

Harnessing Active Force: The Pumping Mechanism of Child's Swing Motion

Rong Li

Research Center for Industries of the Future, Westlake University, Hangzhou, Zhejiang 310030, China and Key Laboratory of Coastal Environment and Resources of Zhejiang Province, School of Engineering, Westlake University, Hangzhou, Zhejiang 310030, China

Weicheng Cui

Research Center for Industries of the Future, Westlake University, Hangzhou, Zhejiang 310030, China and Key Laboratory of Coastal Environment and Resources of Zhejiang Province, School of Engineering, Westlake University, Hangzhou, Zhejiang 310030, China

ABSTRACT

Life mechanics, an emerging field, focuses on the self-organizing motions manipulated by the mind within living systems. This study introduces the concept of 'active force', generated by mind-body-environment interactions, as a fundamental driver underlying these self-organizing movements. As an example, we propose a new set of control equations to model the self-pumping swing motion by incorporating the active force into Newton's second law. With this new mechanical framework, we inversely derived the total (i.e., responsive) active force due to the body-environment interaction from the child's swing motions with rapid standing and squatting movements. It revealed a pulse-like pattern of the total active force along the swing length, driving changes in the radial speed and swing length. This force counteracts the resistance and propels the swing, which is not attainable by the stone. Consequently, the active force serves as the foundational principle for self-organization in living systems, offering a novel mechanical approach for understanding and predicting extraordinary movements (e.g., sports and rehabilitation) regulated by the mind (e.g., nervous system) in biological systems.

Keywords: Mind-body interaction, Active force, Newton's Second Law, Swing, Pulse

INTRODUCTION

Dyson, a physicist, remarked that the twenty-first century may be the century of biology [1]. Life, the most intricate of complex systems, is usually defined as a system that exhibits many nontrivial movements, including responsiveness, energy transformation, metabolism, growth, reproduction, and evolution [2]. Understanding these living movements poses the greatest challenge in modern science [3, 4]. Accepting the axiom that force is the only reason for the change in body movement, Schrödinger's seminal question "What is life?" [5] can be reframed in the context of Newtonian mechanics: can we construct a mechanical model that describes nontrivial movements inherent in life?

Traditional Newtonian mechanics, which views the human body as the mechanical sum of its parts, overlooks the complexity of mind-manipulating interactions and emergent behaviors. From the late 19th century, it became clear that viewing life merely as a machine was insufficient for understanding phenomena, such as embryonic cell development. For instance, Driesch's experiments suggest that cells have an inherent ability to adapt to changing environments [6, 7]. Further studies led to the birth of the modern system theory in the 1930s [8]. The "vitality" and "entelechy" postulated by early vitalists [9] found modern interpretation in the concept of "self-organization" in complex system science [10, 11]. However, despite the development of numerous phenomenological differential equations and theories for complexities and lives [11-15], mechanical descriptions of the dynamics of self-determined (i.e., by mind) movement in life remain rare. This void in physicists' understanding signals the need for the development of life mechanics [16].

In response to this need, we propose the concept of "active force", an internal force and its direct response arising from mind-body-environment interactions, as an integral part of life mechanics. Recently, studying the autonomous motions in living and engineering systems has led to a conceptual innovation related to "active" mechanics, for instance, "active matter" in physics [17] and "active cell mechanics" in biomechanics [18], reminiscent the "action potential" in electrophysiology [19]. Therefore, the active force is a straightforward conceptual development along this concept series to define the mechanical driver of autonomous motions, which ranges from cells, fish, birds, and people. Indeed, the concept of active force has been mentioned in a minority of literature, for instance, in experimental analyses of certain skeletal and muscular motions [20, 21] as well as in active cell mechanics [18], where it acts as the (stochastic) force driving self-organization behaviors. However, its mechanical study is still in its infancy, possibly due to its vague mechanical definition, calculation, and measuring complexity.

Therefore, there remains a substantial gap in understanding the generation mechanisms, temporal patterns, and physiological significance of the active force. To fill this gap, our new general system theory (NGST) [22-25] begins a preliminary study into its generation mechanisms, classification, and mechanical representation in Newton's laws. It presents unified mechanics incorporating this active force from mind-body-environment interactions and the passive forces arising from external interactions with other objects or the inanimate matter-matter interactions independent of the mind. In other words, whether the force is active or passive only depends on whether it is generated by mind-body interaction. This classification doesn't violate Newton's laws; thus, the concept of active force is not a violation but rather an extension of classical mechanics to incorporate the dynamics of living objects under the living state. Our previous work on NGST has also shown that the active force must compensate for the energy dissipated by resistance in a changing environment through doing work [26, 27].

In this study, we examined the dynamics of a simple pendulum system to illustrate the necessity of an active force to explain the observed phenomena in swing motions. Specifically, by comparing the motion patterns of a child and a stone of equivalent weight, we demonstrate that an active force that extends beyond the conventional forces of gravity and friction is introduced by a child to generate motion patterns that diverge from those of a lifeless swing.

This difference originates from the mind manipulating internal interactions and relative motions between elements (e.g., leg muscles in swing motion or walk) of the body, which induce a subsequent extra body-environment interaction, propelling the center of mass motions of the swing. Our findings underscore the importance of incorporating an active force into Newton's second law and the mechanical analysis of the mind-body-environment interaction as a fundamental paradigm in active life mechanics.

GENERAL FRAMEWORK OF NEWTON'S SECOND LAW WITH ACTIVE FORCE

Consider the mechanics of a particle within a multiparticle system situated in an Earth-fixed, non-inertial coordinate system (as depicted in Fig.1). This model is grounded in the perception that our planet is in motion, an understanding that dates back to the era of Galileo Galilei. In this context, the governing equation for each particle is derived from Newton's second law:

$$\frac{d^2}{dt^2} m_i r_i = F_i^P + F_i^A + F_i^D, \quad (1)$$

where m_i and r_i are the mass and displacement vectors of the i th particle, respectively, F_i^P , F_i^A and F_i^D are the passive-driven force, active force, and dissipative force, respectively. Note that the vectors are indicated by Roman letters.

