

A Theoretical Study of Solenoid Inductive Effect on Propagation Delay in Closed Feedback Control

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ABSTRACT

Solenoid is widely and commonly used electro-mechanical devices in industrial processes and engines for hydraulic, fuel and pneumatic controls. The speed, reliability and response time of solenoids can be used to characterize their performance especially where propagation delay can cause instability in the system. In this paper, we model the dynamic characteristics of a proportional solenoid, and investigate its effect on the overall response time in closed loop. The transfer function of the proportional translational motion solenoid at different positions of the plunger is derived. Appropriate boundary conditions are applied to solve transients and steady state current and deduce the response time of the system. At different resistance and inductance, the results show that the rising and settling times of a solenoid system is inversely to the values of Resistance-Inductance (R-L) pair. The lower the values of R-L pair, the higher the rising and settling times. The transient current is shown to be inversely proportional to the values of R-L pair. Solenoid with lowest resistance has the highest transient current flowing through it. These indicate that the selection of solenoid R-L parameters are important consideration in closed loop control system designs.

Keywords: Solenoids, Propagation delay, Resistance-Inductance (R-L), Transients, Closed loop, Kirchoff's law, Transfer Function.

INTRODUCTION

Feedback control systems are widely used class of control systems with wide area of applications from home and offices to industries and manufacturing processes. Control systems can be either open loop system or a closed loop system [1],[2] (Canete et al., 2018; Phillips et al., 1990). The performance of a controller (digital or analogue) is the response time between when a command is issued and when the control action is taken place. Although, there are many indicators that can be used to measure controller's performance (Sternfield and Gates, 1950) [3]. Time delay affects controller performance and stability of the control system. Timely information is crucial in modern process plants, digital controllers widely used for their flexibility, speed and efficiency to deliver precise control signals to actuators to alter the behaviour of a plant or processes. Discrete-time processors are computers that sample

analogue data through its input and discretized into a finite but quantifiable values for digital processing. The results of the computations are converted back to the analogue form to correct or maintain the position of an actuator.

Digital control system works on difference equation and discrete-time processing. The study of physical and engineering systems usually requires the use of mathematical model that describes the static and dynamical behaviour of any physical systems.

Different mathematical models have been proposed for modelling the dynamic response of control systems closed loop control systems. Many physical systems can be modelled using the linear time-invariant (LTI) approach in both continuous and discrete time domains (Canete et al., 2018; Phillips et al., 1990) [1],[2]. LTI combines the properties of linearity and time-invariants to simplify mathematical modelling, analysis and synthesis.

Linearity imposes the condition on the control system that the output is linearly related to the output (Dinuzza, 2013; Moudgalya, 2007) [4],[5]. LTI has been applied analyze stability and performance requirements in absolutely distinguishable discrete dynamic systems (Rosa and Silvestre, 2011) [6]. In digital control theory, modeling, designing, analysis and implementation is done in discrete time domain (Tahir et al., 2018; Moudgalya, 2007) [5],[7]. Linear time-invariant (LTI) model has been used for synthesis of automated digital controller to guarantee safety and stability in automatic control systems (Abate et al., 2017) [8]. LTI model of analyzing controllers and control systems represents a class of wide range of dynamical system that has applications in different domains especially in continuous time LTI. However, the use of LTI in digital control system design and synthesis present a new challenge as a result of the effect of finite representation of signal and quantization noise during digital to analogue or vice versa conversions (Abate et al., 2017) [8]. Khargonekar et al. (1985), The analysis of robust control of linear time-invariant plant using both time-varying and time-invariant, stated that time varying controller provides better performance over time-invariant ones for large classes of control problems [9]. In process plant and manufacturing industries, digital controls such as the programmable logic controller (PLC) drive electro-mechanical actuators to effect control characteristics. Actuators are electromechanical devices that converts electrical energy into mechanical energy using magnetic field as an intermediary force.

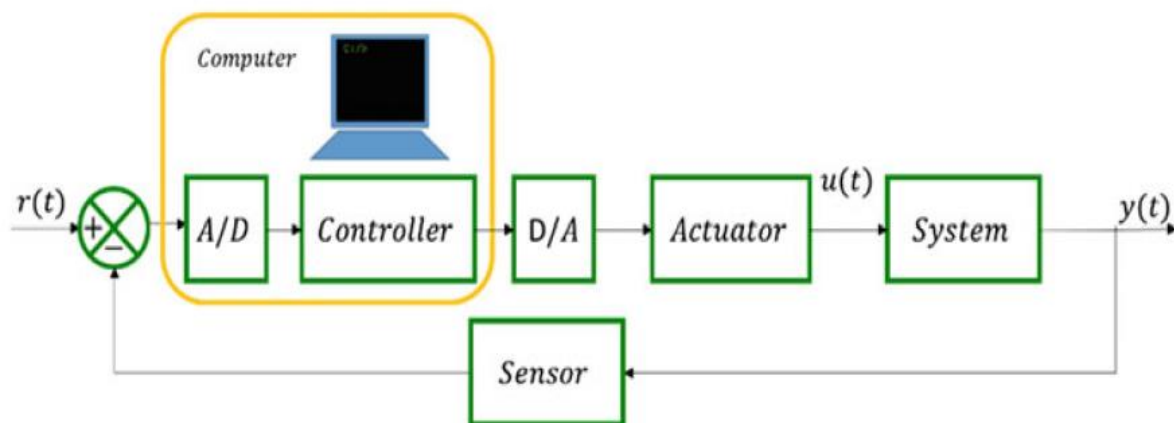


Fig.1: Discrete time closed loop automatic control system [1] (Canete et al., 2018).

