

The Fluid Dynamics of Conduit Hydrodynamic Entrance Effects Explained: A Rebuttal Paper

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ABSTRACT

Fluid flow in closed conduits is a field of study that has been characterized in the published literature, up until recently, by contradictions, false assertions and sometimes even more egregious misrepresentations of what the Laws of Nature dictate. The issue of conduit hydrodynamic entrance effects, in particular, has been studied and published on, extensively, over the last 100 years but this endeavor has yielded more heat than light. A recent publication (2023) appeared in the prestigious journal of Fluid Mechanics entitled, “Hydrodynamics of finite-length pipes at intermediate Reynolds numbers”, in which the entire paper was dedicated to this oftentimes misunderstood phenomenon. We challenge the author’s conclusions, herein, and in so doing, we demonstrate that their measured data is fully consistent with the recently published “Quinn’s Law of Fluid Dynamics in Closed Conduits” and, consequently, we provide an alternative explanation which establishes a comprehensive understanding of the physics underlying the so-called conduit hydrodynamic end-effects, a necessary consideration in this study, but one which the authors surprisingly chose to ignore. Furthermore, we validate our analysis in the context of the two classical published studies of (a), Nikuradze, for his sand-roughened pipes, and (b), the Princeton Super Pipe, for smooth walled pipes, both of which are considered contemporary gold standards in the field of fluid dynamics in closed conduits.

Keywords: pipe flow boundary layer, transition to turbulence, pipe flow, Wall Effects, Pipe Entrance Effects.

INTRODUCTION

This paper is a rebuttal to the publication appearing in the Journal of Fluid Mechanics (Pomerenk et al 2023). Since this rebuttal is based upon an analysis which, in turn, is based upon the recently published “Quinn’s Law of Fluid Dynamics” (Quinn 2019,2019, 2020,2020,2020,2023), we will briefly explain why the Quinn Fluid Flow Model (QFFM) is superior to any current extant fluid flow model when considering conduit hydrodynamic entrance effects. This is because the QFFM contains an expression for the Pressure/Flow relationship like none other, to the extent that it is universally valid over the entire fluid flow regime. In addition, it is equally valid for conduits packed with particles and, accordingly, contains terms for all elements of hydrodynamic wall-effects which include, the viscous boundary layer, wall roughness and entrance-effects. Additionally, it conforms to the teachings of the Forchheimer fluid model (Forchheimer 1901), regarding the relationship between fluid velocity and pressure drop outside of the laminar flow regime. We recommend that the reader look to the original publications of the QFFM contained in the references herein for the details

of all the relevant governing equations. Thus, we can write the teaching of the QFFM in its dimensionless form as;

$$P_Q = k_1 + k_2 C_Q \quad (1)$$

Where, P_Q represents dimensionless pressure drop, k_1 and k_2 are universal constants and C_Q represents the dimensionless fluid flow term.

In order to make it more convenient for the reader, we include herein our Table 1, which contains the nomenclature and governing equations of the QFFM.

Table 1 Nomenclature and Governing equations

Empty Conduit			Empty Conduit		
Category	Symbols	Formula	Category	Symbols	Formula
A. Conduit Independent Variables			H. Flow Rate Range		
Conduit diameter	D	= Independent variable	Volumetric fluid flow rate	q	= Independent variable
Conduit length	L	= Independent variable	Fluid superficial velocity (flux)	μ_s	= $4q/(\pi D^2)$
Conduit inner wall roughness	k	= Independent variable	I. Fluid Dynamics		
Conduit hydrodynamic entrance length	L_E	= Independent variable	Modified Reynolds number	R_{em}	= $4q d_c \rho_f / (\eta \pi D^2)$
B. Particle Independent Variables			Quinn number (fluid current)	Q_N	= δR_{em}
Measured particle diameter	d_{pm}	= D	J. Wall-Effect		
Particle sphericity	Ω_p	= Independent variable	Initial boundary layer	β_0	= $k_1/(k_2 Q_N + k_1)$
Particle porosity	ϵ_p	= Independent variable	Primary wall-effect	W_1	= $\beta_0^{(1/3)}/\tau$
Number of particles	n_p	= n_{pq}	Conduit Hydrodynamic entrance radius	r_e	= L_E/L
C. Fluid Independent Variables			Secondary wall-effect	W_0	= $2\pi L_E/L$
Fluid absolute viscosity	η	= Independent variable	Residual secondary wall-effect	W_{0R}	= $W_0 W_1^{(1/2)}$
Fluid density	ρ_f	= Independent variable	Conditional		= if $W_{0R} < 0$; $W_{0R} = 0$
D. Conduit Dependent Variables			Relative wall roughness	k_{dc}	= k/d_c
Spherical particle diameter equiv.	d_p	= $\Omega_p d_{pm}$	Secondary wall-effect	W_2	= $30k_{dc}^{(1/3)}$
Empty conduit volume equiv.	n_{pq}	= $3D^3 L / (2d_p^3)$	Residual secondary wall-effect	W_{2R}	= $W_2 W_1^{(1/2)}$
Architectural design coefficient	γ	= $n_{pq} D/L$	Conditional		= if $W_{2R} < 0$; $W_{2R} = 0$
Particle fraction	p_f	= (n_p/n_{pq})	Net wall-effect	W_N	= $W_1 + W_{0R} + W_{2R}$
External conduit porosity	ϵ_0	= $1 - n_p/n_{pq}$	Wall-effect norm. coeff.	λ	= $1 + W_N$
Internal conduit porosity	$\epsilon_p p_f$	= $\epsilon_p n_p/n_{pq}$	Normalized fluid current	C_Q	= λQ_N
Total conduit porosity	ϵ_t	= $(1 - p_f) + \epsilon_p p_f$	K. Dimensional permeability		
E. Hypothetical Q Channel			Kinetic fitting coefficient	a	= $8\delta^3 \lambda \rho_f L / (\pi^2 D^4 d_c)$
Hypothetical Q channel diam.	d_c	= $d_p / (\text{abs}(p_f))$	Viscous fitting coefficient	b	= $1024\delta \eta L / (3D^3 d_c^2)$
F. Q Channel permeability			Differential pressure drop	ΔP	= $aq^2 + bq$
Universal Constants			Hydraulic gradient	$\Delta H/L$	= $\Delta P / (\rho_f g L)$
Universal constant	π	= 22/7	Time to displace conduit volume	t_0	= $\pi D L / (4q) \epsilon_t$
Gravity acceleration	g	= 981	Conduit wall shear stress	τ_w	= $\Delta P d_c / 4L$
Permeability constants			Conduit frictional velocity	μ_f	= $(\tau_w / \rho_f)^{(1/2)}$
Drag norm. coefficient.	r_h	= $(\pi d_p^3) / ((\pi d_p^3)/4)$	Instantaneous boundary layer	β	= $k_1/(k_2 \lambda Q_N + k_1)$
Universal Viscous Constant	k_1	= $(4\pi/3) r_h^2$	Boundary layer thickness	BLT	= $\beta / \tau_w d_c / 2$
Norm. channel circumference	ϕ_h	= $2\pi r_h$	L. Dimensionless Permeability		
Universal kinetic constant	k_2	= $1/\phi_h$	Instantaneous angular frequency	ω	= λ/ϕ_h
Viscous angular frequency	ω_0	= k_2	Quinn's Law of Fluid Dynamics	P_Q	= $k_1 + k_2 C_Q$
G. Conduit Vector Dot Product			Kinetic friction factor norm.	P_K	= P_Q/Q_N
Conduit porosity kinetic coeff.	δ	= $1/\epsilon_0^3$	Dimensionless permeability	Θ	= Q_N/P_Q
Conduit tortuosity	τ	= δ/τ_f	Viscous friction factor	f_r	= $P_Q \phi_h$

