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Laser Doppler Sensor of the Angular Speed Of Rotation of a Disc with a Retroreflective Surface

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ABSTRACT

A method for measuring angular speed of rotation of a disc is considered, taking into account features of retroreflecting surface based on micro glass beads. Method involves single-beam object probing, detection of scattered radiation and spectral analysis of the recorded photocurrent. A mathematical model of the proposed method is described. Results of physical simulation are presented in form of plots, showing dependencies of received signals Doppler shift on the frequency of rotation. Analysis of changes in registered photocurrent was done, taking into account changes due to the vary of the angular velocity of the disc rotation, the angle of inclination of the disc surface to the incident beam, the beam diameter, and the position of the illuminated spot on the disc.

Keywords: Laser speckle-velocimetry; Doppler frequency; Retro reflective sheeting; Microscopic glass beads.

INTRODUCTION

Various purposes laser remote sensing devices are widely used today for solving applied problems [1,2]. A significant number of them are based on the analysis of scattered from the probed object radiation, which forms a speckle structure [3,4]. Laser sensors for the speed of an object are commonly referred to as speckle velocimeters. [5-7]. To increase the power of the signal scattered in the direction of the laser source, a retroreflective surface (RRS) is applied as a coating to the object under study in the form of a regular grid of tetrahedral microprisms or a set of chaotically located microspheres [8]. The feature analysis of objects speckle velocimetry of in this formulation of the problem was previously carried out for the case of rectilinear motion of an object [9], as well as for the case of irradiation of a rotating cylindrical object [10]. In continuation of these works, this article examines the regularities of measuring the angular velocity of disc rotation.

Main part

Under irradiation of moving objects with an RRS, the beats of frequency-shifted scattered beams are observed experimentally due to the Doppler effect [9,10]. This manifests itself in the

spectrum of the photocurrent obtained during the detection of scattered radiation (Fig. 1). The position of the maximum and its width depend on the parameters of the object's motion, as well as on the conditions of irradiation and detection. Using the approach described in these works, let us analyze the regularities of the appearance of Doppler frequencies in the photocurrent spectrum when a rotating disc is irradiated.

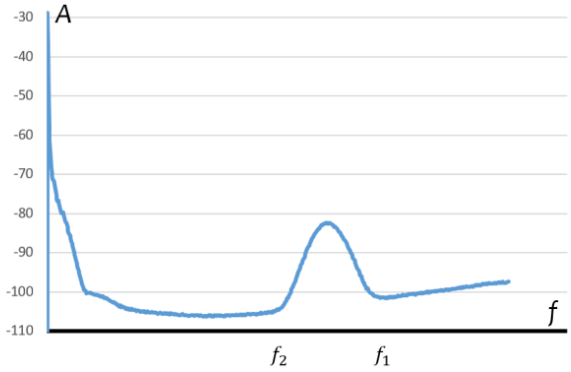


Figure 1. Spectrum of the photocurrent of detected radiation scattered from moving object with RRS

We will assume that a disk with a reflective surface in the form of a set of chaotically located microspheres rotates with a cyclic frequency ν.

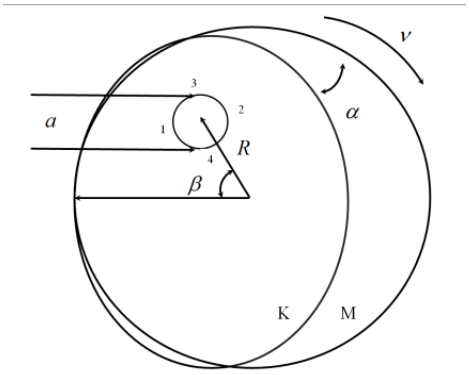


Figure 2. Beam can fall at the right angle or aslant to the surface of the disc

The disk is illuminated by a laser beam of diameter a with a plane wavefront (Fig. 2). The center of the beam is located at a distance R from the center of the disc and at an angle β measured from the direction to the left edge of the disc, as shown in Fig. 2. The beam falls at right angle to the surface of the disc when it is in position K or aslant when the disc is in position M, while the plane M is deflected from the plane K by an angle α.

