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SERVICES FOR SCIENCE AND EDUCATION

Approbation of the Algorithm of Construction of Criteria of Similarity of a Physical Process to A Technical Problem

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ABSTRACT

The developed general algorithm for constructing a set of similarity criteria of a physical process is applied to the technical problem of measuring the thickness of a layer of material transported along a linear conveyor line. Following the algorithm and using chain diagrams, the corresponding set of systems of linear algebraic equations with respect to homogeneity indices is found. Excluding degenerate and equivalent systems of equations, the set of basic similarity criteria consisting of two equal half-sets of direct and inverse criteria is found. Derived similarity criteria are determined by the basic criteria using the Cayley tables. The obtained set of similarity criteria includes known results.

Keywords: Modelling, Similarity criteria, Degree complex, Chain diagrams, Caley tables.

INTRODUCTION

The similarity criteria establish the conditions of equivalence of a natural and modelled physical process [1,2]. It is believed that modelling of physical processes in solid and deformable body mechanics, continuum mechanics, relativistic mechanics, etc. [3,4] is allowed in the class of homogeneous functions [5]. Homogeneous functions are represented by stacked complexes of initial independent arguments [6]. The dimensionality of the original independent arguments and the function is determined by the choice of the system of units. When replacing the original dimensional independent arguments by new variables of dimensionless form, the physical process is described in the new variables in an equivalent way, invariant to the used system of units.

Finding new dimensionless arguments (similarity criteria) through the original independent arguments reduces to a linear programming problem [7], solved here by the combinatorial method [8, 9]. The general algorithm for constructing a set of similarity criteria for a physical

process in the class of homogeneous functions is formulated in [10] and involves the following operations:

1. Studying the physical process and establishing the independent arguments $x_i\ (i = 1,\ 2,..., n)$ defining the desired function $y = f(x_1, x_2,..., x_n)$;
2. Selection of the system of units with, $e_j\ (j = 1,\ 2,..., k)$ independent arguments and the desired function;
3. Finding the desired function in the class of homogeneous functions;
4. Representation of the desired function by a degree complex of independent arguments with corresponding unknown homogeneity indices: $a_i\ (i = 1,\ 2,..., n)$;
5. Representation of the dimensionality of the desired function $[y]$ and independent arguments $[x_i]$ in the adopted system of measurement units e_j ;
6. Representation of the dimensionality of the degree complex in the adopted system of units of measurement;
7. Transformation in an equivalent way of the identity equality of the dimensionality of the desired function and the degree complex to a system of k algebraic equations with respect to n unknown indices of uniformity;
8. Formation by means of chain diagrams of the set of choices of k basic variables and $(n-k)$-free variables among n unknown homogeneity indicators as the number of combinations of C_n^k provided that $k < n$;
9. Finding for each variant a solution to the system of linear algebraic equations for the basis variables with respect to the free variables;
10. Exclusion in the original stack of baseline variables respectively for each of the variants;
11. Grouping in a stacked set of independent arguments under the degree sign of the free variables for each of the options;
12. Formation of dimensionless sets of arguments for each of the variants Π_j^m, taken as the main similarity criteria of the natural and modelled physical process;
13. Establishing derived similarity criteria from the basic ones, using Cayley tables according to the chosen rational arithmetic operation.

In this paper, the proposed algorithm for constructing a set of similarity criteria is tested on a well-known example of a pneumatic measuring device for thickness control of flat parts [2].

PROBLEM STATEMENT

The structural scheme of the pneumatic thickness gauge (Fig. 1) described in [2] is considered.

