

Modeling the Fourth Power Law Response of Viscoplastic Materials

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ABSTRACT

A standard way to parametrize a steady structured shock going through a viscoplastic material, is by plotting the stress jump across the wave ($\Delta\sigma$) as function of its maximum strain rate ($\dot{\epsilon}_{\max}$) plotted on a logarithmic scale. Doing this we usually get a straight line, which means that $\dot{\epsilon}_{\max} \propto (\Delta\sigma)^\beta$. Grady and coworkers noticed in the 1980s that for many materials $\beta=4$. Since then, several workers in the field tried to understand this unusual behavior, but they were not able to arrive at any plausible explanation. Recently, Grady revisited his fourth power law with an extensive review, including an outline of various mechanisms proposed as possible explanations of this law. But still, none of these mechanisms is able to explain the fourth power law response. In view of this background, we use here the fourth power law to calibrate the strain rate component of the constitutive response of Al 6061-T6. To this end, we integrate the steady viscoplastic equations (developed long ago) describing the propagation of a plane wave from the elastic precursor level to the stress plateau level. We find that in order to reproduce the fourth power law, the plastic deformation rate has to depend on the deviator overstress by: $\dot{d}_p \propto (s - \frac{2}{3}Y)^\alpha$, with $\alpha=2.38$. But for high shock levels, when the elastic precursor is overdriven, using this value of α leads to a higher value of the slope β , close to $\beta=6$. This result has yet to be verified experimentally.

INTRODUCTION

When a planar shock enters a viscoplastic material (like in a planar impact), a structured quasi-steady wave is formed after some propagation distance. It includes an elastic precursor wave and a plastic wave. The elastic precursor level is known as the Hugoniot elastic limit (HEL). The plastic wave is a structured shock. The mechanics of quasi-steady viscoplastic waves became clear in the 1960s [1,2], and some years later such waves were observed in planar impact tests [3,4]. From the results of tests reported in [4] it followed that Al 6061-T6 responds by a fourth power law curve. In the 1980s, this work was continued and expanded with many more tests on various materials, and the main conclusion was that the fourth power law response is almost universal [5-8].

The fourth power law response is expressed in terms of $\Delta\sigma$ and $\dot{\epsilon}_{\max}$, where

$\Delta\sigma = \sigma_{\max} - \sigma_{\text{HEL}}$, σ is the longitudinal stress and ϵ is the longitudinal strain. Plotting pairs $\Delta\sigma, \dot{\epsilon}_{\max}$ on a log scale, one gets a straight line represented as:

$$\dot{\epsilon}_{\max} \propto (\Delta\sigma)^4 \quad (1)$$

An example of the fourth power law response of Al 6061-T6 taken from [9] is reproduced in Fig.1.

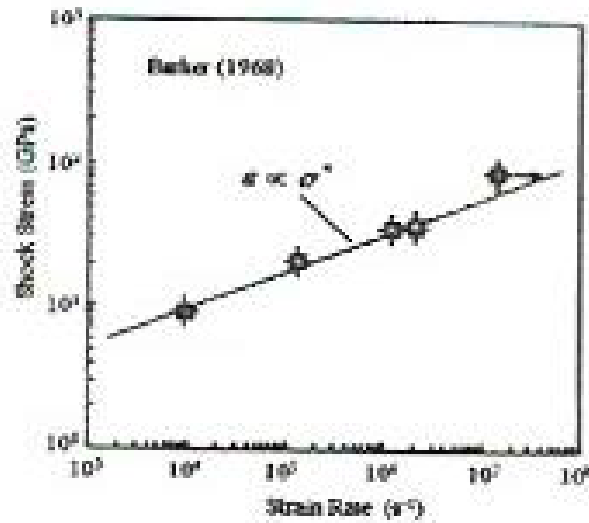


Figure 1. Fourth power law response of Al 6061-T6, from [9]

Grady and coworkers discovered the universality of the fourth power law and also found materials for which the exponent is less than 4. Recently Grady revised the subject with an extensive review [9]. From the review we learn that: 1) Since the fourth power law has been discovered, the main research effort on the subject was to find the mechanism responsible for such a behavior; 2) many mechanisms were proposed, but none was proved to be the right one, and the nature of the fourth power law response is still not understood; 3) wave code simulations using standard viscoplastic models to reproduce the fourth power law response were unsuccessful, probably because artificial viscosity usually overshadows the effects of real viscosity.