A general block diagram of a closed loop control system is shown in Fig.1 where the $r(t)$ is the reference input signal in time domain, $u(t)$ is the response of the actuator as a function of the controller's action and $y(t)$ is the controlled variable consequence of the actuator in the control loop.

In this paper, we focus on the performance of the actuator (Solenoid) and its effect on the propagation delay of the control system. Solenoid is basically an electromechanical system that mechanically open or close an orifice when energized with electrical energy. The device can be modelled as a resistance and variable inductance connected in series. The mechanical section is represented in terms of forces namely the damping force, the force due to electrical energy, the gravitational force (mg) and the force stored in the string. Plunger interferes with the inductance of the coil as it moves through the sea of magnetic field inside the solenoid (Xu et al., 2017, Schimpf, 2013; Zieba, 2003) [10],[11],[12].

This work develops a mathematical model of Solenoid actuator at different positions to study transient and steady state characteristics of the device. This paper is organised in four (4) sections. Section II presents the theory of Solenoid, models, and the derivation of the solenoid characteristics at different plunger position. In section III, the methods and approach used in the modelling and simulations are explained. The results of the modelling using MATLAB presented and discussed in Section IV. Section V.

THEORETICAL ANALYSIS

In this work, we consider linear time invariant Single Input Single Output (SISO) closed loop control systems in our system analysis and modelling. In control theory, Laplace transform is an integral transformation theory that has provided a powerful mathematical technique for analysing system-based problems by transforming differential equations into algebraic equations (Redhwan et al., 2020) [13].

Laplace transform is expressed as:

$$\mathcal{L}[f(t)] = \int_0^{\infty} f(t)e^{-st} dt \quad (1)$$

Using the Laplace transform equation, differential operators can be translated into algebraic equations.

$$\mathcal{L}[g'(t)] = \int_0^{\infty} g'(t)e^{-st} dt = sG(s) - g(0) \quad (2)$$

$$\mathcal{L}[g''(t)] = \int_0^{\infty} g''(t)e^{-st} dt = s^2G(s) - sg(0) - g'(0) \quad (3)$$

Laplace theory is a very convenient tool in the description of any linear time invariant control systems and led to the concept of transfer function (Jaksic and Rusjan, 2008, Olivi, 2006) [14],[15]. The transformation theory simplifies analysis for linear systems. This allow system harmonic identification and system stability to be studied.

The Linear time invariant system can be described in the following equation,

$$y(t) = \int_{-\infty}^0 h(t - \tau)u(\tau)d\tau + \int_0^{\infty} h(t - \tau)u(\tau)d\tau \quad (4)$$

Where $y(t)$ is the output response of the systems, $u(\tau)$ is the reference input signals and $h(t)$ is the impulse response of the system,

The transformed version of Eq. (4) is given as,

$$Y(s) = H(S)U(s) \quad (5)$$

The transfer function of a system which is the ratio of Laplace transformation of the output to the Laplace transformation of the input can be deduced from Eq. (5).

$$H(S) = \frac{Y(s)}{U(s)} \quad (6)$$

Transfer function analysis describes the dynamics of a systems. It relates the system's output to the input as described in Eq. (5) and Eq. (6). Once the transfer function is known, the system behavior can be determined for an arbitrary input (Jaksic and Rusjan, 2008) [14].

MATERIALS AND METHODS

Transfer Function

In this section, solenoid is modelled as an equivalent R-L circuit as shown in Fig.3. The mechanical component of the solenoid, the plunger, is represented by the mass, M. The applied voltage, V_{in} , energizes the solenoid to actuate the plunger.