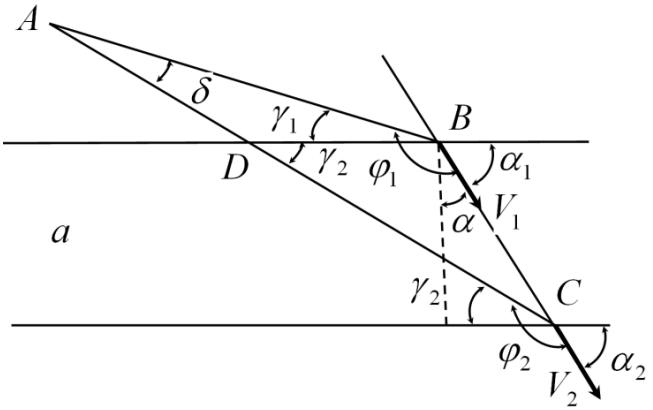


Figure 3. Detecting radiation scatterd from different points of the disc

Figure 3 shows that when detecting radiation at point A, which is at a distance L from the disk, the rays coming from points B and C located on the illuminated part of the disc, in the general case, have different velocities, also have an angular divergence δ determined by the probing conditions. Using the designations in Figure 3, we note some relationships between the angles that are going to be useful in subsequent calculations. As such, from ΔABD it follows, that $\gamma_2 - \gamma_1 = \delta$, while from ΔABC it follows, that $\varphi_2 - \varphi_1 = \delta$. Since $\varphi_1 = \pi - \alpha_1 + \gamma_1$ and $\varphi_2 = \pi - \alpha_2 + \gamma_2$, then $\varphi_2 + \varphi_1 = 2\pi - (\alpha_2 + \alpha_1) + (\gamma_1 + \gamma_2)$. Note also that $\alpha_1 = \alpha_2 = \frac{\pi}{2} - \alpha$ and $\sin\frac{\alpha_1 + \alpha_2}{2} = \cos\alpha$.

Analyzing Fig. 2, it can be noted that the Doppler shift between two points on the disc can be caused by two factors: the difference in the angular position on the disc in the angle β (points 1 and 2) while having an equal radius, and also the difference in velocities at the same angle β (points 3 and 4). Let us first consider the former case, when points 1 and 2 have different coordinates β, i.e. $\beta_1 = \beta - \Delta\beta$, $\beta_2 = \beta + \Delta\beta$, where $\Delta\beta = \frac{a}{2R}$.

In this case, it follows from Figure 3 that $V_1 = V\sin\beta_1$ and $V_2 = V\sin\beta_2$, considering the diameter of the laser beam a to be small in comparison with the radius R, it can be noted that

$\sin\beta_1 = \sin(\beta - \Delta\beta) = \sin\beta - \Delta\beta\cos\beta$

$\sin\beta_2 = \sin(\beta + \Delta\beta) = \sin\beta + \Delta\beta\cos\beta$.

Taking into account the comments made, it is possible to determine the Doppler shift for the beam scattered from points 1 and 2, respectively, as follows:

$$f_1 = f\left[1 - \frac{V_1}{c}(\cos\alpha_1 - \cos\varphi_1)\right]$$

$$f_2 = f\left[1-\frac{V_2}{c}\left(\cos\alpha_2-\cos\varphi_2\right)\right],$$

where the frequency of light f is linked with its wavelength λ by the relation $f = c/\lambda$, where c is a speed of light. Then the range of Doppler frequencies that are observed in the spectrum of the radiation detected at point A have the form

$$f_1 - f_2 = -\frac{fV_1}{c}\left(\cos\alpha_1-\cos\varphi_1\right)+\frac{fV_2}{c}\left(\cos\alpha_2-\cos\varphi_2\right)=$$
$$=-\frac{fV\ \sin\beta_1}{c}\left(\cos\alpha_1-\cos\varphi_1\right)+\frac{fV\ \sin\beta_2}{c}\left(\cos\alpha_2-\cos\varphi_2\right)=$$
$$=\frac{fV}{c}\left[-\sin\beta_1\cos\alpha_1+\sin\beta_2\cos\alpha_2+\sin\beta_1\cos\varphi_1-\sin\beta_2\cos\varphi_2\right]$$

