R}_h(P_{(h; -1, p)})$ ;
     $J(h) = J(h) \cup \underline{R}_h(P_{(h; -1, p)})$ ;          /*  $P_{(h; -1, p)}$  is regarded as an  $SP$  of  $J(h)$  */
  }
  if ( $G_{(h^*, -1, a^*)} \in J(h)$  and  $G_{(h, 0, b)} \in A_h(G_{(h^*, -1, a^*)}, 0)$  exist) { Go to  $PG$ -part; }
  while (unexamined  $P_{(h; -1, p)}$  exists in  $J(h)$ ) {
    Select  $Q_{(h; 0, q)} \in A_h(P_{(h; -1, p)}, 0)$  arbitrarily;
    Do the block below for each  $h^* (\neq h)$  {
       $\Psi =$  set of copies at position  $P_{(h; -1, p)}$  in  $R_{h^*}(I_0(h^*))$ ;
      while (unexamined  $P_{(h^*, 0, p^*)}$  exists in  $\Psi$ ) {
        If ( $Q_{(h^*, -1, q^*)} \in E_{-1}(h^*)$  at position  $Q_{(h; 0, q)}$  satisfies  $Q_{(h^*, -1, q^*)} \notin J(h^*) \cap P_{(h^*, 0, p^*)} \in A_{h^*}(Q_{(h^*, -1, q^*)}, 0)$ ) {
          Add  $Q_{(h^*, -1, q^*)}$  to  $J(h^*)$  as a candidate for an  $SP$  and attach  $\{P_{(h; -1, p)}, P_{(h^*, 0, p^*)}, Q_{(h; 0, q)}, M_h\}$ 
          to  $Q_{(h^*, -1, q^*)}$  as an  $HP, GP, PGP$  and a predecessor accompanying the  $SP$ ;
        }
      }
    }
    if (candidates for  $SP$ s were added to  $J(h^*)$  but  $J(h^*) \notin \Theta$ ) {Put  $J(h^*)$  in  $\Theta$  as its last element; }
  }
}

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Figure 4: Reachable set calculation (RC) part

Then, if $\{a^*, b\}$ exist and satisfy $G_{(h^*, -1, a^*)} \in J(h)$ and $G_{(h, 0, b)} \in A_h(G_{(h^*, -1, a^*)}, 0)$, the PG -part starts, i.e. Proposition 1 ensures that $\underline{M}_h(0)$ can reach goal attitude $(G_{(h^*, -1, a^*)}, G_{(h, 0, b)})$. But otherwise, $P_{(h; -1, p)}$ is regarded as an HP , if $P_{(h^*, 0, p^*)} \in R_{h^*}(I_0(h^*))$ exists at position $P_{(h; -1, p)}$ ($h^* \neq h$). Namely as in Proposition 3, $Q_{(h^*, -1, q^*)}$ exists corresponding to arbitrarily selected $Q_{(h; 0, q)} \in A_h(P_{(h; -1, p)}, 0)$, and M_h and M_{h^*} in attitudes $(P_{(h; -1, p)}, Q_{(h; 0, q)}, *)$ and $(Q_{(h^*, -1, q^*)}, P_{(h^*, 0, p^*)}, *)$ can exchange W . Therefore, to include points to which $\underline{M}_{h^*}(0, B)$ can reach by receiving W from $\underline{M}_h$, $Q_{(h^*, -1, q^*)}$ is added to $J(h^*)$ as a candidate for an SP . Also, to memorize attitudes to exchange W , $\{P_{(h; -1, p)}, P_{(h^*, 0, p^*)}, Q_{(h; 0, q)}, M_h\}$ are attached to $Q_{(h^*, -1, q^*)}$ as an HP, GP, PGP and a predecessor.

After that, $J(h^*)$ is put in Θ again if candidates for SP s were added to $J(h^*)$ despite it is not in Θ , because joint reachable sets are removed from set Θ after they are expanded. Here, although $X_{(h^*, -1, x^*)} (\neq Q_{(h^*, -1, q^*)})$ may exist corresponding to $X_{(h; 0, x)} \in A_h(P_{(h; -1, p)}, 0)$, it is not necessary to add $X_{(h^*, -1, x^*)}$ to $J(h^*)$ as in Proposition 3.