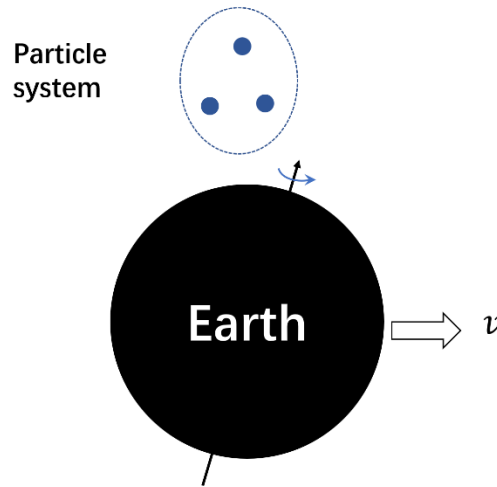


Fig. 1: A schematic representation of a N-body system in an earth-fixed coordinate system. It means that we only consider the motions of the particle systems relative to the observer on the earth.

Eq. (1) embodies Newton's axiom that force is the agent of the motion change. The dynamics of nonliving objects can be adequately described by the passive-driven forces F_i^P and dissipation forces F_i^D . These passive-driven forces were generated from other objects, including particles and the earth, in the system we studied. The most common passive-driven forces in the macroscopic world are gravity ($F_i^G = GmM/r^2$) and the static electromagnetic force, that is, $F_i^M = q_i(E + v_i \times B)$, where E is the electronic field, B is the magnetic field strength, q_i and $v_i = \dot{r}_i$ are the charge and velocity of the particle, respectively. These forces

can be expressed as a derivative of the generalized potential: $F_i^P = -\nabla U + d(\partial U / \partial \dot{r}_i) / dt$, where the generalized potential is $U = V_G + q\phi - q\mathbf{A} \cdot \mathbf{v}$. Here, V_G is the gravity potential and ϕ and $\mathbf{A}$ are the scalar and vector potentials of the electromagnetic field, respectively. This analysis employs an Earth-fixed coordinate system and implicitly assumes the validity of Newton's second law in a non-inertial coordinate system. According to the NGST ontology [26, 27], we must abandon the assumption of inertial coordinate systems because they do not exist for human observers. However, the origin and expression of F_i^D are typically complex. The classical linear friction ($F_i^D = -k_i v_i$) and its corresponding Rayleigh dissipation function $D = \sum_i k_i v_i^2 / 2$ [28] is only a particular case (i.e., $n = 1$) of the general formula, that is, $F_i^D = -k_i v_i^n$. Although complex, friction is always defined along the inverse direction of the velocity.

In addition, a new type of force, referred to as the "active force," was introduced to explain the initiating movement changes in living entities [22]. As introduced in the Introduction section, the active force is the internal force directly generated by the mind-body interaction in a living system. In contrast, the passive forces arise from external interactions with other objects or the internal but inanimate matter-matter interactions independent of the mind. Therefore, the critical distinction between active and passive forces is whether directly generated by the mind-body interaction. Thus, any entity possessing a mind can exert an active force; the separation of the mind from the body signifies the death of a living organism. After death, a living object no longer exhibits an active force.

Thus, two immediate questions arise. How can the internal active force generate the center of mass motions, and how can this active force be calculated? From our daily experiences, humans exhibit active forces during their self-determined movements. For example, humans and other animals can walk, run, and swing by using both the internal active force and the body environment interactions. To generate these self-determined movements, one generally utilizes the mechanism of mind-body-environment interaction: the mind issuing biosignal (e.g., Nerve impulse) to modulate the interactions between elements of the body and their relative motions. Subsequently, the body exerts extra force changes to the environment, which in turn applies a reactive force that propels the body into the center of mass motion. This tripartite interaction is essential to two kinds of active forces for the self-determined movements of the living system. Compared to the baseline state of no motion or external stimulation, we define the force change of the interactions between body elements as the "internal active force" and the force change of the body-environment interaction as the "responsive active force". Thus, for each part of the body, we can obtain a decomposition of the active force

$$F_i^A = F_i^I + F_i^R, \quad (2)$$

where F_i^I is the internal active force and F_i^R is the responsive active force. The mind-body interaction mainly determines the former, while the environment constrains the latter. Therefore, active force is interpreted as a mind-body-environment interaction. Without the support of environment, the active force could not be generated.

However, calculating these active forces is a significant challenge, as it is governed by the unpredictable free will of the mind, as well as the complex mind-body-environment interaction. We suggest initially adopting a model that explains this phenomenon qualitatively and subsequently developing methods to quantify the active force. This study presented an early effort and some initial results. In general, everyday human movements such as walking, running, and swinging are inherently complex, necessitating rigorous modeling of the interactions and movements of various body parts [29-31]. However, this paper aims not to explore modeling too many complex motions but to clarify the differences between active forces in living systems and those in inanimate bodies. Therefore, our primary focus in this study was on the motion of the center of mass for its simplicity of degree of freedom and the more straightforward measurement of the responsive active force compared to the internal active force. In this case, the internal active forces of different body parts cancel with each other (i.e., $\sum_i F_i^I = 0$ following Newton's third law), while the summation of the responsive active force from the environment to the body is nonzero. Thus, in line with the definition of the center of mass coordinates ($R = \sum_i m_i r_i / \sum_i m_i$), Newton's second law can be obtained from the summation of Eq. (1) as,

$$M \frac{d^2}{dt^2} R = F_p + F_a + f, \quad (3)$$

where $M = \sum_i m_i$ is the total mass, $F_p = \sum_i F_i^P$ is total passive (environment-to-body) force at the baseline state of the body, $f = \sum_i F_i^D$ is the total passive-driven force, total active-driven force, and the total dissipation force, respectively. $F_a = \sum_i F_i^A = \sum_i F_i^R$ is the total active force of the body relative to the baseline state, equaling the total responsive active force since $\sum_i F_i^I = 0$. In this paper, we employ Eq. (2) to extract the total (responsive) active force driving the swing's motion, deferring the elucidation of internal active forces to future investigations.

THE RESPONSIVE ACTIVE FORCE FOR PUMPING A SWING BY A LIVING SYSTEM

This section explores how an active force propels a swing, mainly by comparing the dynamic differences (displacement, velocity, and force) between the swing motions of a child and a stone. This comparison enables us to quantitatively derive the temporal pattern of the responsive active force, revealing the essential role of active force in explaining the motions of living systems.