THE WALL EFFECT TERM λ

We define the term $C_Q = \lambda Q_N$ where both λ and Q_N are dependent variables. It will be appreciated that our equation (1) has to do with fluid flow through closed conduits, which means there has to be a wall involved, i.e., the wall of the conduit confines the fluid and any obstacles in the path of the fluid, to the free space contained within the conduit. Thus, we have present in our methodology, the so-called “wall effect”. Accordingly, we can say that within the QFFM, the

term λ deals with the wall effect. We can re-express equation (1) to identify the role of λ as follows:

$$P_Q = k_1 + k_2 \lambda Q_N \quad (2)$$

We should point out, at this point, that conduit hydrodynamic entrance effects are a subset of the overall wall-effect phenomenon, but this term is typically omitted from most general equations because, in practice, a practitioner will always choose a sufficiently long conduit to ensure that entrance effects are eliminated. This, in turn, makes sense since entrance effects typically detract from the overall conduit performance and add nothing of value for the practitioner. In the case of chromatographic columns, for instance, end-effects totally destroy the ability of the packed conduit to do meaningful separations. This means that conduit end-effects are more of an academic interest than anything a typical practitioner is interested in. Nonetheless, the QFFM being a rigorous theoretical development based upon the fundamentals of the physics underlying fluid flow in closed conduits, easily accommodates this element of wall effect and, in so doing, teaches that there are just five elements to the wall effect:

1. primary wall effect (W_1);
2. secondary wall effects (W_0 and W_2);
3. residual secondary wall effects (W_{0R} and W_{2R});
4. net wall effect (W_N) and finally,
5. wall normalization coefficient (λ).

The net wall effect is the sum of the primary and residual secondary wall effects, $W_N = (W_1 + W_{0R} + W_{2R})$, and will only manifest in measured pressure drop values where kinetic contributions are significant. In other words, the net wall effect has a relatively small effect at very low flow rates (laminar flow).

The wall normalization coefficient $\lambda = (1 + W_N)$, and enters the pressure flow relationship through the kinetic term. Accordingly, the λ parameter has a relatively small effect when the flowrate is low, i.e., when the kinetic contributions are negligible and the value of Q_N is very small, i.e., less than unity, say.

THE PRIMARY WALL EFFECT (W_1)

The primary wall effect has the symbol W_1 and is a derivative of two distinct parameters. Firstly, W_1 is a derivative of the dimensionless viscous boundary layer $\beta_0 = (k_1/k_1 + k_2 Q_N)$, where $k_1 = (64\pi/3)$ and $k_2 = (1/8\pi)$, and are the universal constants displayed in equation (1) above. $Q_N = \delta R_{em}$ is the Quinn number. The symbol R_{em} stands for the well-known modified Reynolds number and $\delta = 1/(1 - n_p/n_{pq})^3$.

Secondly, the formula for the primary wall effect is $W_1 = (\beta_0^{(1/3)} / \tau)$. It is, therefore, also a derivative of the tortuosity factor, $\tau = \delta\gamma$ where $\gamma = (n_{pq}D/L)$ is a structural feature of the flow embodiment under study (packed or empty conduit), where n_{pq} is the volume of the empty conduit expressed in terms of number of particle equivalents having a diameter of d_p , the average spherical particle diameter equivalent.

