

Understanding the So Called “Anomalous Thermal Strengthening” of Elastic Precursor Decay Tests

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ABSTRACT

Some time ago a Russian shock physics group performed planar impact tests on thin metal targets (1mm or less) at different temperatures. They were able to observe the elastic precursor wave amplitudes at different target thicknesses and for different target temperatures. They concluded from the test results that the elastic precursor wave decay rate was lower for higher test temperatures, or as they phrased it, dynamic strength increases with temperature. Such a behavior is counter intuitive, as we are used to believe from our experience with quasistatic viscoplastic response, that strength should decrease with temperature. Accordingly, these results of the Russian group were referred to by them as "Anomalous Thermal Strengthening". Our purpose here is to reproduce such test results using a hydrocode. To this end we do the following: 1) we change the way our hydrocode computes dynamic viscoplastic response, from the usually used radial return approach to the more appropriate overstress approach; 2) we compute a planar impact example and reproduce an elastic precursor decay behavior; 3) we show how the quasistatic part of the response can be separated from dynamic part; and 4) we make the quasistatic and the dynamic response parts depend separately on temperature, and in this way we're able to reproduce the elastic precursor wave increase with temperature, as observed in the above mentioned tests.

INTRODUCTION

Planar impact tests have been used for over 80 years to study the dynamic response of materials to high pressure shocks. In such tests a thin plate disc (the projectile) is accelerated against a thicker plate disc (the target). A shock, or a structured shock, then propagates through the target. By monitoring the velocity history of the back surface of the target on the axis, it is possible to acquire information on the pressure (related to the equation of state) and on the shear stress (related to the strength model) response of the target material. The dynamic response of viscoplastic materials in planar impact situations has been described and understood since the 1960s. Herrmann et al. [1] and later Partom [2] used the overstress approach to account for the experimentally observed phenomena of elastic precursor decay, and of structured viscoplastic shocks observed in planar impact tests. Dynamic response to impact (planar and non-planar) is usually simulated with a hydrocode (or a wavecode), and hydrocodes use as a rule the radial return approach to calculate shear stresses of dynamic elastoplastic or viscoplastic response. Using this approach (which we later call the flowstress approach), the rate dependence of the flowstress is accounted for by making it dependent on the effective strain rate. As a result, hydrocodes are not able to reproduce the phenomenon of elastic precursor decay. Instead, they produce an elastoplastic wave structure right from the

start. As outlined in the abstract, we adapt our hydrocode here to calculate shear stresses using the overstress approach. In this way we are able to reproduce precursor decay in planar impact situations. We then show how the so called anomalous thermal strengthening, as observed in [3], can be modeled by using two separate temperature influences: 1) increasing temperature decreases the quasistatic yield stress, and 2) increasing temperature decreases plastic deformation rate (thereby increasing the stress) for a given overstress situation. So, by employing the overstress approach to dynamic viscoplasticity instead of the flowstress approach, we're able to obtain thermal strengthening of the elastic precursor wave, similar to that obtained from tests in [3].

In the next section we describe the tests performed in [3], and in the section after that we explain for completeness what is meant by the flowstress approach and by the overstress approach to dynamic plasticity.

ANOMALOUS THERMAL STRENGTHENING

Kanel and his group [3] performed many planar impact tests on thin targets of different materials, and with those tests they used different temperatures up to very close to the melting temperatures. In those tests they measured the amplitude of the wave coming out of the free boundary, and deduced the level of the elastic precursor wave. The results they obtained were unexpected and amazing, considering their understanding of viscoplastic response. They were trained to consider viscoplastic response in terms of the flowstress approach, and according to this approach strength, and therefore also the precursor wave amplitude, should have decreased with the increase of temperature. This is why they regarded the outcome of those tests as anomalous.

In contrast we show later that when we treat the problem with the overstress approach and not with the flowstress approach, we get results that are in agreement with the tests. According to the overstress approach, only the quasistatic strength decreases with temperature. What goes up with temperature is the resistance to dislocation motion, caused by the increased thermal motion of the atomic network. As a result, dislocation velocity decreases, and so does the plastic deformation rate. This then leads to a slower decay of the elastic precursor wave.

The so-called anomaly that the Kanel group discovered is therefore not an anomaly. What the tests show is that the rate of decay of the elastic precursor wave decreases with temperature, but not that strength increases with temperature.

