

Shear Modulus and Yield Stress Change with Pressure and Temperature

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ABSTRACT

It is well known that the shear modulus (G) and the yield stress (Y) of metals increase with pressure (P) and decrease with temperature (T). Steinberg [1], in his popular compendium of dynamic material properties, assumes for $Y/Y_0(P,T)=G/G_0(P,T)$ linear relations based on tests at ambient conditions. But recent tests of high-pressure dynamic loading of certain metals yielded results that generally deviate from Steinberg's equations. Here we use a different approach to estimate $G/G_0(P,T)$. As a first approximation we let G/G_0 follow from the assumption of constant Poisson ratio (ν). This leads to $G/G_0=K/K_0$, where K is the isentropic bulk modulus. With this assumption we compute the longitudinal sound speed of tantalum along its principal Hugoniot curve, and compare the result to recent measurements. There is a slight disagreement which we correct by assuming (second approximation) that Poisson's ratio decreases slightly with pressure, and increases slightly with temperature. As K is always available in a hydrocode run from the equation of state, so are therefore also G/G_0 and Y/Y_0 .

INTRODUCTION

Derivatives of the shear modulus (G) with respect to pressure (P) and temperature (T) at ambient conditions have been determined for many materials. Steinberg [1] extrapolates these derivatives to high pressure and temperature by:

$$\frac{G}{G_0} = 1 + AP \left(\frac{\rho}{\rho_0} \right)^{-\frac{1}{3}} - B(T - T_0)$$
$$A = \frac{1}{G_0} \left(\frac{\partial G}{\partial P} \right)_0 \quad B = -\frac{1}{G_0} \left(\frac{\partial G}{\partial T} \right)_{RT}$$
(1)

Steinberg, as well as others before him, also assumed that:

$$\frac{Y}{Y_0}(P, T) = \frac{G}{G_0}(P, T)$$
(2)

where Y is the yield stress. In [2] and [3] they determine changes of yield stress and shear modulus of copper and tantalum at high pressure dynamic loading, and the results generally deviate from the predictions of Steinberg [1].

One way to determine $G(P,T)$ is by assuming that Poisson ratio (ν) does not change, or changes only slightly, with pressure and temperature. Assuming (as a first approximation) that $\nu(P,T)=\nu_0=\text{constant}$, we get:

$$\frac{G}{K} = \frac{1-2\nu}{1+\nu} = \text{Const.}$$

$$K = \rho c_b^2 \quad (3)$$

where K is the isentropic bulk modulus and c_b is the sound speed. Eq. (3) leads to:

$$\frac{G}{K} = \frac{G_0}{K_0} \quad \frac{G}{G_0} = \frac{K}{K_0} \quad (4)$$

In any dynamic situation simulated by a hydrocode, ρ, c_b and therefore also K are known for any (x,t) . Therefore, K/K_0 and using Eq. (4) also G/G_0 can be determined directly without need to evaluate their separate dependence on P and T .

Generally, it is known that Poisson's ratio decreases slightly with pressure and increases slightly with temperature. For our second approximation we therefore assume that:

$$\nu = \nu_0 + \nu_P P + \nu_T (T - T_0) \quad (5)$$

Where the coefficients ν_P and ν_T are quite small. Eq. (5) is a reasonable approximation only as long as the state (P,T) is away from melting. When (P,T) is close to melting, we may use the relation:

$$\nu = \nu_0 + \nu_P P + \nu_T (T - T_0) + \nu_m(P)(T - T_0)^2 \quad (6)$$

where $\nu_m(P)$ is determined from:

$$\nu_m(P) = \frac{0.5 - \nu_P P - \nu_T [T_m(P) - T_0]}{[T_m(P) - T_0]^2} \quad (7)$$

and where $T_m(P)$ is the melting curve. Here we use Eq. (5) as we compare to test results only up to 50GPa, which is far away from melting.

One way to determine ν_P and ν_T for a certain metal is through planar impact tests, in which the longitudinal sound speed c_L is measured [4,5]. c_L is given by:

$$c_L^2 = c_b^2 + \frac{4}{3} c_s^2$$

$$c_s^2 = G/\rho \quad (8)$$

where the bulk sound speed c_b is known from the equation of state, and where c_s is the shear wave speed. Measuring c_L yields c_s and therefore also G .

In what follows we check (calibrate) our approach against data for c_L along the principal Hugoniot curve of tantalum measured in [5]. We do this only up to 50 GPa, because at that stress there is a break in the c_L curve.