W_1 Has Its Maximum Value $W_1 = 5.33$ Approx

There is just one operating condition when the value of the primary wall effect has its maximum value of 5.33 approx. This condition exists when D/d_p has its minimum value of unity [$(D/d_p) = 1.0$], which only occurs in a conduit packed with particles of free space wherein the particles are fully porous ($\varepsilon_p = 1$), i.e., an empty conduit, and when the value of Q_N is very small. In this scenario, the tortuosity coefficient, τ , is relatively small which results in a thick viscous boundary layer when the value of Q_N is very small (Laminar flow). As the value of Q_N increases, however, the viscous boundary layer starts to dissipate which reduces its thickness and eventually approaches a value of zero at very large values of Q_N (fully developed turbulence).

W_1 Manifestation

The primary wall effect W_1 will only manifest when the value of (D/d_p) is very small, such as in an empty conduit, and when kinetic contributions are significant. At very high values of Q_N , on the other hand, where the boundary layer is totally depleted, the value of $W_1 = 0$, and thus will not manifest at all. Additionally, when D/d_p is very large, say greater than 30, which only occurs in a conduit packed with solid particles, the tortuosity coefficient, τ , is very large and this reduces the thickness of the boundary layer to an infinitesimally small value, hence the value for $W_1 = 0$ approx., at all values of Q_N .

It will be appreciated that $W_1 = W_N$, the net wall effect, for all values of Q_N in a smooth walled empty conduit.

The Secondary Wall Effects (W_0 and W_2)

The secondary wall effect W_2 is a derivative of the sand-grain roughness, k , and the diameter of the hypothetical fluid channel d_c . It is proportional to the ratio of the two where $W_2 = 30 k_{dc}^{(1/3)}$ and $k_{dc} = k/d_c$ where k is the sand grain roughness and $d_c = d_p / [\text{abs}(n_p/n_{pq})]$ is the diameter of the hypothetical fluid channel. The value of 30 represents a constant of proportionality. It will be appreciated that d_p stands for the average spherical particle diameter equivalent of the particles in the conduit, either packed with solid particles ($\varepsilon_p < 1$) or particles of free space ($\varepsilon_p = 1$), i.e., an empty conduit, where ε_p stands for the *particle* porosity. In an empty conduit, $d_p = D$.

The secondary wall effect W_0 represents the wall (conduit) hydrodynamic entrance effects and is a function of the hydrodynamic entrance radius $r_c = L_E/L$ where L_E is the conduit hydrodynamic entrance length. In fact, $W_0 = 2\pi r_c$, is the conduit hydrodynamic entrance circumference which, in turn, represents the minimum circumference magnitude above which the fluid profile is stable, i.e., it takes the typical parabolic shape associated with laminar flow. In other words, the magnitude of L_E represents the minimum conduit length necessary to eliminate the unstable flow associated with conduit hydrodynamic entrance effects.

The Residual Secondary Wall Effects (W_{0R} and W_{2R})

The residual secondary wall effects are a derivative of both the primary and secondary wall effects. Indeed, they are a derivative of the difference between the two, i.e., $W_{2R} = W_2 - W_1^{(1,2)}$ and $W_{0R} = W_0 - W_1^{(1,2)}$.

The Net Wall Effect (W_N)

The net wall effect is the sum of the primary wall effect and the residual secondary wall effects, i.e., $W_N = W_1 + W_{0R} + W_{2R}$. It is the net wall effect W_N that influences the measured pressure drop.

The Wall Normalization Coefficient (λ)

We define the wall normalization coefficient $\lambda = (1 + W_N)$. This is the parameter that accounts for the impact of wall effect in fluid dynamics. When there is no wall effect, i.e., $W_N = 0$, the value of $\lambda = 1.0$.

METHODS AND ANALYSIS

We shall now apply the teaching of the QFFM to the data set contained in the Pomerenk et al paper. Included in our Table 2, we show the QFFM calculated data for all 25 conduits presented in this study.

Table 2 Conduit hydrodynamic entrance effect identified

Conduit No.	π <u>22</u>	D	L	η	L_E	r_c <u>L_E</u>	W_0 <u>$2\pi r_c$</u>
	7					L	
	none	cm	cm	poise	cm	none	none
21	3.14	0.14	17.80	0.0104	17.80	1.00	6.29
31	3.14	0.14	30.80	0.0103	30.80	1.00	6.29
11	3.14	0.14	5.20	0.0104	12.88	2.48	15.57
GLYC 12	3.14	0.254	5.10	0.0797	5.10	1.00	6.29
32	3.14	0.254	30.60	0.0089	30.60	1.00	6.29
22	3.14	0.254	17.90	0.0103	24.66	1.38	8.66
12	3.14	0.254	5.10	0.0102	17.75	3.48	21.88
33	3.14	0.381	30.30	0.0103	30.70	1.01	6.37
GLYC 23	3.14	0.381	17.80	0.0460	30.54	1.72	10.78
23	3.14	0.381	17.80	0.0104	31.92	1.79	11.27
GLYC 13	3.14	0.381	5.20	0.0797	19.13	3.68	23.12
13	3.14	0.381	5.20	0.0104	22.26	4.28	26.91
3	3.14	0.381	1.95	0.0103	18.24	9.35	58.80
GLYC 3	3.14	0.381	1.95	0.0753	22.98	11.78	74.07
GLYC 34	3.14	0.5055	30.70	0.0507	30.67	1.00	6.28
44	3.14	0.5055	61.30	0.0106	61.30	1.00	6.29
54	3.14	0.5055	90.00	0.0106	90.00	1.00	6.29
34	3.14	0.5055	30.70	0.0102	49.94	1.63	10.23
24	3.14	0.5055	17.80	0.0103	40.76	2.29	14.39
14	3.14	0.5055	5.10	0.0102	29.31	5.75	36.12
GLYC 14	3.14	0.5055	5.10	0.0797	30.84	6.05	38.01
4	3.14	0.5055	1.90	0.0102	23.50	12.37	77.74
GLYC 4	3.14	0.5055	1.90	0.0753	29.56	15.56	97.79
GLYC 45	3.14	0.63754	61.30	0.0507	61.24	1.00	6.28
55	3.14	0.63754	82.20	0.0102	107.57	1.31	8.23
45	3.14	0.63754	61.30	0.0107	80.60	1.31	8.26
35	3.14	0.63754	30.70	0.0103	57.92	1.89	11.86
GLYC 25	3.14	0.63754	17.80	0.0507	46.05	2.59	16.26
25	3.14	0.63754	17.80	0.0102	46.03	2.59	16.25
15	3.14	0.63754	5.20	0.0104	35.72	6.87	43.18
GLYC 15	3.14	0.63754	5.20	0.0797	38.21	7.35	46.19
5	3.14	0.63754	2.00	0.0103	27.82	13.91	87.43
GLYC 5	3.14	0.63754	2.00	0.0753	32.80	16.40	103.09
56	3.14	0.7747	90.00	0.0107	111.16	1.24	7.76
46	3.14	0.7747	61.30	0.0106	86.48	1.41	8.87
36	3.14	0.7747	30.90	0.0101	63.67	2.06	12.95
26	3.14	0.7747	17.60	0.0104	48.43	2.75	17.30
GLYC 26	3.14	0.7747	17.60	0.0797	50.59	2.87	18.07
GLYC 16	3.14	0.7747	5.10	0.0753	44.54	8.73	54.90
6	3.14	0.7747	1.80	0.0102	35.53	19.74	124.07
GLYC 6	3.14	0.7747	1.80	0.0753	39.17	21.76	136.78
16	3.14	0.7747	5.10	0.0102	200.73	39.36	247.40
Plastic 1	3.14	0.257	18.90	0.0106	24.89	1.32	8.28
Plastic 2	3.14	0.53	19.50	0.0106	44.12	2.26	14.22
Bubble tea	3.14	1.117	23.00	0.0106	81.06	3.52	22.15

