

The Golden Mean Number System: Its Platonic Roots, Paradigmatic Symmetry and Quantum Parameters

Scott Olsen

Professor Emeritus of Philosophy and Religion
College of Central Florida, Ocala, Florida

M. S. El Naschie

Distinguished Professor of Physics and Engineering
Department of Physics, Faculty of Science
University of Alexandria, Alexandria, Egypt

ABSTRACT

The *golden mean number system* may well be the most powerful tool in the physicist/mathematician's arsenal. It emerges naturally out of Plato's two principles, the One and the Indefinite Dyad of the Greater (Φ) and the Lesser (ϕ or $1/\Phi$). Herein we unpack its structural backbone in the golden series of exponential powers, ..., ϕ^7 , ϕ^6 , ϕ^5 , ϕ^4 , ϕ^3 , ϕ^2 , ϕ , 1, Φ , Φ^2 , Φ^3 , Φ^4 , Φ^5 , Φ^6 , Φ^7 , ..., along with its perfect combinatorial properties of addition and subtraction in growth and diminution, as well as, through multiplication and division via application of the modular Φ . We unravel the underlying *paradigmatic symmetry* of any given golden power serving simultaneously as geometric, arithmetic and harmonic means. And in the process, we reveal how the quantum parameters, including the pre-quantum particle, pre-quantum wave, Einstein spacetime, Unruh temperature, Hardy entanglement and the Barbero-Immirzi parameter, emerge naturally within Plato's famous *Republic* similes of the Sun, Divided Line and Cave. The *golden mean number system* has now reemerged most completely and successfully in the E-Infinity theory.

Keywords: golden section, One and Indefinite Dyad, the Greater and the Lesser, continuous geometric proportion, geometric mean, arithmetic mean, harmonic mean, pre-quantum particle, pre-quantum wave, Einstein spacetime, Unruh temperature, Hardy entanglement, Barbero-Immirzi parameter, Plato's Sun, Divided Line and Cave.

INTRODUCTION

"The *golden mean number system* is the only number system which includes all integers, rational and irrational numbers in addition to zero and countable as well as uncountable infinities in a natural way and does not break down anywhere. It is the mathematics of the constructor theory and the *lingua franca* of nature and consequently it is the foundation and unification of rational existence." [1,24]

The starting and the ending points of this paper are the mathematical and philosophical *golden mean number system* of Plato, that great Pythagorean philosopher who was an initiate of the ancient Egyptian (Saitic) and Greek (Eleusinian) mystery traditions. Under an oath of strict silence (regarding the mysteries' central revelation of continuous "golden" proportional

symmetry) with the penalty of death, Plato brilliantly concealed (and yet revealed for the astute student/reader) the profound truth in his *Dialogues* when viewed in conjunction with his principles of the One and Indefinite Dyad. [2,3,4] The teachings were carried forward very subtly through an oral tradition (consistent with Plato's *agrapha dogmata* – unwritten lectures, including his *On the Good*) only to later emerge quite forcefully in the famous eclectic Greco-Egyptian school, the Museum of Alexandria. There Egyptian, Persian, Indian, Chinese, Greek, Roman and Arabic civilizations were melded together to form an international shrine of science, art and wisdom. The school's library was even known at one point to have in excess of 400,000 papyrus scrolls, the largest repository of recorded information, knowledge and wisdom in the ancient world. Unfortunately, it suffered successive decimation at the hands of various political and religious extremists, the scrolls eventually serving as fuel for the public baths. [5] Forced underground once again, aspects of the doctrine reemerged later amongst various Pythagoreans, Platonists, number theorists, the Tübingen platonic school in Germany, and finally most fully and pragmatically in the 20th and 21st centuries' through the brilliant development of E-Infinity theory. [1-4,6-15,18,24-29]

And now, the present authors, along with their E-Infinity collaborators, [24-29] are focused on achieving a wider recognition of the doctrine. One of us (Olsen) established quite clearly that the *golden mean number system* is derived from Plato. [4,2,3] And the other (El Naschie) has been using the resulting system to literally crack open the deepest paradoxes of modern-day physics, certainly from at least 1994 when the seminal paper was published, *Is Quantum Space a Random Cantor Set with a Golden Mean Dimension at the Core?* [6] [7-15]

PLATONIC BASIS FOR THE GOLDEN MEAN NUMBER SYSTEM, AND THE EMERGENCE OF QUANTUM PARAMETERS IN PLATO'S THREE REPUBLIC SIMILES