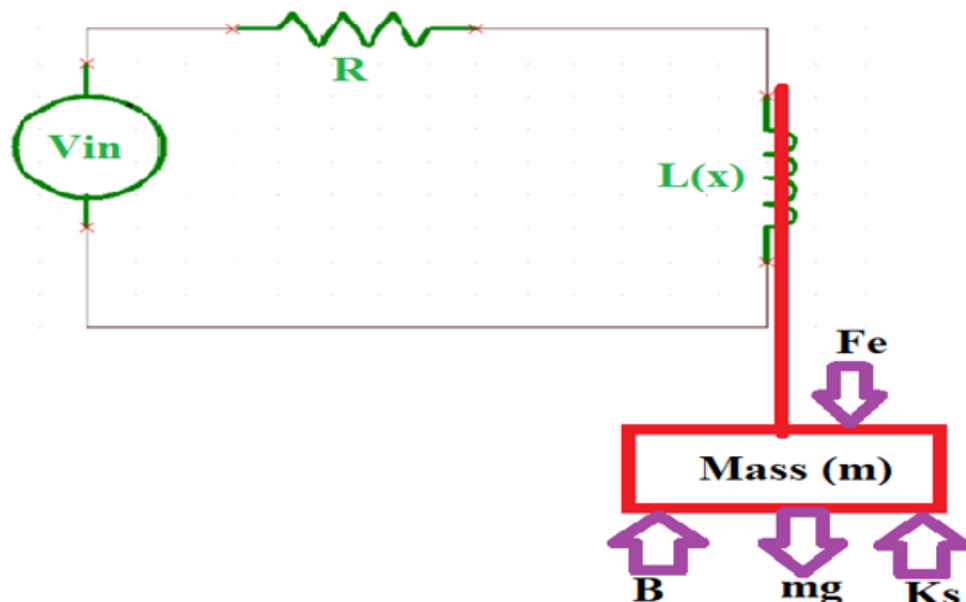


Fig.2. Solenoid R-L equivalent circuit

Applying the Kirchoff's voltage (KVL) to the R-L equivalent circuit shown in Fig.3. The basic assumption here is that inductance is treated as a variable component of the system.

$$V_{in} = Ri + V_L \quad (7)$$

$$V_{in} = Ri + \frac{d}{dt}(L(x).i) \quad (8)$$

$$V_{in} = Ri + L(x)\frac{di}{dt} + i\frac{dL(x)}{dt} \quad (9)$$

$$L(x)\frac{di}{dt} + Ri = V_{in} - i\frac{dL(x)}{dt} \quad (10)$$

Eq. (10) can be represented in terms of speed and change in inductance as,

$$L(x)\frac{di}{dt} + Ri = V_{in} - i.v_a\frac{dL(x)}{dx} \quad (11)$$

where $L(x)$ is the inductance as a function of length, v_a , is the speed of the movable armature, V_{in} is the input supply voltage, R is the resistance of the winding and i is the current flowing through the windings.

To solve equation (11), we considered three (3) cases.

Case 1. Applying conditions, $t = 0$, $v_a = 0$ and $\frac{dL(x)}{dx} = 0$, to Eq. (5) yields,

$$L\frac{di}{dt} + Ri = V_{in} \quad (12)$$

Eq. (12) can be re-written in s-domain as,

$$LSI(s) - I(0) + RI(s) = V_{in}(s) \quad (13)$$

With initial condition, $I(s) = 0$

$$LSI(s) + RI(s) = V_{in}(s) \quad (14)$$

$$I(s)(LS + R) = V_{in}(s) \quad (15)$$

Transfer function is given as;

$$G(s) = \frac{I(s)}{V(s)} = \frac{1}{LS + R} \quad (16)$$

Case 2. Consider a situation where the plunder travels from $x = 0$ to $x = l$. At $x = l$, $v_a = 0$. The variation in inductance can be approximated as (Schimpf, 2013),

$$L(x) = L_0 e^{-\frac{\alpha}{l}x} \quad (17)$$

Then Eq. (12) becomes,

$$L_0 e^{-\alpha} \frac{di}{dt} + Ri = V_{in} \quad (18)$$

Applying Laplace transform to Eq. (18) yields,

$$I(s)(L_0 e^{-\alpha} s + R) = V_{in}(s) \quad (19)$$

$$\left(G(s) = \frac{I(s)}{V(s)} = \frac{1}{L_0 e^{-\alpha} s + R} \right) \quad (20)$$

Case 3. Consider a situation where the plunger travels between $0 < x < l$ and $v_a > 0$. Differentiating Eq. (17) yields,

$$\frac{dL(x)}{dx} = -\frac{\alpha}{l} L_0 e^{-\frac{\alpha}{l}x} \quad (21)$$

Substituting Eq. (21) into Eq. (11) yields,

$$L_0 e^{-\frac{\alpha}{l}x} \frac{di}{dt} + Ri = V_{in} + i \cdot v_a \left(\frac{\alpha}{l} L_0 e^{-\frac{\alpha}{l}x} \right) \quad (22)$$

$$L_0 e^{-\frac{\alpha}{l}x} \frac{di}{dt} + \left(R - v_a \frac{\alpha}{l} L_0 e^{-\frac{\alpha}{l}x} \right) i = V_{in} \quad (23)$$

$$L_0 e^{-\beta x} \frac{di}{dt} + (R - v_a \beta L_0 e^{-\beta x}) i = V_{in} \quad (24)$$

$$\beta = \frac{\alpha}{l} \quad (25)$$

$$I(s)[L_0 e^{-\beta x} s + (R - v_a \beta L_0 e^{-\beta x})] = V_{in}(s) \quad (26)$$

$$\left(G(s) = \frac{1}{[\xi s + (R - v_a \beta \xi)]} \right) \quad (27)$$

$$\xi = L_0 e^{-\beta x} \quad (28)$$

Electrical Current in Solenoid

In this section, we derive the current by considering the type of input source with respect to the cases discussed earlier.

