

Detonation Velocity as Function of Curvature from Diameter Effect Data

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ABSTRACT

$D_n(k)$ (normal detonation velocity as function of curvature) is usually calibrated from rate stick breakout data. Using this approach, it is often found that sections of the $D_n(k)$ curve, obtained from sticks of different diameters, do not agree at overlapping k ranges. We therefore ask if it is possible to calibrate $D_n(k)$ from diameter effect data, and show that this is indeed possible. We use for that an indirect calibration procedure: 1) assume a $D_n(k)$ curve; 2) for different values of the steady rate stick detonation velocity D_s , integrate the two front shape ODEs until the boundary angle is reached, and record the resulting diameter d ; and 3) compare the resulting D_s, d pairs to the experimental diameter effect curve. We show here that by appropriately correcting the assumed $D_n(k)$ curve, we can easily get agreement with the experimental diameter effect curve, thereby completing the calibration procedure. As a byproduct we show that the $D_n(k)$ curve must have a limit curvature, or otherwise the failure stick diameter is not reproduced. Finally, we apply the same procedure to predict the size effect (like diameter effect, but for nonstick geometries) and the breakout curves for other steady detonations.

INTRODUCTION

The model called DSD (= Detonation Shock Dynamics) makes it possible to calculate propagation of a steady or quasi-steady diverging detonation front, independent of the hydrodynamics behind it [1,2]. According to DSD, what determines the normal detonation velocity (D_n) at a point on the detonation front is the mean curvature (k) of the front at that point. To apply the DSD model, we therefore need to measure or calibrate the relation $D_n(k)$ for the considered explosive. In addition, what determines the front shape as a whole is also the boundary angle θ_b (= angle between the front and the normal to the boundary), specific to the explosive and the inert material behind the boundary. Many times, DSD is applied to situations that are not quite steady-state. When this is the case, normal detonation velocity at a point on the front is still determined by $D_n(k)$, but the boundary angle condition is treated as a limit condition: $\theta_b \geq \theta_L$ (= Limit angle).

The standard practice of calibrating $D(k)$ and θ_L is from steady detonation in cylinders (or rate sticks). In a typical test one measures the steady axial detonation velocity D_s , the breakout curve from the far end of the stick $t(r)$, and the boundary angle θ_b . Tests like this are performed for several stick diameters (d), and each of them covers a certain range of the curvature k with some overlap. The procedure for extracting $D_n(k)$ and θ_b from such rate stick test data is as follows: 1) convert the breakout curve $t(r)$ to a front shape curve $z_{ex}(r)$, using the measured axial detonation velocity D_s ; 2) curve fit $z_{ex}(r)$ to obtain $z(r)$. Such curve fitting is needed because we have to differentiate $z(r)$ twice to obtain the curvature k , and the end result is quite sensitive

to experimental noise and to the curve fit function; 3) differentiate $z(r)$ once to get θ , $D_n = D_s \cos \theta$, and the perpendicular curvature component $k_2 = \sin \theta / r$; 4) differentiate twice to get the curvature component k_1 and the mean curvature $k = (k_1 + k_2) / 2$. We call this kind of approach Differential Calibration (= DIFC). Using DIFC we encounter three problems: 1) results are sensitive to the curve fit function; 2) $D_n(k)$ sections extracted from tests with different stick diameters do not connect to a single curve [3,4]; 3) the slope of $z(r)$ changes rapidly near the boundary, and the error in extracting θ_b is large.

On this background we ask the following: 1) is it possible to assume $D_n(k)$ and θ_b and calculate from them the diameter effect $D_s(d)$?; 2) if yes, can we go further and calibrate $D_n(k)$ to match the measured diameter effect. We call this calibration procedure Integral Calibration (= INTC). In [5] we've already demonstrated that INTC might work.

In what follows we first perform integral calibration to match diameter effect data from [3,6]. We then apply the results to predict the diameter effect (or size effect) and breakout curves for other steady detonations.