The Governing Equation for Pumping A Swing

Applying Eq. (3) elucidates the differences between the swing motions of a child and a stone, considering only simple pendulum motions within a vertical two-dimensional plane. Fig. 2 illustrates the simple pendulum system: (a) represents a classical case with a non-living stone, whereas (b) substitutes the stone with a child of equivalent weight. In the stone case, L denotes the length of the rigid massless rod, m denotes the mass of the stone, and θ represents the angle of the rod along the vertical axis. Furthermore, three forces acted at the center of the mass of the stone. First, gravity, $F_g = mg$ acts in the downward vertical direction. Second, a passive tension force, T_p acts along the rod owing to the balance of gravity and centrifugal force toward the frictionless pivot. Third, a friction force f resists the motion of

the stone through the medium, which is assumed to be proportional to the velocity of the stone with a coefficient of friction b in the present work for simplicity. Hence, Newton's second law is expressed as follows:

$$m \frac{d\mathbf{v}}{dt} = \mathbf{F}_g + \mathbf{T}_p - b\mathbf{v}. \quad (4)$$

Because the passive tension force $\mathbf{T}_p$ is balanced by gravity and centrifugal force, the pendulum length is constant, that is, $L = L_0$. Therefore, it is straightforward to derive the equation for circumferential motion $v_{\perp} = L\dot{\theta}$ from Eq. (4) as

$$mL\ddot{\theta}(t) = -mg \sin \theta(t) - bL\dot{\theta}(t). \quad (5)$$

If we assume that the initial angle is very small, a general solution can be obtained as

$$\theta(t) = e^{-\frac{bt}{2m}}(c_+ e^{i\omega t} + c_- e^{-i\omega t}), \quad (6)$$

where $\omega = \sqrt{g/L_0 - b^2/4m^2}$ is the frequency of the damped pendulum, and c_+ and c_- are two coefficients determined by the initial conditions. Here, the time-dependent coefficient $e^{-bt/2m}$ reveals that the pendulum motion is a damped oscillation that decays with time and finally ceases. That is, it is a passive motion that depends on the external pumping force.

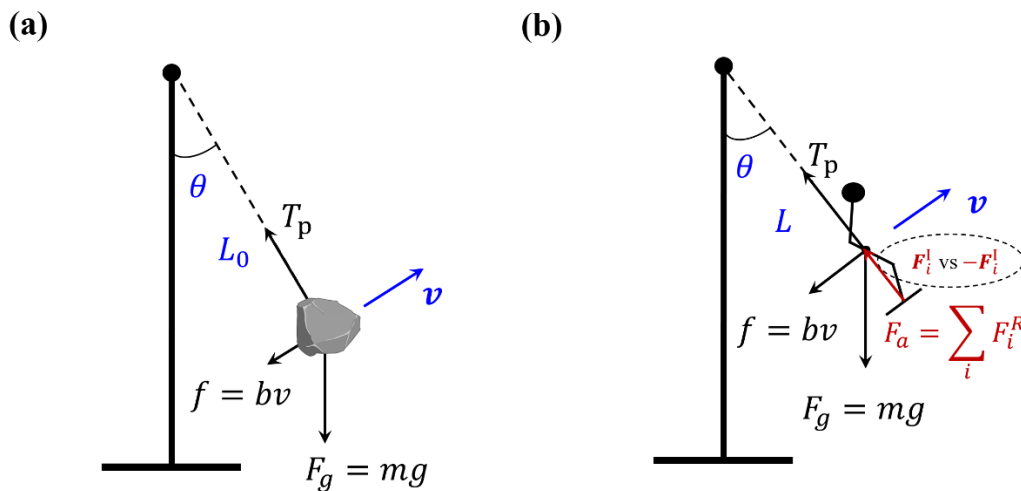


Fig. 2: (a) A force analysis of a simple pendulum of a lifeless stone. (b) A force analysis of a living pendulum, i.e., a child named Bob. $T_p = mg \cos \theta(t) + mv_{\perp}^2/L$ —represents the swing rope tension owing to the passive balance of gravity and centrifugal force. f is the external friction force. F_a is the responsive active force. A child's active motion on the swing originates from the mind issuing signals that modulate the interactions between elements (e.g., leg muscle) of the body, which is termed as the internal active forces. The terms F_i^L and $-F_i^L$ represent a pair of internal active forces acting in opposite directions, indicated in the dashed circle. Subsequently, the body exerts extra force changes to the environment, which in turn applies a reactive force (the responsive active force F_a) that propels the body

into the center of mass motion.

Conversely, Fig. 2 (b) depicts a child, Bob, as the pendulum, where the system behavior is influenced not only by external driving and damping forces but also by an active force F_a acting in a distinct direction. This introduces a fundamental distinction between living beings and inanimate stones. In a prior analysis of an inanimate pendulum, the system was considered to be closed. However, Bob, being a person, introduces potential matter and information exchange with the environment through breathing, drinking, eating, and excreting. Nonetheless, focusing solely on Bob's swing motion allows us to overlook his matter and information exchanges with the environment. This simplification reveals four forces depicted in Fig. 2 (b): the total responsive active force F_a , external gravitational force $F_g = mg$, external tension force T along the rod, and external friction force f .

Establishing a relationship between the active (internal) and passive (external) forces is crucial. The total force exerted on an object, F_{total} , is defined as the sum of the active and passive forces (F_a and F_p respectively). Thus, F_a can be expressed as $F_a = F_{\text{total}} - F_p$. For an inanimate pendulum, $F_a = 0$ and $F_{\text{total}} = F_g + T_p - bv$, where T_p —representing the swing rope tension owing to the passive balance of gravity and centrifugal force, which is defined as $T_p = mg \cos \theta(t) + mv_{\perp}^2/L$.