Considering case 1. For a constant DC voltage, $V_{in}(t) = V$ and

$$V_{in}(s) = \frac{V}{s} \quad (29)$$

For an AC supply voltage, $V_{in}(t) = A \sin(\omega t)$ and

$$V_{in}(s) = \frac{A\omega}{s^2 + \omega^2} \quad (30)$$

Substituting Eq. (29) into Eq. (16) and applying partial fraction to obtain the inverse of the Laplace transform yields,

$$i(t) = \frac{V}{R} \left(1 - e^{-\frac{R}{L}t} \right) \quad (31)$$

Eq. (31) gives the transient current.

Similarly, substituting Eq. (30) into Eq. (16) yields,

$$I(s) = \frac{A\omega}{(s^2 + \omega^2)(Ls + R)} \quad (32)$$

$$I(s) = \frac{A\omega}{(s + j\omega)(s - j\omega)(Ls + R)} \quad (33)$$

By applying partial fraction and taking the Laplace inverse, we have:

$$i(t) = \frac{A\omega}{2\omega L \left(j\frac{R}{L} - \omega \right)} e^{j\omega t} - \frac{A\omega}{2\omega L \left(j\frac{R}{L} + \omega \right)} e^{-j\omega t} + \frac{A\omega}{L \left(\frac{R^2}{L^2} + \omega^2 \right)} e^{-\frac{R}{L}t} \quad (34)$$

Considering case 2. Substituting Eq. (29) and Eq. (30) into Eq. (20) yields the following set of equations,

$$I(s) = \frac{V}{s(L_0 e^{-\alpha} S + R)} \quad (35)$$

$$I(s) = \frac{A\omega}{(s^2 + \omega^2)(L_0 e^{-\alpha} S + R)} \quad (36)$$

From Eq. (35) and Eq. (36), the inverse of the Laplace transforms yields,

$$i(t) = \frac{V}{R} \left(1 - e^{-\frac{R}{L}t + \alpha} \right) \quad (37)$$

$$i(t) = \frac{A}{2\omega(j\omega L_0 e^{-\alpha} - R)} e^{-j\omega t} - \frac{A}{2\omega(j\omega L_0 e^{-\alpha} + R)} e^{j\omega t} + \frac{A}{\omega R} + \frac{A\omega(L_0 e^{-\alpha})^2}{(R^3 + \omega^2(L_0 e^{-\alpha})^2 R)} e^{-\frac{R}{L_0 e^{-\alpha}}t + \alpha} \quad (38)$$

Considering case 3. Eq. (33) and Eq. (35) are substituted into Eq. (27), we obtain the following set of equations,

$$I(s) = \frac{V}{s[\xi s + (R - v_a \mu \xi)]} \quad (39)$$

$$I(s) = \frac{A\omega}{(s^2 + \omega^2)[\xi s + (R - v_a\mu\xi)]} \quad (40)$$

The inverse of the Laplace transforms of Eq. (39) yields,

$$i(t) = \frac{V}{(R - v_a\mu\xi)} \left(1 - e^{-\frac{(R - v_a\mu\xi)}{\xi}t} \right) \quad (41)$$

And the obtained inverse of Eq. (40) is results to,

$$i(t) = \frac{A}{2\omega[j\omega\xi - (R - v_a\mu\xi)]} e^{-j\omega t} - \frac{A}{2\omega[j\omega\xi + (R - v_a\mu\xi)]} e^{j\omega t} + \frac{A}{\omega R} + \frac{A\omega\xi^2}{[(R - v_a\mu\xi)^3 + \omega^2\xi^2(R - v_a\mu\xi)]} e^{-\frac{(R - v_a\mu\xi)}{\xi}t} \quad (42)$$

In each of the cases, current responses for both DC and AC are derived to simulate the input to output responses of a linear actuated solenoid.

RESULTS AND DISCUSSIONS

In this section, the mathematical modelling is simulated and validated using MATLAB. The transients, step responses and impulse responses of the solenoid are considered. Three (3) cases considered in the modelling of solenoid characteristics, and, in each case, the electric current behavior is obtained.

System Characteristics at Rest

The systems characteristics and dynamics are obtained from Eq. (16) by substituting the values of the resistance and inductance pairs as shown in table 1.