G(P) FOR TANTALUM ALONG THE PRINCIPAL HUGONIOT

In Fig. 1 we show $c_L(P)$ data from [5] up to 50GPa and our curve-fit to it. (At 50GPa there is a kink in the data curve which the authors of [5] attribute to a phase change).

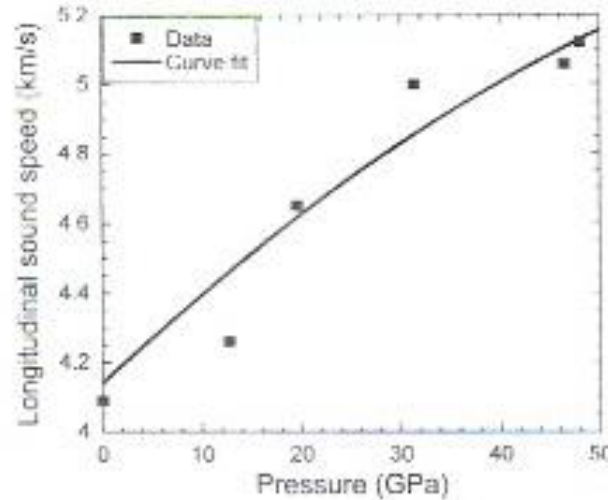


Figure 1. Data of longitudinal sound speed of tantalum up to 50GPa along the Hugoniot curve. Taken from [5].

We see from Fig. 1 that on this scale there's some scatter in the data. The curve fit is a parabola, starting at $c_L(0)=4.14\text{km/s}$.

For computing the Hugoniot curve we use the usual Uu straight line. For tantalum we use the parameters from [1]:

$$\begin{aligned} U &= c_0 + Su \\ c_0 &= 3.4\text{km/s} \quad S = 1.2 \end{aligned} \quad (9)$$

where U is the shock velocity and u is the particle velocity. The Hugoniot curve in the PV plane is then:

$$\begin{aligned} P_h &= \rho_0 c_0^2 \frac{\varepsilon}{(1 - S\varepsilon)^2} \quad \varepsilon = 1 - \frac{V}{V_0} \\ V &= 1/\rho \quad \rho_0 = 16.69\text{gr/cc} \end{aligned} \quad (10)$$

where ρ is density and V is specific volume. We use the usual Gruneisen equation of state referred to the Hugoniot with constant $\rho\Gamma$, where Γ is the Gruneisen parameter. The equation of state is:

$$E(P, V) = E_h(V) + \frac{V_0}{\Gamma_0} [P - P_h(V)]$$

$$E_h = \frac{1}{2} P_h (V_0 - V) \quad (11)$$

where E is the specific internal energy, and the index h refers to the Hugoniot curve. It is easy to verify that:

$$c_b^2 = V^2 \frac{P + \partial E / \partial V}{\partial E / \partial P} \quad (12)$$

and we do not detail here the expressions for the partial derivatives of Eq. (12) along the Hugoniot curve. We first calculate $c_L(P)$ along the Hugoniot curve assuming that $v=v_0=\text{const.}$ We show the result of this calculation in Fig. 2, and compare it there to the data curve-fit shown in Fig. 1.

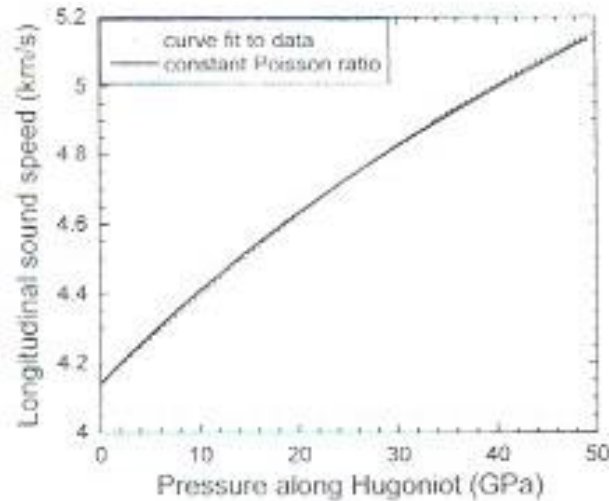


Figure 2. Longitudinal sound speed of tantalum up to 50GPa.
Points: curve fit to data from [5].
Full curve: calculated with constant Poisson ratio.

We see from Fig. 2 that even for a constant Poisson ratio the agreement is quite good, especially in view of the scatter in the data.