"It is in this way, when they preserve the standard of the mean that all their works are good and beautiful.... The Greater [Φ] and the Lesser [$1/\Phi$ or ϕ] are to be measured in relation, not only to one another [$\Phi:\phi$ or $\phi:\Phi$], but also to the establishment of the standard of the mean [$\Phi:1:\phi$ or $\phi:1:\Phi$].... [T]his other comprises that which measures them in relation to the moderate, the fitting, the opportune, the needful, and all the other standards that are situated in the mean between the extremes." [Plato, *Statesman* 284a1-e8; bracketed material inserted by author]

In an earlier work [2] it was established that Plato did indeed have (as espoused by Aristotle [16]) two principles, the One (or Good) and the Indefinite Dyad (the Greater and Lesser, Excess and Deficiency, or the More and the Less). And most significantly, that the Greater referred to the major form of the golden ratio (i.e. $\Phi \approx 1.618033...$) and the Lesser referred to the minor form of the golden ratio (i.e. $\phi \approx 0.6180339...$). And from these two principles, the One and the Indefinite Dyad, Plato derived his ontology and epistemology. In addition, when the One is placed in relation to the Greater and Lesser, it becomes apparent that the One is itself the "golden" geometric mean between the Lesser and Greater $\phi:1:\Phi = \phi:1::1:\Phi$. And this of course comports with Plato's assertion at *Timaeus* 31b-32a that "continuous geometric proportion" is how separate, even disparate items are bonded or united together (e.g. the immaterial and material, being and becoming, mind and body). And as is clearly demonstrated [4], Plato's ontology and epistemology of the Intelligible and Sensible realms, as reflected in his three

Republic similes of the Sun (502d-509c), Divided Line (509d-511e) and Cave (514a-521b), are rooted in continuous geometric proportion (see figures 1 and 2)

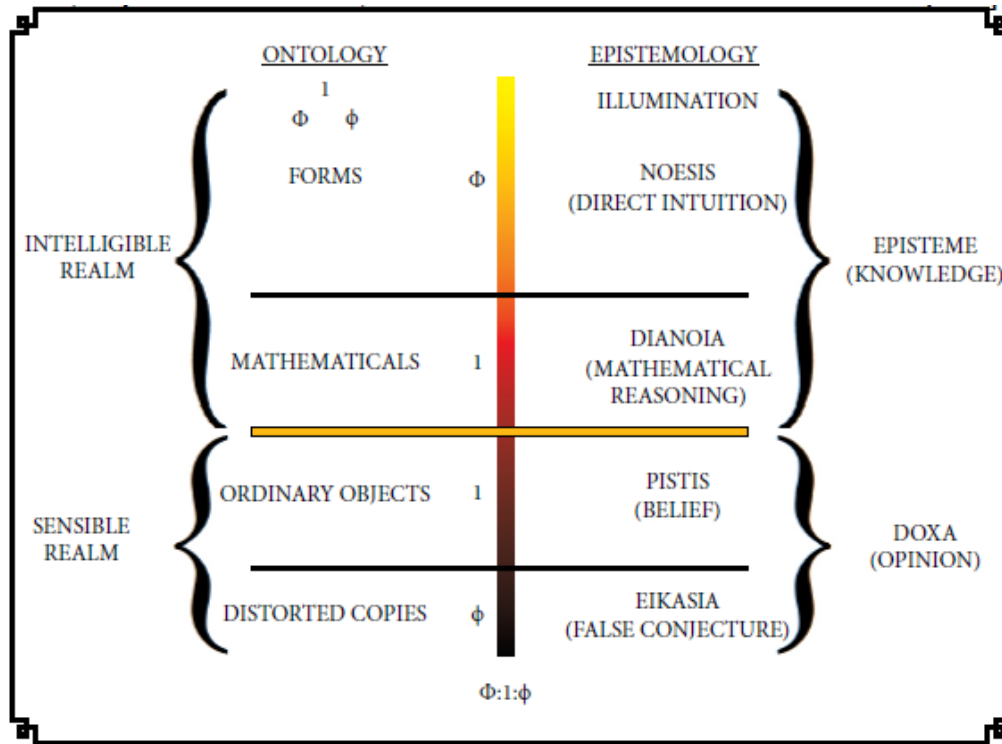


Figure 1. Plato's Divided Line showing his ontology and epistemology [4]