Table 1. R-L theoretical values

	Inductance, L (H)	Resistance, R (Ω)	DC voltage (V)	AC voltage (V)
System 1	0.5	100	24	$240\sin\omega t$
System 2	0.78	200	24	$240\sin\omega t$
System 3	0.98	230	24	$240\sin\omega t$

Eq. (16) gives the step unit function, and using the values in table 1, we plotted and compared the response and determine the effect of inductance and resistance on the performance of Solenoids (System 1, System 2 and System 3). The step function provides information about time delay between when an action is initiated and when the control action is executed. Fig.4 compares the rising times of three (3) solenoids at the moment of step input, when the plunger position is $x=0$. The plots indicate that the system with the lowest R-L combination has the largest rising times of 0.011 Sec compared to other solenoids. Conversely, the largest R-L combination produced the lowest rise time of 0.00936 Sec. The rising time is proportional to the magnitude of R-L as shown in Fig.4. Rising time of an actuator has a significant effect on the performance of closed loop (feedback) control system.

The settling time of solenoids in table 1 is shown in Fig.5. As expected, the settling time describes the steady state of the solenoids. For solenoids with higher values of R-L, their rising and settling times are higher compared to solenoids with lower resistance values. The settling

time of the solenoid with the highest values of R-L of 0.0196 Sec and the solenoid with the lowest values of R-L pair is 0.0167 Sec. The solenoid with average R-L values has a settling time of 0.0153 Sec as shown in Fig.5. As discussed in the previous section, both rising and settling time affects the performances of actuators in feedback control system. This dynamic characteristic of a system affects the overall propagation delay and stability. From the simulation results, system1(Solenoid) represents the system with the largest propagation delay while system 2 has the lowest propagation delay.

At plunger position, $x=0$, for impulse response in fig.6, the curves exhibit a negative slope and decrease exponential at a rate depending in the values of resistance and inductance. The solenoid with the lowest R-L value (System 1) decreases slower than System 2 and System 3 while System 3 decayed faster than System 1 and System 2. Thus, System 3 has the tendency to quickly restore back to its steady state faster than System 1 and System 2 after a brief disturbance.

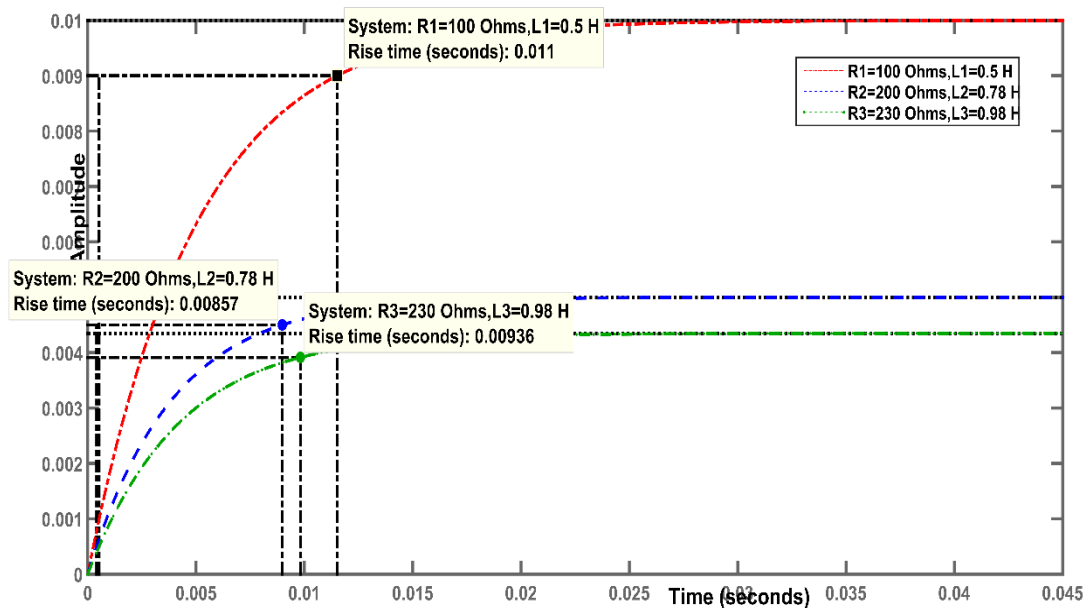


Fig.4. Step unit response of the solenoid at plunger position $x=0$ showing rise time for different values of resistance, R and inductance, H.