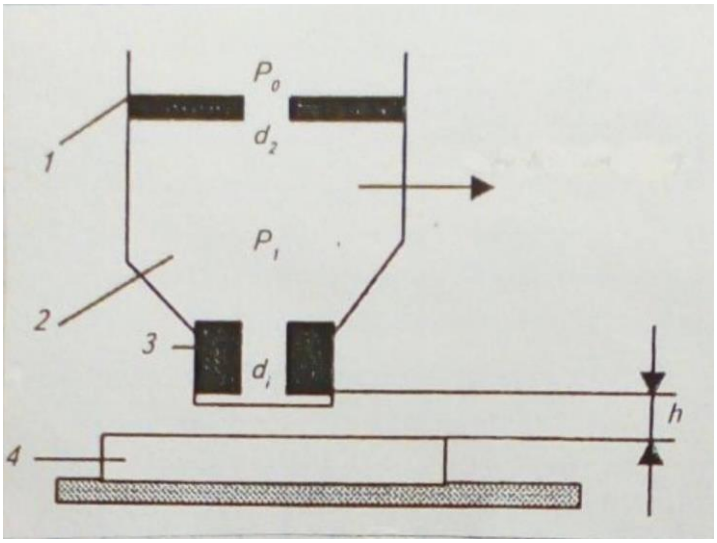


Fig: 1: Structural diagram of the pneumatic thickness gauge.

d_2 - diameter of the input throttle (1);
P_1 - pressure in the measuring chamber (2);
d_1 - diameter of the output throttle (3);
P_0 - inlet pressure;
h - distance between the throttle (3) and the part (4).

It is assumed that the pressure measured in the device P_1, depends on the geometrical parameters d_1, d_2, h, inlet pressure P_0 and compressibility of air - g, ρ, i.e..

$$P_1 = f\left(P_0, h, d_1, d_2, g, \rho\right)$$

Here the dimensionality of the function and the independent arguments is expressed by the units: M-mass, L - Length, T-time, i.e.

$$[P_0] = [P_1] = M \cdot L^{-1} \cdot T^{-2};$$
$$[h] = [d_1] = [d_2] = L;$$
$$[g] = L \cdot T^{-2};$$
$$[\rho] = M \cdot L^{-3}.$$

As a result of the dimensionality analysis in [2], the following dimensionless combinations are obtained:

$$\frac{h}{d_1}, \frac{d_2}{d_1}, \frac{d_1 \cdot g \cdot \rho}{P_0}.$$

Obviously, this solution is not exhaustive. Therefore, it seems reasonable to find the set of all dimensionless arguments-criteria of similarity describing the considered physical process in an equivalent way, invariant to the used system of units.

SOLUTION ALGORITHM

In order to unify the algorithm, convenient notations are introduced for function, independent arguments and units of measurement:

$$y = P_1,\ x_1 = P_0,\ x_2 = h,\ x_3 = d_1,\ x_4 = d_2,\ x_5 = g,\ x_6 = \rho$$

$$e_1 = M,\ e_2 = L,\ e_3 = T.$$

Then we obtain:

$$\begin{aligned}
&[P_0] = [P_1] = e_1 \cdot e_2^{-1} \cdot e_3^{-2} = [y] = [x_1];\\
&[h] = [d_1] = [d_2] = e_2 = [x_2] = [x_3] = [x_4];\\
&[g] = e_2 \cdot e_3^{-2} = [x_5];\\
&[\rho] = e_1 \cdot e_2^{-3} = [x_6].
\end{aligned}$$

We represent the desired function by a power complex of independent arguments with corresponding unknown homogeneity indices $a_i\ (i = 1, 2, 3, 4, 5, 6)$:

$$y = C \cdot x_1^{a_1} \cdot x_2^{a_2} \cdot x_3^{a_3} \cdot x_4^{a_4} \cdot x_5^{a_5} \cdot x_6^{a_6}$$

or, passing to dimensions, we obtain:

$$[y] = [x_1]^{a_1} \cdot [x_2]^{a_2} \cdot [x_3]^{a_3} \cdot [x_4]^{a_4} \cdot [x_5]^{a_5} \cdot [x_6]^{a_6}$$

We express the dimensionality of the desired function and independent arguments in the adopted system of measurement units:

$$e_1^1 \cdot e_2^{-1} \cdot e_3^{-2} = \left(e_1 \cdot e_2^{-1} \cdot e_3^{-2}\right)^{a_1} \cdot (e_2)^{a_2} \cdot (e_2)^{a_3} \cdot (e_2)^{a_4} \cdot \left(e_2 \cdot e_3^{-2}\right)^{a_5} \cdot \left(e_1 \cdot e_2^{-3}\right)^{a_6}$$

