



High-Rate Stress Upturn of Brittle Materials

Y. Partom

18 HaBanim, Zikhron Ya'akov 3094017, ISRAEL

ABSTRACT

High-rate stress upturn of ductile materials at rates between 10^3 - 10^4 /s has been known since the 1980s, and previously we've shown how to model this behavior based on our overstress approach to dynamic viscoplasticity. It turns out that brittle materials undergo high-rate stress upturn as well, but at a lower rate of 1-10/s. Most available data on high-rate stress upturn of brittle materials are for concrete (from obvious reasons), and they are usually represented by the quantity DIF as function of strain rate, where DIF=Dynamic Increase Factor. Here we model high-rate stress upturn of brittle materials using our overstress response model. Our overstress model for brittle materials includes: 1) damage onset curves for both compression and tension; 2) damage accumulation rate relation as function of the damage onset overstress; 3) damage onset reduction relation as function of the damage level, all the way down to the fully damaged state; 4) rate of plastic flow of the fully damaged material, which is what we usually use for a granular material; and 5) a finite upper limit for the damage accumulation rate. This last item stems from the finite rate of fracturing of brittle materials and from the finite speed of crack growth, and it turns out that this is the feature that leads to the high-rate stress upturn response. To demonstrate how the model works we compute several examples in cylindrical symmetry, with different ratios of radial to axial velocities.

INTRODUCTION AND MODEL DESCRIPTION

It is widely accepted that brittle materials have a damage onset threshold curve ($=S_D$) in the P-S plane, where P is pressure and S is the equivalent or effective (von Mises) stress. A damage onset threshold curve is used, for instance, by the JH brittle materials response model and by its derivatives [1, 2]. When the state point of a computational cell (or control volume) of a brittle material is within its damage onset threshold curve, its response is elastic. When the state point reaches the damage onset threshold curve, the material does not fail instantaneously, but only starts to accumulate damage (cracks and pores), and the state point may therefore protrude out of it. Such an approach to material response is known as an overstress approach [3, 4]. In general, the overstress approach to dynamic response of materials acknowledges the fact that various kinds of material-response changes (like the change from an intact to a fully damaged material considered here) are not instantaneous, but happen at a finite rate. For high-rate loadings, such changes may therefore lag behind the loading process and produce an overstress situation. When the state point of a computational cell protrudes out of the damage onset threshold curve, various types of cracks and pores are being formed there. In a macroscopic response model, we represent the extent of cracking and pore formation by a scalar variable called Damage and denoted by D (as did many others before). D is usually defined between 0 and 1, 0 for the intact and 1 for the fully damaged material, and between these values the material is partially damaged. The fully damaged material behaves like a granular material. It flows plastically in response to shear loading, and because of grain-to-grain friction, its flow

stress increases with pressure. For the partially damaged material. the rate of damage increase is determined by the amount of overstress relative to the current damage onset threshold curve (of the considered computational cell). To increase the rate of damage accumulation as function of D , we assume (similar to the JH models) that as D increases, the damage onset threshold curve goes down towards the flow stress curve, and for lack of any concrete information, we assume for that a linear relation:

$$\begin{aligned} S_D(P) &= (1-D)S_F(P) + DS_I(P) \\ S_I(P) &= S_0 + A_I P \quad S_F(P) = A_F(P) \end{aligned} \quad (1)$$

where the index I stands for Intact, the index F stands for Fully damaged, $S_D = \text{DOT}(D)$, $S_F = \text{FSC}$, $S_I = \text{DOT}(D=0)$, DOT=Damage Onset Threshold, FSC=Flow Stress Cueve, and A_I , A_F are material coefficient to be determined from tests.

In addition to damage accumulation resulting from shear stress, we also assume damage accumulation thresholds for compression and for tension. For damage accumulation rate in shear, we assume (because of lack of any concrete information) a linear dependence on the amount of overstress:

$$\dot{D} = A_{DS}(S - S_D) \quad \text{for } S > S_D \quad \text{and} \quad D < 1 \quad (2)$$

and we assume similar relations for damage accumulation rates in compression and in tension (see later). According to Eq. (2), the rate of damage accumulation is unlimited. But the rate of damage accumulation depends on the rate of crack formation and on the speed of crack growth, and these two processes are rather slow (compared to the rate of loading in shock loading situations). We therefore limit the rate of damage accumulation by:

$$\dot{D} \leq \dot{D}_{\max} \quad (3)$$

As we show later, it turns out that this rate limitation has a crucial role in modeling high-rate stress upturn of these materials. In Fig. 1 we show a schematic plot of the various curves defined above for both compression and tension of brittle materials.

