

One Simple Cosmological Implication of Light Transmission in Hubble's Law

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ABSTRACT

Hubble's law gives a great indication that the Universe expands and will do so in the near future. In this "Paper" we try to extract information from Hubble's law and so to retrace its forward and backward steps. The most "striking" result for the universe is that the "theory" predicts a maximum radius of the universe and a maximum life-time. In addition, one can predict if there is causality in between two points (t_1, d_1) and (t_2, d_2) of space time.

Keywords: Universe "radius", Universe "life-time", Cosmology, FLRW-metric, "Causality".

INTRODUCTION

If the life-time of the Universe is supposed to be 1yr (real time ~ 10 Gyrs) then the observation time of Hubble's law is 3,15 sec (real time ~ 1000 yrs).

Therefore, the present observation time is 10^{-7} times, the "real" life time of the Universe.

In the rest $(1-10^{-7})$ time – interval, the law may be altered and also it is not sure if the Universe is going to have the same behavior – For instance the expansion may stop or start to contract.

One the other hand, what is different from the "3,15 sec" of observation to the "1yr" life-time? The answer is probably nothing! Therefore, we will presently trust Hubble's law and accept it trying in parallel from its evolution to "see" if it gives reasonable results (For instance in the initial stages of the Universe) - This will be an indirect but strong confirmation of its validity.

In the "usual" ("normal") theories of physics one handles a set of equations (usually differential) and from this point, after the solution, he tries to "support" theoretically an experiment or an observation.

On the contrary, if one knows Hubble's law which is an observational-experimental fact, then he is called, together with other information, probably to "predict" the past and the future of the Universe.

If someone is at a point of the Universe and decides to move through a "constant" direction then he is probably moving at the direction of a geodesic (What is a geodesic? [1, 2]) which is the curve of the shortest "distance" between two points "inside" a certain $(3+1)$ manifold.

Of course, we may do not know the manifold itself of the space-time but we know its dimension which is $(3+1)$ - space-time.

This manifold $(3+1)$ to be not confused with the $R^3 \times R$ structure of the Euclidean affine space-time of classical Mechanics.

The last one may be a tangent manifold of the first, at least at the points where gravity is not very strong.

He who has decided to make the “trip” along a “constant” vector, he will after tenths of billions of years, return to the same space point. This exactly shows the peculiarity of the $3+1$ space, especially the three-dimensional space. [Strange geometry (General Relativity)].

This reveals one more fact: According to Hubble’s Law all the far galaxies move, as the Doppler effect says, far from the observational point and of one another.

What happens? Are we in a privileged point of the Universe? [like for instance with the geocentric planetary system] or as it happens, the same thing is true for all the points of the Universe? Therefore, one can say, every point of the Universe is a center of the Universe.

The dimensionality $(3+1)$ of the manifold and the manifold itself are so complicated that cannot accept their visualization although the dimensions of the usual space time $R^3 \times R$ is also four-dimensional.

Hopefully there is a paradigm (example) of a $(2+1)$ manifold that resembles the $(3+1)$ manifold of the Universe: Let’s imagine an elastic toy-bubble which is blowed in by a kid for instance. If we write points at a lot of places on the bubble then if the kid starts to blow, the spots will all move one apart the other.

In this toy the spots are the galaxies (and in general the Universe objects) and the bubble is the space-time (manifold) or more loosely Hubble’s Law. One more thing has to be mentioned somebody who is on the bubble has no experience of the “third” (radial – 3D) direction. He is an ant in a surface.

MAIN THEME

- A. In the late 1910’s and 1920’s Hubble’s law was introduced experimentally (observationally) by Edwin Hubble (Using Doppler effect) and theoretically (Using Einstein’s Equations) by Georges Lemaitre. The contribution of Vesto Slipher (far galaxies red - shift) not also to be forgotten.

This results in, that far Galaxies (mainly quasars) display red shift in their spectra and the more far they are the stronger the shift to the red (which is always red) which finally means departure.

Finally, the formula

$$v = H \cdot d \quad (1)$$

was proposed where d is the radial distance from the observation point and v is the radial velocity of the [far] galaxy. [Actually, for far galaxies there is only radial distance and radial velocity due to the isotropic (and homogeneous) expansion of space – time which in our case is expressed by Hubble's law].