Let us transform the power complex in an equivalent way:

$$e_1^1 \cdot e_2^{-1} \cdot e_3^{-2} = e_1^{a_1 + a_6} \cdot e_2^{-a_1 + a_2 + a_3 + a_4 + a_5 - 3a_6} \cdot e_3^{-2a_1 - 2a_5}$$

We reduce the identity equality of the dimensionality of the sought function and the stele complex to a system of algebraic equations concerning the unknown homogeneity indices:

$$e_1 : a_1 + a_6 = 1;$$
$$e_2 : -a_1 + a_2 + a_3 + a_4 + a_5 - 3a_6 = -1;$$
$$e_3 : -2a_1 - 2a_5 = -2;$$

or

$$e_1 : a_1 + a_6 = 1;$$
$$e_2 : a_1 - a_2 - a_3 - a_4 - a_5 + 3a_6 = 1;$$
$$e_3 : a_1 + a_5 = 1.$$

Form a set of options $C_6^3 = 20$ of choosing three baseline variables and three free variables among six unknown homogeneity indicators, using chain diagrams:

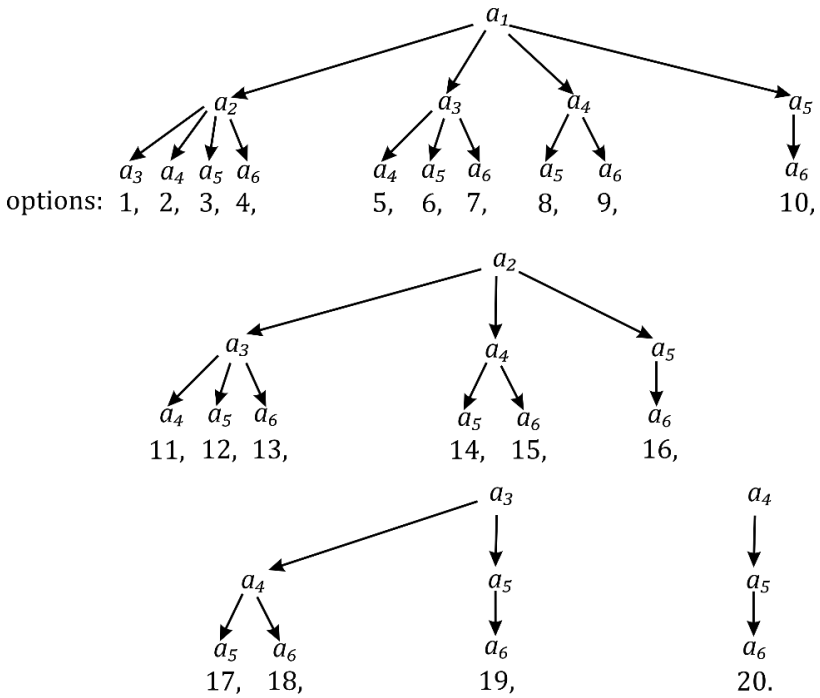


Figure 2. Value diagrams of multiple solution options.

Find solutions to the system of linear algebraic equations for the constructed set of variants. Variants 1, 2, 5, 11, 12, 13, 14, 15, 17, 18 correspond to degenerate systems of equations. Variants 3 and 4, 6 and 7, 8 and 9 are equivalent. The following similarity criteria correspond to Option 3:

$$\frac{d_1}{h}, \frac{d_2}{h}, \frac{h \cdot g \cdot \rho}{P_0};$$

Option 6:

$$\frac{h}{d_1}, \frac{d_2}{d_1}, \frac{d_1 \cdot g \cdot \rho}{P_0};$$

Option 8:

$$\frac{h}{d_2},\ \frac{d_1}{d_2},\ \frac{d_2 \cdot g \cdot \rho}{P_0};$$

Option 10:

$$\frac{h \cdot g \cdot \rho}{P_0},\ \frac{d_1 \cdot g \cdot \rho}{P_0},\ \frac{d_2 \cdot g \cdot \rho}{P_0};$$