[Proposition 2]: Each connected component C of $J(h)$ includes point P as an SP , at which sub-distance $d^*(P) = 0$ holds. But C does not have multiple SP s, and $d^*(Q) > 0$ for any $Q \in C$ except P .

(Proof) $J(h)$ is the union of connected components $R_h(P_k)$ ($k = 1, \dots, K$), each $R_h(P_k)$ is expanded from P_k and $R_h(P_k)$ is not calculated if $J(h)$ includes points in it already. Therefore, connected component C of $J(h)$ has a single SP and $d^*(Q) > 0$ for any $Q \in C$ if it is not an SP . Q.E.D.

[Proposition 3]: M_{h^*} can receive W held by $\underline{M}_h$ in attitude $(P_{(h; -1, p)}, *)$ if and only if $P_{(h^*; 0, p^*)} \in R_{h^*}(I_0(h^*))$ exists at position $P_{(h; -1, p)}$, and if $P_{(h^*; 0, p^*)} \in R_{h^*}(I_0(h^*))$ exists, $Q_{(h^*; -1, q^*)}$ exists for arbitrary $Q_{(h; 0, q)}$ in $A_h(P_{(h; -1, p)}, 0)$ and $\underline{M}_{h^*}$ in attitudes $(Q_{(h^*; -1, q^*)}, P_{(h^*; 0, p^*)}, *)$ can receive W from $\underline{M}_h$. Here, $X_{(h^*; -1, x^*)}$ exists also for $X_{(h; 0, x)} (\neq Q_{(h; 0, q)})$ in $A_h(P_{(h; -1, p)}, 0)$, but it is not necessary to add $X_{(h^*; -1, x^*)}$ to $J(h^*)$ if $Q_{(h^*; -1, q^*)}$ is in it already.

(Proof) To receive W from $\underline{M}_h$ in attitude $(P_{(h; -1, p)}, *)$, $M_{h^*}(1, B)$ must be at position $P_{(h; -1, p)}$, i.e. $R_{h^*}(I_0(h^*))$ includes $P_{(h^*; 0, p^*)}$. On the other hand, if $P_{(h^*; 0, p^*)} \in R_{h^*}(I_0(h^*))$, because attitude $(P_{(h; -1, p)}, Q_{(h; 0, q)})$ of $\underline{M}_h(0)$ is collision free for $Q_{(h; 0, q)} \in A_h(P_{(h; -1, p)}, 0)$, $Q_{(h^*; -1, q^*)}$ exists and $\underline{M}_{h^*}(0)$'s attitude $(Q_{(h^*; -1, q^*)}, P_{(h^*; 0, p^*)})$ is also feasible, i.e. $P_{(h^*; 0, p^*)} \in A_{h^*}(Q_{(h^*; -1, q^*)}, 0)$ and M_{h^*} can receive W . About $X_{(h; 0, x)} (\neq Q_{(h; 0, q)})$ in $A_h(P_{(h; -1, p)}, 0)$, sequence $\{Y_{j(h; 0, y(j))}; 1 \leq j \leq J\}$ of 0-connective points from $Y_{1(h; 0, y(1))} = X_{(h; 0, x)}$ to $Y_{J(h; 0, y(J))} = Q_{(h; 0, q)}$ exists in $A_h(P_{(h; -1, p)}, 0)$, and for $Y_{j(h^*; -1, y^*(j))}$ corresponding to each $Y_{j(h; 0, y(j))}$, $\underline{M}_{h^*}(0)$'s attitude $(Y_{j(h^*; -1, y^*(j))}, P_{(h^*; 0, p^*)})$ is feasible. Namely, $P_{(h^*; 0, p^*)} \in A_{h^*}(Y_{j(h^*; -1, y^*(j))}, 0)$, and $A_{h^*}(Y_{j(h^*; -1, y^*(j))}, 0)$ and $A_{h^*}(Y_{j+1(h^*; -1, y^*(j+1))}, 0)$ have connecting point pair $\{P_{(h^*; 0, p^*)}, P_{(h^*; 0, p^*)}\}$. Then, $X_{(h^*; -1, x^*)}$ and $Q_{(h^*; -1, q^*)}$ are (-1)-reachable, and $X_{(h^*; -1, x^*)}$ is added to $J(h^*)$ at a time when $J(h^*)$ is expanded from $Q_{(h^*; -1, q^*)}$. Q.E.D.