We denote

$$G=\cos\alpha_1\left(\sin\beta_2-\sin\beta_1\right)=\sin\alpha 2\Delta\beta\cos\beta=\frac{a}{R}\sin\alpha\cos\beta$$

and

$$J=\left(\sin\beta-\Delta\beta\cos\beta\right)\cos\varphi_1-\left(\sin\beta+\Delta\beta\cos\beta\right)\cos\varphi_2=$$
$$=\sin\beta\left(\cos\varphi_1-\cos\varphi_2\right)-\Delta\beta\cos\beta\cos\varphi_1-\Delta\beta\cos\beta\cos\varphi_2=$$
$$=\sin\beta\left(\cos\varphi_1-\cos\varphi_2\right)-\Delta\beta\cos\beta\left(\cos\varphi_1+\cos\varphi_2\right)$$

Then

$$f_1-f_2=\frac{fV}{c}\left(G+J\right)=\frac{fV}{c}\left(A+S-T\right),$$

where

$$S=\sin\beta\left(\cos\varphi_1-\cos\varphi_2\right)=\sin\beta\left(-2\sin\frac{\varphi_1+\varphi_2}{2}\sin\frac{\varphi_1-\varphi_2}{2}\right)=$$
$$=2\sin\beta\sin\frac{2\pi+\left(\gamma_1+\gamma_2\right)-\left(\alpha_1+\alpha_2\right)}{2}\sin\frac{\delta}{2}=2\sin\beta\sin\frac{\delta}{2}\sin\left(\pi+\frac{\gamma_1+\gamma_2}{2}-\frac{\alpha_1+\alpha_2}{2}\right)=$$
$$=-2\sin\beta\sin\frac{\delta}{2}\sin\left(\frac{\gamma_1+\gamma_2}{2}-\frac{\alpha_1+\alpha_2}{2}\right)=-2\sin\beta\sin\frac{\delta}{2}\left(\sin\frac{\gamma_1+\gamma_2}{2}\cos\frac{\alpha_1+\alpha_2}{2}-\sin\frac{\alpha_1+\alpha_2}{2}\cos\frac{\gamma_1+\gamma_2}{2}\right)=$$
$$=-2\sin\beta\sin\frac{\delta}{2}\left(\sin\frac{\gamma_1+\gamma_2}{2}\sin\alpha-\cos\alpha\right);$$
$$T=\Delta\beta\cos\beta\left(\cos\varphi_1+\cos\varphi_2\right)=\Delta\beta\cos\beta\left(2\cos\frac{\varphi_1+\varphi_2}{2}\cos\frac{\varphi_1-\varphi_2}{2}\right)=$$
$$=2\Delta\beta\cos\beta\cos\frac{\delta}{2}\cos\frac{2\pi-\left(\alpha_2+\alpha_1\right)+\left(\gamma_1+\gamma_2\right)}{2}=2\Delta\beta\cos\beta\cos\left(\pi+\frac{\gamma_1+\gamma_2}{2}-\frac{\alpha_1+\alpha_2}{2}\right)=-$$
$$=2\Delta\beta\cos\beta\left(\cos\frac{\gamma_1+\gamma_2}{2}\cos\frac{\alpha_1+\alpha_2}{2}+\sin\frac{\gamma_1+\gamma_2}{2}\sin\frac{\alpha_1+\alpha_2}{2}\right)=-2\Delta\beta\cos\beta\left(\sin\alpha+\sin\frac{\gamma_1+\gamma_2}{2}\cos\alpha\right)$$