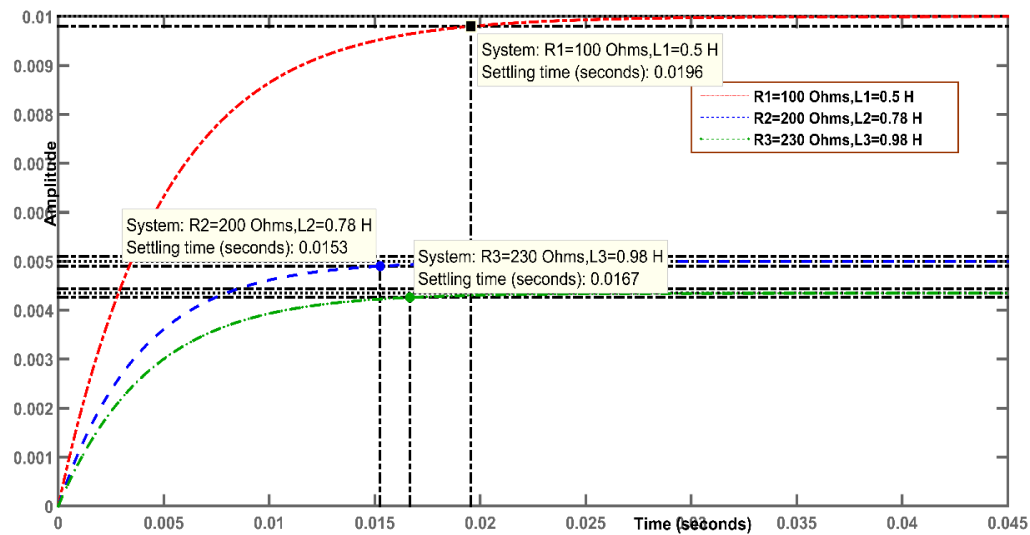


Fig.5. Step unit response of the solenoid at plunger position $x=0$ showing settling for different values of resistance, R and inductance, H .

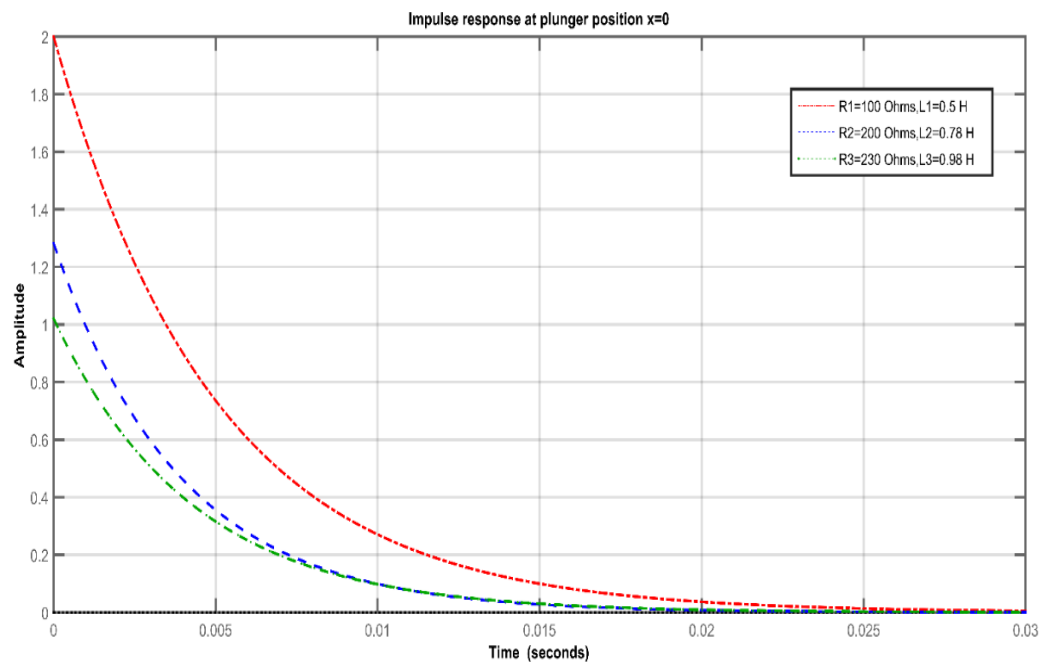


Fig.6. Impulse response of the solenoid at position $x=0$.

Transient Response

From Eq. (31), the transient response of a solenoid at the instance of activation is obtained for a constant voltage, E . We obtained the transient characteristics of each of the solenoids. Fig.7 shows the transient current of each of the solenoids.