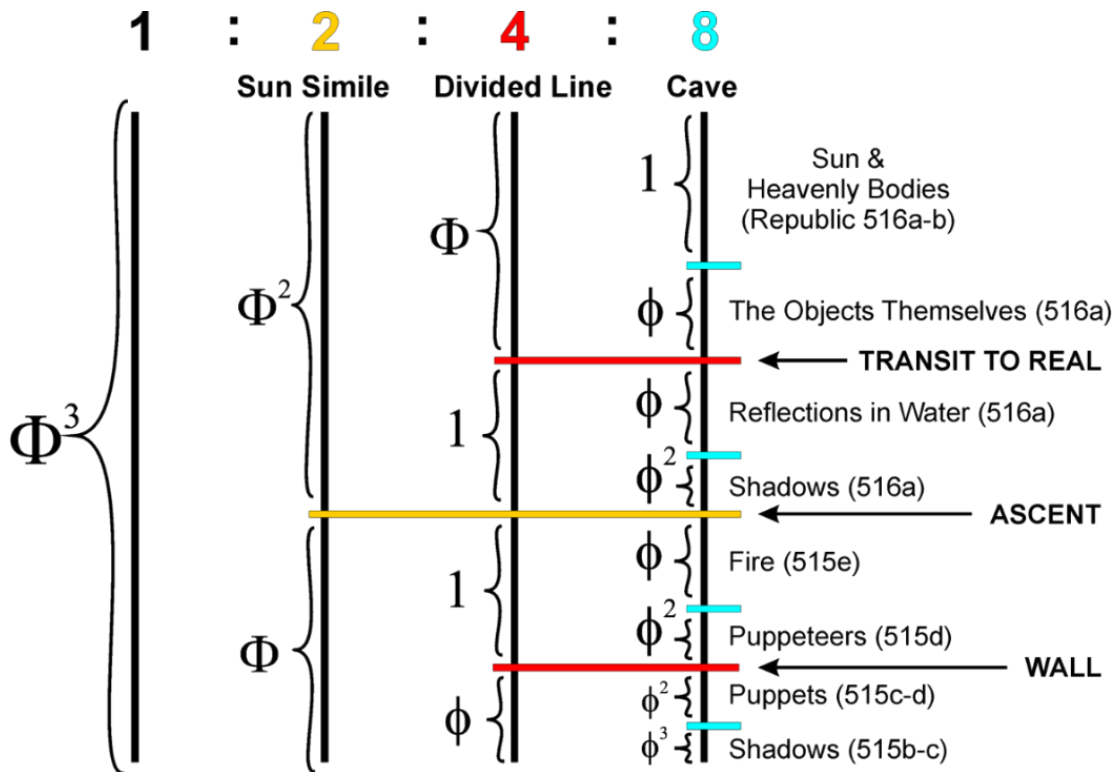


Figure 2. Plato's Republic similes in "golden" geometric proportion [4]

In Figure 3 below, by beginning with the golden sectioning of the One, we show how the quantum parameters naturally emerge as the fractal golden powers.

