



GammaV EOS for Detonation Products

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Abstract

The standard way to start the derivation of an equation of state for detonation products is by defining a trial function $P_s(V)$ along the principal isentrope of the products (where P_s is pressure and V is specific volume), with parameters to be determined from a cylinder expansion test. If that specific function turns out not to be good enough, a different function needs to be tried. We propose here a different approach which is to use instead the function $\gamma_s(V)$, where γ_s is the adiabatic gamma along the principal isentrope, as defined by Eq. (2) below. By choosing γ_s we don't have to assume the form of the function from the outset. Instead, we define the function by a set of points ($n \geq 4$), and we're able to define the amplitudes of these points recursively. In what follows we first write down the equations for determining the values of γ_{si} for the specific volumes V_i , and then work out an example with $n=4$. We show that even for $n=4$ the reproduction of test data is quite good, and also that the results are not sensitive to the exact choice of the values of V_i .

INTRODUCTION

The standard way to define an equation of state (EOS) for detonation products is: 1. define the principal isentrope $P_s(V)$, which is the isentrope descending from the Chapman-Jouguet (CJ) state (the free planar detonation state), where P =pressure and V =specific volume. Usually $P_s(V)$ is defined to be analytically integrable; 2. Calculate $E_s(V)$, which is the internal energy along the principal isentrope, by evaluating the integral:

$$E_s(V) = \int_{\infty}^V -P_s(V) dV \quad (1)$$

3. Calculate the adiabatic gamma $\gamma_s(V)$ by:

$$\gamma_s(V) = -\frac{V}{P_s} \frac{dP_s(V)}{dV} \quad (2)$$

4. Define the EOS as a Gruneisen EOS referred to the principal isentrope and given by:

$$E - E_s(V) = \frac{V}{\Gamma(V)} [P - P_s(V)] \quad (3)$$

where Γ is known as the Gruneisen parameter. Many times, the dependence of Γ on V is not known, and a constant value for Γ is assumed. Several such EOSs have been proposed during the years, and from those the most popular has been the JWL EOS [1-7], which is given by:

$$P_s = A \exp(-\rho_0 R_1 V) + B \exp(\rho_0 R_2 V) + C(\rho_0 V)^{-(1+w)} \quad (4)$$

$$w = \Gamma$$

where A , B , C , R_1 , R_2 , w are parameters to be calibrated from test data. A common way to calibrate these parameters is from: 1. the CJ state data (the detonation products state behind a plane detonation front), usually D_{CJ} (where D_{CJ} is the detonation shock velocity), and from the sonic CJ condition. These are given by:

$$\frac{V_{CJ}}{V_0} = \frac{\gamma_{CJ}}{\gamma_{CJ} + 1}$$

$$\text{or : } \gamma_{CJ} = \frac{V_{CJ}}{V_0 - V_{CJ}} \quad (5)$$

$$P_{CJ} = \rho_0 D_{CJ}^2 (V_0 - V_{CJ}) = \frac{\rho_0 D_{CJ}^2}{\gamma_{CJ} + 1}$$

and from 2. the expanding cylinder test data $u(r-r_0)$, where u is the radial velocity of the inner surface of the expanding cylinder shell of a cylinder test. This common way of calibration has several drawbacks: 1. the parameters A , B , C , R_1 , R_2 of the function $P_s(V)$ need to be determined

simultaneously; and 2. when a form for $P_s(V)$ is adopted and the fit to data that follows is not satisfactory, a different form has to be looked for.

To remedy these drawbacks, we propose here a different approach to finding an appropriate function for $P_s(V)$. Instead of assuming beforehand what this function should be, we specify the function $\gamma_s(V)$, and derive $P_s(V)$ from it. We specify $\gamma_s(V)$ as a piecewise linear curve (V_i, γ_{si}) , and we show that in this way the values of γ_{si} can be determined recursively.

In what follows we first write down the equations for determining the piecewise linear curve V_i, γ_{si} , and then present an example with $i=1,4$.

$\gamma_s(V)$ BASED EOS FOR DETONATION PRODUCTS

Let $V_{CJ} < V < V_\infty$ correspond to the range r_0 to r covered in a cylinder test, where for $V(r)$ we use the approximate relation:

$$\frac{V}{V_{CJ}} = \left(\frac{r-h}{r_0-H} \right)^2 ; \quad h = H \frac{r}{r_0} \quad (6)$$

where r_0 is the initial cylinder radius, r is the current cylinder radius, H is the initial cylinder thickness, and h is the current cylinder thickness. V_∞ is outside the range of r of the cylinder test, and $V_1=V_{CJ}$. To fit $\gamma_s(V)$ by a piecewise linear curve we need to have $n \geq 3$ (number of points of this piecewise linear curve), so that we can have $V_n=V_\infty$. The following example is with $n=4$. For $V < V_1$, $\gamma_s = \gamma_1$. For $V > V_n$, $\gamma_s = \gamma_n$. For any straight-line section:

$$\gamma = -\frac{V}{P} \frac{dP}{dV} = a + bV$$

$$a_{i-1,i} = \frac{\gamma_{i-1} V_i - \gamma_i V_{i-1}}{V_i - V_{i-1}} \quad (7)$$

$$b_{i-1,i} = \frac{\gamma_i - \gamma_{i-1}}{V_i - V_{i-1}}$$

$$P = A_{i-1,i} V^{-a_{i-1,i}} \exp(-b_{i-1,i} V)$$

$$A_{i-1,i} = P_{i-1} (V_{i-1})^{a_{i-1,i}} \exp(b_{i-1,i} V_{i-1}) \quad (8)$$

We see that the coefficients $A_{i-1,i}$ are determined recursively from left to right starting at V_1 , where:

$$V_1 = V_0 \frac{\gamma_1}{\gamma_1 + 1}$$

$$P_1 = \frac{\rho_0 D_{CJ}^2}{\gamma_1 + 1} \quad (9)$$

For $V < V_1$

$$P = P_1 \left(\frac{V}{V_1} \right)^{-\gamma_1}$$

$$E = E_1 + \frac{PV - P_1 V_1}{\gamma_1 - 1} \quad (10)$$

and for $V > V_n$:

$$P = P_n \left(\frac{V}{V_n} \right)^{-\gamma_n}$$

$$E = \frac{PV}{\gamma_n - 1} \quad (11)$$

$$E_n = \frac{P_n V_n}{\gamma_n - 1}$$

and $E_s(V)$ is determined by integrating from right to left:

$$\begin{aligned}
E_s(V) &= E_n + \int_{V_n}^V -P_s(V) dV \quad \text{for } V \geq V_{CJ} \\
E_{CJ} &= E_s(V_{CJ}) \\
E_s &= E_{CJ} + \frac{P_s V - P_{CJ} V_{CJ}}{\gamma_{CJ} - 1} \quad \text{for } V \leq V_{CJ}
\end{aligned} \tag{12}$$

But $P_s(V)$, as derived above, is not analytically integrable. We therefore do the integration numerically and create a table. Then, when using these results in a hydrocode, we interpolate for $P_s(V)$ from the table.

To construct the full EOS referred to this isentrope, we need also to know $\Gamma(V)$. For that we assume $\Gamma(V)$ to be piecewise linear between V_{CJ} and V_n . We get:

$$\begin{aligned}
\Gamma &= \Gamma_{CJ} \quad \text{for } V \leq V_{CJ} \\
\Gamma &= a_\Gamma + b_\Gamma V \quad \text{for } V_{CJ} \leq V \leq V_n \\
\Gamma &= \gamma_n - 1 \quad \text{for } V \geq V_n \\
a_\Gamma &= \frac{\Gamma_{CJ} - \gamma_n}{V_{CJ} - V_n} \\
b_\Gamma &= \frac{\Gamma_{CJ} - \gamma_n}{V_{CJ} - V_n}
\end{aligned} \tag{13}$$

The value of Γ_{CJ} that we use in Eq. (13) we determine to reproduce the derivative $k = dD_{CJ}/d\rho_0$ by using what is known as the Jones-Stanukovitch-Manson relations, which are:

$$\begin{aligned}
\Gamma_{CJ} &= \frac{\alpha_{CJ}}{1 + \alpha_{CJ}} \gamma_{CJ} \\
\alpha_{CJ} &= \frac{\gamma_{CJ} - 1 - 2k}{1 + k} \\
k &= \frac{\rho_0}{D_{CJ}} \frac{dD_{CJ}}{d\rho_0} = \frac{d \ln D_{CJ}}{d \ln \rho_0}
\end{aligned} \tag{14}$$

and for Γ_n we use $\Gamma_n = \gamma_n - 1$.

From the energy jump condition:

$$E_{CJ} = \frac{E_0}{\rho_0} + \frac{1}{2} P_{CJ} (V_0 - V_{CJ}) = \frac{1}{2} P_{CJ} (V_0 - V_{CJ}) + Q \tag{15}$$

It follows that the heat of detonation Q is determined when the principal isentrope is known.

So, if Q is known or specified, one of the γ_i values follows. Usually, we determine γ_n from Q .