As it also comes out from the formula (1), H is a constant which turns out to be between $50 \sim 100$ km/sec/Mpc with more dominant value

$$H_0 \approx 80 \text{ km/sec/Mpc} \approx 2,6 \cdot 10^{-18} \text{ sec}^{-1} \quad (2)$$

- B. Suppose two space – points A and B and time t at space distance d . At a later time t' the distance is going to be $d' = d(t')$ where $d' = d(t') > d(t) = d$ ($t' > t$).

This happens for any two space – time points. How this is possible to happen for any two points? The answer is that the geometry of the Universe is radically different than our experience which is this of the $R^3 \times R$ space time.

The Euclidean (or better affine) space time can only be realized locally in the Universe and probably can only be realized there where there is no strong gravitational field.

This is the reason why we cannot “get rid of” theories like “General Relativity” that take into account the whole “Universe” and not only local “slices” of it.

- C. Since by Hubble's law $v = H \cdot d$ [5, 6] the velocity of the far galaxies (also) must obey the laws of physics and since they are constituted by material objects, we must necessarily have $v \leq c$ which implies:

$$d < \frac{c}{H} \quad (3)$$

i.e., a maximum value for d :

$$d_{max} = \frac{c}{H} \quad (4)$$

For a value of H which is given by (2) we get $d_{max} \approx 12,24$ Gly. However, the most far Galaxy observed is at a distance $d_{max} \approx 13,8$ Gly [6].

Solving inversely, we get for this Hubble constant

$$H \approx 70,956 \text{ km/sec/Mpc} \quad (5)$$

In a later section we will try to make clear what is meant by d_{max} .

- D. The radial distance of a far Galaxy is d and therefore its radial velocity is

$$v = \frac{d}{dt}(d) \quad (6)$$

Combining with Hubble's law we get the expression

$$d(t) = d_0 e^{Ht} \quad (7)$$

where $d_0 = d(0)$. In equation (7) there is a "small" uncertainty: What is time t ? if there is at least one motionless point in the Universe the "naked" time t has a meaning, otherwise no for this simple "theory".

E. Combining eqs (7) and (1) we have

$$v = Hd_0 e^{Ht} \quad (8)$$

Since we have $v < c$ (for the same reasons as in (3)) then $t < t_{max}$ where

$$t_{max} = \frac{1}{H} \ln\left(\frac{c}{Hd_0}\right) = \frac{1}{H} \ln\left(\frac{d_{max}}{d_0}\right) \quad (9)$$

which gives the "life time" of the Universe (it is of order H^{-1}) which is approximately 13,8 Gyrs. The life time of Universe changes but not dramatically as d_0 changes which is the size of the Universe at $t=0$ (The "present" time of the Universe). Since the function $\ln|x|$ increases very slowly as d_0 decreases it is right to consider t_{max} approximately as H^{-1} .

However as d_0 decreases and especially as $d_0 \rightarrow 0^+$, then $t_m \rightarrow +\infty$ which means that the smaller the initial size, the largest the life time.

The case $d_0 = 0$ cannot be handled but it gives a hint that in this case $t_m \rightarrow \infty$ which finally means that the smaller the initial size of the Universe, the larger its future!

Finally, this probably means that the Universe cannot have been started by a point and subsequently it has an end.

F.

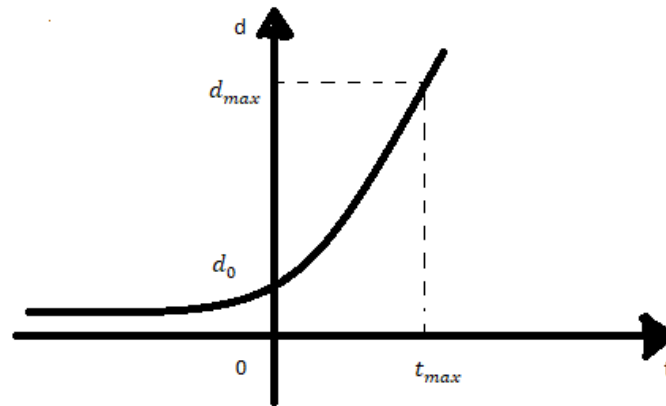


Figure 1

G. The function $d = d(t)$ with its end at $t = t_m, d = d_m$. Then a constant situation arises.

It is known from cosmology, FLRW metric and the resulting General Relativity Eqs. that

$$\left(\frac{\dot{\alpha}}{\alpha}\right)^2 = H^2 = H_0^2 (\Omega_{oR}\alpha^{-4} + \Omega_{oM}\alpha^{-3} + \Omega_{oK}\alpha^{-2} + \Omega_{o\Lambda}) \quad (10)$$