INTEGRAL CALIBRATION

As outlined above, to perform integral calibration we need the diameter effect curve $D_s(d)$, and the boundary angle θ_b . For $D_s(d)$ we use data from [3,6], as shown in Fig. 1. The value to use for θ_b is somewhat uncertain, as demonstrated in [3]. We performed reactive flow calculations of detonation in sticks (not shown here), and extracted from them the boundary angle. We obtained $\theta_b = 0.5$ radian, which is at the lower end of the range given in [3], and this is what we use here. We show later, that instead of using θ_b we can use the boundary front lag Δz , which is determined from rate stick tests with less uncertainty. Also, as θ_b changes rapidly near the boundary, integral calibration is not sensitive to it.

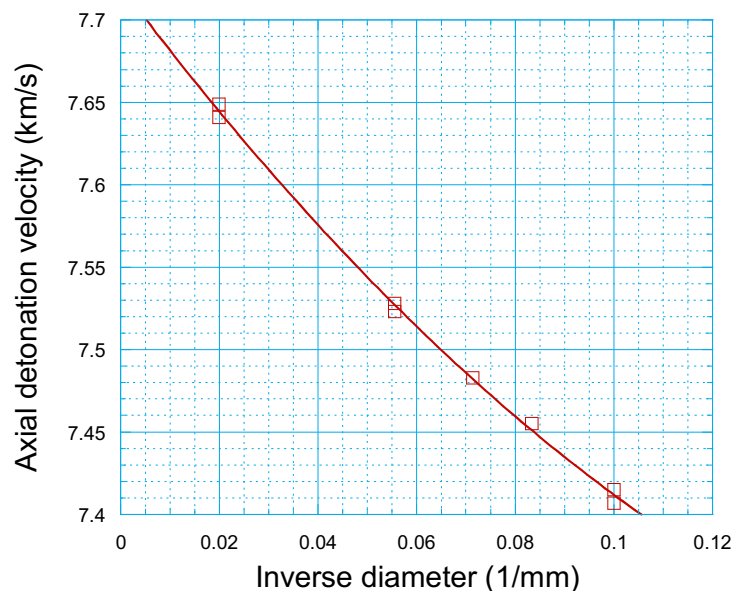


Figure 1. Diameter effect data from [3,6].

The procedure to calculate $D_s(d)$ from $D_n(k)$ and θ_b is: 1) choose a value for D_s . In fact, we go through a D_s range in steps ΔD , starting from D_{CJ} ; 2) integrate the detonation front equations along the front, starting from the axis. These equations are:

$$\begin{aligned}\frac{dz}{dr} &= z' \\ \frac{d}{dr}(z') &= -k_1 [1 + (z')^2]^{\frac{3}{2}}\end{aligned}\quad (1)$$

where:

$$\theta = -\arctan(z')$$

$$D_n = D_s \cos \theta$$

$$k = k(D_n)$$

$$k_2 = \sin \theta / r$$

$$k_1 = 2k - k_2 \quad (2)$$

Eqs. (1) are a system of two first order ODEs, and we integrate them numerically from $\theta=0$ to $\theta=\theta_b$, where $r=a$ and $d=2a$. The pairs (D_s, d) thus obtained are on the diameter effect curve corresponding to the $D_n(k)$ (or $k(D_n)$) relation assumed.

Initially we assumed for $D_n(k)$ a straight line $D_n = D_{CJ}(1 - bk)$, with $b=1\text{mm}$. In Fig. 2 we compare the $D_s(d)$ relation obtained to the data points from Fig. 1. We see that: 1) the calculated curve is convex upwards, has a horizontal tangent and goes above the data points near $1/d=0$; 2) at high $1/d$ values the calculated curve goes below the data points. We conclude that $D_n(k)$ needs to have a higher slope initially and a lower slope later on. The easiest way to achieve that is by using a bi-linear curve. After some trials we ended up with the following curve:

$$\begin{aligned}D_n &= D_{CJ}(1 - b_1 k) \quad ; \quad 0 \leq k \leq k_b \\ D_n &= D_b(1 - b_2 k) \quad ; \quad k \geq k_b \\ D_b &= D_{CJ} \frac{1 - b_2 k}{1 - b_1 k} \\ k_b &= 0.14 / \text{mm} \\ b_1 &= 1.65\text{mm} \quad b_2 = 0.56\text{mm}\end{aligned}\quad (3)$$

In Fig. 3 we show the agreement obtained with the curve defined by Eqs. (3).

