

Area Theorem of Infection Curves for Large Basic Reproduction Number

Takesi Saito

Department of Physics, Kwansei Gakuin University
Sanda 669-1337, Japan

ABSTRACT

A remarkable theorem is established in the SIR model in Epidemiology, that is, when the basic reproduction number α is larger than 5, the area of any infection curve is constant given by $1/c$, where c is the removed ratio. From this theorem we see that as α gets bigger, the infection curve raises sharply, but its half width decreases inversely. Accordingly, the wave of Omicron will end with a shorter life-time than that of the wave of Delta, because α for Omicron is larger than Delta's which is regarded as $5 < \alpha < 9$. A rough estimation of the 5-wave area of infection in Japan tells us that the 6-wave will reach a peak on early in Feb., then it will be controlled around early in March.

Keywords: Variants of SARS-COV-2, SIR model, Epidemiology, Molecular Biology

INTRODUCTION

The new COVID-19 variant 'Omicron' is spreading across the world. Its speed is very high since it is extremely infectious. Fortunately, however, the number of deaths has remained low. In a previous article [1], we have considered, based on the SIR equation [2], [3-14] in Epidemiology, the reason why the death toll due to such a highly infectious virus is so small. In the present article we would like also to consider how long-time such infections are living on.

We derive an area theorem from the SIR equation, that is, "the area S of any infection curve is constant given by $1/c$, c being the removed ratio, when the basic reproduction number α is larger than 5." From this theorem we see that as α gets bigger, the infection curve raises sharply, but its half width decreases inversely. Accordingly, the wave of Omicron ($\alpha > 9$) will end with a shorter life-time than that of the wave of delta ($\alpha = 5 \sim 9$). As another example, Omicron in South Africa appeared on 11/24/2021, raised sharply at the middle of Dec., then turned out to decrease suddenly.

FORMULATION AND RESULTS

Let us start with the 3rd equation of SIR model [2],

$$dR(t)/dt = cI(t), \quad (1)$$

where $R(t)$ is the removed number and $I(t)$ the infective number, and c is the removed ratio. We integrate Eq.(1) over t from 0 to infinity under the boundary condition, $R(0)=0$, then it follows:

$$R(\infty) = c \int_0^{\infty} I(t) dt = cS. \quad (2)$$

Here, S is a whole area of function $I(t)$ of t from 0 to infinity. Thanks to the well-known formula in the SIR model,

$$\alpha = -\frac{\ln [1-R(\infty)]}{R(\infty)}, \quad (3)$$

where α is the basic reproduction number, it is possible to obtain $R(\infty)$ against α . In order to see this more easily, let us put $R(\infty) = 1 - \varepsilon$, $\alpha \varepsilon \ll 1$. Substituting this into Eq. (3), we get $\varepsilon = e^{-\alpha} e^{\alpha \varepsilon} = e^{-\alpha} + 0(\alpha \varepsilon)$. (4)

In Fig. 1, numerical values of ε against α are given. We see that ε can be regarded as zero for $\alpha > 5$, hence $R(\infty) = 1$ for $\alpha > 5$. The error is $0(e^{-5} = 0.0076)$. This shows from Eq.(2) that the area S of any infection curve takes a definite value of $1/c$ for $\alpha > 5$.

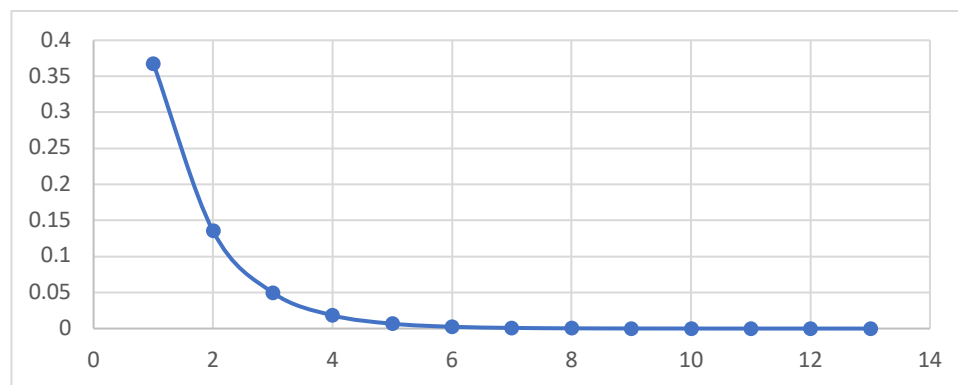


Fig. 1. Numerical values of ε against α

In Table 1, we give numerical values of $I(T)$, infection number at the peak $t=T$, and for half width against α . Here we have used the formula in the SIR model

$$I(T) = 1 - \frac{1}{\alpha} - \frac{\ln \alpha}{\alpha}. \quad (5)$$

One can see from Table 1 that as α gets bigger, $I(T)$ increases, while the half-width decreases inversely.

Accordingly, the 6-wave(Omicron) will go to the end faster than the 5-wave(Delta), because the Omicron is regarded to have $\alpha > 9$ while the Delta has $9 > \alpha > 5$, hence the half-width of the former becomes narrower than that of the latter. A rough estimation of the 5-wave area in Japan [15] tells us that the 6-wave will reach a peak on early in Feb., then it will be controlled around early in March.