Path Generation Part

PG-part calculates loci of arms of manipulators to bring object W to attitude (G^*, G) , where Fig. 5 assumes that $J(g)$ includes $G^*(g; -1, a^*)$ at position G^* when the RC-part ended (i.e. $\underline{M}_g$ brings W to the goal).

$L_{(-1, h^*)}$ and $L_{(0, h^*)}$ in the figure are loci of $M_{h^*}(0, B)$ and $M_{h^*}(0, T)$, and for initial value g of h^* , $G^*(g; -1, a^*)$ and $G_{(g; 0, b)} \in A_{h^*}(G^*(g; -1, a^*), 0)$ are put in X_1 and Y_1 as goals of $L_{(-1, g)}$ and $L_{(0, g)}$. Then, $L_{(-1, g)}$ is extended to $Q_{(g; -1, q(g))}$ at which $d^*(Q_{(g; -1, q(g))}) = 0$ holds by tracing (-1)-connective points in $J(g)$ so that sub-distance $d^*(\cdot)$ decreases, where $Q_{(g; -1, q(g))}$ exists as an SP of $J(h)$ as in Proposition 2.

Besides, $\{P_{(g; 0, p(g))}, P_{(h; -1, p(h))}, Q_{(h; 0, q(h))}\}$ accompany $Q_{(g; -1, q(g))}$ as a GP , HP and PGP , or $S_{(g; 0, s(g))}$ accompanies $Q_{(g; -1, q(g))}$ ($= S^*(g; -1, s^*(g))$) as a GP as in Proposition 4. Therefore, $\underline{M}_g$ and predecessor $\underline{M}_h$ in attitudes $(Q_{(g; -1, q(g))}, P_{(g; 0, p(g))}, *)$ and $(P_{(h; -1, p(h))}, Q_{(h; 0, q(h))}, *)$ can exchange W . Or $(S^*(g; -1, s^*(g)), S_{(g; 0, s(g))}, *)$ is an attitude of $\underline{M}_g$ that is grasping W in attitude $\{S^*, S\}$.

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 $h^* = g;$ 
 $X_1 = G^*(g; -1, a^*); Y_1 = G(g; 0, b); \text{ /* } G(g; 0, b) \in A_g(G^*(g; -1, a^*), 0) \text{ */}$ 
while (true) {
  Construct  $L_{(-1, h^*)} = \{X_1, \dots, X_k, \dots, X_{K(h^*)} = Q_{(h^*; -1, q(h^*))}\}$  as a locus of  $M_{h^*}(0, B)$  by tracing (-1)-connective points in  $J(h^*)$  from  $X_1$  so that  $d^*(X_k)$  decreases with  $k$ ; /*  $Q_{(h^*; -1, q(h^*))}$  is an SP in  $J(h^*)$  */
   $\{P_{(h^*; 0, p(h^*))}, P_{(h; -1, p(h))}, Q_{(h; 0, q(h))}, M_h\} = \{GP, HP, PGP, \text{predecessor}\}$  accompanying  $Q_{(h^*; -1, q(h^*))}$ ;
   $Z = \text{union of FASs of } M_{h^*}(0) \text{ at points on } L_{(-1, h^*)};$ 
  Calculate  $d_Z(Y, P_{(h^*; 0, p(h^*))})$  as the distance in  $Z$  from  $P_{(h^*; 0, p(h^*))}$  to  $Y$  for each  $Y$  in  $Z$ ;
  Construct  $L_{(0, h^*)} = \{Y_1, \dots, Y_r, \dots, Y_{R(h^*)} = P_{(h^*; 0, p(h^*))}\}$  as a locus of  $M_{h^*}(0, T)$  by tracing 0-connective points in  $Z$  from  $Y_1$  so that  $d_Z(Y_r, P_{(h^*; 0, p(h^*))})$  decreases with  $r$ ;
  Construct sequence  $\{Y_{R(h^*)}, Y_{R(h^*)+1}, \dots, Y_u, \dots, I_0(h^*)\}$  of 0-connective points in  $E_{h^*}(0)$  so that  $d(Y_u, I_0(h^*))$  decreases with  $u$ ;
   $L_{(0, h^*)} = \{Y_1, \dots, Y_{R(h^*)}, Y_{R(h^*)+1}, \dots, I_0(h^*)\};$ 
  if  $(Q_{(h^*; -1, q(h^*))})$  is a point at position  $S^*$  {quit this loop; }
   $X_1 = P_{(h; -1, p(h))}; Y_1 = Q_{(h; 0, q(h))};$ 
   $h^* = h;$ 
}
Calculate locus  $L_{(n, h)}$  of  $M_h(n, T)$  for each  $h$  and  $n$  ( $1 \leq n \leq N$ ) based on  $L_{(n-1, h)}$ ; /* start point of  $L_{(n, h)}$  is  $I_n(h)$  and its end point is an arbitrary point in  $A_h(D, n-1)$ , where  $D$  is the 1st element on  $L_{(n-1, h)}$  */
Construct paths of manipulators based on arrays  $\{L_{(-1, 1)}, L_{(N, 1)}\}, \dots, \{L_{(-1, H)}, L_{(N, H)}\}$ ;