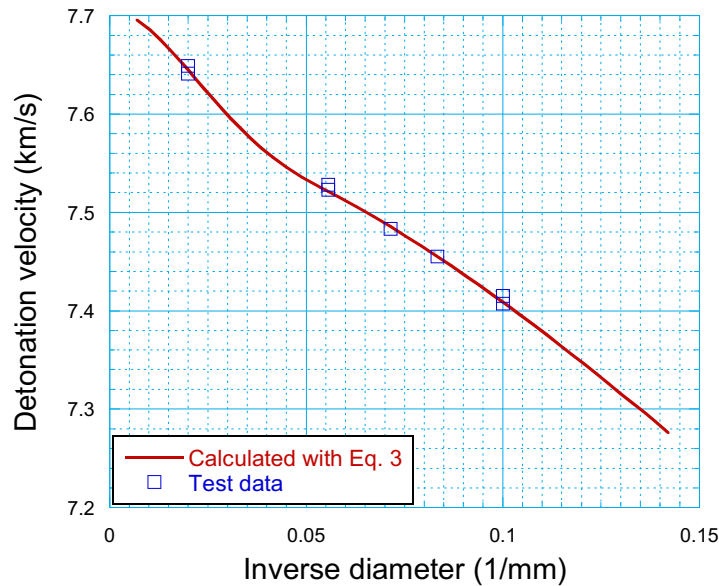


Figure 3. Calculated $D_s(1/d)$ curve, using the bi-linear $D_n(k)$ curve of Eq. (3).

From Fig. 3 we see that the agreement is quite good, but something is still missing. The calculated curve goes on even beyond the failure diameter ($d_f=8\text{mm}$). The only way to avoid a steady solution below d_f is for $D_n(k)$ to turn back at a failure or limit curvature k_f . In [8] we show a $D_n(k)$ curve that we calculated with our reactive flow model [7] for quasi-steady spherical symmetry. It has a limit curvature and a lower branch. In Fig. 4 we reproduce this curve together with the bi-linear $D_n(k)$ curve from Fig. 3, to which we added a straight-line lower branch. Using the calibrated upper branch with the straight-line lower branch, we are able to reproduce the diameter effect data, including the failure diameter.

In Fig. 5 we show the front lag data Δz taken from [3]. We see from Fig. 5 that the front lag Δz can be represented by a straight line. As mentioned above, Δz can be used instead of θ_b to end the integration along the front. At the end of every integration step we check if $z_0 - z(r) \geq \Delta z(2r)$. When it does, we have reached the diameter corresponding to the assumed value of D_s .

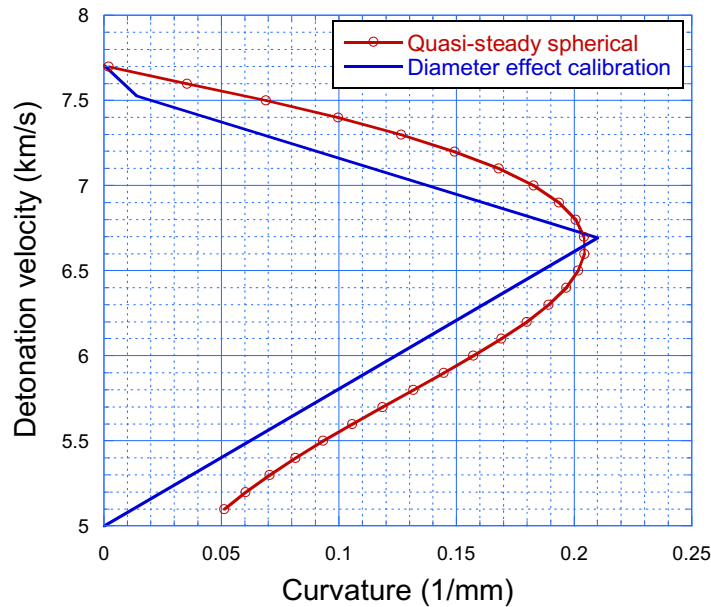


Figure 4. Bi-linear $D_n(k)$ from Eq. (3) with an added lower branch, compared to $D_n(k)$ calculated in [8].