As a result, we get

$$f_1 - f_2 = \frac{fV}{c}\left[\begin{array}{l}\frac{a}{R}\sin\alpha\cos\beta - 2\sin\beta\sin\frac{\delta}{2}\left(\sin\frac{\gamma_1+\gamma_2}{2}\sin\alpha - \cos\alpha\right) + \\ 2\Delta\beta\cos\beta\cos\frac{\delta}{2}\left(\sin\alpha + \sin\frac{\gamma_1+\gamma_2}{2}\cos\alpha\right)\end{array}\right] =$$

$$= \frac{fV}{c}\left(\begin{array}{l}\frac{a}{R}\sin\alpha\cos\beta - 2\sin\beta\sin\frac{\delta}{2}\sin\frac{\gamma_1+\gamma_2}{2}\sin\alpha + 2\sin\beta\sin\frac{\delta}{2}\cos\alpha \\ +2\Delta\beta\cos\beta\cos\frac{\delta}{2}\sin\alpha + 2\Delta\beta\cos\beta\cos\frac{\delta}{2}\sin\frac{\gamma_1+\gamma_2}{2}\cos\alpha\end{array}\right)$$

When obtaining this expression, the trigonometric relationships between the angles specified above were taken into account. In addition, a small value of the angle γ was assumed on the basis of considerations of obtaining a high radiation power of the scattered RRS.

Similar to the approach used in [9,10], we distinguish two terms:

$$\delta f = \frac{fV}{c}\sin\alpha\left(\frac{a}{R}\cos\beta - 2\sin\beta\frac{a}{2L}\sin\frac{\gamma_1+\gamma_2}{2} + \frac{a}{R}\cos\beta\right) = \frac{fV}{c}\sin\alpha\left(\frac{2a}{R}\cos\beta - \frac{a}{L}\sin\beta\sin\frac{\gamma_1+\gamma_2}{2}\right)$$

and

$$\Delta f = \frac{fV}{c}\cos\alpha\left(\frac{a}{R}\cos\beta\sin\frac{\gamma_1+\gamma_2}{2} + \frac{a}{L}\sin\beta\right)$$

Since

$$\frac{fVa}{c} = \frac{2\pi\nu Ra}{\lambda}$$

it means

$$\delta f = \frac{2\pi\nu a}{\lambda}\sin\alpha\left(2\cos\beta - \frac{R}{L}\sin\beta\sin\frac{\gamma_1+\gamma_2}{2}\right) \quad (1)$$

$$\Delta f = \frac{2\pi\nu a}{\lambda}\cos\alpha\left(\cos\beta\sin\frac{\gamma_1+\gamma_2}{2} + \frac{R}{L}\sin\beta\right) \quad (2)$$

The first of them is proportional to $V\sin\alpha$, which means that it takes into account the contribution of the radial component of the object's velocity. Accordingly, the second value is proportional to $V\cos\alpha$ and takes into account the component of the movement velocity orthogonal to the incident beam. As noted in [9,10], the Δf values determine the width of the Doppler frequency contour. In turn, the values of δf also contribute to the expansion of the contour, but, what is important, they also determine the shift of the maximum of the Doppler frequencies relative to the zero value. Note that both components depend on the angular velocity of rotation of the disc, as well as on the beam diameter and angular coordinate β.

Let us now consider the contribution to the spectrum of Doppler frequencies of radiation from points 3 and 4 (Fig. 2) lying on the same radius, i.e. $\Delta\beta = 0$, $R_2 = R_1 + a$. These points will have different velocity modules $V_1 = \varpi R_1 \sin\beta$ and $V_2 = \varpi R_2 \sin\beta$. Note also that in this case $\alpha_1 = \alpha_2$, $\varphi_1 = \varphi_2$, $\varphi_2 = \pi - \alpha_1 + \frac{\gamma_1 + \gamma_2}{2}$, $cos\alpha_1 = \sin\alpha$.