FLOWSTRESS AND OVERSTRESS APPROACHES TO DYNAMIC VISCOPLASTICITY

Dynamic viscoplastic response of ductile materials is usually modelled by assuming a constitutive law for the flowstress (Y):

$$Y = Y(P, T, \varepsilon^p, \dot{\varepsilon}) \quad (1)$$

$$Y^2 = \frac{3}{2} s_{ij} s_{ij}$$

where P =pressure, T =temperature, ε^p =effective plastic strain, $\dot{\varepsilon}$ =effective strain rate, s_{ij} =stress deviator. The initial value of Y denoted by Y_0 is called yield stress. The second equation of (1) is

a closed surface in deviatoric stress space, known as the flow surface. With the flowstress approach the current stress point is always inside or on the flow surface.

Most (if not all) hydrocodes use the flowstress approach, which works in the following way. For any computational cell and for each time cycle, the new stress deviator is determined by a two-step algorithm introduced by Wilkins [4] and known as radial return. Step 1 calculates an intermediate stress deviator s_{ij}^* and a flowstress Y^* , assuming that the current deformation rate is fully elastic. For $Y^* \leq Y$, the deformation is indeed elastic, and $s_{ij} = s_{ij}^*$. For $Y^* > Y$, the new stress deviator is on the intersection of the flow surface Y and the ray from s_{ij}^* to the origin, and is given by:

$$\beta = Y/Y^* ; s_{ij} = \beta s_{ij}^* \quad (2)$$

To evaluate Y , we need the new values of ϵ^p and $\dot{\epsilon}$. These values are not available at this stage, and previous values are used instead. This often introduces noise into the solution. It can be shown, that for an unchanging flow surface, radial return is equivalent to the flow rule:

$$d_{ij}^p = \lambda s_{ij} \quad (3)$$

where d_{ij}^p is the plastic deformation rate, and λ is a positive scalar. But when the flowstress changes by Eq. (1), radial return is not equivalent to Eq. (3). Eq. (1) expresses isotropic hardening/softening. But it can be generalized to express also kinematic hardening and/or shape change. This has no effect on the radial return algorithm, except that in some extreme cases the ray to the origin may not cross the flow surface. Once the new s_{ij} are obtained, plastic deformation rate follows from:

$$\begin{aligned} \delta_{ij}^p &= d_{ij}^p = \delta_{ij} - \delta_{ij}^e \\ \delta_{ij}^e &= s_{ij} / 2G(P, T) \\ (\dot{\epsilon}_{eff}^p)^2 &= \frac{2}{3} d_{ij}^p d_{ij}^p \end{aligned} \quad (4)$$

where G is the shear modulus, and δ_{ij} is the deformation rate deviator. Finally, the stress deviator is corrected for rigid body rotation by:

$$\dot{s}_{ij} = s_{i\alpha} \omega_{\alpha j} + s_{j\alpha} \omega_{\alpha i} \quad (5)$$

where ω_{ij} is the spin tensor.

We see that the flowstress approach is simple and easy to apply, and is therefore used in most if not all hydrocodes. Upon loading (deviatoric stress increase) from the flow surface, the current state becomes the new flow surface, so that unloading by the flowstress approach is always elastic. In this respect the overstress approach (described next) is quite different from the flowstress approach.

The overstress approach distinguishes the quasistatic yield stress (that sets stationary dislocations into motion) from the current stress (that maintains the motion of already moving

dislocations), which may be higher than the quasistatic yield stress. The quasistatic yield stress is:

$$Y_{qs} = Y_{qs}(P, T) \quad (6)$$

and the effective plastic deformation rate is given by:

$$\begin{aligned} d_{eff}^p &= d_{eff}^p (\sigma_{eq} - Y_{qs}) \\ \sigma_{eq} &= \frac{3}{2} s_{ij} s_{ij} \end{aligned} \quad (7)$$

where $\sigma_{eq} - Y_{qs}$ is the overstress. The plastic deformation rate components are then given by Eq. (3) where λ is:

$$\lambda = \frac{3}{2} d_{eff}^p / \sigma_{eq} \quad (8)$$

The rates of the deviatoric stress components are then:

$$\begin{aligned} \delta_{ij}^e &= \delta_{ij} - d_{ij}^p \\ \dot{s}_{ij} &= 2G(P, T) \delta_{ij}^e + s_{i\alpha} \omega_{\alpha j} + s_{j\alpha} \omega_{\alpha i} \end{aligned} \quad (9)$$

where the first right hand side term in the second equation of (9) is an approximation, that may be good enough for low deviatoric elastic strains.

