

Dynamic Shear Failure Model for Ductile Materials

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ABSTRACT

We propose a model for the shear failure of viscoplastic materials under dynamic loading. We use the overstress approach to dynamic plasticity, and include thermal softening, mechanical damage softening, strain hardening and pressure hardening. We demonstrate the model performance in torsion of a thin-walled tube. We introduce an initial yield stress reduction perturbation at the tube mid-section, and a shear band is formed there and then develops all the way to material failure. At the end we propose a scheme for applying our dynamic shear failure model to a computational cell of a macroscale hydrocode computation.

INTRODUCTION

Traditionally, failure of ductile materials is defined in terms of the effective plastic strain to failure $\varepsilon_{\text{Peff}}$. We believe that this definition follows from years of experience in testing the response of materials to quasistatic uniaxial loading in tension. In these tests a small rod of the tested material is extended slowly by a tension machine until it is torn apart, and its strain upon failure is recorded. As the strain to failure of materials, determined in this way, has been for many years the most available piece of information on their failure process, it has been adopted (in our opinion) as the failure criterion. But it seems not appropriate to use the quasistatic strain to failure of rods in tension as a general viscoplastic failure criterion, from the following reasons: 1) putting a viscoplastic material rod under quasistatic tension usually creates a neck in the rod before it fails, and there is no way to deduce from that the strain to failure for a different geometry, where a neck is not formed; 2) a rod under tension is in a uniaxial stress state, and it's plausible to assume that for different states of stress the strain upon failure would be different. For example, a group from MIT, which is leading in the field, assumes that the effective plastic strain to failure depends on the isotropic stress and on the Lode angle [1]. But they still base their criterion on quasistatic tests.

We're interested here mainly in failure of ductile materials under dynamic loading. The response of materials to dynamic loading is usually modelled by hydrocodes, or by explicit finite element codes. We're generally accustomed to hydrocodes, in which material failure in a computational cell is reached when both shear strength and deviator stress are set to zero (which is like putting in the cell a non-viscous fluid at the existing density). To this end we need a failure criterion that depends on the loading conditions, but not on the macroscopic geometry. The first situation to cause failure that comes to mind is that of substantial tension which causes neighbouring atoms to get apart from each other to a distance beyond their attraction distance. But to create this failure mode requires to load the material to a very high stress at a very high rate. Because if the stress level or the loading rate are not high enough, other failure mechanisms have enough time to become dominant. These other failure mechanisms are usually related to various modes of instability of the plastic flow. Three such instability modes

are: 1) formation of a neck or of necks within thin bodies like rods, plates or shells. A neck results from the localization of plastic flow in a small partial volume of a thin body. The neck volume becomes thinner as plastic deformation proceeds, which finally leads to failure by one or both of the two other instability mechanisms. Again, neck formation may only happen within thin bodies; 2) cavitation, or spontaneous growth of pores, an instability mechanism that may take place only under substantial isotropic tension at high tensile triaxiality. Materials usually contain small voids. In the presence of substantial isotropic tension, such voids may cross their cavitation threshold and grow spontaneously. For pure isotropic tension this may happen at triaxiality values between 3 and 4. In the presence of shear stress, the cavitation threshold decreases, and cavitation may happen even under low isotropic pressure, as long as one of the principal stress components is tensile. The common way to cause dynamic cavitation experimentally is by reflecting a shock from a free boundary (spall test). In other dynamic situations the cavitation threshold is not even approached. When the cavitation threshold is crossed dynamically somewhere in the material, cavities there soon grow spontaneously, and the material between the cavities becomes very thin and finally fails by other failure mechanisms; 3) formation of shear bands. A shear band (or an adiabatic shear band) may form dynamically as a result of localization of plastic flow. It's customary to assume that such a localization takes place because of strength reduction resulting from heating. This requires a dynamical situation (short duration) to avoid the escape of heat created by the plastic flow. Localization of plastic flow may start somewhere if strength there is initially somewhat lower (perturbation), or if it becomes lower during the deformation process. During plastic flow the perturbation may strengthen as a result of plastic heating (as strength decreases with temperature), and may weaken (strength increasing) because of strain hardening and pressure hardening. If the combined result of these effects is that strength decreases, this constitutes an instability mechanism. As strength decreases, the rate of plastic flow increases, which increases the rate of heating, which decreases strength even more. This leads to localization of the plastic flow field and formation of a shear band. The plastic deformation of the material layer that forms a shear band may reach very high values of strain. Because of heat conduction out of a shear band, its thickness is finite, usually from several microns to several tens of microns. As a result of its fast plastic flow, a shear band heats up toward melting. But usually much before melting the shear band material undergoes substantial mechanical damage (voids and cracks) and fails.