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Figure 5: Path generation (PG) part

About locus $L_{(0, g)}$ of $M_g(0, T)$ or $M_g(1, B)$, the part from $P_{(g; 0, p(g))}$ to $G(g; 0, b)$ is constructed based on $L_{(-1, g)}$, i.e. it is extended from $Y_1 = G(g; 0, b)$ by tracing 0-connective points in set Z so that distance $d_Z(\cdot, P_{(g; 0, p(g))})$ decreases, where Z consists of points in FASs of $M_g(0)$ at points on $L_{(-1, g)}$ and $d_Z(Y, P_{(g; 0, p(g))})$ is the distance in Z from $P_{(g; 0, p(g))}$ to Y . The part from $P_{(g; 0, p(g))}$ to $I_0(g)$ is constructed by simply tracing 0-connective points in $E_0(g)$ so that distance $d(\cdot, I_0(g))$ decreases.

Then, loci of predecessor M_h is constructed, i.e. because h^* is replaced with h , and $P_{(h; -1, p(h))}$ and $Q_{(h; 0, q(h))}$ are put in X_1 and Y_1 as the HP and PGP, $L_{(-1, h^*)}$ and $L_{(0, h^*)}$ configure loci of $M_h(0, B)$ and $M_h(0, T)$. Loci of other manipulators are constructed by iterating the same process until $L_{(-1, h^*)}$ includes a point at position S^* .

Locus $L_{(n, h)}$ ($1 \leq n \leq N$) of $M_h(n, T)$ is calculated based on $L_{(n-1, h)}$ for each h and n as same as in Fig.2, where $L_{(n, h)}$ starts from $I_n(h)$ and terminates at an arbitrary point in the FAS of $M_h(n)$ at the 1st point on $L_{(n-1, h)}$. Finally, paths of manipulators are constructed by using arrays $\{L_{(-1, 1)}, L_{(N, 1)}\}, \dots, \{L_{(-1, H)}, L_{(N, H)}\}$, and Proposition 4 holds.

[Proposition 4]: When collisions among arms of manipulators that exchange object W are ignored, Fig. 4 determines the existence of paths of manipulator chain $\{M_1, \dots, M_H\}$ that conveys W from attitude $\{S^*, S\}$ to attitude $\{G^*, G\}$, and Fig. 5 calculates paths if they exist.

(Proof): If the chain can bring W from attitude $\{S^*, S\}$ to $\{G^*, G\}$, apparently Fig. 5 is activated, i.e. $M_g, G^*(g; -1, a^*)$ and $G(g; 0, b)$ exist and $G^*(g; -1, a^*) \in J(g)$ and $G(g; 0, b) \in A_g(G^*(g; -1, a^*), 0)$ hold. Also, M_k which corresponds to M_g can grasp W in attitude $\{S^*, S\}$, i.e. $S^*(k; -1, s^*(k)) \in J(k)$ and $S_{(k; 0, s(k))} \in A_k(S^*(k; -1, s^*(k)), 0)$ exist and $M_k(1, B)$ can move from $I_0(k)$ to $S_{(k; 0, s(k))}$. On