For a living pendulum, $F_a \neq 0$. The active force, as we discussed in Sec. 2, bifurcates into two categories: one instigating mutual motion within subsystems of the living entity (the internal active force) and the other driving the overall center of mass movement (the responsive active force). The former originates from the interaction forces among subsystems, independent of external forces, whereas the latter necessitates external responsive forces from the environment. In our scenario, where Bob is considered a living pendulum, the significance of the latter interaction is apparent, manifesting as the force between Bob and the swing. This implies that the active force within the human body incites a responsive active force, F_a . It is applied from the trapeze to Bob to catalyze the active movement of the person's center of mass. Therefore, the total external tension force T along the rod can be divided into two categories, i.e., the “passive” tension force T_p from the response to the gravity and the responsive active force F_a from the reaction to the external active force. Consequently, Newton's second law applied to Bob's center of mass motion is:

$$m \frac{dv}{dt} = F_g + T_p + F_a - bv. \quad (7)$$

In the left-hand side of the equation, variations are observed in the components of the velocity vector and in the orientations of the radial and tangential axes with respect to the fixed coordinate system. Therefore, the radial and tangential components of dv/dt are $dv_r/dt - v_{\perp} \dot{\theta}(t)$ and $dv_{\perp}/dt + v_r \dot{\theta}(t)$. The active force in Eq. (7) represents the swing's reaction to a person, which can be oriented in any direction, yet is constrained by a maximum limit owing to the finiteness of Bob's mind-body interaction strength. We examined a straightforward scenario in which a person can move vertically while maintaining a stable stance on the board. In this case, the relationship $T_p = mg \cos \theta(t) + mv_{\perp}^2/L$ consistently applies.

Conversely, in the extreme case where a person disengages from the board, T_p is equal to zero. Furthermore, any supplementary force between the individual and the swing originates from the active force. Consequently, Bob possesses the capacity to move radially (where L is not a constant) and in circumferential directions (where θ is not a constant):

$$m \frac{dv_r}{dt} = m\ddot{L}(t) = F_a^{\parallel} - b\dot{L}(t), \quad (8)$$

$$m \frac{dv_{\perp}}{dt} = m \frac{d[L(t)\dot{\theta}(t)]}{dt} = -mv_r\dot{\theta}(t) - mg \sin \theta(t) - bL(t)\dot{\theta}(t) + F_a^{\perp}, \quad (9)$$

where $-mv_r\dot{\theta}(t)$ comes from the radial axes change, $F_a^{\parallel}$ and $F_a^{\perp}$ are the responsive active forces in the radial and circumferential directions, respectively.

In Eq. (8), the radial motion is solely driven by the active force. By stating $L(t) = L_0 + l(t)$ where $l(t) = 0$ at $t \leq 0$, we can simplify these two equations as follows:

$$\ddot{l}(t) = \frac{F_a^{\parallel}}{m} - b\dot{l}(t). \quad (10)$$

$$\ddot{\theta}(t) = -\frac{g \sin \theta(t)}{L_0 + l(t)} - \left[\frac{b}{m} + \frac{2\dot{l}(t)}{L_0 + l(t)}\right]\dot{\theta}(t) + \frac{F_a^{\perp}/m}{L_0 + l(t)}. \quad (11)$$

Eqns. (10) and (11) govern the general behavior of a living pendulum, requiring a solution to a system of nonlinear, nonhomogeneous ordinary differential equations. The active force yields a non-trivial active motion compared with the motion of a non-living pendulum. Significant differences include the following: (1) Active motion can result in radial movement, while passive pendulum motion primarily permits circumferential movement. (2) Active motion can sustain or augment the circumferential motion amplitude, known as the body's self-pumping swing [30, 32-37], whereas passive pendulum motion can only decrease and rest. Indeed, these active motion patterns rely entirely on the active force patterns, that is, the magnitudes and temporal patterns of $F_a^{\parallel}$ and $F_a^{\perp}$. However, calculating this active force from a specific mind-body-environment interaction mechanism is a challenging problem to solve. At present, only the inversion method can ascertain the active force from the measured trajectory, assuming Newton's second law as a universal law for any dynamics, and the resistance can be determined from prearranged experiments. As rudimentary examples to investigate the relationships between swing motions and active force, we will examine two simple modes of human pumping swing devoid of circumferential responsive active force ($F_a^{\perp} = 0$) in the subsequent two subsections. The underlying logic is to select an appropriate motion pattern specifying the person's vertical movement, that is, $l(t)$ is a function of $\theta(t)$, enabling us to solve $\theta(t)$ from Eq. (11). We then derived the radial active force $F_a^{\parallel}$ from Eq. (10).

Pumping a playground swing is a very popular sport, especially for children. Its dynamic mechanism has been the subject of research for more than half a century of research [30, 32-

37]. It can be considered as a coupled oscillator system composed of a swing and a human. Typically, there are two pumping strategies: pumping from a standing position (as depicted in Fig. 3) and a seated position. In the former instance, the person stands at the lowest point and crouches at the highest point during the swing motion. Each stand-crouch cycle enhances the swing amplitude. The analysis demonstrated that each crouch-stand cycle provides a swing with an energy boost from the rider. In the latter scenario, the person abruptly rotates their body around the end of the swing chain. The amplitude of the swing increases as these rotations elevate the rider slightly above the highest level.

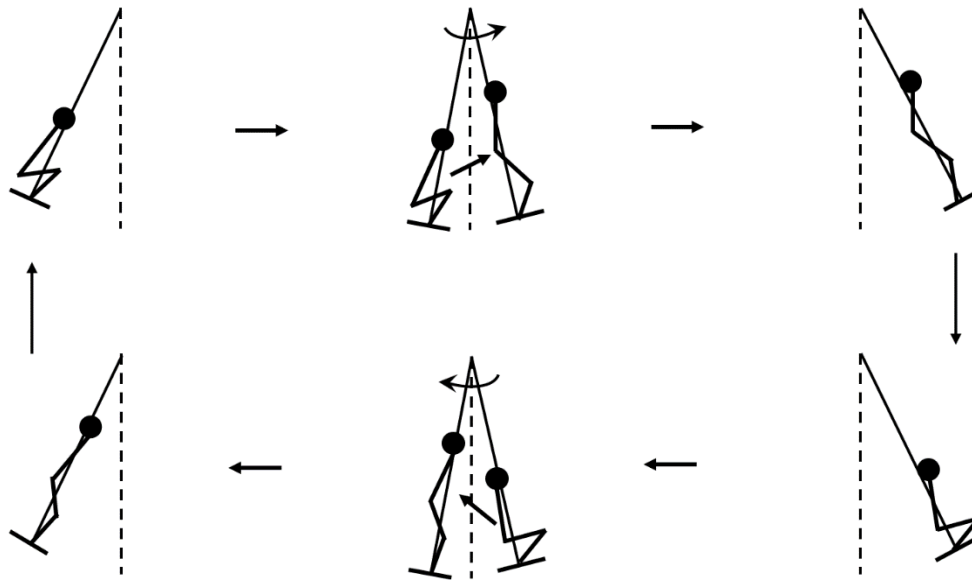


FIG. 3: Strategy for pumping a swing while standing, adapted from Ref. [38]. The child stands up near the lowest point and crouches down near the highest point during the swing motion.