Option 16:

$$\frac{d_1}{h},\ \frac{d_2}{h},\ \frac{P_0}{h \cdot g \cdot \rho};$$

Option 19:

$$\frac{h}{d_1},\ \frac{d_2}{d_1},\ \frac{P_0}{d_1 \cdot g \cdot \rho};$$

Option 20:

$$\frac{h}{d_2},\ \frac{d_1}{d_2},\ \frac{P_0}{d_2 \cdot g \cdot \rho}.$$

From where it follows that the obtained set of basic similarity criteria consists of pairwise inverse combinations of the initial parameters, i.e. here it is reasonable to introduce the following notations:

$$\pi_1 = \frac{d_1}{h},\ \pi_2 = \frac{d_2}{h},\ \pi_3 = \frac{d_1}{d_2},\ \pi_4 = \frac{h \cdot g \cdot \rho}{P_0},\ \pi_5 = \frac{d_1 \cdot g \cdot \rho}{P_0},\ \pi_6 = \frac{d_2 \cdot g \cdot \rho}{P_0};$$

$$\pi_1^{-1} = \frac{h}{d_1},\ \pi_2^{-1} = \frac{h}{d_2},\ \pi_3^{-1} = \frac{d_2}{d_1},\ \pi_4^{-1} = \frac{P_0}{h \cdot g \cdot \rho},\ \pi_5^{-1} = \frac{P_0}{d_1 \cdot g \cdot \rho},\ \pi_6^{-1} = \frac{P_0}{d_2 \cdot g \cdot \rho}.$$

The set of derived similarity criteria can be found from the resulting basic set using, for example, the multiplication operation in the Cayley table. Fragments of the Cayley table and the corresponding derived similarity criteria are illustrated below:

Table №1

•	π_1	π_2	π_3	π_4	π_5	π_6
π_1	π_1^2	$\frac{d_1 \cdot d_2}{h^2}$	$\frac{d_1^2}{h \cdot d_2}$	π_5	$\frac{d_1^2 \cdot g \cdot \rho}{h \cdot P_0}$	$\frac{d_1 \cdot d_2 \cdot g \cdot \rho}{h \cdot P_0}$
π_2	$\frac{d_1 \cdot d_2}{h^2}$	π_2^2	π_1	π_6	$\frac{d_1 \cdot d_2 \cdot g \cdot \rho}{h \cdot P_0}$	$\frac{d_1^2 \cdot g \cdot \rho}{h \cdot P_0}$
π_3	$\frac{d_1^2}{h \cdot d_2}$	π_1	π_3^2	$\frac{d_1 \cdot h \cdot g \cdot \rho}{d_2 \cdot P_0}$	$\frac{d_1^2 \cdot g \cdot \rho}{d_2 \cdot P_0}$	π_5
π_4	π_5	π_6	$\frac{d_1 \cdot h \cdot g \cdot \rho}{d_2 \cdot P_0}$	π_4^2	$\frac{d_1 \cdot h \cdot g^2 \cdot \rho^2}{P_0^2}$	$\frac{d_2 \cdot h \cdot g^2 \cdot \rho^2}{P_0^2}$

π_5	$\frac{d_1^2 \cdot g \cdot \rho}{h \cdot P_0}$	$\frac{d_1 \cdot d_2 \cdot g \cdot \rho}{h \cdot P_0}$	$\frac{d_1^2 \cdot g \cdot \rho}{d_2 \cdot P_0}$	$\frac{d_1 \cdot h \cdot g^2 \cdot \rho^2}{P_0^2}$	π_5^2	$\frac{d_1 \cdot d_2 \cdot g^2 \cdot \rho^2}{P_0^2}$
π_6	$\frac{d_1 \cdot d_2 \cdot g \cdot \rho}{h \cdot P_0}$	$\frac{d_1^2 \cdot g \cdot \rho}{h \cdot P_0}$	π_5	$\frac{d_2 \cdot h \cdot g^2 \cdot \rho^2}{P_0^2}$	$\frac{d_1 \cdot d_2 \cdot g^2 \cdot \rho^2}{P_0^2}$	π_6^2