Taking into account the Doppler effect, we have

$$f_1 = f\left[1 - \frac{V_1}{c}\left(\cos\alpha_1 - \cos\varphi_1\right)\right], \quad f_2 = f\left[1 - \frac{V_2}{c}\left(\cos\alpha_2 - \cos\varphi_2\right)\right]$$

and the Doppler frequency range can be determined by the relationship

$$f_1 - f_2 = -\frac{fV_1}{c}\cos\alpha_1 + \frac{fV_2}{c}\cos\alpha_2 + \frac{fV_1}{c}\cos\varphi_1 - \frac{fV_2}{c}\cos\varphi_2$$

We group the terms and carry out the necessary transformations

$$-\frac{f\varpi R_1 \sin\beta}{c}\sin\alpha + \frac{f\varpi R_2 \sin\beta}{c}\sin\alpha = \frac{f\varpi\sin\beta}{c}\sin\alpha\left(R_2 - R_1\right) = f\frac{2\pi\nu a}{c}\sin\alpha\sin\beta$$

$$\frac{f\varpi R_1 \sin\beta}{c}\cos\varphi_1 - \frac{f\varpi R_2 \sin\beta}{c}\cos\varphi_2 = -f\frac{2\pi\nu a}{c}\sin\beta\sin\alpha - f\frac{2\pi\nu a}{c}\sin\beta\cos\alpha\sin\frac{\gamma_1 + \gamma_2}{2}$$

Then

$$f_1 - f_2 = -\frac{2\pi\nu a}{\lambda}\cos\alpha\sin\beta\sin\frac{\gamma_1 + \gamma_2}{2} = \Delta f_1 \qquad (3)$$

Let us analyze the obtained relationships. First, we note that expression (3) does not contain terms proportional to the radial component of the velocity of motion of points on the disc, which means that the considered points contribute only to the value of Δf.

Further, it should be noted that in Fig. 2 only the extreme positions of the points on the illuminated part of the disc are marked. Therefore, the value of the maximum range of the Doppler frequency shift was estimated here, the contribution of radiation from all points inside the spot do not exceed the estimated maximum value of the Doppler frequency shift. The value of Δf itself is determined by the bigger value of the two components which are defined by the relationships (2) and (3), but, in any case, does not exceed their sum.

Analyzing relations (1), (2) and (3), it can be noted that the components δf and Δf can be used to solve the practical problem of measuring the angular velocity of the disc. Apparently, to solve this problem, it is advisable to use the value of δf, that characterizes the position of the maximum of the Doppler frequency contour, for which it is necessary to deflect the beam from the normal to the disc surface.

The regularities described as a result of the calculations were verified experimentally on the setup described in [9,10], a flat disc with an RRS was used as the probing object. In particular,

a linear increase in the shift of the maximum's position of the Doppler frequency contour δf was recorded with an increase in the angle α or the angular frequency of disc's rotation v, which corresponded to the results of calculations (relation 1). Then, with an oblique incidence of the beam, the dependence of the increase in the value of δf with increasing radius R at $\beta = 90°$ and the absence of such a dependence at $\alpha = 0°$ were recorded.

Analyzing relation (1), it can be noted that the magnitude of the Doppler shift depends on the angle α of the beam incidence on the disc surface. This angle remains constant as the disc rotates if the surface of the disc remains flat. If the disc is curved, then when it rotates with a frequency v, the angle α changes periodically. Therefore, it is expected that the magnitude of the Doppler shift periodically changes with the rotation of the disc in accordance with the change in the angle α.