EXAMPLE

We demonstrate the performance of our GammaV EOS for the special case of $n=4$. As a $u(r-r_0)$ cylinder test data curve we use our own JWL calculation results. In this way we're avoiding uncertainties related to the strength model of the cylinder test copper cylinder. The V_i values we're using are: $V_2=0.75$, $V_3=1.75$, $V_4=2.75$ cc/g. The calibrated γ_i values are: $\gamma_1=\gamma_{CJ}=2.75$, $\gamma_2=4.1$, $\gamma_3=1.8$, $\gamma_4=\gamma_\infty=1.46$, and for these values of γ the value of Q is 3.731 kJ/g.

In Fig. 1 we show the curve $\gamma_s(V)$ obtained from the above values along the principal isentrope, and we compare this curve to the JWL $\gamma_s(V)$ curve.

In Fig. 2 we compare the cylinder outer boundary radial velocity as function of its radial displacement obtained from our GammaV EOS, to that obtained from the JWL EOS that we used to create our 'experimental' data.

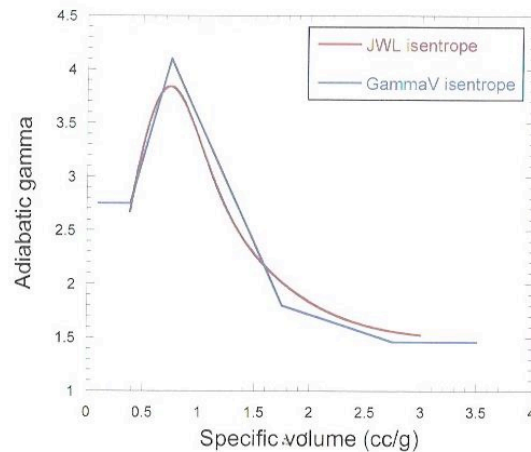


Figure 1. The $\gamma_s(V)$ function (value of γ along the principal isentrope) from GammaV EOS with 4 points on the principal isentrope, compared to the JWL EOS.

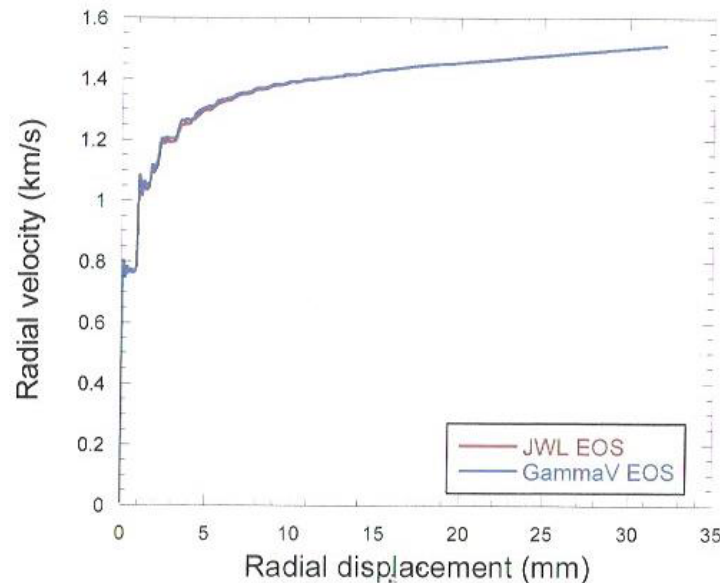


Figure 2. Radial velocity of the outer cylinder surface as function of its radial displacement, from the GammaV and the JWL EOSs.

In Fig. 3 we show that changing γ_4 from 1.46 to 1.36, and changing Q correspondingly from 3.371 to 3.935, the cylinder radial velocity $u(r)$ stays almost unchanged.

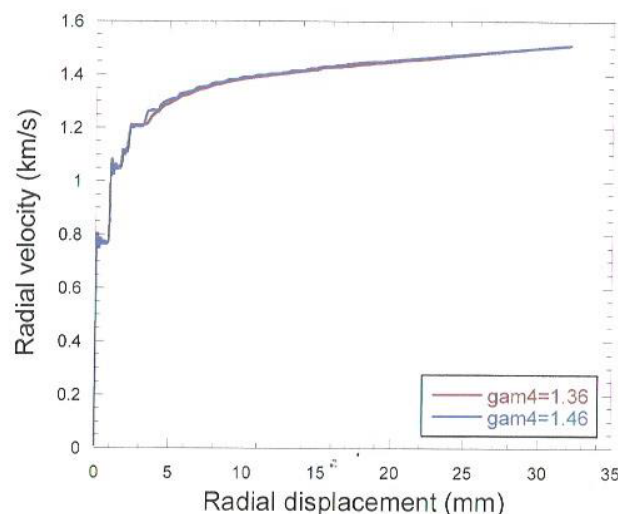


Figure 3. Radial velocity of the outer cylinder surface as function of its radial displacement for two values of γ_4 .

We see from Fig. 3 that the influence of changing the value of γ_4 from 1.36 to 1.46 is minimal. We may conclude that both Q and γ_n cannot be determined from cylinder test data alone. To determine them we need some other independent information in addition.