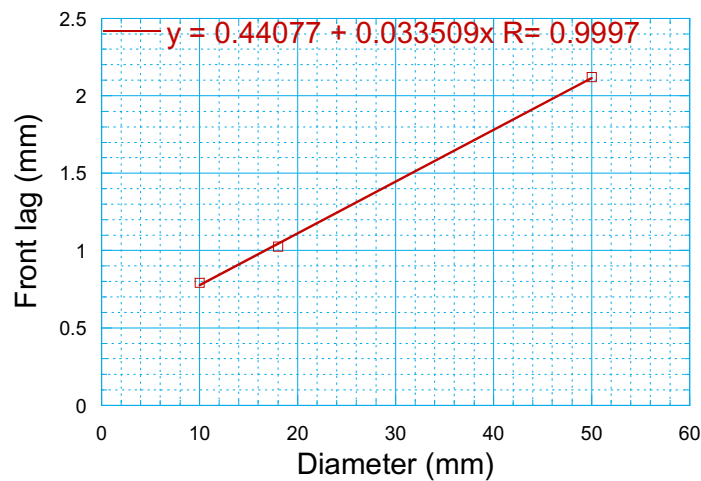


Figure 5. Front lag data from [3].

OTHER STEADY DETONATIONS

Next, we apply the same procedure to other 2D steady detonations. The first is detonation in a hollow cylinder (or tube), $r_{in} \leq r \leq r_{out}$. There are no data from detonations in tubes, maybe because the products in the hole collapse on the axis and may form a jet. The equations to integrate along the front are Eqs. (1) as before. The only difference is that we start the integration from $r=r_{in}$, and the boundary angle is the same at both boundaries. In Fig. 6 we show $D_s(d)$ curves for $r_{in}=1$ and 2mm and $r_{out}=9$ mm, compared to the $D_s(d)$ curve for a solid cylinder with $d=18$ mm. In Fig.7 we show breakout curves $t(r)$ for the same three cases.

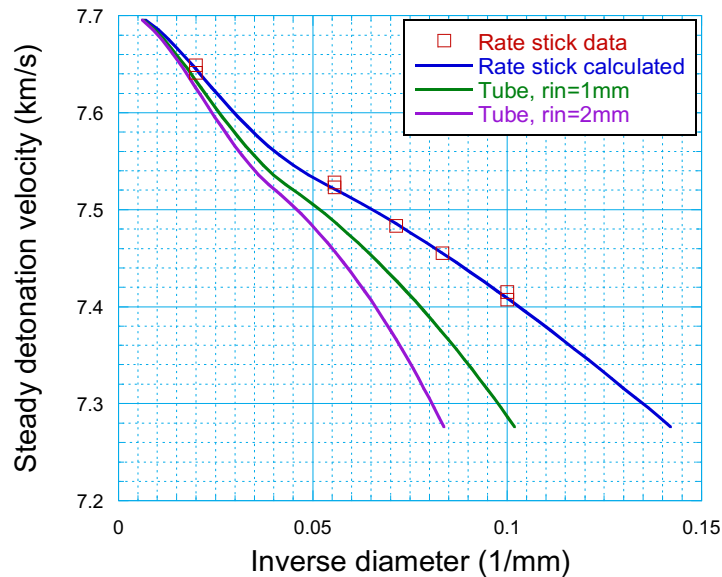


Figure 6. Calculated $D_s(1/d)$ for tubes, compared to a solid cylinder.