[Actually the First Friedmann Eq] [3] where α is the scaled radius of the Universe, Ω_{oR} is the radiation density (today), Ω_{oM} the matter (dark + baryonic) density, Ω_{oK} is the “spatial curvature” density and $\Omega_{o\Lambda}$ is the cosmological constant where:

$$1 = \Omega_{oR} + \Omega_{oM} + \Omega_{oK} + \Omega_{o\Lambda} \quad (11)$$

Various cases have been studied from time to time and place to place. We will focus in one: this for which

$$\Omega_{o\Lambda} \gg \Omega_{oR}\alpha^{-4}, \Omega_{oM}\alpha^{-3}, \Omega_{oK}\alpha^{-2} \quad (12)$$

Known as the Λ – dominant (or dark – energy – dominant) equation for which we have $\left(\frac{\dot{\alpha}}{\alpha}\right)^2 \approx H^2 \approx H_0^2 \Omega_{o\Lambda}^2$ which implies $\frac{\dot{\alpha}}{\alpha} \approx H_0 \Omega_{o\Lambda}^{\frac{1}{2}}$ and subsequently:

$$a(t) \approx \alpha_i e^{H_0 \Omega_{o\Lambda}^{\frac{1}{2}} t} \quad (13)$$

From equation (11) $\Omega_{o\Lambda} \approx 1$ & $\Omega_{oR} = \Omega_{oM} = \Omega_{oK} = 0$ and since $a(t) = \frac{d(t)}{d_{scale}}$ we get $d(t) \approx d_0 e^{H_0 t}$ which is “exactly” equation (7).

Therefore the Λ – dominant eq. (dark energy) can explain Hubble’s law. However, things are not “so good” as they seem by a first glance: Looking at the limitation (12), they are

valid only if $\alpha \gg 1$ which means that the eq. (7) is true only in this regime and is not suitable for the initial moments of the Universe.

On the other hand if we are sure that Hubble's law is valid (for instance from observation) then we are sure that we are far from the initial "space - time - point" of the Universe.

- H. Let's suppose two space time points A and B where $A = (t_1, d_1)$ and $B = (t_2, d_2)$ with $t_2 > t_1$ and $d_2 > d_1$. Since they have both departed by initial surface " d_0 " we must have

$$d_1 = d(t_1) = d_0 e^{Ht_1} \quad (14a)$$

$$d_2 = d(t_2) = d_0 e^{Ht_2} \quad (14b)$$

Please note that the time t_1 is not the same like t_2 while we have supposed a "simultaneous" existence. Let us also suppose that a light signal moves from point A to point B.

Is it ever going to reach B? The velocity of light is very large but also the point B moves at a very high rate (exponentially) and therefore the above statement is not sure.

As the light travels from A to B the point B is going to be replaced at the point B' at which the adoption of the photon will be made. Also, at "this time" the point A will have been replaced to the point A'.

Totally we can write

$$d_1' = d(t_1') = d_0 e^{Ht_1'} : A' \quad (15a)$$

$$d_2' = d(t_2') = d_0 e^{Ht_2'} : B' \quad (15b)$$

So we eventually have a light transition from the point A to the point B'

I.

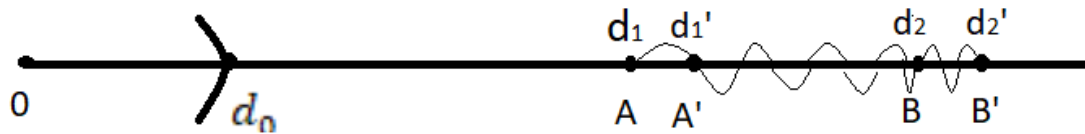


Figure 2

$$\begin{array}{ccc} t_1' > t_1 & t_2' > t_2 & t_2 > t_1 \\ d_1' > d_1 & d_2' > d_2 & d_2 > d_1 \end{array}$$