In what follows we concentrate on shear failure, which is the most common failure mode out of the three outlined above. We model a shear failure process of a thin-walled tube loaded in torsion, using a 1D computation procedure. The tube may represent a macroscopic computational cell of dimensions 0.1 to 1mm long. It is a continuum computation with heat conduction, and with cells that are 1 μ m long, and the computation reproduces the formation of a shear band quite reliably. We obtain from it the shear stress, shear strain and temperature in the shear band all the way to failure. From this we compute the average equivalent stress and the average effective strain in the whole tube all the way to full failure. We see from the results that usually, the strain to full failure is rather large. Comparing to failure data in the literature, we conclude that usually, material in the shear band fails much before the temperature there reaches melting. We assume that much before melting, the material in the shear band accumulates mechanical damage (voids and cracks) that lead to failure before melting. In what follows we represent mechanical damage by a schematic damage accumulation model. We

assume that starting from some (assumed) plastic strain, damage starts to accumulate, and that shear stress and strength decrease according to the amount of accumulated damage.

Next we describe our 1D model for shear band formation and for shear failure prediction. Then we show results of computational examples with thermal softening, mechanical damage, strain hardening and pressure hardening.

Finally we discuss how to implement our model in a hydrocode.

1D MODEL FOR SHEAR BAND FORMATION

We model a thin-walled tube loaded in torsion, similar to Duffy's tests [2]. We've described this model before [3,4], but for the sake of completeness we outline it here again.

We load the thin-walled tube in torsion by specifying the tangential particle velocities u_1 and u_{n+1} on the longitudinal boundaries ($i=1$ is the first boundary and $i=n+1$ is the last one) of the tube in opposite directions. We assume that the loading and the response is 1D, that the motion and the material variables change only along the tube, and that circumferential motion is simultaneous. Also, as the tube is thin walled, we assume that there are no changes in the radial direction. We aim at the formation of a shear band at the longitudinal mid-section of the tube. We're therefore not counting on the spontaneous formation of a shear band, and introduce a perturbation in the mid section of the tube. The perturbation is in terms of the initial strength, and in most computations, we put the initial strength at the mid-section as 0.99 of the initial strength of all other cells. Also, in most computations we put this reduced strength in three neighbouring cells at the tube mid-section, although it is possible to put it just in a single cell at the center. At $t=0$ the tube is stress free and the temperature is 300K. The temperature conditions at the tube longitudinal boundaries are of zero gradient. We ignore heat conduction at the radial boundaries, as we're dealing with a rather low temperature, and with short times. We also ignore the radial heat leakage from the shear band, although the temperature there is relatively high.

The equations of motion of our tube are:

$$\begin{aligned}\rho \dot{u} &= -\frac{\partial \tau}{\partial z} \\ \dot{\gamma} &= -\frac{\partial u}{\partial z} \\ \rho C_p \dot{T} &= k \frac{\partial^2 T}{\partial z^2} + \beta \tau \dot{\gamma}_{\text{eff}}^p\end{aligned}\quad (1)$$

W here u =tangential particle velocity, ρ =density (and we ignore here its changes as the isotropic stress is zero or low), τ =tangential shear stress at the tube cross section, z = coordinate along the tube, γ =tangential shear strain at the tube cross section, T =temperature, C_p =specific heat at constant pressure (and we ignore here its change with temperature), k =coefficient of heat conduction, β =Taylor-Quinny coefficient (which quantifies the part of plastic work converted to heat), and γ_{eff}^p =effective plastic strain.