Table 1. Numerical values of $I(T)$, infection number at the peak, and half width against α

α	2.5	5	7	9	11	13
$I(T)$	0.23	0.5	0.6	0.64	0.69	0.73
half width	3.29	1.7	1.3	1.15	1.12	1.07

CONCLUDING REMARKS

A remarkable theorem has been established in the SIR model in Epidemiology, that is, when the basic reproduction number α is larger than 5, the area of any infection curve is constant given by $1/c$ with error $0(0.0076/c)$, where c is the removed ratio.

The SIR equations for S , I and R can be solved by means of EXCEL, if the basic reproduction number α and the removed ratio c are given. In Fig.2 we draw such curves for $\alpha = 5$ and $c=1$, where S is black, susceptible), I is red, infective and R is blue, removed. We also draw them for $\alpha=10$ and $c=1$ in Fig.3. These cases are typical examples for the 5 wave and the 6 wave in Japan. According to the area theorem, both areas of red curves in Fig.2 and Fig.3 are equal with each other, though the latter curve raises more sharply than the former. A rough estimation of the 5-wave area in Japan tells us that the 6-wave will reach a peak on early in Feb., then it will be controlled around early in March.

The area theorem is also useful to check whether both data of infections, say, Delta and Omicron, are right or not from the view point of the SIR equation. When both areas of infections are equal with each other, both data are right, but when they are not, the data are not so.

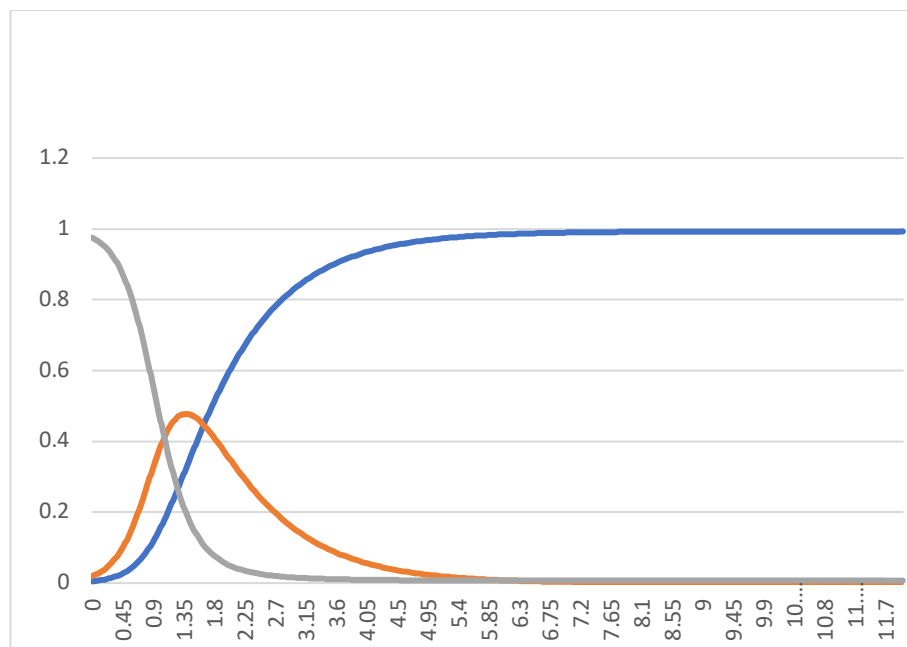


Fig. 2. Curves for S (black, susceptible), I (red, infective) and R (blue, removed) for $\alpha = 5$ and $c=1$

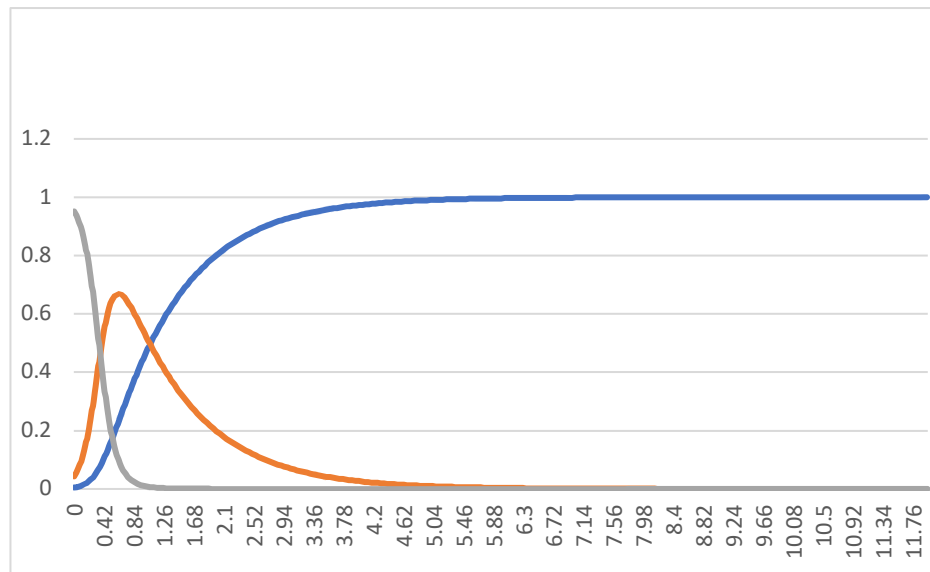


Fig. 3. Curves for S(black, susceptible), I(red, infective) and R(blue, removed) for $\alpha = 10$ and $c=1$

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