the other hand, the path of the chain is calculated without fail once Fig. 5 starts as below, and this means that Fig. 4 determines the existence of paths. About Fig. 5, sequence $L_{(-1, g)}$ of (-1) -connective points in $J(g)$ from $G^*(g; -1, a^*)$ reaches $Q_{(g; -1, q(g))}$ at which $d^*(Q_{(g; -1, q(g))}) = 0$ holds, because each connected component of $J(g)$ has a single SP as in Proposition 2. Besides, because an SP is an RP or a point at position S^* , $\{P_{(g; 0, p(g))}, P_{(h; -1, p(h))}, Q_{(h; 0, q(h))}\}$ accompany $Q_{(g; -1, q(g))}$ as a GP , HP and PGP , or $Q_{(g; -1, q(g))}$ coincides with $S^*(g; -1, s^*(g))$ and $S_{(g; 0, s(g))}$ at position S accompanies $S^*(g; -1, s^*(g))$ as a GP . Therefore, $\underline{M}_g$ and predecessor $\underline{M}_h$ in attitudes $(Q_{(g; -1, q(g))}, P_{(g; 0, p(g))}, *)$ and $(P_{(h; -1, p(h))}, Q_{(h; 0, q(h))}, *)$ can exchange W , or $(S^*(g; -1, s^*(g)), S_{(g; 0, s(g))}, *)$ is $\underline{M}_g$'s attitude that grasps W in attitude $\{S^*, S\}$. Because $P_{(g; 0, p(g))}$ or $S_{(g; 0, s(g))}$ respectively satisfies $P_{(g; 0, p(g))} \in R_g(I_0(g))$ or $S_{(g; 0, s(g))} \in R_g(I_0(g))$ as a GP , sequence $L_{(0, g)}$ of 0 -connective points in $E_0(g)$ from $I_0(g)$ to $P_{(g; 0, p(g))}$ or $S_{(g; 0, s(g))}$ is also constructed. Then, Proposition 1 ensures that a path of $\underline{M}_g$ from initial attitude to attitude $(P_{(g; 0, p(g))}, *)$ or $(S_{(g; 0, s(g))}, *)$, and a path of $\underline{M}_g$ from $(Q_{(g; -1, q(g))}, P_{(g; 0, p(g))}, *)$ or $(S^*(g; -1, s^*(g)), S_{(g; 0, s(g))}, *)$ to $(G^*(g; -1, a^*), G_{(g; 0, b)}, *)$ are constructed. Paths of other manipulators are constructed by iterating the same process, *i.e.* connected component of $J(h)$ that includes $P_{(h; -1, p(h))}$ as an HP also includes an SP , and because $J(g)$ is expanded from $J(k)$ which includes $S^*(k; -1, s^*(k))$ as an SP , finally sequence $L_{(-1, k)}$ in $J(k)$ reaches $S^*(k; -1, s^*(k))$. Q.E.D.

About the efficiency, the RC and PG -part require computation volumes of $o(H \cdot N \cdot D \cdot U)$ and $o(H \cdot N \cdot D)$, where D and U are maximum numbers of points in a workspace and a FAS , and although the RC -part may calculate (-1) -reachable sets for multiple SP s to construct each joint reachable set $J(h)$, the number of points in $J(h)$ does not exceed V .

The possibility of collisions among arms of $\underline{M}_h^*$ and predecessor $\underline{M}_h$ that exchange W can be reduced substantially if SP s are not added to $J(h^*)$ when they are near to $\underline{M}_h$'s base arm position. In fact, arms of 2 manipulators $\underline{M}_1$ and $\underline{M}_2$ that jointly grasp objects do not collide when distances between points in each $E_n(h)$ ($h = 1$ or 2) are defined so that they become large when the points are in the vicinity of other manipulator's base arm position (this means arms are prone to being away from other manipulator) [5].

A Scheme for Mobile Manipulators

Mobile manipulator M consists of base vehicle V and manipulator M , where N -arms of M are numbered so that the base arm, which is fixed on V , becomes the 1st one. The workspace of V is a 2-dimensional floor and positions in it are approximated by points in a 2-dimensional grid.

Then, if V is holonomic, it is trivial to apply BFA to $\underline{M}$ by considering V as the 0-th arm. Also, although collision checks between V and obstacles become cumbersome, volumes of computations are suppressed because V moves in the 2-dimensional floor. But base vehicles are usually not holonomic as shown in Figs. 6, where V which is heading to point X_1 at point X_0 must visit $X_1, X_2, \dots, X_7, X_8$ to move to X^* , because V cannot change its direction instantly.