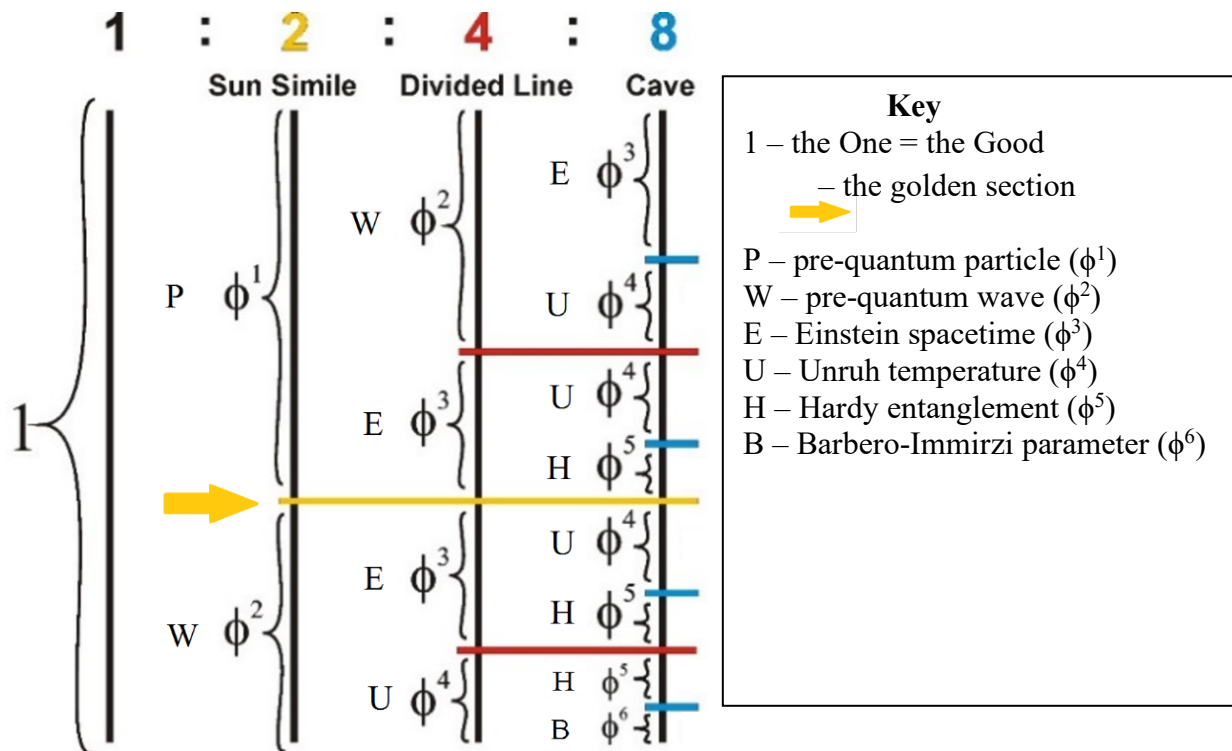
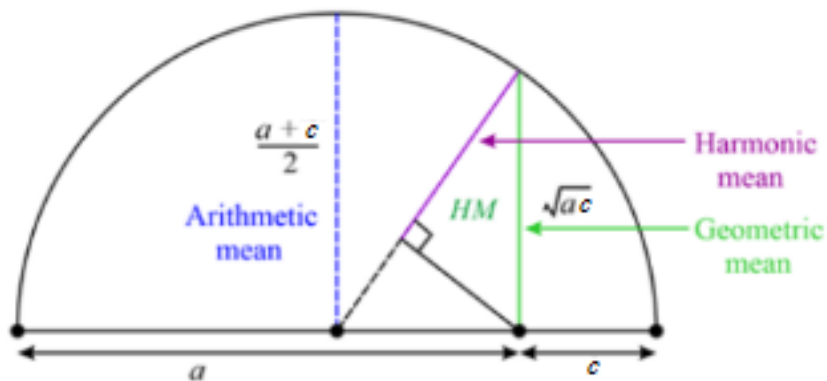


Figure 3. Republic similes with quantum parameters

THE THREE MEANS OF TWO GIVEN NUMBERS



Let us denote the three well-known means of any two numbers (a and c) using the chart below:

AM is the arithmetic mean of a and c : $AM = \frac{a+c}{2}$ or $2AM = a+c$.

GM is the geometric mean of a and c : $GM = \sqrt{ac}$ or $GM^2 = ac$.

HM is the harmonic mean of a and c : $HM = \frac{2ac}{a+c} = \frac{2GM^2}{2AM} = \frac{GM^2}{AM}$.

There is a well-known relation [17] between the means that can be found by substitution:

$$AM \otimes HM = (GM)^2$$

$$HM = (GM)^2 / AM.$$

Using this substitution relation above, one of us [El Naschie, 18] derived a “golden” harmonic mean via the Barbero-Immirzi parameter [24-29] that connects the fundamental work of David Gross and Edward Witten to that of Lee Smolin [19] and Carlo Rovelli [20,9]. By establishing that D=10 superstrings and loop quantum gravity are simply two sides of a “harmonic” golden coin, he was able show that their respective pioneers are talking about the same idea using different mental pictures.