As shown in Table 2, the data is arranged according to conduit diameter and presented in sequence from smallest to largest value of end effects (W_0). By arranging the data in this format, one can immediately assess the impact of L_E as well as that of the absolute viscosity. We point out that the authors of the paper did not provide the details of both the absolute viscosity (η) and the density of the fluids used (ρ_f). Rather their spreadsheet contained only the ratio of the two expressed as the kinematic viscosity. This will not affect our values for the fluid water, since its density is unity. It is clearly obvious from Table 2 that the largest end-effects occur in the combination of the shortest conduits and the largest conduit diameters, i.e., the largest value of $W_0 = 247$ which represents a conduit of diameter 0.7747 cm and length 5.10 cm. The fluid absolute viscosity value also plays a role by reducing the end effects as its value increases. Thus, we conclude from Table 2, that the value of $Wo = 2\pi L_E/L$ is the governing physical parameter which quantifies this effect and, therefore, is the normalization coefficient necessary to account for this phenomenon.

Having now identified the magnitude of the end-effects (W_0) we can now proceed to incorporate this parameter into the overall parameter W_N , the net wall effect, as shown in Table 3.

Table 3 Data Normalized for Wall-Effect

	Con.	Fluid	Cond.														Reported	QFFM
Conduit	π	q	D	L	η	R_{em}	P_Q	C_Q	Q_N	Θ	W_1	L_E	W_0	W_2	W_N	λ	Δh	Δh
ID	22					n_k	f_x	λQ_N	δR_{em}	$\frac{1}{P_K}$	$\beta_0^{(1/3)}$		$\frac{2\pi L_E}{30k_{dc}^{(1/3)}}$	$W_1 + W_{0R} + W_{2R}$	$(1 + W_N)$			
	7					n_r	4			P_K	τ		L					
I.D.	none	mL/sec	cm	cm	poise	none	none	none	none	none	none	cm		none	none	none	cm	cm
31	3.14	0.06	0.14	30.80	0.0103	57	69	45	7	0.10	5.33	30.80	6	0.00	5	6.33	3.00	2.37
11	3.14	0.18	0.14	5.20	0.0104	154	78	279	19	0.25	5.32	12.88	16	0.00	13	14.46	2.00	1.25
21	3.14	0.11	0.14	17.80	0.0104	99	70	78	12	0.18	5.32	17.80	6	0.00	5	6.32	3.00	2.47
32	3.14	0.59	0.254	30.60	0.0089	332	77	261	42	0.54	5.29	30.60	6	0.00	5	6.29	2.20	1.96
GLYC 12	3.14	0.43	0.254	5.10	0.0797	27	68	22	3	0.05	5	5.10	6	0.00	5	6.33	2.00	1.88
12	3.14	0.62	0.254	5.10	0.0102	306	99	795	38	0.39	5.29	17.75	22	0.00	20	20.79	1.00	0.50
22	3.14	0.63	0.254	17.90	0.0103	307	79	290	38	0.49	5.29	24.66	9	0.00	7	7.56	2.00	1.41
Plastic 1	3.14	0.44	0.257	18.90	0.0106	207	74	185	26	0.35	5.31	24.89	8	0.00	6	7.17	1.00	0.97
33	3.14	0.80	0.381	30.70	0.0103	259	75	204	32	0.43	5.30	30.70	6	0.00	5	6.30	1.00	0.59
3	3.14	3.32	0.381	1.95	0.0103	1,076	376	7,770	135	0.36	5	18.24	59	0	57	57.76	1.00	0.77
13	3.14	2.32	0.381	5.20	0.010375	746	163	2,412	93	0.57	5.24	22.26	27	0.00	25	25.85	0.90	0.63
23	3.14	1.33	0.381	17.80	0.0104	426	89	543	53	0.60	5.27	31.92	11	0.00	9	10.19	1.00	0.67
GLYC 13	3.14	0.92	0.381	5.20	0.0797	38	71	106	5	0.07	5	19.13	23	0.00	21	22.00	1.00	0.83
GLYC 23	3.14	0.92	0.381	17.80	0.0460	67	70	81	8	0.12	5	30.54	11	0.00	9	9.67	1.80	1.63
GLYC 3	3.14	2.04	0.381	1.95	0.0753	91	100	826	11	0.11	5	22.98	74	0	72	72.94	1.00	0.92
4	3.14	6.22	0.50544	1.90	0.0102	1,534	652	14,713	192	0.29	5.15	23.50	78	0.00	76	76.74	1.00	0.78