In view of this background, we use here the following approach: 1) we're not trying to understand the origin and nature of the fourth power law response; 2) instead, we assume that the fourth power law exists, and use it to calibrate strain rate hardening; and 3) we do this by integrating directly the system of ODEs (Ordinary Differential equations) that represents quasi-steady 1D viscoplastic flow.

STEADY VISCOPLASTIC FLOW

The ODEs for 1D steady viscoplastic flow were originally developed by Duvall and coworkers [1], and later by Herrmann [2]. These equations are:

$$\begin{aligned}\dot{\sigma} &= \frac{2Gd^p}{c_1^2/U^2 - 1} \\ \dot{u} &= \frac{\dot{\sigma}}{\rho_0 U} \quad \dot{\epsilon} = \frac{\dot{\sigma}}{\rho_0 U^2}\end{aligned}\tag{2}$$

where σ =stress, u =particle velocity, $\epsilon=1-V/V_0$ =strain, V =specific volume, G =shear modulus, U =plastic wave speed, c_1 =dilatational sound speed, $\rho=1/V$ = mass density, and d^p = plastic

deformation rate. It is easy to see that Eqs. (2) describe the Raleigh line, in agreement with the fact that steady structured shocks obey the Hugoniot relations.

It is possible to develop Eqs. (2) in this form only by assuming a single compression curve for the viscoplastic material (ignoring entropy changes). And this is a good approximation below say 30GPa, where dealing with plastic waves is relevant. For higher shock levels the influence of plastic flow would be just a small perturbation.

For the compression curve we use the usual shock C,S parametrization:

$$P = \rho_0 C^2 \frac{\varepsilon}{(1 - S\varepsilon)^2} \cong \rho_0 C^2 \varepsilon (1 - 2S\varepsilon) \quad (3)$$

For the shear modulus we assume:

$$G = G_0 (1 + G_1 \varepsilon) \quad (4)$$

The dilatational sound speed is given by:

$$\rho_0 c_1^2 = \frac{dP}{d\varepsilon} + \frac{4}{3} G \quad (5)$$

The plastic wave speed is:

$$\rho U^2 = \frac{\sigma_{\max} - \sigma_{\text{HEL}}}{\varepsilon_{\max} - \varepsilon_{\text{HEL}}} \quad (6)$$

And finally, the plastic deformation rate is given as a function of the overstress from the quasi-static flowstress:

$$d^p = F(s - s_0) = F\left(s - \frac{2}{3} Y\right) \quad (7)$$

where s is the shear (or deviator) stress, and s_0 is the value of s at the HEL level. s_0 should include the influence of strain hardening, pressure hardening and temperature softening, when these influences are of importance. In what follows we ignore these causes of hardening/softening.

What we do next is to calibrate the function F of Eq. (7) from the fourth power law. To make the calibration useful for a general 3D stress state, we need to express quantities in Eq. (7) in terms of effective (or equivalent) quantities. From the definitions of effective quantities, it follows that:

$$\begin{aligned} d_{\text{eff}}^p &= d^p & \sigma_{\text{eff}} &= Y & s &= \frac{2}{3} Y \\ \therefore s &= \frac{2}{3} \sigma_{\text{eff}} \end{aligned} \quad (8)$$

Substituting into Eq. (7) we get:

$$d_{\text{eff}}^p = F\left[\frac{2}{3}(\sigma_{\text{eff}} - Y)\right] \quad (9)$$

In Fig. 2 we show a schematic plot of the stress history in a regular steady plastic wave, and in Fig. 3 we show a similar plot for an overdriven plastic wave. A regular plastic wave is when $U < U_{\text{HEL}}$, and an overdriven plastic wave is when $U > U_{\text{HEL}}$. We see from Fig. 3 that when the elastic

wave is overdriven, the plastic wave is not S shaped, its maximum rate is near that of the elastic wave, and it is attached to the elastic wave. We haven't seen in the literature data of a plastic wave overdriving an elastic precursor.