UNCOVERING THE *PARADIGMATIC SYMMETRY* OF ANY GIVEN GOLDEN POWER, ACTING SIMULTANEOUSLY AS A GEOMETRIC, ARITHMETIC AND HARMONIC MEAN

“...the more deeply we probe Nature’s secrets the more profoundly we are driven into Plato’s world of mathematical ideals as we seek our understanding.” – Sir Roger Penrose [30]

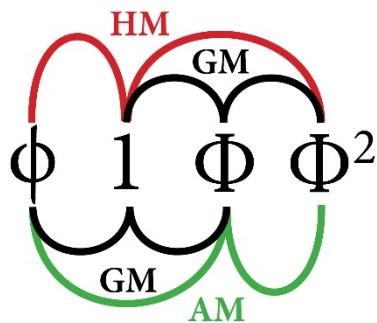


Figure 4. Harding diagram

It was during the writing of our new book, *A Grand Unification of Science, Arts and Consciousness: Rediscovering the Pythagorean Plato’s Golden Mean Number System*, [4] that one of us (Olsen) uncovered the underlying *paradigmatic symmetry* arising out of Plato’s One and Indefinite Dyad. What led to this discovery was that several years earlier, Lance Harding had been studying the geometric relations involved in Le Corbusier’s *The Modulor* [21] and found a variety of significant geometric (GM), arithmetic (AM) and harmonic (HM) mean relationships surrounding the One and Indefinite Dyad and their exponential powers in the golden series. His findings [22] included, amongst others, the following highly

meaningful relationship: see figure 4, Harding diagram.

Then more recently, Adam Tetlow, unaware of Harding’s discovery, pointed out his own finding regarding the three means along a portion of the golden series of exponential powers:

see

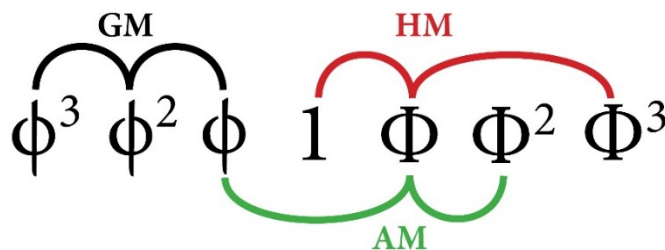


Figure 5. Tetlow diagram

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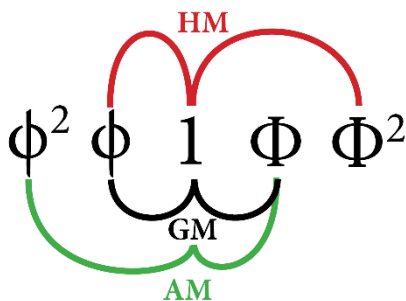


Figure 6. Olsen diagram of paradigmatic symmetry

Reflecting on the observations of both Harding and Tetlow, and perceiving the significance of the fact that these mean relationships maintain their internal structure and can slide back and forth along the golden series, Olsen realized that the One is simultaneously the “golden” geometric, arithmetic and harmonic means of the Indefinite Dyad

when the Greater squared and Lesser squared are included in the golden powers under consideration: see figure 6, Olsen diagram of *paradigmatic symmetry*.

Following Plato's assertion regarding continuous geometric proportion at *Timaeus* 31b-32a, the most important mean here is the fact that One is the geometric mean between (and therefore, bonding together) the Greater and the Lesser through continuous geometric proportion. But this also "means" that the One actually plays the role of the Golden Mean, i.e. One is the Golden Mean between the Greater and the Lesser. We maintain that the Golden Mean is the geometric mean par excellence of all geometric means. Plato understood very well that It is the archetypal Form or Standard for all geometric means. Furthermore, notice the interesting bilateral symmetry of the geometric mean combined with "mirrored" asymmetry of the arithmetic and harmonic means that arises in the following sequence of means associated with the *paradigmatic symmetry*, see figure 7.