We see that the overstress approach is expressed in terms of rate equations. These need to be integrated for each computational cell and for each time step separately, which can be done with a standard ordinary differential equation solver like the Runge-Kutta solver. Many times, a single integration step over a computational time step may be sufficient for that.

The deviatoric rate equations are coupled to the equation of state (EOS) both ways. First, the deviatoric part needs P and T, determined from the EOS. Second, plastic work is added to the volumetric internal energy, thereby influencing P, T and V (specific volume). The rate equation for P is:

$$\begin{aligned} E &= E(P, V) ; \quad \dot{E} = -P\dot{V} + \dot{W}^p \\ \dot{W}^p &= V s_{ij} d_{ij}^p \\ \therefore \dot{P} &= -\frac{P + \partial E / \partial V}{\partial E / \partial P} \dot{V} + \frac{\dot{W}^p}{\partial E / \partial P} \end{aligned} \quad (10)$$

where $\dot{V}$ during a time step is known from mass conservation, and T can be determined from the temperature EOS $T(P, V)$.

Eq. 7, which we call the flow function, is the main equation of the overstress approach. Its meaning is:

- In a dynamic situation the stress point is allowed to move out of the quasistatic yield surface, hence the term overstress.
- When this happens, the stress point tends to fall back (or relax) onto the quasistatic yield surface at a rate that increases with the amount of overstress.
- The rate of relaxation determines the strain rate sensitivity of the deformation.
- During plastic flow, the stress point is mostly outside the quasistatic yield surface.
- Also, plastic flow may continue upon unloading, and there is no noise from abrupt changes between elastic unloading and plastic reloading, as often happens with the flowstress approach.
- We use for the flow function (Eq. (7)) the simplest forms possible:

$$\begin{aligned} d_{eff}^p &= A(\sigma_{eq} - Y_{qs}) \\ d_{eff}^p &= A(\sigma_{eq} - Y_{qs})^\alpha \end{aligned} \quad (11)$$

The flow function, which reflects dislocation motion through the crystal lattice, determines both strain and strain rate hardening/softening. For a given value of A , when the total deformation rate is higher, there is more elastic deformation rate, and the stress increases (strain rate hardening). Also, for a given total deformation rate, when A decreases (with $\varepsilon^{p_{eff}}$), elastic strain rate increases (strain hardening).

In the next section we compute examples for which we use the overstress approach to show that the elastic precursor wave may increase when the sample temperature is increased.

EXPLAINING THE SO CALLED “ANOMALOUS THERMAL STRENGTHENING” OBSERVATION

As mentioned above, planar impact tests on thin metal targets (1mm or less) and at different temperatures have been reported in [3]. In these tests the authors focused on the elastic precursor wave amplitudes and observed that for higher temperatures the rate of elastic precursor decay is slower. Their own interpretation was that dynamic strength increases with temperature, and accordingly they referred to the phenomenon as “Anomalous Thermal Strengthening”.

Planar impact tests have been performed for over 70 years to study the dynamic response of materials to high pressure shocks. In such tests a thin plate disc (the projectile) is accelerated against a thicker plate disc (the target). A shock, or a structured shock, then propagates through the target. By monitoring the velocity histories of the back surface of the target on the axis in those tests, it is possible to acquire information on the pressure (EOS) and on the shear stress (strength) response of the target material.

The response of viscoplastic materials to planar impact has been described and understood since the 1960s [1,2]. Dynamic response to impact (planar or non-planar) is usually simulated with a hydrocode (or wavecode), and as stated above, hydrocodes usually use the flowstress approach to calculate the shear response. As a result, hydrocodes are not able to reproduce the

phenomenon of elastic precursor decay in planar impact. We’ve shown this by running a planar impact problem using the flowstress approach.