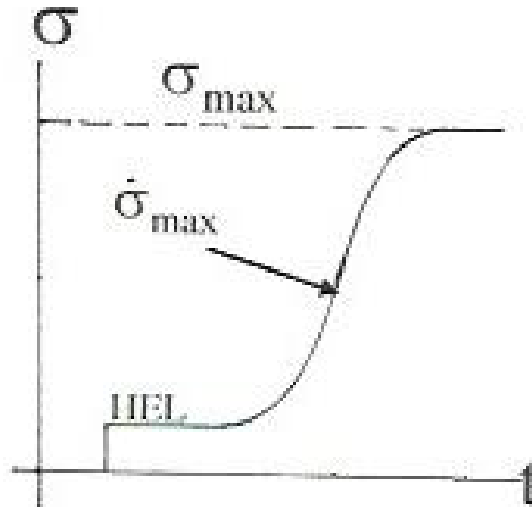


Figure 2. Schematics of a regular plastic wave

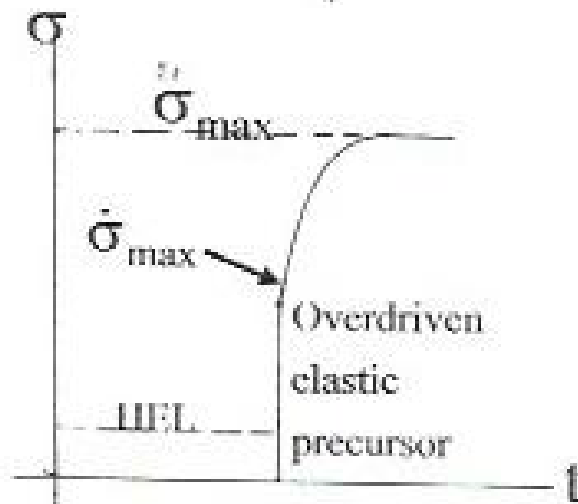


Figure 3. Schematics of an overdriven plastic wave

COMPUTATIONS

For the function F in Eq. (7) we use:

$$d^p = A \left(s - \frac{2}{3} Y \right)^\alpha$$

$$s = \sigma - P \quad (10)$$

which is a two-parameter relation. We calibrate A and α for Al 6061-T6 to fit the curve given in Fig. 1. We use the following parameters, all taken from Steinberg's EOS and strength compendium [10]:

$$\begin{aligned} \rho_0 &= 2.703 \text{ gr / cc} & C &= 5.24 \text{ km / s} \\ S &= 1.40 & G_0 &= 27.6 \text{ GPa} \\ G_1 &= 4.84 & Y &= 0.28 \text{ GPa} \end{aligned} \quad (11)$$

We integrate Eqs. (2), using the standard 4th order Runge-Kutta ODE solver.

To calibrate A and α we made several integration-runs with a constant value of A and different values of α . We found that α controls the slope in Fig. 1, and that to get $\beta=4$, we need to use $\alpha=2.38$. We then adjusted A to bring the two curves together. In Fig. 4 we compare computed points to the experimental curve. The parameters used in the computations are: A=0.033 (GPa, μs) and $\alpha=2.38$.

We see from Fig. 4 that the agreement between tests and computations is almost perfect.

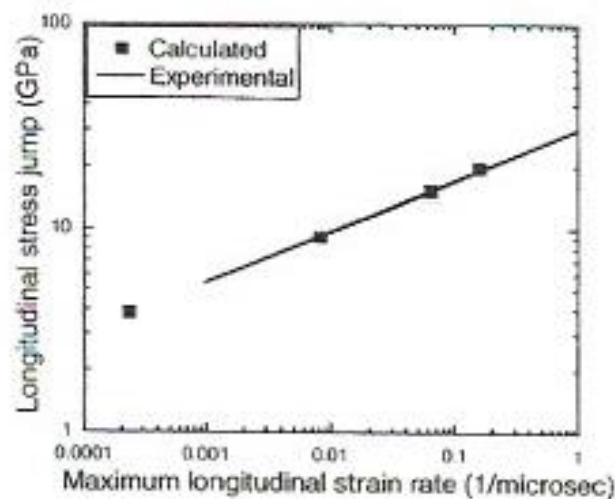


Figure 4. Calculated points compared with the experimental curve of Fig. 2

Having calibrated the parameters A and α of Eq. 10, we now use them to compute steady plastic waves in the overdriven range (as in Fig. 3). We show the results in Fig. 5.