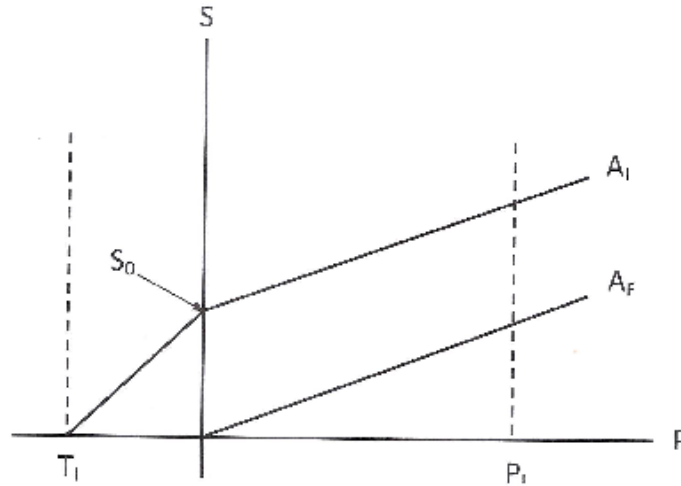


Figure 1. Schematic plot showing the various threshold lines of the S(P) (shear-pressure) plane of brittle materials.

It is well known [5-7], that the flow stress of ductile materials (mainly metals) increases dramatically at strain rates beyond $10^4/s$. This type of response is known as High-Rate Stress Upturn (HRSU). It turns out that brittle materials may also exhibit a HRSU response, but at lower strain rates of about 1-10/s. Data on HRSU for brittle materials is mainly available for concrete [8-10], which is a typical brittle material. These data are usually expressed in terms of the quantity DIF (= Dynamic Increase Factor), which is the ratio of the dynamic flow stress to the quasistatic flow stress. In what follows we use our overstress model described above to compute the response of concrete to various loading paths in the shear-pressure plane of Fig. 1. We show that increasing the maximum rate of damage accumulation pushes the rate of stress upturn to higher values.

COMPUTATIONS

We compute the response of a single cell of a brittle material (concrete) (which we regard as an 0D computation). The cell is a box for which the y and z directions are identical (cylindrical symmetry). The initial box dimensions are a_x and $a_y=a_z$. The box boundaries that go through the origin are at rest. To the other boundaries we apply the constant velocities u_x and $u_y=u_z$, so that:

$$a_x = a_x + u_x \Delta t \quad a_y = a_y + u_y \Delta t \quad (4)$$

The deformation rate tensor components are:

$$d_x = -\frac{u_x}{a_x} \quad d_y = -\frac{u_y}{a_y} \quad (5)$$

The volume deformation rate, the volume strain rate, and the volume strain that are usually used for parametrization of the shock Hugoniot curve are:

$$\begin{aligned} d_v &= d_x + 2d_y \\ \dot{\varepsilon} &= -(1-\varepsilon)d_v \quad \varepsilon(t + \Delta t) = \varepsilon(t) + \dot{\varepsilon} \Delta t \end{aligned} \quad (6)$$

From the strain ε we compute the pressure P by the shock Hugoniot equation (ignoring the small difference between the isentrope and the Hugoniot at the relatively low pressures that we consider):

$$P = \rho_0 C_H^2 \frac{\varepsilon}{(1 - S_h \varepsilon)^2} \quad (7)$$

where ρ_0 is the initial density, and C_H , S_H are the straight line Hugoniot parameters in the shock velocity-particle velocity plane. The deviatoric stress components and the equivalent stress we compute by:

$$\begin{aligned} \delta_x &= \frac{2}{3}(d_x - d_y) \quad \delta_y = -\frac{1}{3}(d_x - d_y) \\ \dot{s}_x &= 2G \delta_x \quad \dot{s}_y = 2G \delta_y \\ s_x(t + \Delta t) &= s_x(t) + \dot{s}_x \Delta t \quad s_y(t + \Delta t) = s_y(t) + \dot{s}_y \Delta t \\ S &= \frac{3}{2}(s_x^2 + 2s_y^2)^{1/2} \end{aligned} \quad (8)$$

where G is the shear modulus. When the state point is out of the current damage onset threshold, the rate of damage accumulation is:

$$\begin{aligned} \dot{D} &= A_{DS}(S - S_D) \\ \dot{D} &= \dot{D} + A_{DP}(P - P_D) \quad \text{for } P > P_D \\ \dot{D} &= \dot{D} + A_{DT}(T_D - P) \quad \text{for } T_D > P \\ \dot{D} &\leq \dot{D}_{\max} \quad \text{for } D < 1 \end{aligned} \quad (9)$$

where P_D and T_D (see Fig.1) are the Damage Onset Thresholds for compression and tension. and where we use the following parameters: $\rho_0=2.4\text{g/cc}$, $C_H=2.0\text{km/s}$, $S_H=1.5$, $G=5\text{GPa}$, $S_0=50\text{MPa}$, $A_I=A_F=0.1/\text{GPa}$, $P_D=50\text{MPa}$, $T_D=-10\text{MPa}$, $A_{DS}=A_{DP}=A_{DT}=0.0001/\text{GPa}/\mu\text{s}$, $\dot{D}_{\max} = 0.0004/\mu\text{s}$.

RESULTS

In Fig. 2 we show six different loading paths in the $S(P)$ plane, along which we compute examples of the performance of our model. They are all in the range of u_y/u_x between 0 and -0.8. When $u_y/u_x > -0.5$ we get compression, and otherwise we get tension. For all these loading paths, the effective strain rate when the material becomes fully damaged, is around 1/s. For

each loading path we perform several runs with different values of u_x , and we stop each run when the material gets fully damaged. We then plot the equivalent stress as function of the effective strain rate at the end of these runs. In Fig. 3 we show results for two of the loading paths shown in Fig. 2.

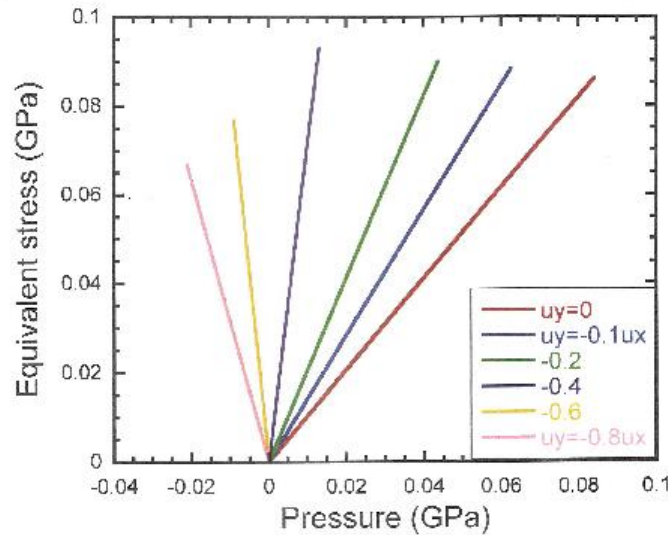


Figure 2. Different loading paths in the S(P) plane for different values of u_y/u_x .

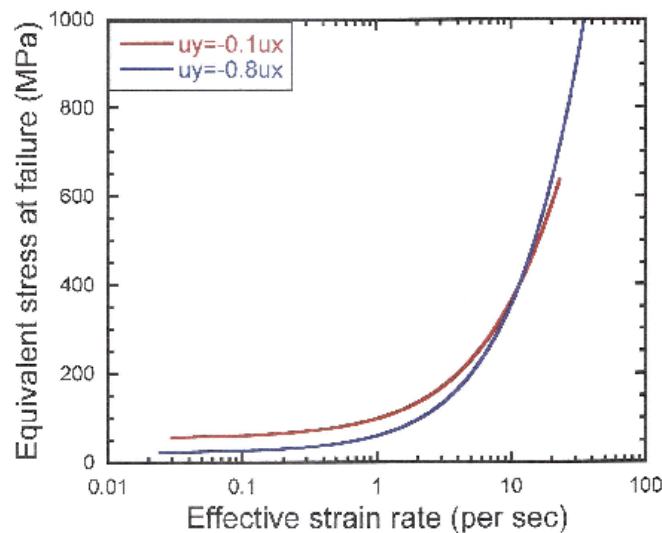


Figure 3. Equivalent stress as function of effective strain rate for two of the loading paths shown in Fig. 2.

We see from Fig. 3 that: 1) for different values of u_y/u_x we get similar but not identical response curves; 2) there is a fast change of curvature beyond a strain rate of about 1/s; and 3) the slopes at high strain rates agree with slopes obtained in tests [8-10]. In Fig. 4 we show the effect of the maximum damage accumulation rate for the loading path with $u_y/u_x = -0.1$. We express the ordinate of Fig. 4 by the variable DIF=Dynamic Increase Factor, commonly used in test results.