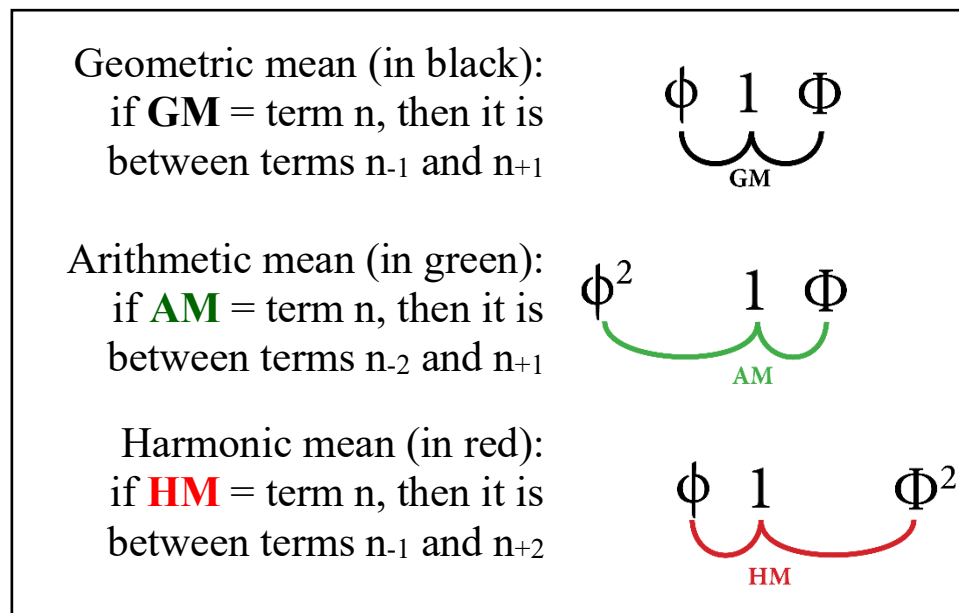


Figure 7: Unpacked paradigmatic symmetry displaying mirrored asymmetries of AM and HM

This is of course in addition to the fact that the golden section which gives rise to the golden ratios is itself an asymmetric cut, thus in another sense marrying together symmetry and asymmetry.

Also notice below (in figure 8) that the *paradigmatic symmetry* can be centered on any golden power, as done here with the Greater (Φ), Hardy entanglement (ϕ^5), and the Barbero-Immirzi parameter (ϕ^6), respectively.

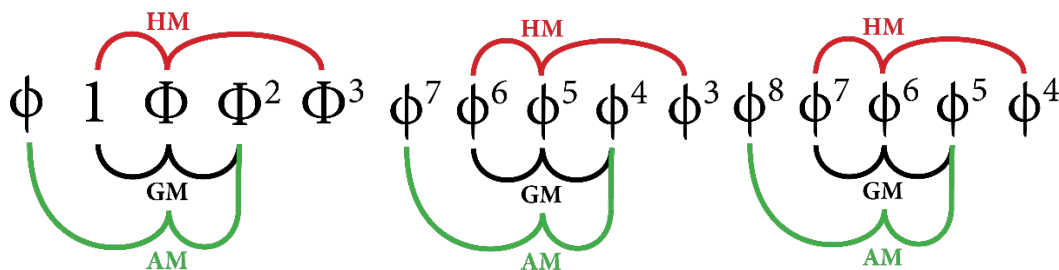


Figure 8. All three means centered on Φ , ϕ^5 or ϕ^6 , respectively, each example displaying an instantiation of paradigmatic symmetry

Finally, let us note here that MIT physicists, led by Pablo Jarillo-Herrero discovered that bi-layered graphene twisted by “about 1.1 degrees” allows electrons to become easily entangled and superconductive [31,32,33]. We (the authors) predict they will eventually discover this angle is precisely $\phi^{12} \times 360^\circ = 1.118023205\dots$ degrees. [4] This should give rise to the discovery of further quantum parameters as illustrated below in our Olsen-El Naschie entanglement (ϕ^{12}) centered *paradigmatic symmetry*.

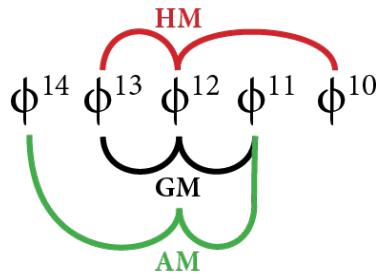


Figure 9. Olsen-El Naschie entanglement (ϕ^{12}) centered paradigmatic symmetry.

CONCLUSION

“The essential features of [quantum interconnectedness] are that the whole universe is in some way enfolded in everything, and that each thing is enfolded in the whole.” - David Bohm [23]