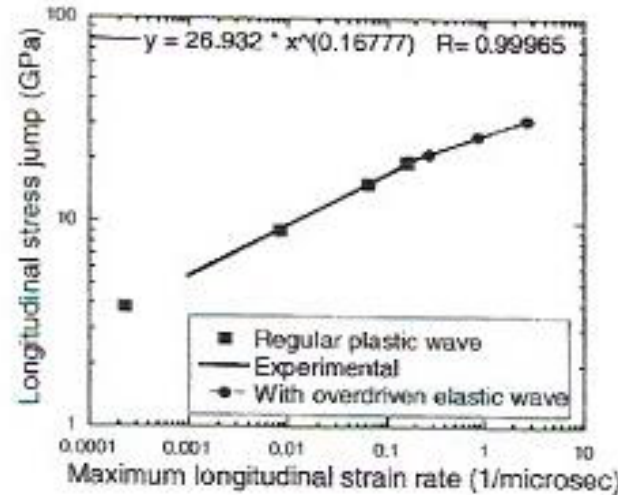


Figure 5. Calculated curve in the regular and in the overdriven ranges, compared with the experimental curve of Fig. 2

We see from Fig.5 that in the overdriven range the slope is higher, about 6. To our knowledge, this higher slope has not been observed experimentally, probably because experiments were not conducted in the overdriven range.

SUMMARY

Impact tests of viscoplastic materials lead to steady viscoplastic waves, and for such waves a relation $\dot{\epsilon}_{\max}(\Delta\sigma)$ can be extracted, where $\dot{\epsilon}_{\max}$ is the maximal strain rate in the wave, and $\Delta\sigma$ is the stress jump from the elastic precursor to the wave plateau (see Fig. 2). For many materials tests show that this relation is a power law of the form: $\dot{\epsilon}_{\max} = B(\Delta\sigma)^\beta$, where $\beta=4$ to a good approximation. This result is known as the fourth power law.

There are many works on this subject in the literature. In those works, researchers tried two things: 1) to understand the origin of this law in terms of microscale mechanisms; and 2) to be able to reproduce this kind of response by running a wavecode, and using established viscoplastic models.

According to Grady's recent review on the subject [9], the current situation is: 1) an understanding or an explanation of the fourth power law response has not been found; and 2) the fourth power law response has not been reproduced by running a wavecode, probably because artificial viscosity (used to smear shocks) overshadows the plastic wave.

In this work we use the steady 1D viscoplastic ODEs developed in [3,4], to calibrate the strain rate hardening of the viscoplastic response of Al 6061-T6 from Barker's data [3,4] (reproduced in Fig. 1). We do not run a wavecode. Rather, we integrate numerically the system of ODEs representing a 1D steady viscoplastic response. In this way we're able to calculate the details of the plastic wave rather quickly, and to any desired accuracy.

The plastic deformation rate appearing on the right-hand side of the first of Eqs. 2, and given by Eq. 10, has two adjustable parameters. The exponent α determines directly the slope β of the

curve in Fig. 1. We find that to get $\beta=4$ (as found in tests), we need to have $\alpha=2.38$. Also, the coefficient A in Eq. (10) shifts the curve in Fig.1 up and down, and we find that to get agreement with Barker's data we need to have $A=0.033$ (GPa, μs).

Barker's data are for regular (not overdriven) plastic waves. Computing overdriven plastic waves with the same values of A and α , we find that the slope increases to about 6 (see Fig. 5). This result is yet to be checked experimentally.

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