For the viscoplastic response we use our overstress approach [5], which is different from the more common approach, that we call the flowstress approach. With the overstress approach the yield stress is not influenced by strain rate hardening. Instead, strain rate may cause the shear stress to protrude out of the yield surface. When this happens, plastic strain rate increases with the amount of protrusion, and in this way the stress level is drawn back towards the current yield surface. As we show later, using the overstress approach is advantageous when modeling the formation of shear bands. Using the overstress approach, we compute the effective plastic strain rate by:

$$\begin{aligned}\dot{\gamma}_{\text{eff}}^p &= A_p \left(\frac{\tau_{\text{eq}}}{Y} - 1 \right)^{\alpha_p} & \text{for } \tau_{\text{eq}} > Y \\ \dot{\gamma}_{\text{eff}}^p &= 0 & \text{for } \tau_{\text{eq}} \leq Y \\ \tau_{\text{eq}} &= \sqrt{3} |\tau|\end{aligned}\quad (2)$$

where Y =yield stress, A_p and α_p are material parameters, and as usual, yield stress increases with pressure and decreases with temperature. In addition, Y may decrease as a result of damage accumulation, especially at the shear band. We should also emphasize here, that the expression for $\dot{\gamma}_{\text{eff}}^p$ in Eq. (2) we've calibrated in the past [5] from the so called σ^4 law of D. Grady [6]. We obtained there that the exponent α_p is almost the same for many metals and is equal to 2.4. We refer to Eq. (2) by the name flow curve. Inverting Eq. (2), it may be interpreted as expressing strain rate hardening caused by the overstress.

As mentioned above and discussed below, the yield stress Y may undergo various processes of hardening and softening. These include: 1) pressure hardening (Y increases with pressure); 2) strain hardening (Y increases with plastic strain and then stabilizes); 3) thermal softening (Y decreases with temperature until it reaches zero at the melting temperature); 4) mechanical softening (defects of various kinds are created in the material, mainly voids and cracks, which we generally refer to as damage). As a result, the material may flow like a granular material. In addition, the coefficient A_p in Eq. (2) may undergo changes by several factors and thereby influence the rate of change of plastic strain. The coefficient A_p may change with pressure, temperature, plastic strain and mechanical damage. These changes influence plastic strain, and if we invert Eq. (2), we may interpret them as hardening/softening of the overstress. It follows that with the overstress approach we may account separately for hardening/softening of the yield stress and of the equivalent stress. In the examples shown later we don't use this capability as we don't have concrete information on the relation between A_p and the amount of hardening/softening of the overstress.

The rates of change of plastic and elastic strains are given by:

$$\begin{aligned}\dot{\gamma}^p &= \frac{\sqrt{3}}{2} \dot{\gamma}_{\text{eff}}^p \text{sign}(\tau) \\ \dot{\gamma}^e &= \dot{\gamma} - \dot{\gamma}^p \\ \dot{\tau} &= 2G\dot{\gamma}^e\end{aligned}\quad (3)$$

where the upper index e means “elastic”, and where G =shear modulus. For the shear modulus we assume (from lack of concrete information) that it changes in the same way as the yield stress:

$$\frac{G}{G_0} = \frac{Y}{Y_0} \quad (4)$$

So that the two parameters Y and G become zero together at the melting temperature, or before that when mechanical damage enables the material to flow without resistance.

We divide the length of the thin-walled tube into equal computational cells of length $\Delta z = 1\mu$, as 1μ length cells enable a realistic computation of heat conduction. The cells are labeled by $i=1, n$, and the left boundaries of the cells are labeled the same way. We express the space derivatives of Eq. (1) by finite differences, and we get a system of time dependent ODEs. The rates of change of strain, stress and temperature are defined for the cells (or at the mid cells), so that for each of them there are n equations. The material velocities are defined at cell boundaries, so that we have for them $n-1$ equations. In addition, we add at some point equations for the amount of damage, so that we get altogether a system of $5n-1$ time dependent ordinary differential equations (ODEs). We integrate this system of ODEs with a standard 4th order Runge-Kutta solver.

REFERENCE COMPUTATION WITH THERMAL SOFTENING ONLY

We start our shear band computations with a case that has no hardening, and has only thermal softening. This is our reference case. We expect that in this computation a shear band will start forming just when the shear stress at the perturbed cell passes the yield stress. For the amount of thermal softening, we assume it as usual to be given by:

$$f_T = \left(\frac{T_m - T}{T_m - T_0} \right)^{\alpha_T}$$

$$Y = f_T Y_0 \quad (5)$$

where T_m =melting temperature, and when the exponent α_T is close to 1.