In the child's swing motion, the mind determines the pumping strategies, while the active force derives the body to motion, and environmental constraints provide the control conditions for this driver; all these three factors of mind, active force and environmental constraints are paramount. However, prior research on swing mechanics has primarily concentrated on effective pumping strategies and environmental constraints, such as the pumping mode (standing or seated [34, 38]), the modulation of frequency, and the initial phase [37] under the swing constraints. On the other hand, the force underlying swing pumping, particularly the active force that drives these movements, has been less explored. This indicates that the essential force mechanism generated by the mind-body-environment interaction— has not been adequately addressed. Our study aims to fill this gap by focusing on the (internal or responsive) active force, which directly results from the mind-body-environment interaction and serves as the driver of the swing's active motion. This work mainly focuses on the responsive active force, representing the body-environment interaction. Understanding the temporal evolution pattern of this responsive active force is crucial, as it not only reveals the direct capacity of individuals to drive the center of mass movements but also how the environment responds to the body. Given that the force pattern corresponding to the stand-crouch motion is simpler, we mainly concentrate on pumping from the standing position in this study.

When pumping from a standing position, it can be presumed that only radial active motion is involved. Hence, the circumferential active force is zero, which is denoted by $F_a^\perp = 0$. Consequently, the governing equations for a pump from the standing position are as follows:

$$F_a^\parallel = m[\ddot{l}(t) + b\dot{l}(t)], \quad (12)$$

$$\ddot{\theta}(t) = -\frac{g \sin \theta(t)}{L_0 + l(t)} - \left[\frac{b}{m} + \frac{2\dot{l}(t)}{L_0 + l(t)}\right]\dot{\theta}(t). \quad (13)$$

To solve these equations, we must quantify the specific strategy of the stand-crouch motions. Ideally, standing and crouching motions occur within a short time span. Therefore, a step function can be used to represent abrupt changes in the pendulum length analytically:

$$l(t) = -k[1 + 2H(\theta)H(\dot{\theta}) - H(\theta) - H(\dot{\theta})], \quad (14)$$

where $H(x) = 1$ ($x > 0$) or 0 ($x \leq 0$) is a step (or Heaviside) function. Here, $H(\theta)$ characterizes the temporal pattern of standing up at the minimum angle, whereas $H(\dot{\theta})$ represents the temporal pattern of crouching down at the maximum angle. When $\theta > 0$ and $\dot{\theta} > 0$ or $\theta < 0$ and $\dot{\theta} < 0$, $l(t) = -k$, implying that the child is standing. When $\theta > 0$ and $\dot{\theta} < 0$ or $\theta < 0$ and $\dot{\theta} > 0$, $l(t) = 0$, signifying that the child is crouching. The derivative of $l(t)$ can be calculated as:

$$\dot{l}(t) = -k[2\delta(\theta)H(\dot{\theta})\dot{\theta} + 2H(\theta)\delta(\dot{\theta})\ddot{\theta} - \delta(\theta)\dot{\theta} - \delta(\dot{\theta})\ddot{\theta}], \quad (15)$$

where $\delta(x)$ is the derivative of $H(x)$, equivalent to the Dirac function.

Eqns. (13)– (15) can only be solved using numerical simulations. The Heaviside and Dirac functions must be substituted with finite-function approximations to facilitate this. Moreover, the actual stand-crouch motions transpire in finite time, necessitating a continuous time evolution function. One of the simplest approximations for the step function is the logistic (and pulse) function. Thus, for $H(\theta)$, $H(\dot{\theta})$ and their derivatives, we use

$$H(\theta) = \frac{1}{e^{-\frac{\theta}{c_a}} + 1}, \quad (16 - a)$$

$$\delta(\theta) = \frac{d}{d\theta} H(\theta) = \frac{1}{c_a} \frac{1}{2 + e^{-\frac{\theta}{c_a}} + e^{\frac{\theta}{c_a}}}, \quad (16 - b)$$

$$H(\dot{\theta}) = \frac{1}{e^{-\frac{\dot{\theta}}{c_b}} + 1}, \quad (16 - c)$$

$$\delta(\dot{\theta}) = \frac{d}{d\dot{\theta}} H(\dot{\theta}) = \frac{1}{c_b} \frac{1}{2 + e^{-\frac{\dot{\theta}}{c_b}} + e^{\frac{\dot{\theta}}{c_b}}}, \quad (16 - d)$$

Here, c_a determines the temporal pattern of standing up at the minimum angle, whereas c_b determines the temporal pattern of crouching down at the maximum angle. These two parameters can be independently chosen according to the child's free will.

The Responsive Active Force for Pumping A Swing Without Friction

By employing Eqns. (12)–(16), we can derive the time evolution of several parameters: the angle $\theta(t)$, responsive active force $F_a^{\parallel}$, pendulum length $L(t) = L_0 + l(t)$, vertical height of the mass center of the child $h(t) = L_0 - L(t) \cos \theta$, the circumferential velocity $v_{\perp} = L(t)\dot{\theta}(t)$ and radial velocity $v_{\parallel} = \dot{l}(t)$. The angle and velocities in two directions depict the dynamics of the motion, whereas the alterations in swing length and height represent the spatial state. The responsive active force, which is our primary objective, emerged from these calculations.

To execute this simulation, we must establish the parameters within these equations and initial conditions. Let us assume that the masses of the individual and stone are identical, that is, $m = 20$ kg. The swing's maximum pendulum length, L_0 , was 2 m, and the gravitational acceleration g was 9.81 m/s^2 . We assume the height change of the child is $k = 0.2 \text{ m}$ and zero for the stone. The friction coefficient b , is influenced by the friction between the individual and air, the swing, and between the swing rope and the fixed point.