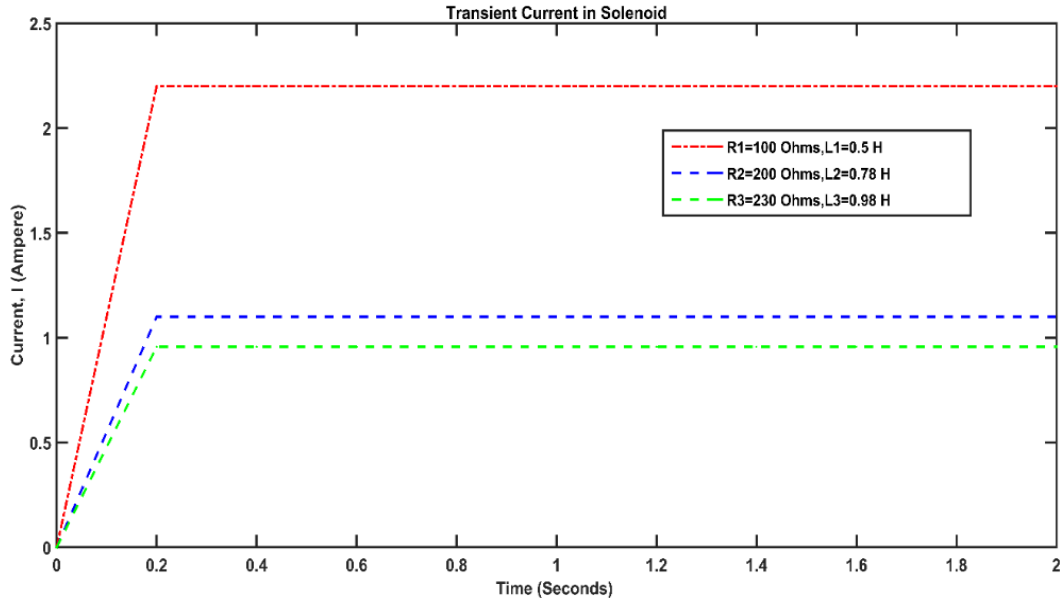


Fig.7. Transient current in solenoid for a constant supply voltage.

From Fig.7, the slopes can be calculated as,

$$\text{Slope} = \frac{I_2 - I_1}{t_2 - t_1} \quad (43)$$

$$\text{Slope1 (System 1)} = \frac{I_2 - I_1}{t_2 - t_1} = \frac{2.25}{0.2} = 11.25 \text{ AS}^{-1} \quad (44)$$

$$\text{Slope2 (System 2)} = \frac{I_2 - I_1}{t_2 - t_1} = \frac{1.2}{0.2} = 6 \text{ AS}^{-1} \quad (45)$$

$$\text{Slope3 (System 3)} = \frac{I_2 - I_1}{t_2 - t_1} = \frac{0.9}{0.2} = 4.5 \text{ AS}^{-1} \quad (46)$$

As shown in Fig.7, System 1 being the one with the lowest values of R-L circuit, has the longest slope of the three Systems. At the instance of DC voltage supply to the solenoid, the current positively raises with time until a steady state is reached at approximately 0.2 Seconds where the current peaks at 2.25A and remain constant throughout. The slope of system 1 is calculated to be 11.25 A/S. System 3 presents the system with the lowest slope of 4.5 A/S at a peak value of 0.9A. System 2 has a slope value of 6 A/S and peak current of 1.2A. System 1,2 and 3 attained steady state at 0.2 Seconds. These results validate the step response system analysis in Figs 4, 5, and 6.

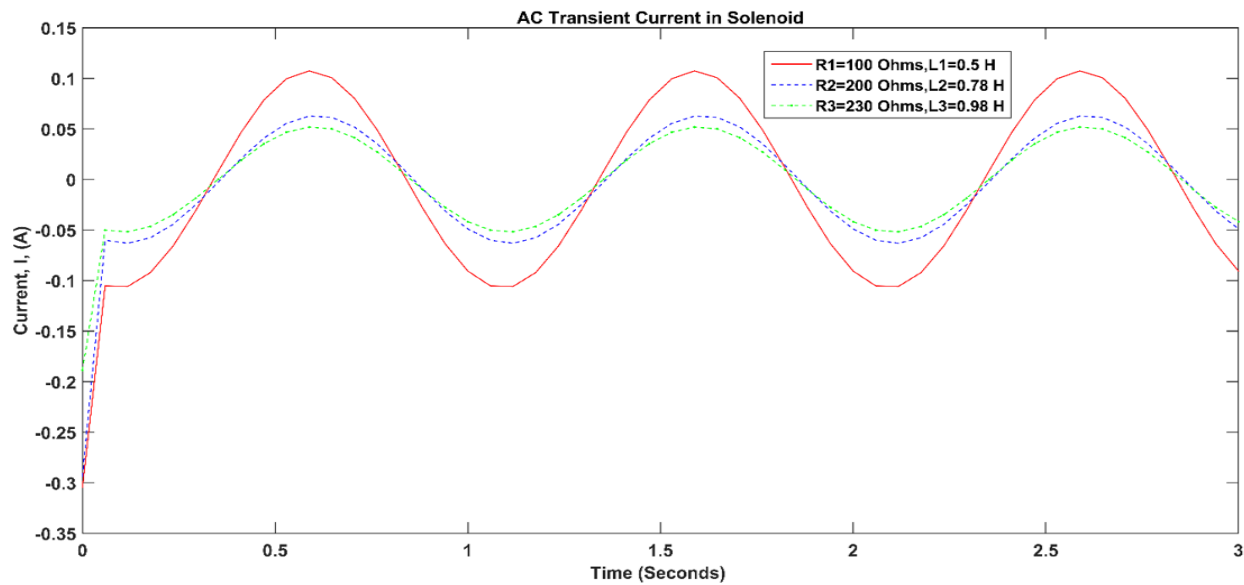


Fig.8. Transient current for arbitrary AC sources.