- J. Combining eqs. (14a) & (15b) and the fact, that light travels the distance $c(t_2' - t_1) = c\Delta t'$ between two instants t_1 & t_2' we have

$$c\Delta t' = \Delta d' = d_2' - d_1 = d_1(e^{H\Delta t'} - 1) \quad (16)$$

Making the substitution

$$\Delta \xi' = H\Delta t' \text{ ("time")} \text{ and } \chi_1 = \frac{Hd_1}{c} \text{ ("distance")}$$

we get

$$\Delta \xi' = \chi_1(e^{\Delta \xi'} - 1) \quad (17)$$

if there is a solution $\Delta \xi' = \Delta \xi'_r$ of eq. 17 then a transition from the point A to the point B' is possible. Otherwise it is not. (The Universe moves "too fast").

The scaled distance χ_1 can be anything: it is enough to be greater than $\chi(d_0)$. Therefore the results of formula (17) are valid for any time t_1 and any distance d_1 provided that $d_1 = d_0 e^{Ht_1}$.

Defining the functions:

$$f_1(x) = e^x - 1 \text{ and } f_2(x) = \frac{1}{\chi} x$$

We can "solve" the equation

$$(e^{\Delta \xi_{r'}} - 1) = \frac{1}{\chi} \Delta \xi_{r'} \quad (18)$$

where (r) stands for the word root meaning the solution of eq. (18).

Making inspection of the two graphs we can relatively easy to recognize that (eq. 18) has one solution if $\chi_1 < 1$, i.e. $d_1 < \frac{c}{H}$ [3]

- K. See Figure 3

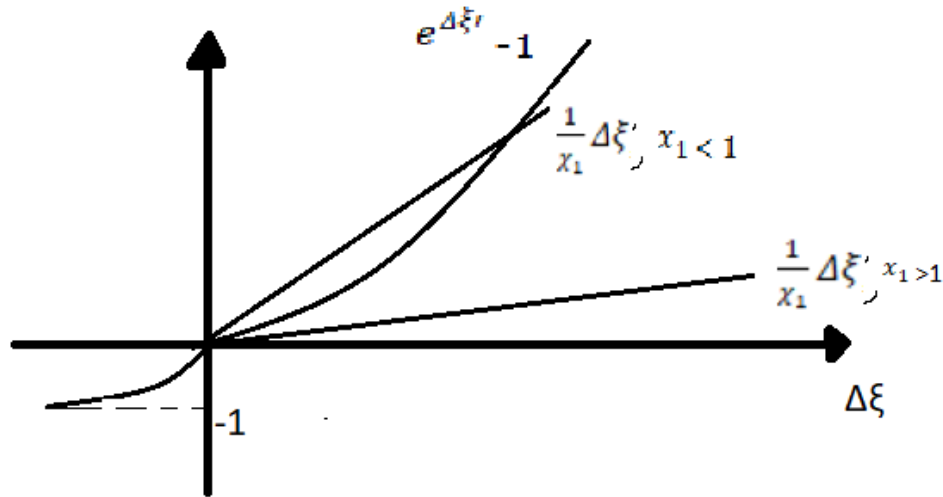


Figure 3

Since $d_0 < d_1 < d_2 < \dots$, if $d_0 > \frac{c}{H}$ the whole Universe cannot accept a light signal solution i.e. it cannot be causally connected.

In general we must have

$$d(t) < \frac{c}{H} \quad (19)$$

in order to have a causal relation. This means that the Universe from every point is extended to the length $L = \frac{c}{H}$. After this length there is no meaning if the Universe exists or there is “another” Universe or anything else since no body can communicate with this part.

- L. It is well – known for the first Friedmann Eq. [3] that $(\frac{\dot{a}}{a})^2 = H_{(a)}^2$, where the right hand side is given by the right hand side of eq. (10) which is positive and therefore so is the left hand side. From this we have

$$\dot{\alpha} = \pm \alpha H(\alpha) \text{ with } H(\alpha) > 0 \quad (20)$$

Since $H(\alpha) > 0$, the unknown function “ α ” of the o.d.e. with the positive sign is increasing and since $H(\alpha)$ has no roots we have

$$\begin{array}{c} t \rightarrow \infty \\ \alpha(t) \text{ -----} \rightarrow \infty. \end{array}$$

On the other hand the o.d.e. for the same “ but opposite” reasons is decreasing and

$$\begin{matrix} t \rightarrow \infty \\ \alpha(t) \text{-----} \rightarrow 0. \end{matrix}$$

From this analysis, one should say that the “radius” of the Universe is either increasing or decreasing, in the second case starting from a “point $\alpha_0 > 0$ ”, it goes to zero (shrinks) – excluding the case of a Universe that oscillates between (α_{min}) and (α_{max}) or blows and then shrinks or any other intermediate behavior.