At the same time, we’ve also shown that the so called anomalous thermal softening response can be reproduced with the flowstress approach, by making the strain rate term of Eq. (1) to depend explicitly on temperature (in addition to the usual temperature dependence factor):

$$Y = Y_{qs} Y_P(P) Y_T(T) Y_{\epsilon}(\epsilon_{eff}^p) [1 + C_T(T) \ln \dot{\epsilon}] \quad (12)$$

$$C_T(T) = C_{Tm} + (C_{T0} - C_{Tm})(T_m - T / T_m - T_0)$$

where the index m signifies melting. In Fig. 1 we show stress histories at 1mm into the target from two such runs. The target is stainless steel with 100 cells per mm. The entering shock is 10GPa. We use $Y_{qs}=0.9\text{GPa}$, $G=77\text{GPa}$, $C_{T0}=0.01$ and $C_{Tm}=0.2$. The two runs are with $T=300\text{K}$ and 1000K , respectively.

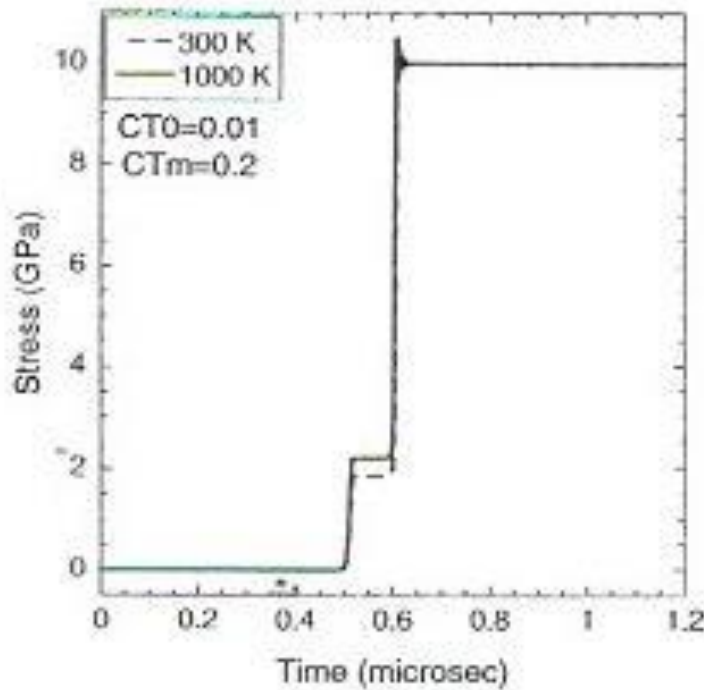


Figure 1. Longitudinal stress history at 1mm into the target, calculated with the flowstress approach and a temperature dependent strain rate coefficient.

We see from Fig. 1 that the precursor level from the run with 1000K is indeed higher. We don’t show it here, but stress history curves at longer distances into the target look the same, except that the precursor wave duration is longer, and precursor decay with distance is not predicted.

We adapted our hydrocode to use the overstress approach for the same planar impact problem. We use Eq. (11) with $\alpha=2.4$ and different values of A.

In Fig. 2 we show stress histories every 0.5mm into the target and with $A=0.1/\text{GPa}/\mu\text{s}$. Elastic

precursor decay is clearly seen.

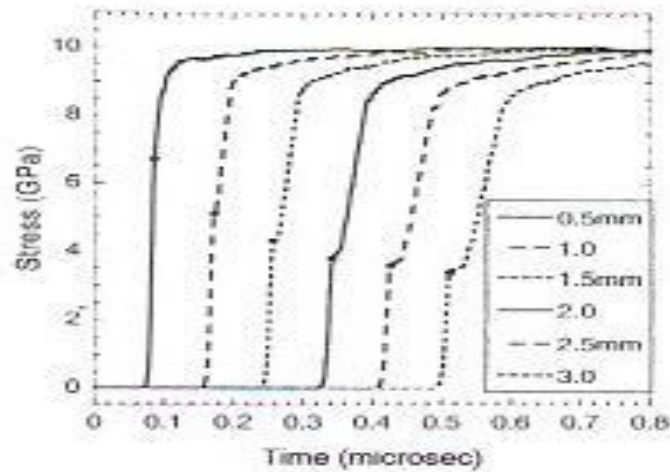


Figure 2. Longitudinal stress histories, every 0.5mm into the target, calculated with the overstress approach.

In Fig. 3 we show the precursor decay curve as function of distance from the run of Fig. 2.