In Fig. 3 we show the results for $G/G_0=K/K_0$ obtained from the calculation of Fig. 2. We compare this result to Steinberg's curve from Eq. (10), with the coefficients given in [1], which are: $A=0.0145/\text{GPa}$, $B=1.3\text{e-}4/K$.

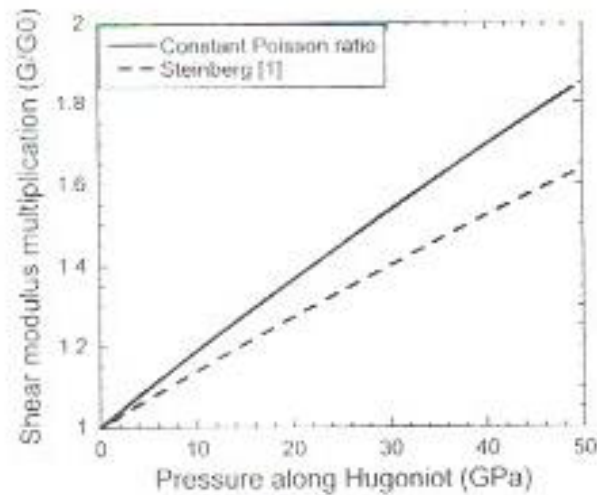


Figure 3. Full line: $G/G_0=K/K_0$ along the Hugoniot curve from the calculation of Fig. 2.
Broken line: Steinberg's curve.

We see from Fig. 3 that Steinberg's line is below our line by about 25% in G/G_0 .

In Figs. 4 and 5 we show the influence of the pressure dependent coefficient ν_p .

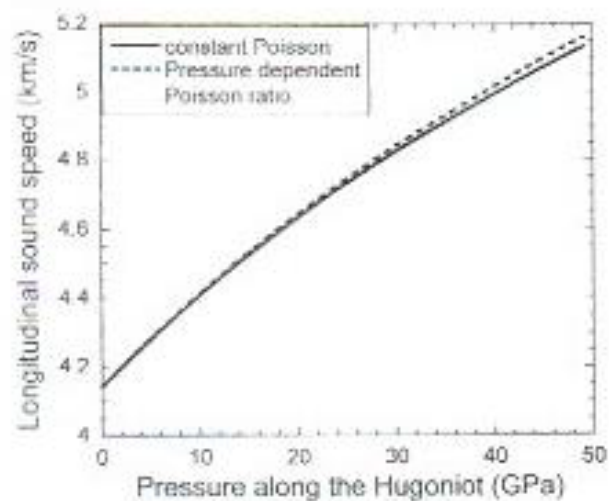


Figure 4. Longitudinal sound speed of tantalum along the Hugoniot curve up to 50GPa. Curve for $\nu_p=-1.e-4/GPa$,
Compared to the curve with constant ν .

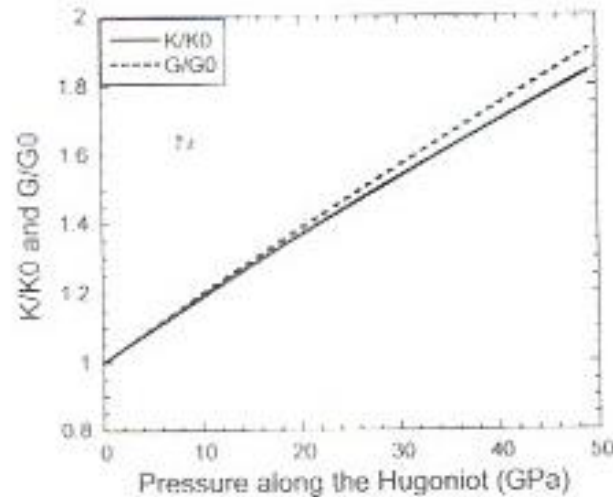


Figure 5. G/G_0 and K/K_0 for tantalum along the Hugoniot curve up to 50GPa, for $v_p = -1.e-4/\text{GPa}$.

From Fig. 4 we see that for $v_p < 0$ (which is the usual case) going up the Hugoniot curve, v decreases slightly and c_L increases slightly.

From Fig. 5 we see that going up the Hugoniot curve, G/G_0 increases above K/K_0 .