We will see that probably things are not so:

The first Friedmann Eq. is:

$$\dot{\alpha}^2 = \alpha^2 H(\alpha) \quad (20\beta)$$

If one solves the algebraic eq. $\Delta^2 = 1$ the solution is $\Delta = \pm 1$.

However, if one solves the same equation with $\Delta = \Delta(t)$ (a function) then the solution (piece – wise constant) is: $\dot{\alpha} = \alpha H(\alpha) \Delta(t)$ where

$$\Delta(t) = \begin{cases} (+1) & t \in (t_{i-1}, t_i) = S_{i-1} \\ (-1) & t \in [t_i, t_{i+1}] = S_i \end{cases} \quad (21)$$

The values ± 1 come alternatively in the function sequence and besides

$$\begin{matrix} S_{i-1} \cap S_i = \emptyset \\ \infty \\ \cup_{i=1} S_i = R^+ \end{matrix} \quad (22)$$

The “ends” of intervals S_i may be chosen in various ways (i.e. they are not unique and may represent appropriately chosen closed, open or semi – open intervals)

By inspection of eq. (21) we see easily that α is increasing in S_{i-1} and decreasing in S_i and so forth a fact that supports the idea of totally or a partially oscillating Universe.

Function $\Delta(t)$ is not only piecewise constant getting the alternative values ± 1 but from a mathematical point of view it is also piece – wise continuous. This is a known “problem” for mathematicians wanting continuous functions and even more differentiable.

This is not a problem: take for instance

$$\begin{matrix} \dot{\alpha}^{(i-1)} = \alpha^{(i-1)} H_{(\alpha)}^{(i-1)} & t \in (t_{i-1}, t_i) \\ \dot{\alpha}^{(i)} = - \alpha^{(i)} H_{(\alpha)}^{(i)} & t \in [t_i, t_{i+1}] \end{matrix}$$

If the i -th eq. is solved then we both know $\alpha_{(t)}^{(i-1)}$ and $\alpha_{(t_i)}^{(i-1)}$. From continuity requirements (of $\alpha(t)$ and not of $\Delta(t)$) we can put $\alpha_{(t_i)}^{(i-1)} = \alpha_{(t_i)}^{(i)}$ so we have the initial condition $\alpha_{(t_i)}^{(i)}$ for the $(i$ -th) o.d.e. which can be solved in principle.

So far, we have seen that the behavior of the “radius” of the Universe depends crucially on the division of subintervals of the total interval $[0, +\infty)$ leading to behaviors like completely increasing completely decreasing, totally or partially oscillating.

Putting it in another way:

The o.d.e., the numbers ± 1 , the initial condition and the set $\{t_i\}_{i=1}^N$ determine the behavior of the function $\alpha(t)$. If we are in the case of “oscillations” what is the underlying physics which determines the set $\{t_i\}_i$? We probably need more physics!

CONCLUSION(S)

Trusting Hubble’s Law as being valid in all stages of the Universe we conclude that:

- a) Using first Friedmann’s eqs. Hubble’s law is compatible with the Λ – dominant Universe.
- b) The Universe (from any point) can only reach a maximum “radius” (of approximately 13,8 Gly) and as one “exceeds” it, the “Causality” is lost i.e. no light signal can exist going further than this “radius”.
- c) The “lifetime” of the Universe is finite unless the radius of “now” is approximately zero ($d_0 \rightarrow 0^+$). In this case the life – time is infinite.
- d) According to Friedmann’s first eq. as it is modified (by pressure and density considerations) the radius of the Universe can:
 - e) Increase monotonically
 - f) Decrease monotonically
 - g) Oscillate partially or totally
 - h) For the case d(iii) we need to know more physics [4].

Acknowledgments

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