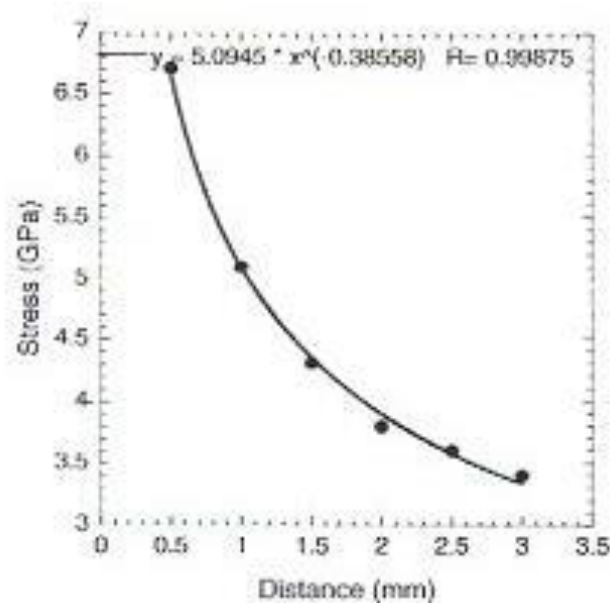


Figure 3. Precursor decay as function of distance from the calculation of figure 2.

In Fig. 4 we show stress history curves at 0.5mm into the target for different values of the coefficient A. For $A=A(T)$ this would be equivalent to curves of different temperatures. We see that for lower values of A (higher temperatures) precursor decay is slower, and precursor stress at a given distance is higher.

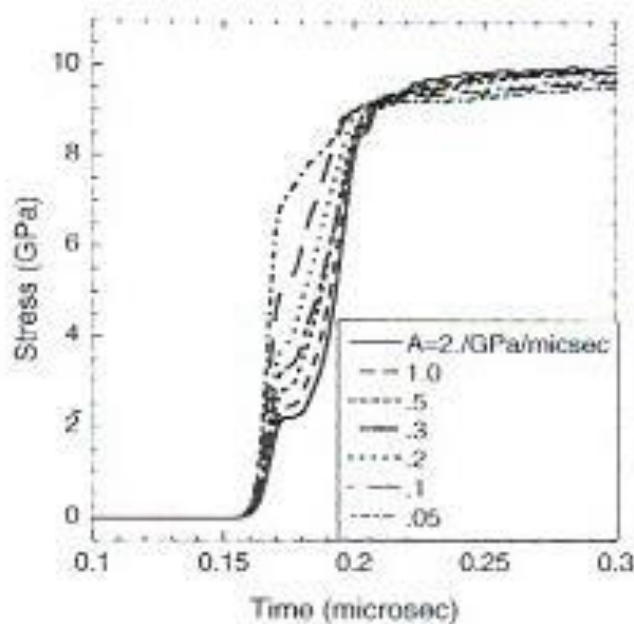


Figure 4. Longitudinal stress histories at 0.5mm into the target for different values of the coefficient A.

SUMMARY

Kanel and his group [3] in Russia performed planar impact tests on thin targets of various materials and at different temperatures, up to the melting temperature. They measured the waves breaking out of the free boundary and deduced from that the amplitude of the elastic precursor wave. The result they obtained was unexpected and even astonishing to them with regard to what they have expected. They have been trained to consider the outcome of dynamic viscoplastic response according to predictions of the flowstress approach to dynamic viscoplasticity, and according to this approach the flowstress should decrease with increasing temperature. But to their astonishment they obtained the opposite result that the elastic wave precursor on the boundary has been increasing with temperature, and they regarded this result as an anomaly.

We suggest here to treat the dynamic viscoplasticity response in the tests by Kanel and his group, using our overstress approach to dynamic viscoplasticity. Doing that we see right away that there's no anomaly whatsoever. According to the overstress approach, only the quasistatic strength decreases with temperature. In contrast, what goes up with temperature is the resistance movement of dislocations, caused by thermal vibrations of the atoms. As a result, dislocation velocity decreases and so does the plastic strain rate, and this slows down the decay of the elastic precursor wave. The level of the measured elastic precursor wave is therefore higher. In the previous section we use the overstress approach to dynamic viscoplasticity to predict such a behavior, and we see the results in Fig. 4.

References

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