Another case is steady detonation in a cylindrical rib $r_{in} \leq r \leq r_{out}$. To write down the equations along the detonation front, we use the following notation: the angle around the rib is φ , the polar coordinates of points on the front are r, φ , the tangent to the front at a point makes an angle θ with the x axis, and α is the angle between the normal to the front and the normal to the radial ray at the considered point. We choose r as the independent variable, and seek to write differential equations for $d\varphi/dr$ and $d^2\varphi/dr^2$. The equations are:

$$x = r \cos \varphi ; y = r \sin \varphi$$

$$\varphi' = \frac{d\varphi}{dr} = \frac{1}{r} \frac{y' \cos \varphi - \sin \varphi}{y' \sin \varphi + \cos \varphi}$$

$$y' = \frac{dy}{dx} = \tan \theta \quad (4)$$

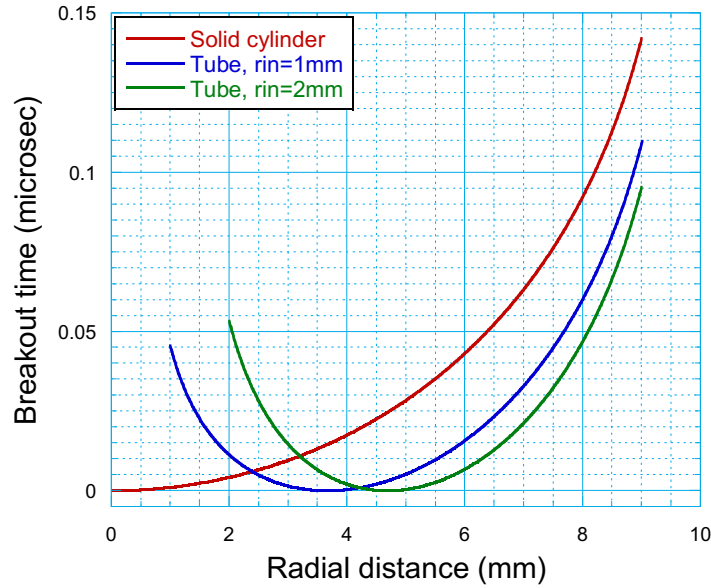


Figure 7. Calculated breakout time curves for tubes, compared to a solid cylinder.

We start the integration at $r=r_{in}$ with $\theta=\theta_b$, so that $\varphi' = \tan \theta_b / r_{in}$. The two ODEs are:

$$\begin{aligned}\frac{d\varphi}{dr} &= \varphi' \\ \frac{d\varphi'}{dr} &= \varphi''\end{aligned}\quad (5)$$

and φ'' is determined according to [9] as follows: k_1 (the front curvature in the r, φ plane) is expressed in polar coordinates in terms of $dr/d\varphi$, and $d^2r/d\varphi^2$ and is given by:

$$\begin{aligned}-k_1 &= \frac{r^2 + 2(r')^2 - r r''}{[r^2 + (r')^2]^{3/2}} \\ \therefore r'' &= \frac{1}{r} \left\langle k_1 [r^2 + (r')^2] + [r^2 + 2(r')^2] \right\rangle\end{aligned}\quad (6)$$

In Eq. (6) we need to convert from r', r'' to φ', φ'' , and this is done by:

$$\begin{aligned}r' &= \frac{1}{\varphi'} \\ \varphi'' &= -\frac{r''}{(r')^3}\end{aligned}\quad (7)$$

As we deal here with a cylindrical rib, $k_2=0$ and $k=k_1/2$ or $k_1=2k$. The normal detonation velocity is thus given by:

$$y' = \frac{r\varphi' \cos \varphi + \sin \varphi}{-r\varphi' \sin \varphi + \cos \varphi}$$