14	3.14	5.24	0.50542	5.10	0.0102	1,299	294	5,707	163	0.55	5.17	29.31	36	0.00	34	35.11	1.00	0.79
24	3.14	3.85	0.5055	17.80	0.0103	938	129	1,564	117	0.91	5.22	40.76	14	0.00	12	13.35	1.00	0.91
34	3.14	3.09	0.5056	30.70	0.0102	759	102	870	95	0.93	5.23	49.91	10	0.00	8	9.17	1.00	0.98
44	3.14	4.04	0.5055	61.30	0.0106	962	97	747	120	1.24	5.21	61.30	6	0.00	5	6.21	2.00	2.51
54	3.14	1.44	0.5055	90.00	0.0106	341	78	268	43	0.55	5.29	90.00	6	0.00	5	6.29	1.00	1.06
GLYC 14	3.14	2.30	0.50543	5.10	0.0797	73	80	335	9	0.11	5	30.84	38	0.00	36	36.90	0.90	0.75
GLYC 34	3.14	0.78	0.5055	30.70	0.0507	39	68	31	5	0.07	5	30.67	6	0.00	5	6.33	0.90	0.82
GLYC 4	3.14	4.41	0.50552	1.90	0.0753	147	138	1,779	18	0.13	5	29.56	98	0	96	96.67	1.00	0.86
Plastic 2	3.14	5.00	0.53	19.50	0.0106	1,136	142	1,872	142	1.00	5.19	44.12	14	0.00	12	13.19	1.00	1.20
5	3.14	11.14	0.63762	2.00	0.01027	2,164	997	23,377	270	0.27	5	27.82	87	0	85	86.50	1.00	0.90
15	3.14	9.13	0.63747	5.20	0.0104	1,749	434	9,226	219	0.50	5.12	35.72	43	0.00	41	42.20	1.00	0.84
25	3.14	7.37	0.63766	17.80	0.0102	1,448	177	2,758	181	1.02	5.16	46.03	16	0.00	14	15.25	1.10	0.93
35	3.14	5.94	0.63749	30.70	0.0103	1,149	129	1,557	144	1.11	5.19	57.92	12	0.00	10	10.84	1.10	0.96
45	3.14	3.97	0.63750	61.30	0.0107	739	94	666	92	0.99	5.24	80.60	8	0.00	6	7.21	1.00	0.96
55	3.14	3.33	0.63768	82.20	0.0102	649	90	580	81	0.90	5.26	107.57	8	0.00	6	7.16	1.00	0.99
GLYC 15	3.14	6.76	0.63750	5.20	0.0797	170	105	955	21	0.20	5	38.21	46	0.00	44	45.08	1.00	1.16
GLYC 25	3.14	3.39	0.63763	17.80	0.0507	134	77	253	17	0.22	5	46.05	16	0.00	14	15.15	0.90	0.93
GLYC 45	3.14	1.42	0.63756	61.30	0.0507	56	69	44	7	0.10	5	61.24	6	0.00	5	6.33	1.00	1.19
GLYC 5	3.14	9.10	0.63747	2.00	0.0753	241	190	3,079	30	0.16	5	32.80	103	0.00	101	101.99	1.00	1.02
6	3.14	14.74	0.77468	1.80	0.0102	2,378	1,523	36,617	297	0.20	5	35.53	124	0	122	123.15	1.00	0.74
16	3.14	6.22	0.7747	5.10	0.0102	1,001	1,294	30,850	125	0.10	5.21	200.73	247	0.00	245	246.37	1.00	0.76
26	3.14	13.45	0.7745	17.60	0.0104	2,117	239	4,322	264	1.11	5.08	48.43	17	0.00	15	16.34	1.10	1.07
36	3.14	11.02	0.7747	30.90	0.0101	1,792	174	2,683	224	1.29	5.12	63.67	13	0.00	11	11.98	1.00	1.08
46	3.14	7.86	0.774	61.30	0.0106	1,220	115	1,199	153	1.33	5.18	86.48	9	0.00	7	7.85	1.00	1.06
56	3.14	6.29	0.775	90.00	0.0107	965	99	812	121	1.21	5.21	111.16	8	0.00	6	6.73	1.00	1.08
GLYC 16	3.14	11.82	0.7745	5.10	0.0753	258	136	1,738	32	0.24	5	44.54	55	0	53	53.80	1.00	1.11
GLYC 26	3.14	6.63	0.7748	17.60	0.0797	137	79	290	17	0.22	5	50.59	18	0.00	16	16.96	1.00	1.32
GLYC 6	3.14	13.64	0.7746	1.80	0.0753	298	268	5,050	37	0.14	5	39.17	137	0	135	135.69	1.00	0.89
Bubble	3.14	26.76	1.117	23	0.0106	2,887	372	7,671	361	0.97	5.00	81.06	22	0.00	20	21.26	1.00	1.01

As shown in Table 3, we have provided all the relevant calculations for the data set based upon the teaching of the QFFM. This includes the value of λ which is the overall normalization coefficient for all conduit wall effects. Before we discuss the implications of λ for this measured data set, however, we want to demonstrate the validity of our theoretical analysis by showing our correlation between the author's measured values and the QFFM calculated values. This correlation is shown in Fig. 1 for a selected number, i.e., 6, of the total of 45 conduits.