At the root of all the deep questions within physics lies the number system one employs in the process. Together with our E-Infinity theory collaborators, we have found that the *golden mean number system* is indeed the *lingua franca* of nature. Its backbone emerges naturally as the golden series of exponential powers out of Plato’s principles of the One and Indefinite Dyad of the Greater (Φ) and Lesser (ϕ). And it all begins with the golden section, or as the ancient Egyptians called it, the primordial scission. It is highlighted by its naturally recursive nature, similar to but more profound, than its derivative Fibonacci series of numbers. The latter being perfectly additive and approximately geometric, whereas the former is both perfectly additive and perfectly geometric. Furthermore, it harbors the most stunning internal structure of *paradigmatic symmetry* linking all aspects of the golden powers together in an incredible symphony of interdependence. And finally, we have shown how the quantum mechanical parameters of the pre-quantum particle (ϕ), pre-quantum wave (ϕ^2), Einstein spacetime (ϕ^3), Unruh temperature (ϕ^4), Hardy entanglement (ϕ^5) and Barbero-Immirzi parameter (ϕ^6) can be naturally and effectively aligned within Plato’s three similes of the Sun, Divided Line and Cave. This is the process of how the whole universe is fractally and holographically enfolded into each and every part. The *golden mean number system* is in fact the *lingua franca* of nature and holds the key to unpacking the fractal nature of the universe – penetrating into its outer fabric and inner mysteries. It has reemerged most completely and satisfactorily in the modern era of high energy physics and cosmology through the stunning simplicity of the many computational successes of E-Infinity theory. In the end, what we have on display is literally the answer to the greatest philosophical question of all: “How does the One become the Many?”

References

- [1] Mohamed S. El Naschie, Complementing the Deep Ideas of Deutsch-Marletto Constructor Theory with the Computational Power of the Pythagorean-Plato Number System. *European Journal of Applied Sciences*, 10(1) (2022). <https://doi.org/10.14738/aivp.101.11792>
- [2] Scott Olsen, The Indefinite Dyad and the Golden Section: Uncovering Plato's Second Principle, *Nexus Network Journal: Architecture and Mathematics*, 2002a, 4, No. 1 (2002) p. 97-110.
- [3] Scott Olsen, Golden proportional symmetry and the divided line – solving the platonic puzzles in one fell swoop, *Symmetry: Culture and Science*, Volume 32, Number 2, (2021) pp. 161-164. https://doi.org/10.26830/symmetry_2021_2_161
- [4] Scott Olsen, Leila Marek-Crnjac, Ji-Huan He and Mohamed S. El Naschie. *A Grand Unification of Science, Arts and Consciousness: Rediscovering the Pythagorean Plato's Golden Mean Number System*, Kindle Press (2021) 184 pages.
- [5] David Fideler, Introduction: Cosmopolis, or the New Alexandria, *Alexandria: The Journal of the Western Cosmological Traditions*, Vol. 2, ed. David Fideler, Phanes Press (1993).
- [6] Mohamed S. El Naschie, Is Quantum Space a Random Cantor Set with a Golden Mean Dimension at the Core? *Chaos, Solitons & Fractals*, 4(2) (1994) pp. 177-179.
- [7] Mohamed S. El Naschie, L. Da Vinci's Turbulence, Multi-dimensional golden spinors, Prandtl's Boundary layer and quantum mechanical spin are all implied by the golden mean number system. *European Journal of Applied Science*. Vol. 9, No 6 (2021) pp. 134-136.
- [8] Mohamed S. El Naschie, A review of E-Infinity theory and the mass spectrum of high energy particle physics, *Chaos, Solitons & Fractals*, Vol. 19, Issue 1 (2004) pp. 209-236.
- [9] Mohamed S. El Naschie, Proving superstring theory using loop quantum mechanics (gravity), *Chaos, Solitons & Fractals*, Volume 26 (2005) pp. 43-45.
- [10] Mohamed S. El Naschie, Elements of a New Set Theory Based Quantum Mechanics with Applications in High Energy Quantum Physics and Cosmology, *International Journal of High Energy Physics*, Volume 4, Issue 6 (2019) pp. 65-74.
- [11] Mohamed S. El Naschie and Leila Marek-Crnjac. Set Theoretical Foundation of Quantum Mechanics, NASA's EM Drive Technology and Minimal Surface Interpretation of the State Vector Reduction of the Quantum Wave Collapse, *Chaos and Complexity Letters*, Volume 12, No. 2 (2018) pp. 85-100.
- [12] Mohamed S. El Naschie, From Nikolay Umov $E = kmc^2$ $E = \gamma mc^2$ via Albert Einstein's to the Dark Energy $E = (21/22)mc^2$ Density of the Cosmos, *World Journal of Mechanics*, Volume 8, No. 4 (2018) pp. 73-81.
- [13] Mohamed S. El Naschie, Symmetria Massima of the Fractal M-Theory Via the Golden Mean Number System - A New Language for A Deep Dialogue between Man and Nature, *International Journal of Artificial Intelligence and Mechatronics*, Volume 7, Issue 3 (2018) ISSN 2329-5121. pp. 11-14.
- [14] Mohamed S. El Naschie, The quantum gravity Immirzi parameter - A general physical and topological interpretation. *Gravitation and Cosmology*, Vol. 19, Issue 3 (2013) pp 151-155. Note: it is more accurate to refer to this parameter as Barbero-Immirzi parameter.
- [15] Mohamed S. El Naschie, On a Fractal Version of Witten's M-Theory. *International Journal of Astronomy and Astrophysics*, Vol. 6, No. 2 (2016) pp 135-144.
- [16] H.J. Kramer, *Plato and the Foundations of Metaphysics*, ed. and trans., J.R. Caton, New York: State University of New York Press (1990).
- [17] Ilya Nickolaevich Bronstein and Konstantin Adolfovich Semendjaew, *Handbuch der Mathematik (pocket book of mathematics)*, Harri Deutsch Publishing, Zurich und Frankfurt (1965) (in German).
- [18] Mohamed S. El Naschie, Superstrings and Loop Quantum Gravity are Two Sides of the Same Golden Coin, *European Journal of Applied Sciences*, 10(1) (2022) 387-392.