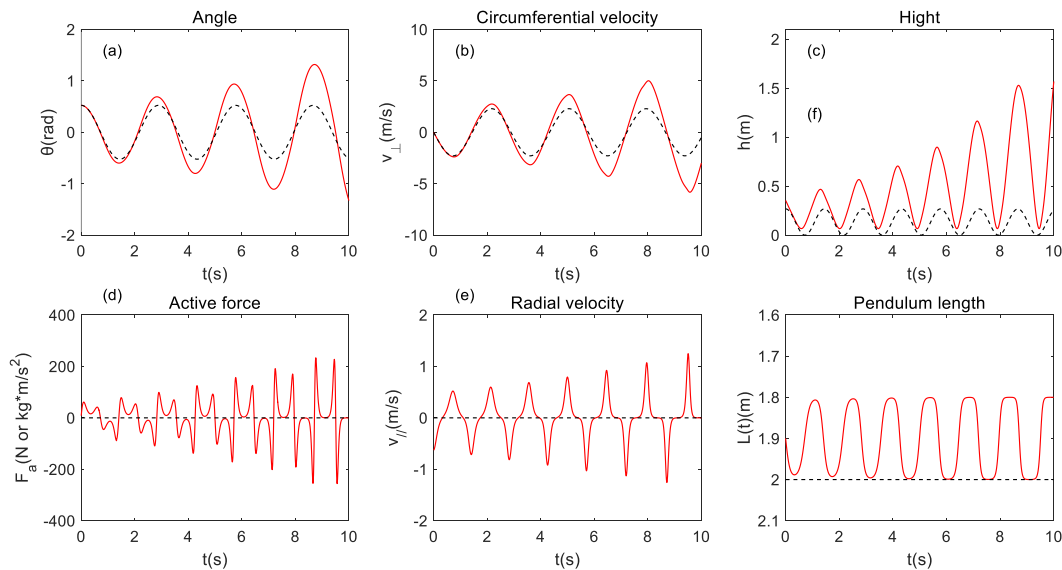


Fig. 4: Active-motion simulations using Eqns. (12-16) for a child pumping a swing without friction, as depicted by the solid red lines. In contrast, the dashed black lines represent the simple pendulum motion of a stone with the same mass as the child. The parameters selected for these simulations are outlined in Table I.

In our study, we account for the linear frictional force at low speeds between the child (or stone) and the air, represented as $f = -bv$. The coefficient b is acknowledged to be variable,

contingent upon the changing conditions of interaction between the child (or stone) and the air. Consequently, b is treated as a free adjustable parameter, facilitating a simplified and flexible model that can adapt to a spectrum of realistic scenarios. The coefficients c_a and c_b , which dictate the patterns of standing and crouching, as well as the initial angle $\theta(0)$ and initial velocity $\dot{\theta}(0)$ can be freely chosen by the child's free will. Consequently, in this study, we treat parameters $b, c_a, c_b, \theta(0)$ and $\dot{\theta}(0)$ as free adjustable parameters.

By substituting these parameters into Eqns. (12)– (16), we can juxtapose the swings of the stone and the child. The simulations for the frictionless scenario are shown in Figure 4. As is commonly understood, the stone (represented by dashed black lines) performs harmonic oscillation, maintaining a constant amplitude throughout each period. Conversely, the child (denoted by solid red lines) generates an enhanced oscillation, with the amplitude increasing in each period. The figures show that the augmentation in the amplitudes of the angle (a) and circumferential velocity (b) aligns with the continuous increase in radial velocity (e), prompted by the child's periodically increasing responsive active force (d). This responsive active force emerged as four pulses per cycle, corresponding to two standing and two crouching instances. More specifically, during the first 1/4 period, the initial angles (a) and initial circumferential velocities (b) for the stone and child were close. However, the child's swift standing motion around (see (d)-(f)) leads to a gradual increase in the angle and the circumferential velocity, reaching peak differences from the stone around 2.0 s and 2.2 s. Consequently, we conclude that in swing motion, the child's active force stimulates the amplification of the swing's amplitude.

Table I: Parameters selection for the stone (zero active force) and the child (finite active force) with zero friction.

System	Mass	Maximum length	Gravitational acceleration	Friction coefficient	Standing height	Pulse angle	Pulse velocity	Initial angle	Initial velocity
	$M(\text{kg})$	$L_0(\text{m})$	$g(\text{kg} \cdot \text{s}^{-2})$	$b(\text{kg} \cdot \text{s}^{-1})$	k	c_a	$c_b(\text{s}^{-1})$	$\theta(0)$	$\dot{\theta}(0)$
Stone	20	2	9.8	0	0	—	—	$\pi/6$	0
Child	20	2	9.8	0	0.2	0.12	0.20	$\pi/6$	0

We proceeded with a quantitative analysis of the energy-pumping mechanism facilitated by the responsive active force. In a frictionless environment, we assume that standing up and crouching down are executed instantaneously. Because the responsive active force solely impacts the radial motion, neither the circumferential momentum (or velocity) nor the angle changes this process. The critical variables that vary are the pendulum length, which shifts from L_0 to $L_0 - k$, and the radial velocity $v_{\parallel} = \dot{l}(t)$. By examining the n th instance of standing up, we can establish a relationship between the maximum swing amplitudes before (θ_{n-1}) and after (θ_n) the actions. The law of energy conservation dictates that the circumferential kinetic energy ($m(v_{\perp}^n)^2/2$) at the lowest point originates from the potential energy decrease ($mgL_0(1 - \cos\theta_{n-1})$) of the child from the maximum swing angle θ_{n-1} . After standing up, this kinetic energy transitions into a potential energy increase ($mg(L_0 - k)(1 - \cos\theta_n)$) of the maximum swing θ_n . As such, the potential energy decreases before standing up is equal to the increase following it,

$$mgL_0(1 - \cos\theta_{n-1}) \approx mg(L_0 - k)(1 - \cos\theta_n). \quad (17)$$

From this relation, we can obtain the n th maximum angle, height, and velocity as:

$$\theta_n \approx \arccos \left[\frac{\cos\theta_{n-1} - k'}{1 - k'} \right], \quad (18)$$

where $k' = k/L_0$. Eq. (18) reveals that the swing amplitude increases every half-cycle according to a function related to $1/(1 - k')$. This reveals that the swing accumulates the net energy from the child's active motions in each pumping cycle. This net energy stems from the work done by the child's active force when standing up, which increases the height by k , over the energy spent when squatting down, which decreases the height by $k\cos\theta_n$. It is worth mentioning that Eq. (18) is obtained based on the neglect of the impact of standing squatting motions. This contribution is $-2\dot{l}(t)\dot{\theta}(t)/L(t)$ for $\ddot{\theta}(t)$ in Eq. (13), revealing that the standing results in an increase for $|\dot{\theta}(t)|$. Therefore, the realistic increase of θ_n (shown in Fig. 4) should be greater the prediction $\theta_n = \arccos[1.11(\cos\theta_{n-1} - 0.1)]$ by substituting parameters in the numerical simulations into Eq. (18).