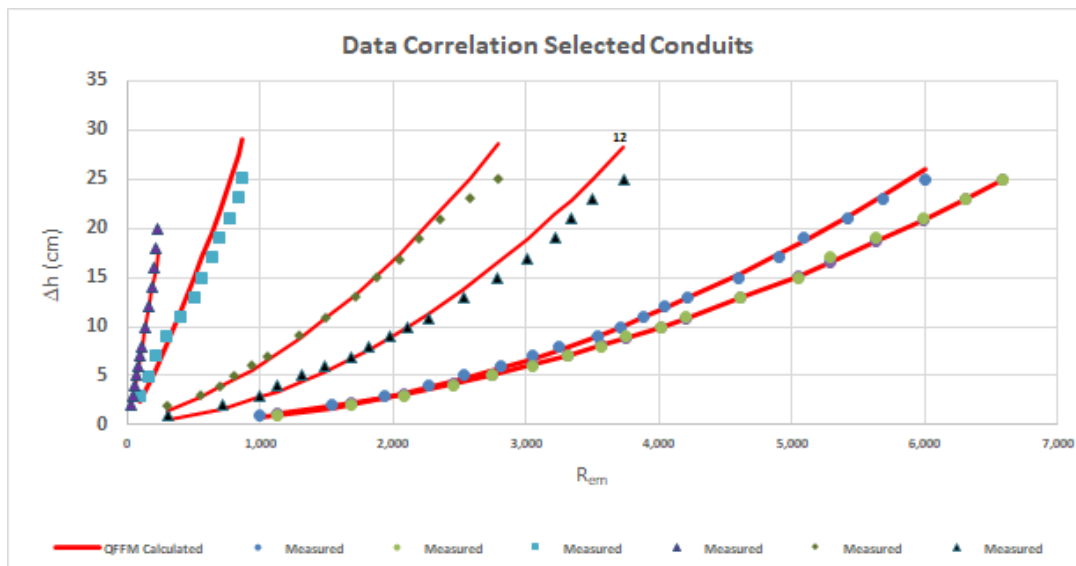


Fig. 1: QFFM correlation with measured data

Note that, as shown in Fig. 1, the correlation is specific and remarkable over the entire range of Reynolds number in the study.

We can now apply the teaching of the QFFM which is designed to highlight the conduit wall effects. This is accomplished in the QFFM by considering the dimensionless permeability of the conduit as a function of Q_N . Accordingly, we show a series of plots of Q , the dimensionless permeability, against Q_N , the fluid current unadjusted for wall effects. These plots are shown in Fig. 2 through Fig. 5.

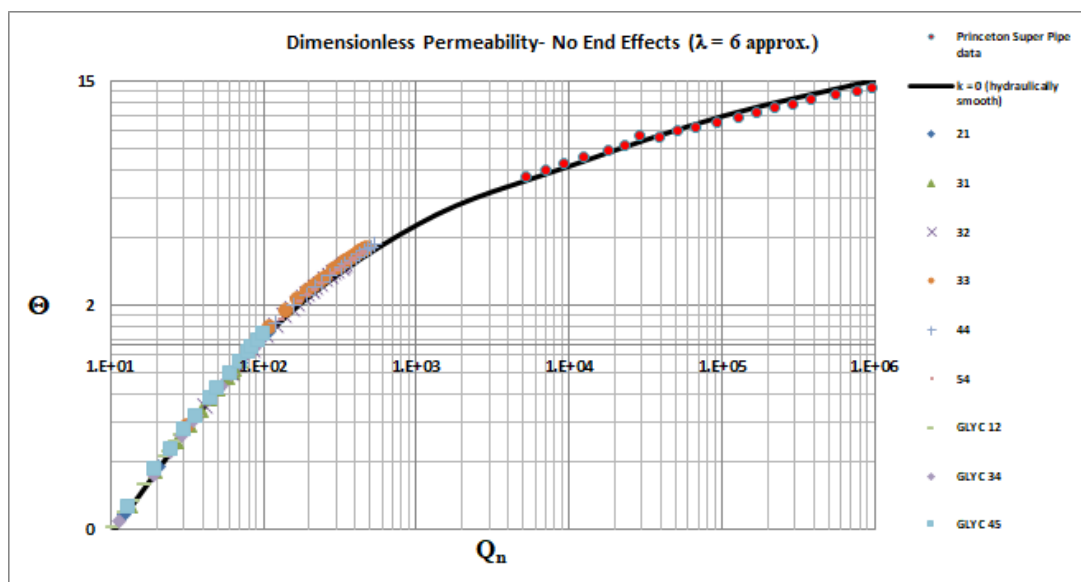


Fig. 2 Conduits with no end-effects

As can be seen in Fig. 2, we have shown all the 9 conduits which are free of end effects. Note that all these conduits fall directly on the same line ($\lambda = 6$ approx.) as do smooth walled conduits

and, for comparison purposes, we include in this plot the data from the Princeton Super Pipe study (McKeon 2005, 2005), which was done at very high values of the Reynolds number.