Where the derived similarity criteria are:

$$\frac{d_1 \cdot d_2}{h^2}, \frac{d_1^2}{h \cdot d_2}, \frac{d_1^2 \cdot g \cdot \rho}{h \cdot P_0}, \frac{d_1 \cdot d_2 \cdot g \cdot \rho}{h \cdot P_0}, \frac{d_2^2 \cdot g \cdot \rho}{h \cdot P_0}, \frac{d_1 \cdot h \cdot g \cdot \rho}{d_2 \cdot P_0}, \frac{d_1^2 \cdot g \cdot \rho}{d_2 \cdot P_0},$$
$$\frac{d_1 \cdot h \cdot g^2 \cdot \rho^2}{P_0^2}, \frac{d_2 \cdot h \cdot g^2 \cdot \rho^2}{P_0^2}, \frac{d_1 \cdot d_2 \cdot g^2 \cdot \rho^2}{P_0^2}.$$

Table №2

•	π_1^{-1}	π_2^{-1}	π_3^{-1}	π_4^{-1}	π_5^{-1}	π_6^{-1}
π_1^{-1}	π_1^{-2}	$\frac{h^2}{d_1 \cdot d_2}$	$\frac{h \cdot d_2}{d_1^2}$	π_5^{-1}	$\frac{h \cdot P_0}{d_1^2 \cdot g \cdot \rho}$	$\frac{h \cdot P_0}{d_1 \cdot d_2 \cdot g \cdot \rho}$
π_2^{-1}	$\frac{h^2}{d_1 \cdot d_2}$	π_2^{-2}	π_1^{-1}	π_6^{-1}	$\frac{h \cdot P_0}{d_1 \cdot d_2 \cdot g \cdot \rho}$	$\frac{h \cdot P_0}{d_2^2 \cdot g \cdot \rho}$
π_3^{-1}	$\frac{h \cdot d_2}{d_1^2}$	π_1^{-1}	π_3^{-2}	$\frac{d_2 \cdot P_0}{d_1 \cdot h \cdot g \cdot \rho}$	$\frac{d_2 \cdot P_0}{d_1^2 \cdot g \cdot \rho}$	π_5^{-1}
π_4^{-1}	π_5^{-1}	π_6^{-1}	$\frac{d_2 \cdot P_0}{d_1 \cdot h \cdot g \cdot \rho}$	π_4^{-2}	$\frac{P_0^2}{d_1 \cdot h \cdot g^2 \cdot \rho^2}$	$\frac{P_0^2}{d_2 \cdot h \cdot g^2 \cdot \rho^2}$
π_5^{-1}	$\frac{h \cdot P_0}{d_1^2 \cdot g \cdot \rho}$	$\frac{h \cdot P_0}{d_1 \cdot d_2 \cdot g \cdot \rho}$	$\frac{d_2 \cdot P_0}{d_1^2 \cdot g \cdot \rho}$	$\frac{P_0^2}{d_1 \cdot h \cdot g^2 \cdot \rho^2}$	π_5^{-2}	$\frac{P_0^2}{d_1 \cdot d_2 \cdot g^2 \cdot \rho^2}$
π_6^{-1}	$\frac{h \cdot P_0}{d_1 \cdot d_2 \cdot g \cdot \rho}$	$\frac{h \cdot P_0}{d_2^2 \cdot g \cdot \rho}$	π_5^{-1}	$\frac{P_0^2}{d_2 \cdot h \cdot g^2 \cdot \rho^2}$	$\frac{P_0^2}{d_1 \cdot d_2 \cdot g^2 \cdot \rho^2}$	π_6^{-2}

Where the reverse criteria are:

$$\frac{h^2}{d_1 \cdot d_2}, \frac{h \cdot d_2}{d_1^2}, \frac{h \cdot P_0}{d_1^2 \cdot g \cdot \rho}, \frac{h \cdot P_0}{d_1 \cdot d_2 \cdot g \cdot \rho}, \frac{h \cdot P_0}{d_2^2 \cdot g \cdot \rho}, \frac{d_2 \cdot P_0}{d_1 \cdot h \cdot g \cdot \rho}, \frac{d_2 \cdot P_0}{d_1^2 \cdot g \cdot \rho},$$
$$\frac{P_0^2}{d_1 \cdot h \cdot g^2 \cdot \rho^2}, \frac{P_0^2}{d_2 \cdot h \cdot g^2 \cdot \rho^2}, \frac{P_0^2}{d_1 \cdot d_2 \cdot g^2 \cdot \rho^2}.$$