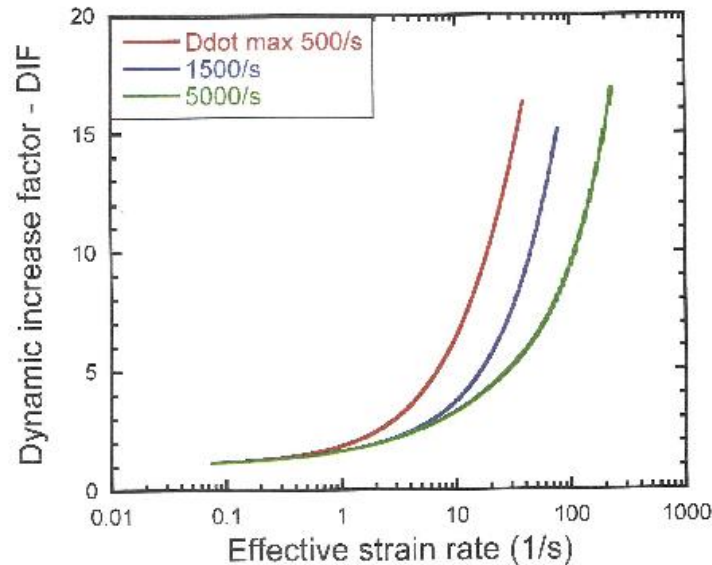


Figure 4. Dynamic increase factor (DIF) as function of strain rate, for different values of the maximum damage accumulation rate, and for the loading path $u_y/u_x = -0.1$.

We see from Fig. 4 that as the maximum rate of damage accumulation increases, the response curves move to higher values of the effective strain rate, and so do the high-rate stress upturn values. $\dot{D}_{\max}$ is therefore the main parameter in the high-rate stress upturn response of brittle materials.

SUMMARY

It is widely accepted that ductile materials (mainly metals) undergo a high-rate stress upturn above a strain rate of about $10^4/s$. From many tests on concrete, it seems that brittle materials undergo a similar upturn, but at lower strain rates of 1-10/s. Here we use an overstress approach model for brittle materials to reproduce such a high-rate stress upturn response. We define a damage onset threshold for shear, compression and tension in the shear-pressure plane. When the state point reaches the damage onset threshold, it may go out of it (overstress), and damage starts to accumulate. When the material is fully damaged it responds like a granular material. We limit the rate of damage accumulation because crack formation and crack growth are rather slow processes. It turns out, and we demonstrate that in Fig. 4, that the maximum rate of damage accumulation controls the strain rate at which the flow stress upturns.

References

1. G.R. Johnson, T.J. Holmquist and S.R. Beissel, Response of aluminum nitride to large strains, high strain rates, and high pressures, J. Appl. Phys. 94, 1639-1647 (2003).
2. T.J. Holmquist and J.R. Johnson, Characterization and evaluation of silicon carbide for high velocity impact, J. Appl. Phys. 97, 093502- 1-12 (2005).
3. Y. Partom, Overstress and flowstress approaches to dynamic viscoplasticity, EPJ Web of conferences 94, 04003 (2015).

4. Y. Partom, Modeling stress upturn at high strain rates for ductile materials, DYMAT 2018 conference proceedings, EPJ web of conferences 183, 01002 (2018).
5. P.S. Follansbee and U.F. Kocks, A constitutive description of the deformation of copper based on the use of the mechanical threshold stress as an internal state variable, *Acta Metall.* 36, 81-93 (1988).
6. H. Couque, the use of direct impact Hopkinson pressure bar technique to describe thermally activated and viscous drag regimes of metallic materials, *Phil. Trans. Royal Soc. A372*, 20130218 (2014).
7. R.W. Armstrong, W. Arnold and F.J. Zerilli, Dislocation mechanics of shock induced plasticity, *Metall. Mater. Trans. A38*, 2605-2610 (2007).
8. S. Lee, K. Kim, J. Park and J. Cho, Pure rate effect on the concrete compressive strength in the split Hopkinson pressure bar, *Int. J. Impact Eng.* 113, 191-202 (2018).
9. L.J. Malvar and C.A. Ross, Review of strain rate effects for concrete in tension, *ACI materials Journal*, 735-739 (1998).
10. D.L. Grote, S.W. Park and M. Zhou, Dynamic behavior of concrete at high strain rates and pressures: I. experimental characterization, *Int. J. Impact Eng.* 25, 869-886 (2001).