The Responsive Active Force for Pumping A Swing with Linear Friction

Next, we explore a more realistic scenario in which both active and frictional forces are at play. A particularly noteworthy situation occurs when the responsive active force counterbalances the frictional force, leading to stable oscillation with a constant amplitude that neither decays nor expands. Such a state represents a stable equilibrium that is sometimes observed in swing sports. An intriguing question is whether our active-force model (Eq. (12) and (13)) can accurately simulate this condition.

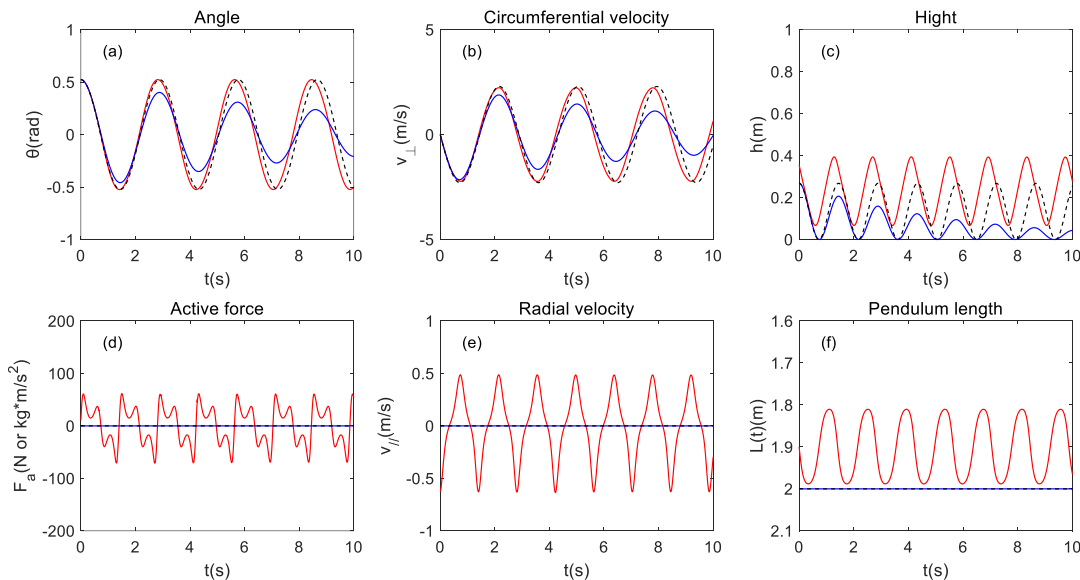


Fig. 5: Active-motion simulations using Eqns. (12-16) showcasing a child pumping a swing in the presence of finite friction, as shown by the solid red lines. To serve as comparisons, the solid blue lines (indicating finite friction) and dashed black lines (representing no friction)

illustrate the simple pendulum motion of a stone with the same mass as the child. The parameters used for these simulations are listed in Table II.

As shown in Fig. 5, the active-force model successfully replicates this scenario. Specifically, for the stone without the active force, the swing amplitude demonstrated a decay (as shown by the solid blue lines) when compared to the harmonic oscillations without friction (represented by dashed black lines). In contrast, for a child exerting an appropriately strong responsive active force (denoted by solid red lines), the swing amplitude remains constant (as seen in images (a) to (c)). This essentially means that, in this situation, the energy pumped from the responsive active force successfully counteracted the damping effect of friction.

Initially, it can be inferred that the frictional force is counterbalanced by the responsive active force, implying that only the net active force minus the frictional force needs to be considered. However, this was not the case. As is evident from Fig. 5, notable differences exist between the balance of the active force and frictional contributions to the swing amplitude (solid red lines) and the simple harmonic motion, where both forces perfectly nullify each other (dashed black lines). These discrepancies are manifested in the smaller period of the former case (2.82 s) by 2% compared with the latter case (2.89 s), despite a less than 1/500 difference between their amplitudes. This is intriguing, as a larger amplitude generally corresponds to a larger period in a scenario devoid of an active force. Nevertheless, according to the simple relation $T = 2\pi\sqrt{L/g}$, the nontrivial smaller period for the child is brought about by the reduction in the pendulum length owing to the standing squatting motions. The root cause of this is the frictional force comprising both the radial and circumferential components, in contrast to the active force, which is exclusively radial. This results in the impracticality of perfectly counteracting the frictional force regardless of the form of the radial active force. The actual scenario involves achieving an energy balance with the radial work executed by the active force compensating for the energy loss induced by the circumferential component of the frictional force.

Table II: The parameters selected for the swing simulations involving a stone (zero active force) and a child (finite active force) under conditions of finite friction, as shown in Fig. 5.

System	Mass	Maximum length	Gravitational acceleration	Friction coefficient	Standing height	Pulse angle	Pulse velocity	Initial angle	Initial velocity
	M (kg)	L_0 (m)	$g(\text{kg} \cdot \text{s}^{-2})$	$b(\text{kg} \cdot \text{s}^{-1})$	k	c_a	$c_b(\text{s}^{-1})$	$\theta(0)$	$\dot{\theta}(0)$
Stone	20	2	9.8	3.64	0	—	—	$\pi/6$	0
Child	20	2	9.8	3.64	0.2	0.12	0.20	$\pi/6$	0

We now turn to the primary parameters that determine the amplitude of the swing propelled by the active force. Holding constant parameters such as the child's mass, swing length, resistance, and initial conditions, we found that the child's center of mass elevation (the height difference between the standing and squatting positions) significantly influences the amplitude. For instance, Fig. 6(a) illustrates that when the standing amplitude surpasses 0.2 meters, the oscillation of the swing gradually increases (as indicated by the blue lines). Conversely, the oscillation diminishes progressively when the amplitude drops below 0.2

meters (indicated by dashed black lines). On the other hand, when the difference in the center of mass height between standing and squatting is fixed at 0.2 meters for each swing (Fig. 6f), the speed of standing (c_a) and squatting (c_b) also affects the amplitude (Fig. 6d). The faster the standing and squatting motions are, the larger the magnitudes are. Therefore, beyond the typical factors influencing both animate and inanimate systems, such as mass, swing length, resistance, and initial conditions, the pivotal determinant of the amplitude in a swing propelled by standing is the difference in the center of mass height between standing and squatting and the standing and squatting speeds.