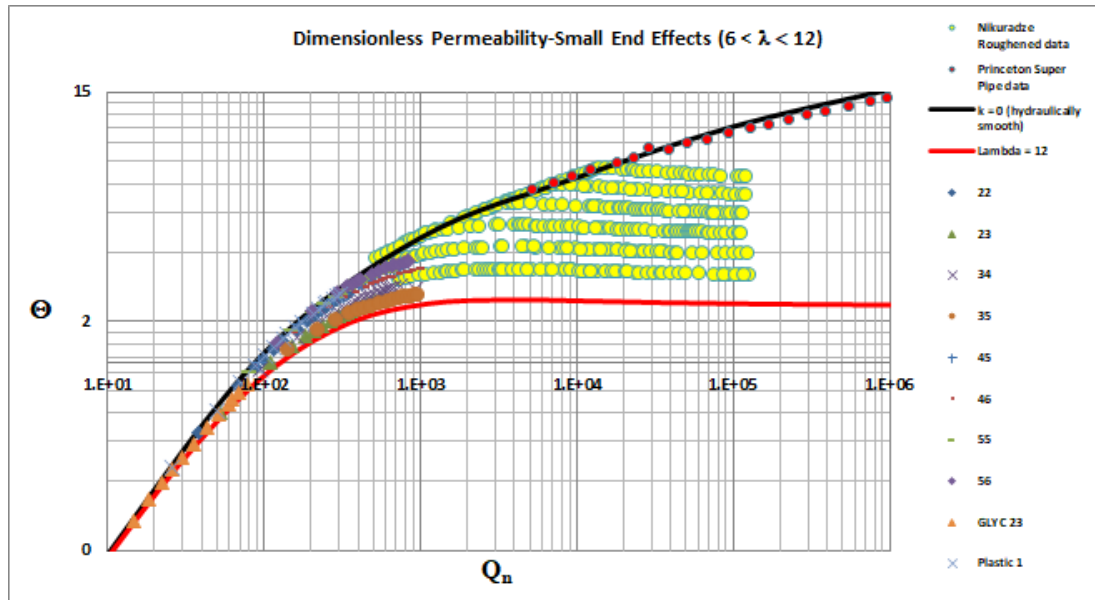


Fig. 3 Conduits with small end effects

As can be seen in Fig. 3, there are 10 conduits which have values of λ falling in the range of 6 – 12 which we designate as small end effects. Note that we have included the data of Nikuradze (Nikuradze 1933) for his sand roughened pipes which, coincidentally, fall in exactly the same range of secondary wall effects, but due to wall roughness rather than end-effects. Nikuradze's data was taken at higher Reynolds number values than the dataset in this paper under study.

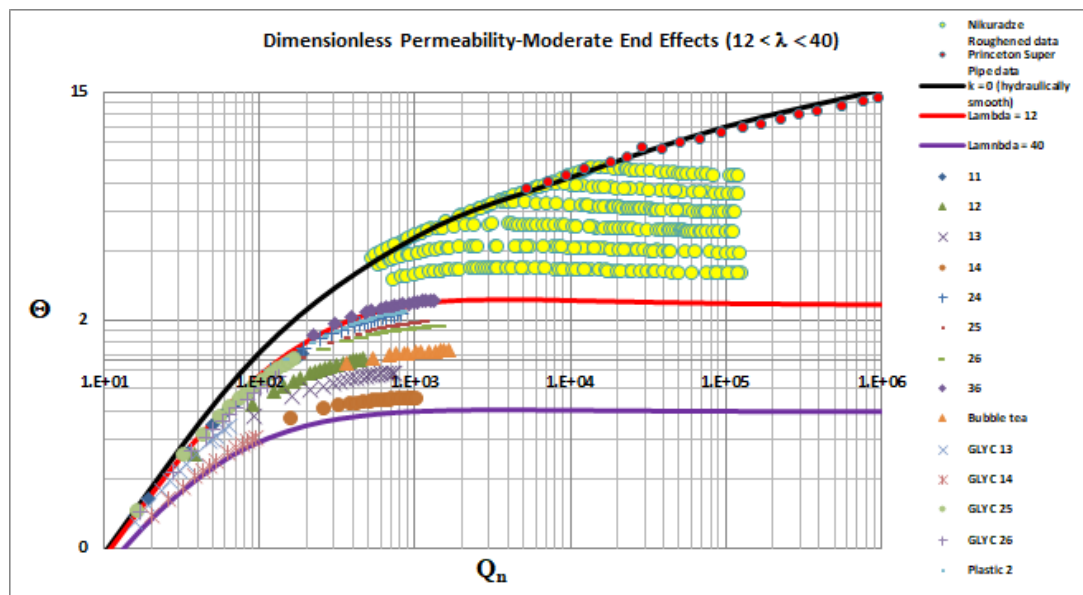


Fig. 4 Conduits with intermediate end effects

As shown in Fig. 4, there are 14 conduits that fall in the range of λ from 12 – 40 which we designate as intermediate end effects.