Table №3

•	π_1^{-1}	π_2^{-1}	π_3^{-1}	π_4^{-1}	π_5^{-1}	π_6^{-1}
π_1	1	π_3	π_2	$\frac{d_1 \cdot P_0}{h^2 \cdot g \cdot \rho}$	π_4^{-1}	$\frac{d_1 \cdot P_0}{h \cdot d_2 \cdot g \cdot \rho}$

π_2	π_3^{-1}	1	$\frac{d_2^2}{h \cdot d_1}$	$\frac{d_2 \cdot P_0}{h^2 \cdot g \cdot \rho}$	$\frac{d_2 \cdot P_0}{h \cdot d_1 \cdot g \cdot \rho}$	π_4^{-1}
π_3	π_2^{-1}	$\frac{h \cdot d_1}{d_2^2}$	1	$\frac{d_1 \cdot P_0}{h \cdot d_2 \cdot g \cdot \rho}$	π_6^{-1}	$\frac{d_1 \cdot P_0}{d_2^2 \cdot g \cdot \rho}$
π_4	$\frac{h^2 \cdot g \cdot \rho}{d_1 \cdot P_0}$	$\frac{h^2 \cdot g \cdot \rho}{d_2 \cdot P_0}$	$\frac{h \cdot d_2 \cdot g \cdot \rho}{d_1 \cdot P_0}$	1	π_1^{-1}	π_2^{-1}
π_5	π_4	$\frac{h \cdot d_1 \cdot g \cdot \rho}{d_2 \cdot P_0}$	π_6	π_1	1	π_3
π_6	$\frac{h \cdot d_2 \cdot g \cdot \rho}{d_1 \cdot P_0}$	π_4	$\frac{d_2^2 \cdot g \cdot \rho}{d_1 \cdot P_0}$	π_2	π_3^{-1}	1

Where the derived similarity criteria are:

$$\frac{d_1 \cdot P_0}{h^2 \cdot g \cdot \rho}, \frac{d_1 \cdot P_0}{h \cdot d_2 \cdot g \cdot \rho}, \frac{d_2^2}{h \cdot d_1}, \frac{d_2 \cdot P_0}{h^2 \cdot g \cdot \rho}, \frac{d_2 \cdot P_0}{h \cdot d_1 \cdot g \cdot \rho}, \frac{d_1 \cdot P_0}{h \cdot d_2 \cdot g \cdot \rho}, \frac{d_1 \cdot P_0}{d_2^2 \cdot g \cdot \rho}.$$

Where the reverse criteria are:

$$\frac{h^2 \cdot g \cdot \rho}{d_1 \cdot P_0}, \frac{h \cdot d_2 \cdot g \cdot \rho}{d_1 \cdot P_0}, \frac{h \cdot d_1}{d_2^2}, \frac{h^2 \cdot g \cdot \rho}{d_2 \cdot P_0}, \frac{h \cdot d_1 \cdot g \cdot \rho}{d_2 \cdot P_0}, \frac{h \cdot d_2 \cdot g \cdot \rho}{d_1 \cdot P_0}, \frac{d_2^2 \cdot g \cdot \rho}{d_1 \cdot P_0}.$$

i.e. every derived similarity criterion corresponds to an inverse derived similarity criterion.

CONCLUSION

The construction of a set of similarity criteria for a physical process is reduced to a linear programming problem, which is solved here by the combinatorial method.

The general algorithm for constructing a set of similarity criteria is tested on a particular example of a known technical problem. It is shown that the obtained set of similarity criteria includes known results. It is established that the set of similarity criteria consists of two equal semi-multiple sets of direct and inverse similarity criteria. Derived similarity criteria are constructed using the basic direct and inverse similarity criteria in the Cayley tables.

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