In detail, when directions to which V is heading are approximated by 16-directions from E to ESE as shown in Fig. 7, V can move from P to A only if its direction is $\{ENE, E, ESE\}$. In the same way, it can move from P to B and P to C only if its direction is $\{ENE, NE, NNE\}$ and $\{NNE, N, NNW\}$, respectively.

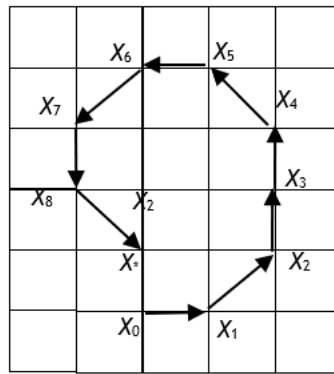


Figure 6: Nonholonomic constraints

In addition, while moving to adjacent points, V can change directions only to adjacent ones, e.g. while moving from P to A , direction E can be changed to $\{ENE, E, ESE\}$. Also, direction ENE at P cannot be maintained, i.e. V which is moving to A can change ENE only to E , and when V moves to B , ENE can be changed only to NE .

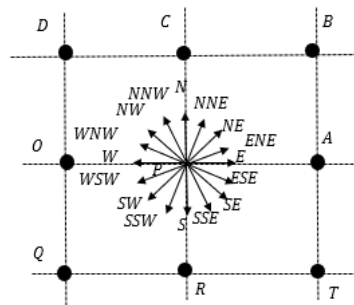


Figure 7: Directions of base vehicle V

The proposed scheme regards base vehicle V as the B -end of its 1st arm, and to distinguish different attitudes of V , it generates copies at each point corresponding to directions of V . For example, if directions of V are approximated as in Fig. 7, 16-copies (e.g. $P_{(0, E)}$, $P_{(0, SW)}$) are generated for each position P in E_0 .

Then, the scheme incorporates nonholonomic constraints by defining adjacencies between these copies as in Fig. 8. Namely, $P_{(0, E)}$ is adjacent only to $\{A_{(0, ENE)}, A_{(0, E)}, A_{(0, ESW)}\}$, but $\{A_{(0, ENE)}, A_{(0, E)}, A_{(0, ESW)}\}$ are not adjacent to $P_{(0, E)}$. Proposition 5 ensures that BFA determines the existence of paths and finds paths if they exist.

[Proposition 5]: Even if V is nonholonomic, BFA determines existence of paths of mobile manipulator $\underline{M}$, and finds paths if they exist, by regarding V as the B -end of the 1st arm and generating copies for each point in E_0 corresponding to different directions of V .

(Proof) Apparently, conditions in Proposition 1 are satisfied if paths of $\underline{M}$ exist. On the other hand, if a 0-reachable set from the initial position includes the goal point, because locus L_0 of V is determined independent of other arms, L_0 is constructed by tracing 0-connective points in E_0 even if the 0-connectivity is asymmetric. Then, L_n ($n > 0$) is constructed based on L_{n-1} without fail because n -connectivity is symmetric. Q.E.D.

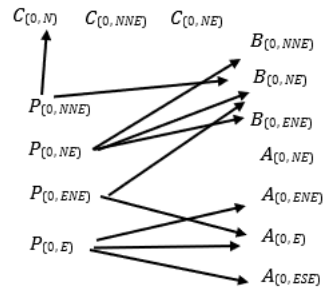


Figure 8: Copies and adjacency between them

Because $\underline{M}$ has N -arms, computation volume of the above scheme is $o(N \cdot D \cdot U)$, where D and U are maximum numbers of grid points in a workspace and each FAS respectively. Although many copies are generated for points in E_0 , increase of the computation volume can be suppressed because V moves on the 2-dimensional floor.

CONCLUSION

As one of future works, performances of the proposed schemes must be evaluated by implementing them. About manipulator chains, the proposed scheme calculates paths by connecting paths of single manipulators. But as a different approach, paths can be calculated by decoupling paths of cooperating 2-manipulators. Namely, manipulators M_h and M_{h^*} that jointly convey object W can be considered as a single manipulator $\underline{M}_{(h, h^*)}$ consists of W and arms of M_h and M_{h^*} [5], and attitudes of M_h and M_{h^*} that exchange W can be extracted from attitudes of $\underline{M}_{(h, h^*)}$ on its path.

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