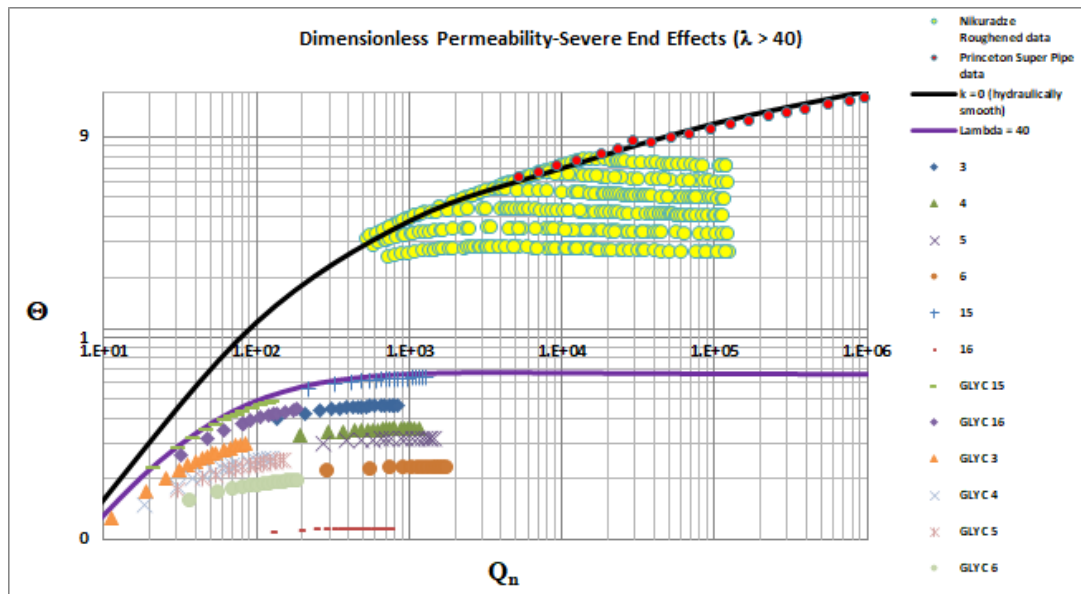


Fig 5 Conduits with severe end effects

As shown in Fig. 5, there are 12 conduits whose values of λ fall in a range greater than 40. We have designated these as severe end effects.

It is obvious from viewing all categories of end effects from small to large, that their impact is more pronounced at higher values of Q_N . It is also apparent from these figures that both end effects and wall roughness effects have exactly the same overall characteristics as the value of Q_N increases. This methodology embedded in the QFFM, differentiates secondary wall effects, which are due to the physical characteristics of the conduit, from the primary wall effect, which is, fundamentally, the formation of the viscous boundary layer, and is due to the physical characteristics of the fluid, i.e., its viscosity and flow characteristics.

VALIDATION

Finally, we conclude our analysis by showing validation of our methods and analysis in the context of the Nikuradze and Princeton Super Pipe studies. We demonstrate this in the form of a plot following Quinn's Law of Fluid Dynamics in Closed Conduits, equation (1), herein, and including all 45 conduits in the author's paper. This is shown as Fig.6 herein.

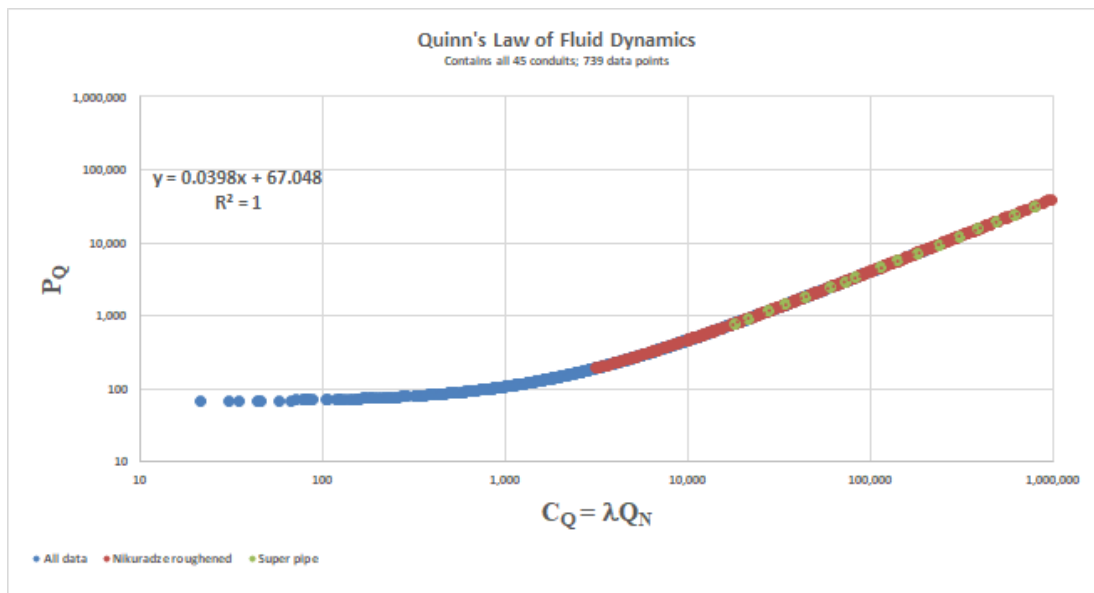


Fig 6 QFFM Data Analysis Validation

As can be seen from Fig. 6, all three data sets fall on the same straight line whose intercept represents the value of k_1 , the viscous universal constant in equation (1), and whose slope represents the value of k_2 , the kinetic universal constant in equation (1). We have shown the plot in a log-log frame of reference to highlight the relative position of each of the three data sets, i.e., the Nikuradze and Princeton data sets were measured at much higher Reynolds number values than the data set under scrutiny in this rebuttal paper. Note also that this frame of reference normalizes for wall-effects which brings the entire data set onto the same straight line.

CONCLUSIONS

In this paper we have used the measured data contained in the (Pomerenk et al 2023) paper to explain the underlying physics responsible for end-effects in fluid dynamics in closed conduits. The following general conclusions can be drawn from this work;

1. Conduit hydrodynamic end-effects is a secondary wall effect in the same way as interior wall roughness.
2. The governing physical parameter underlying end-effects is the conduit hydrodynamic radius defined as $r_c = L_E/L$ where L_E is the conduit hydrodynamic entrance length
3. The governing quantitative factor for end-effects in the overall wall-effect is the corresponding conduit hydrodynamic circumference, defined in the QFFM as the wall end-effects, $W_0 = 2\pi L_E/L$
4. End-effects enter the pressure/flow equation through the kinetic term in the form of the term l which is the overall wall effect normalization coefficient.
5. End-effects become more pronounced at higher Reynolds numbers, which eventually leads to a constant value for Q , the dimensionless permeability.
6. All measured data in this paper conforms to the teachings of equation (1), Quinn's Law.

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