$$\theta = \arctan y'$$

$$\alpha = \theta - \varphi$$

$$D_n = D_\varphi \cos \alpha \quad (8)$$

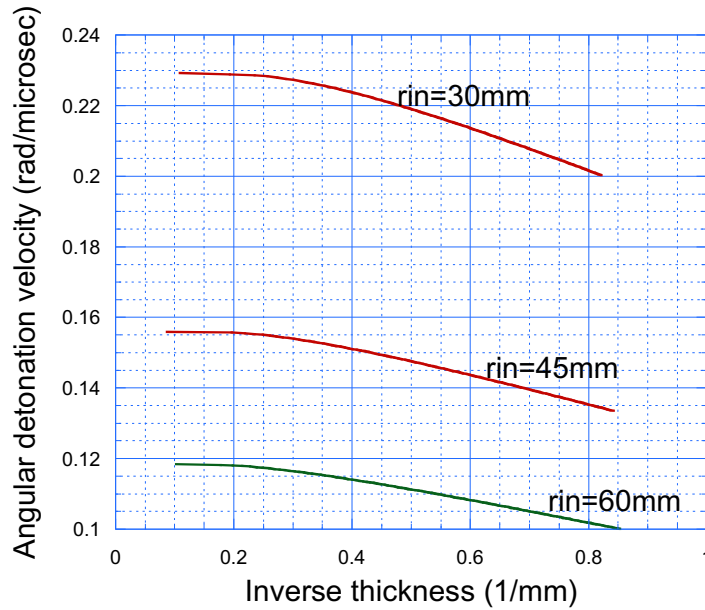


Figure 8. Calculated angular detonation velocities for cylindrical ribs of different internal radii. Ribs thickness is 5mm.

In Fig. 8 we show the size (or rib radius) effect curves, calculated for ribs with $r_{in}=30, 45$ and 60mm . We see from Fig. 8 that for all values of r_{in} there is a rib thickness beyond which there is no steady solution. For all three values of r_{in} this thickness is around 10mm . Beyond this thickness there is no steady solution because the normal velocity cannot exceed D_{CJ} .

In Fig. 9 we show the calculated breakout time curves for the three ribs with a rib thickness of 5mm .

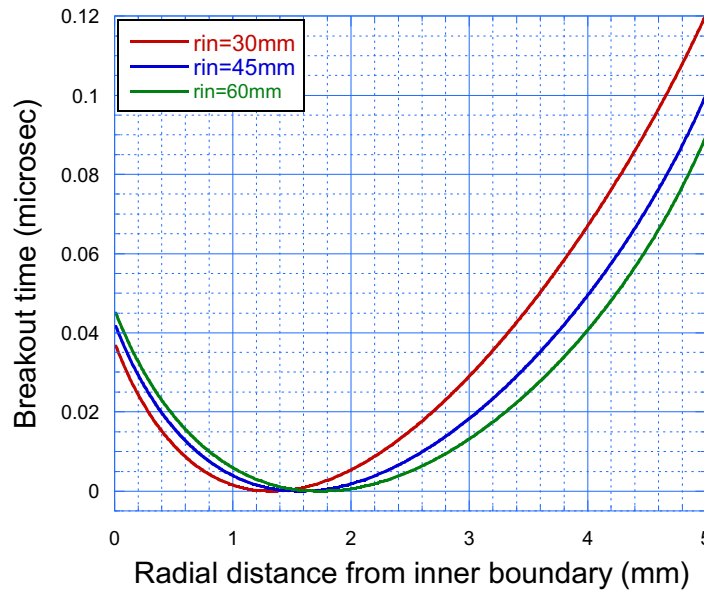


Figure 9. Calculated breakout curves for cylindrical ribs of 5mm thickness.

SUMMARY

The standard way for calibrating a $D_n(k)$ (normal detonation velocity as a function of front curvature) relation is by: 1) performing a rate stick test and measuring the steady detonation velocity D_s and the steady breakout curve; 2) calculating the steady front shape $z(r)$; 3) curve fitting $z(r)$ and 4) extracting a $D_n(k)$ relation from it for a range of k . Doing this for different stick diameters does not usually yield a continuous $D_n(k)$ curve.

We asked ourselves if it's possible to predict the diameter effect curve $D_s(d)$ given the $D_n(k)$ relation and the boundary angle θ_b , and in this way calibrate $D_n(k)$ from the known $D_s(d)$ and θ_b .

We show that this is indeed possible and rather easy to do, and show an example of such a calibration. We also apply the same procedure to predict the size effect (diameter or thickness effect) for other steady detonations.

References

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