Next, we check the influence of having $v_T > 0$. To this end we need to calculate the increase of temperature along the Hugoniot curve. We do this by using the first TdS equation of thermodynamics together with the first law:

$$TdS = dE + PdV$$

$$TdS = C_v dT + \frac{\Gamma_0}{V_0} T dV \quad (13)$$

Equating Eqs. (13) and applying to the Hugoniot curve we get:

$$C_v \frac{dT}{dV} = -\frac{1}{2} P - \frac{1}{2} (V_0 - V) \frac{dP}{dV} - \frac{\Gamma_0}{V_0 T}$$

$$\frac{dP}{dV} = \frac{dP_h}{dV} = -\rho_0^2 c_0^2 \frac{1 + S\varepsilon}{(1 - S\varepsilon)^2} \quad \varepsilon = 1 - \frac{V}{V_0} \quad (14)$$

Integrating Eqs. (14) (with a standard ODE solver), and using for C_v (constant volume heat capacity) 0.15J/gr/K, we get the temperature increase shown in Fig. 6.

In Fig. 7 we see that for $v_T > 0$ (which is the usual case) and going up along the Hugoniot curve, v increases slightly and c_L decreases slightly.

From Fig. 8 we see that going up along the Hugoniot curve as detailed for Fig. 7, G/G_0 decreases below K/K_0 .

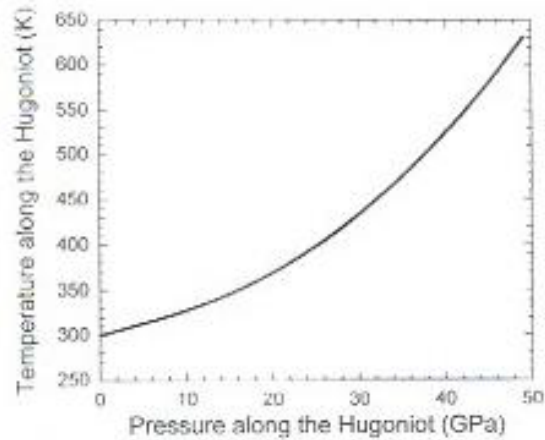


Figure 6. Temperature along the Hugoniot curve of tantalum up to 50GPa.

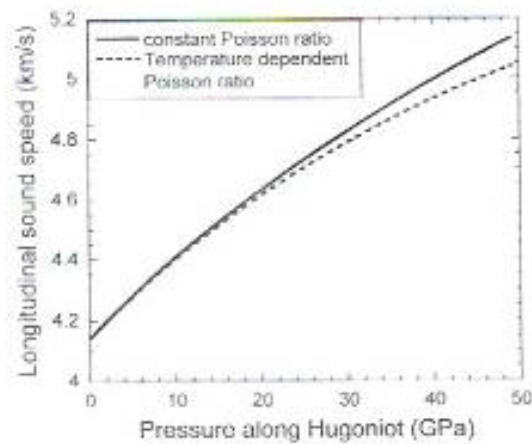


Figure 7. Longitudinal sound speed of tantalum along the Hugoniot curve up to 50GPa. Curve for $\nu_T=5e-5/K$ compared to the curve with constant ν .

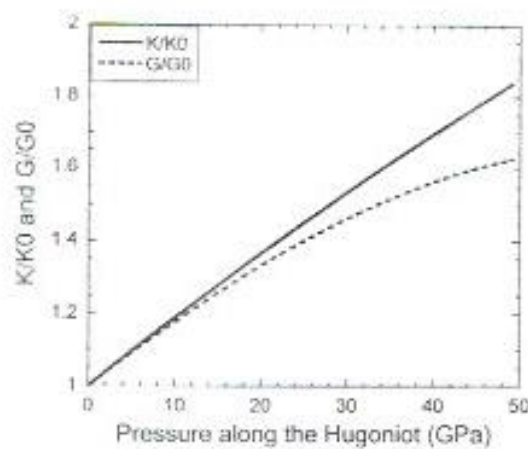


Figure 8. G/G_0 and K/K_0 for tantalum along the Hugoniot up to 50GPa for $\nu_T=5e-5/K$.

SUMMARY

It is well known that the shear modulus and yield stress of metals increase with pressure and decrease with temperature. Steinberg [1] extrapolates pressure and temperature derivatives of the shear modulus at ambient conditions to high pressure and temperature. But indirect determination of strength in high pressure dynamic experiments shows deviations from such an extrapolation.

As a first order approximation we assume here that Poisson's ratio stays constant, which leads to $G/G_0 = K/K_0$, where G is the shear modulus, and K is the isentropic bulk modulus. With this assumption, data along the Hugoniot curve of tantalum are reproduced almost exactly. As a second-degree approximation we let Poisson's ratio decrease slightly with pressure and increase slightly with temperature, and we show the sensitivity to these changes.

One advantage of our approach is that the bulk modulus follows directly from the equation of state, and there's no need to calculate pressure and temperature changes separately.

References

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