<https://doi.org/10.14738/aivp.101.11579>.

[19] Lee Smolin. *Three Roads to Quantum Gravity*. Weidenfeld and Nicholson, London (2009).

[20] David Gross and Carlo Rovelli, String Theory or Loop Gravity?

<https://YouTube.com/watch?V=AUYIIRSPPZW> (2021) (See also Mohamed El Naschie facebook.

<https://www.Facebook.com/profile.php?id=100007609216640>).

[21] Le Corbusier, *The Modulor: A Harmonious Measure to the Human Scale Universally Applicable to Architecture and Mechanics*, London, Faber (1961).

[22] Scott Olsen, *Divine Proportion: The Mathematical Perfection of the Universe*, unpublished MS.

[23] David Bohm and Basil Hiley, *The Undivided Universe: An Ontological Interpretation of Quantum Theory*, Routledge (1968).

[24] Mohamed S. El Naschie, High Energy Physics and Cosmology as Computation, *American Journal of Computational Mathematics*, 6 (2016) pp. 185-189.

[25] Garnet Ord, Transfinite physics - Treading the path of Cantor and Einstein, *Chaos, Solitons & Fractals Volume 25, Issue 4* (2005).

[26] Alexey Stakhov, assisted by Scott Olsen, *The Mathematics of Harmony: From Euclid to Contemporary Mathematics and Computer Science*, World Scientific, Singapore (2009).

[27] Leila Marek-Crnjac, *Cantorian Space-Time Theory: The Physics of Empty Sets in Connection with Quantum Entanglement and Dark Energy*, Lambert Academic Printing (2013).

[28] Peter Weibel, Garnet Ord and Otto E. Rossler, eds., *Spacetime Physics and Fractality: Festschrift in honor of Mohamed El Naschie on the occasion of his 60th birthday*, Springer Verlag, Vienna and New York (2005).

[29] Mohamed S. El Naschie and Scott Olsen, When zero is equal to one: A set theoretical solution of quantum paradoxes, *Fractal Spacetime and Noncommutative Geometry in Quantum and High Energy Physics*, 1(1) (2011) pp. 11-24.

[30] Sir Roger Penrose, *The Road to Reality: A Complete Guide to the Laws of the Universe*, Jonathan Cape, Random House Group Ltd. (2004) p. 1029.

[31] Catherine Zandonella, "Experiments explore the mysteries of 'magic' angle superconductors," *ScienceDaily*, Princeton University (2019). www.sciencedaily.com/releases/2019/07/190731131122.htm.

[32] Mohamed S. El Naschie and Tomasz Kapitaniak, "Soliton chaos models for mechanical and biological elastic chains," *Physics Letters A*, Vol. 147, no. 5,6 (1990).

http://kapitaniak.kdm.p.lodz.pl/papers/1990/125_PhysicsLettersA1990-147-5-6.pdf.

[33] Scott Olsen, *The Golden Section: Nature's Greatest Secret*, Wooden Books, Glastonbury (1986).