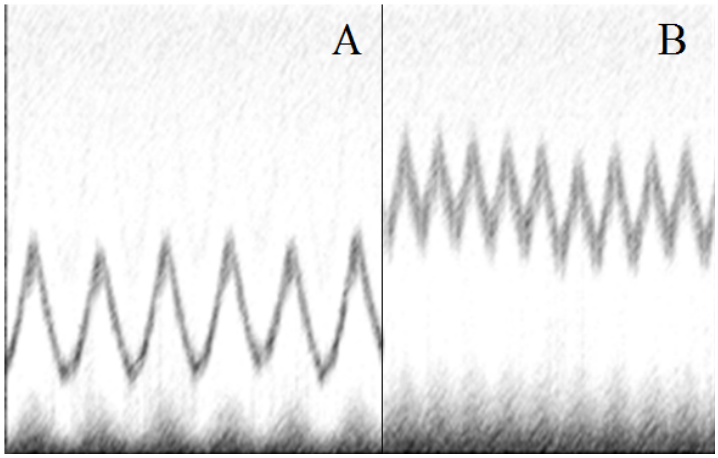


Figure 4.Dependency of Doppler shift over time on angular frequency of disc's rotation

This conclusion has been verified experimentally. Figure 4 shows the dependences of the change in the Doppler shift at two different values of the frequency v. Here, the abscissa shows the time values, and the ordinate shows the Doppler shift values in the detected radiation. Part of figure (A) corresponds to a lower frequency v, part of figure (B) corresponds to a higher value of v. As can be seen from the figure, the magnitude of the Doppler shift periodically changes with the rotation of the non-planar disc, with a larger value of v corresponding to faster changes in the shift frequency, as follows from relation (1). This property of changing the Doppler shift can be used to analyze the quality of the disk surface.

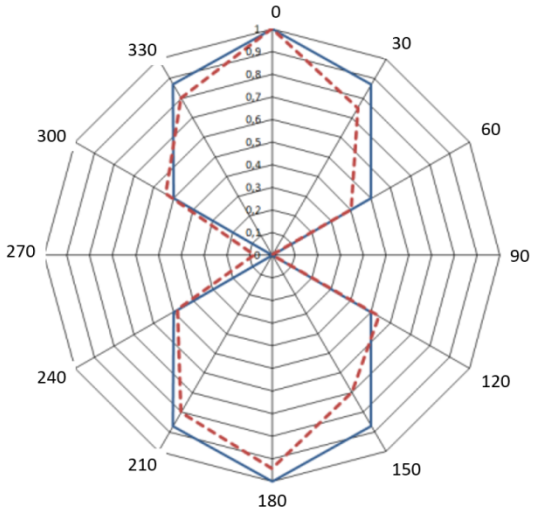


Figure 5. Comparison between experimentally measured and analytically obtained $\delta f(\beta)$

And finally, the dependence of the value of δf for oblique incidence of the beam on the disc as a function of the angle β was experimentally researched. The measurement results are illustrated by the dash line in Fig. 5. in the form of a curve normalized to the maximum value. The continuous line on the same graph shows the results of calculations in accordance with relationship (1). As can be seen from the figure, these dependences of the value of δf on the angle β have a similar character, which indicates the adequacy of the method of theoretical analysis to the physical processes that occur when a rotating disc is probed with a laser beam.

CONCLUSION

As shown by theoretical analysis and experimental studies, when a rotating disc with a retroreflective surface is irradiated by a laser beam, a Doppler frequency shift occurs in the scattered radiation spectrum. This shift can be registered experimentally by detecting scattered radiation and spectral analysis of the photocurrent. The obtained regularities describe both the width of the Doppler frequency contour and the shift of its maximum depending on the angular velocity of the disc rotation, the angle of inclination of the disc surface to the incident beam, the beam diameter, and the position of the illuminated spot on the disc.

The results of calculations and experimental studies obtained in this work are in good agreement, which indicates the adequacy of the mathematical description of the process under study. The linear dependence of the shift of the Doppler frequency contour on the angular velocity of the disc movement at an oblique incidence of the beam on its surface can be of practical use for diagnosing the parameters of the objects motion.

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