Eq.28 provides the solution of Eq.27 in terms of time dependent current using alternating current (AC) sources with arbitrary amplitude, A , to the solenoid. The equation for the AC source signal is given as

$$i(t) = \frac{\alpha}{-2\omega(\omega + j\tau)} e^{-j\omega t} + \frac{\alpha}{-2\omega(\omega - j\tau)} e^{j\omega t} + \frac{\alpha}{(\tau + \omega)(\tau - \omega)} e^{-\tau t} \quad (47)$$

Where,

$$\alpha = \frac{A\omega}{L} \quad (48)$$

$$\tau = \frac{R}{L} \quad (49)$$

Fig.8 shows transient current of solenoids with AC source. The current grows with time and oscillate between well-defined values.

The red line indicates System1, blue is System2 and green is System3 as shown in the legend. At the instance of AC supply, the current in the solenoids raises quickly to a value before oscillating with the same frequency as the voltage supply. The current flowing through System1 oscillates between -0.1 and +0.1 A. The current steeply raises from -0.3A at $t=0$ to about -0.1A where it begins to oscillate with the same waveform as the supply voltage. In System2, the current raises and oscillates between -0.04A and +0.06A. Lastly, in System3, the current raises and oscillates between -0.05A and +0.05A.

In all cases, DC or AC, the value of current in System1 is higher than that of System2 and System3. System3 has the lowest value of steady state current.

System Characteristics with Condition: $0 < x < L$

In this section, the velocity of plunger as it moves from the resting position, $x=0$, to fully open position, $x=L$, is considered. Eq.21 and Eq.34 govern the behavior of the solenoid where the plunger or armature is in motion. Using the lowest R-L values, the dynamic characteristic of solenoid is investigated.

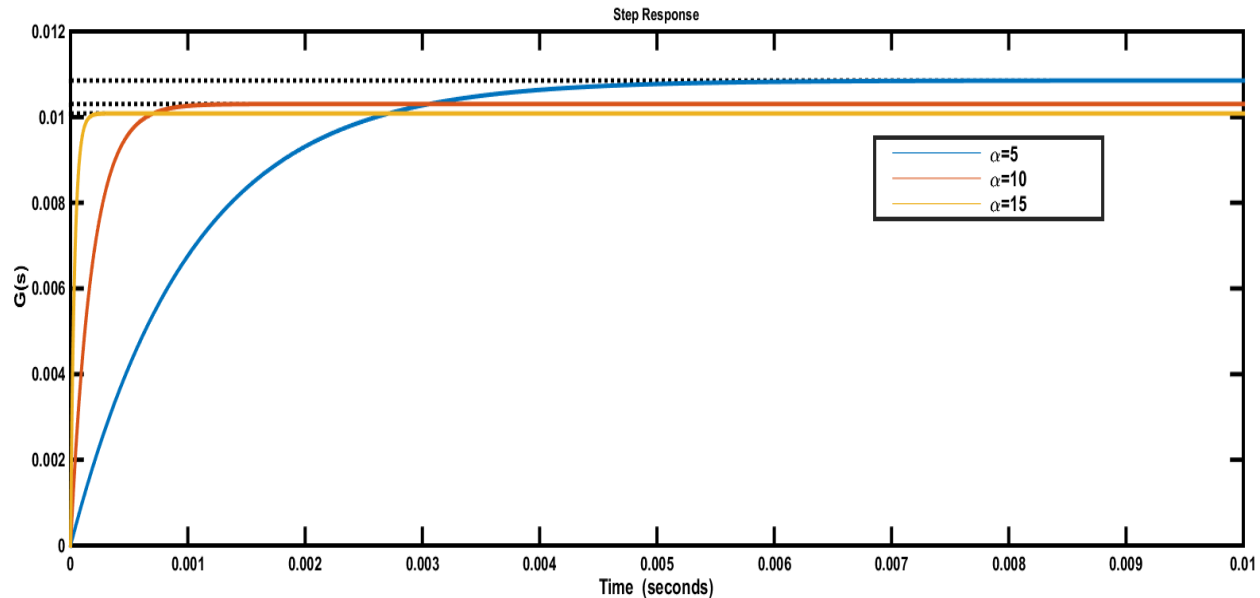


Fig.9. Plots of Solenoid characteristics for moving plunger/Armature at different values of alpha (α).

As shown in the plot, the characteristics is affected by the alpha (α) values which affects the time for the solenoid to attain a steady state. The higher the value of the α , the steeper the slope of the characteristics.

CONCLUSION

In this work, we theoretically established that the resistance-inductance pair values have a significant effect on the performance of a solenoid and propagation delay in a closed loop control system. In each of the cases above, the R-L pair values affect the transient and steady state characteristics of solenoid. The result indicated that solenoid with lower values of R-L pair has the highest Amper per Second and higher impulse response. Thus, the settling time is higher than in systems with higher values of R-L pairs. The higher the settling time increase propagation delay in time sensitive control actions.

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