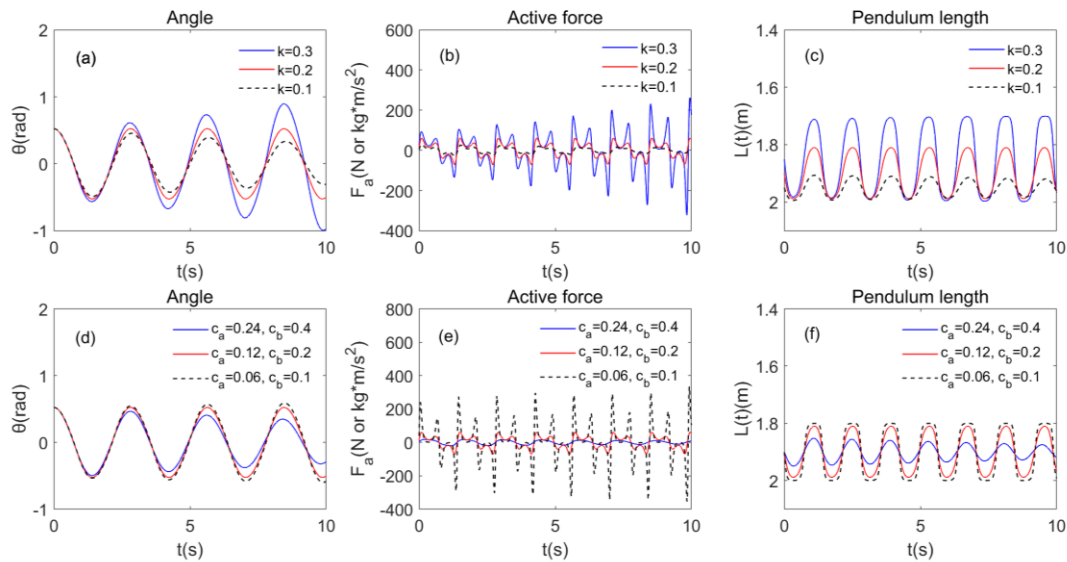


Fig. 6: The amplitude alterations attributable to height (k) variation ((a)-(c)) or speed (c_a and c_b) changes in the standing-squatting motion ((d)-(f)). The adjusted parameters are denoted in legends. Other parameters align with those specified for active motions in Table II.

DISCUSSION AND IMPLICATION

This paper introduces a compelling argument for the existence and necessity of the 'active force' concept to elucidate the dynamics of living objects in the living state, using the human-pumping swing action as an example. This active force, originating from the interactions among the body, mind, and environment, provides a unique contribution to the dynamics of living systems, a facet that conventional physics often disregards. We established a comprehensive mechanical framework applicable to living and nonliving systems by incorporating an active force into Newton's second law. This innovative governing equation was applied to describe the swing motions of a child and an equally weighted stone, focusing on their differences. Our study reveals that children with active force can sustain or augment their amplitude despite friction, whereas a stone's oscillation can only decrease. Quantitative analysis indicated that the energy pumping caused by the responsive active force (i.e., the total active force of the body) performing work in each standing motion was responsible for this phenomenon.

Furthermore, we identified that the responsive active force exhibits a pulse-like pattern, whereas a linear style (e.g., $l = -k\theta$) fails to generate a self-excited oscillation. Therefore,

beyond the typical factors influencing both animate and inanimate systems, such as mass, swing length, resistance, and initial conditions, the pivotal determinant of the amplitude in a swing propelled by standing is the difference in the center of mass height between standing and squatting and the standing and squatting speeds. These findings underscore the existence and necessity of the 'active force' concept for understanding living system dynamics, offering a paradigm and mechanical framework for further exploration into life mechanics [16].

However, this research constitutes a preliminary study of the global (responsive) active force. Future research should first probe further into the quantitative measurements and calculations of both internal (i.e., local) and responsive active forces in the self-determined motions of living beings, such as development, metabolism, and sports in plants, animals, and humans. Such efforts may reveal novel patterns and mechanisms of both active forces as well as their critical quantitative relations, laying a scientific foundation for predicting and controlling life movements. Furthermore, although the current model prioritizes the active force to emphasize its significance, it does not neglect the effects of swing constraints but can address the impacts of both body and environmental constraints. More importantly, by studying the quantitative mechanical relation between the internal and the responsive active forces in future research, we can clarify the physical mechanism of the environmental constraints or, in other words, the body-environment interaction.

It is crucial to remember that the active force discussed herein represents the summation of all the active forces experienced by the human body's center of mass. Actual measurements might necessitate measuring the contact at multiple points between the human body and the interacting object, such as the contact of the feet and hands with a swing. By vectorially synthesizing the forces at multiple points, we can obtain the resultant force at the center of mass, as discussed in this paper. Consequently, further theoretical development should consider the movement of multiple body parts under forces acting at various points on the human body via Eq. (1). In this context, an immediate question would be to investigate the active force pattern in 'pumping a swing from a seated position' and compare it with the results of the present study. This exploration would necessitate considering three subsystems of the human body: the torso, thighs, and lower legs [25,31].

Particularly noteworthy is that the active force is a pivotal control mechanism of the mind over the body during human motion. Optimal movement control is often characterized by the timeliness, strength, and controllability of the active force in response to the mind. Therefore, the practice and measurement of the active force may be a significant topic in the field of sports training and rehabilitation therapy. In this regard, the relationships between active force, bioelectricity, and consciousness present an intriguing topic. The active forces shown in Figs. (4)–(6) exhibit a pulsating characteristic in the time series, reminiscent of the pulsation patterns of bioelectric signals such as ECG and EEG [39]. We hypothesized that this similarity is not coincidental, suggesting that active force indeed embodies the characteristics of life. This is because the active force pattern in the pump swing is a consequence of an individual's conscious decision in their movement mode, which is a product of the brain and heart's electrical activities. In future studies, we will propose simultaneous measurements and mathematical modeling of active force with EEG and ECG signals to investigate their

quantitative correlations, potentially advancing our understanding of the underlying physical mechanisms connecting active force, bioelectricity, and human conscious movement.

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Data Access Statement

All relevant data are within the paper.

Conflict of Interest Declaration

The authors have declared that no competing interests exist.

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