In Fig. 4 we show the relation $Q(\gamma_4)$ when all other parameters stay unchanged.

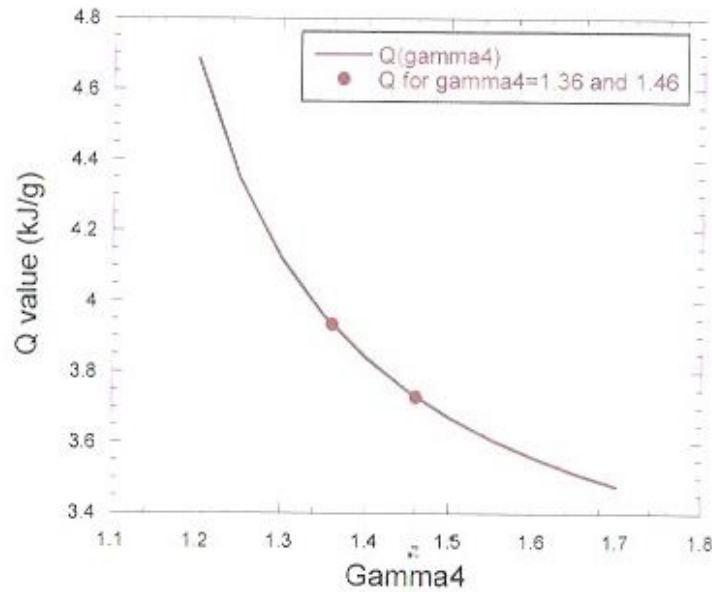


Figure 4. The relation $Q(\gamma_4)$ when all other parameters stay unchanged.

In Fig. 5 we show that the choice of the values V_i can be arbitrary, as long as the spacing between them stays appropriate. We show three $u(r)$ curves, the nominal, with $V_2=0.65$, and with $V_3=2.0\text{cc/g}$.

The parameters of curve 2 are:

$$\gamma_1=\gamma_{CJ}=2.75, \gamma_2=3.9, \gamma_3=1.9, \gamma_4=\gamma_\infty=1.51, Q=3.733\text{kJ/g}$$

The parameters of curve 3 are:

$$\gamma_1=\gamma_{CJ}=2.75, \gamma_2=4.1, \gamma_3=1.15, \gamma_4=\gamma_\infty=1.43, Q=3.730\text{kJ/g}$$

and we see from Fig. 5 that the three curves are practically identical.

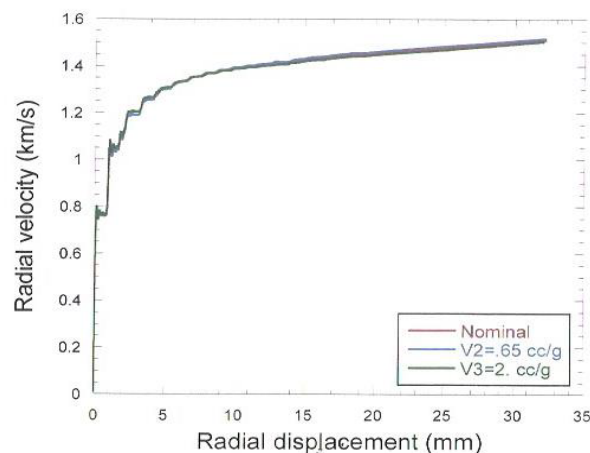


Figure 5. Radial velocity of the outer cylinder surface as function of its radial displacement for different values of γ_2 and γ_3 .

SUMMARY

The standard approach for defining an EOS of detonation products is by assuming the pressure-volume dependence $P_s(V)$ along the principal isentrope, then deriving from it the functions $E_s(V)$ and $\gamma_s(V)$, and finally assuming the form of the EOS based on these functions. A common example of a products EOS derived in this way is the JWL EOS. When a form for $P_s(V)$ is adopted and the fit to data is not satisfactory, a different form has to be looked for. Also, the function $P_s(V)$ and its parameters are assumed simultaneously, and when the fit to data is not satisfactory, a different $P_s(V)$ function with a different set of parameters needs to be tried. To avoid this situation, we propose here a different approach for the determination of the principal isentrope. Instead of assuming $P_s(V)$, we specify the function $\gamma_s(V)$, and derive $P_s(V)$ from it. The advantage of this approach is that $\gamma_s(V)$ can be specified as a piecewise linear curve (V_i, γ_{si}) , and that the values of γ_{si} can be determined recursively. In the previous section we write down the equations for determining the piecewise linear curve V_i, γ_{si} , then we present an example with $i=1,4$ and show that even with such a small number of points we get good results

References

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