The material we use in our computations is aluminum with the following parameters:

$$\rho_0 = 2.7 \frac{\text{g}}{\text{cc}} \quad G_0 = 27.6 \text{ GPa} \quad Y_0 = 0.29 \text{ GPa}$$

$$\beta = 1 \quad C_p = 1 \times 10^{-3} \text{ kJ} / \text{g} / \text{K} \quad k = 2.15 \times 10^{-7} \text{ GPa} \times \text{mm}^2 / \mu\text{s} / \text{K}$$

$$A_p = 1 / \mu\text{s} \quad T_m = 933 \text{ K} \quad \alpha_T = 1 \quad (6)$$

The tube length in the computations is 1mm, and it's divided into 1000 cells of 1μ length. The particle velocities on the boundaries are between 5 and 50 m/s, which is equivalent to an average longitudinal strain between $1e4$ and $1e5$ per second. We perform the ODE integration with a constant time step of 0.2ns, and we validated that this time step is low enough to get smooth and stable results. In the next figures we show results from the reference computations with boundary velocities of 25m/s.

From Fig. 1 we see that: 1) the shear band temperature does not make it to the melting temperature even after a relatively long time; 2) the heated shear band thickness is about 0.2mm, a rather large thickness; 3) the temperature at the boundaries rises substantially, to about 600K.

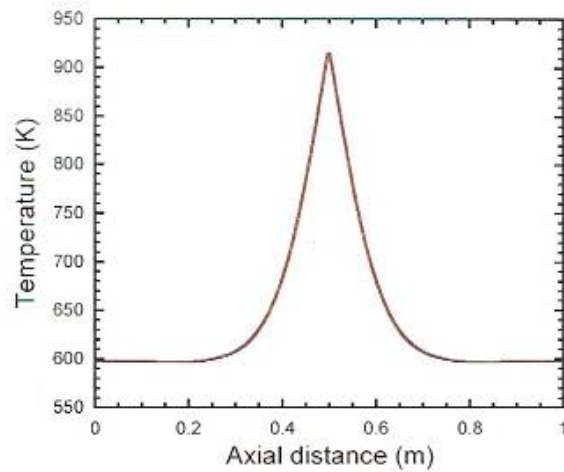


Figure 1. Temperature along the pipe for the reference computation

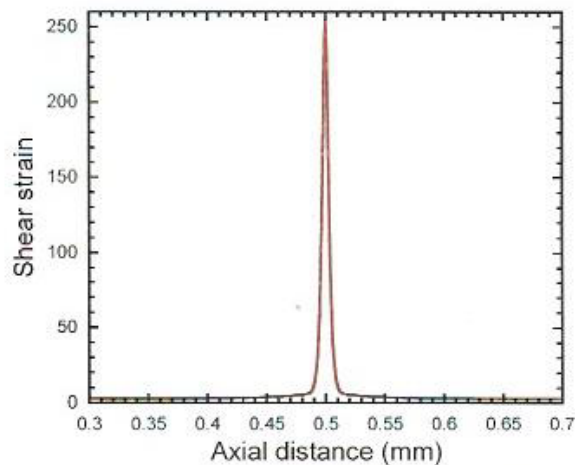


Figure 2. Shear strain along the pipe for the reference computation

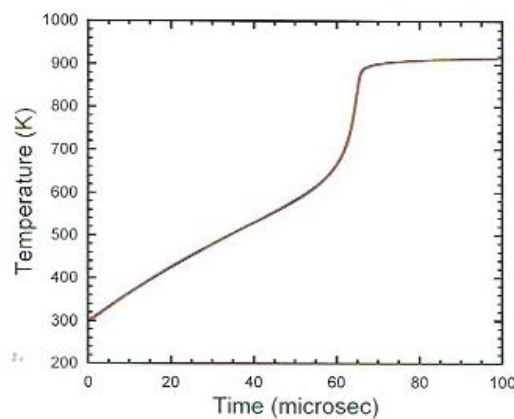


Figure 3. Maximal temperature history in the shear band for the reference computation

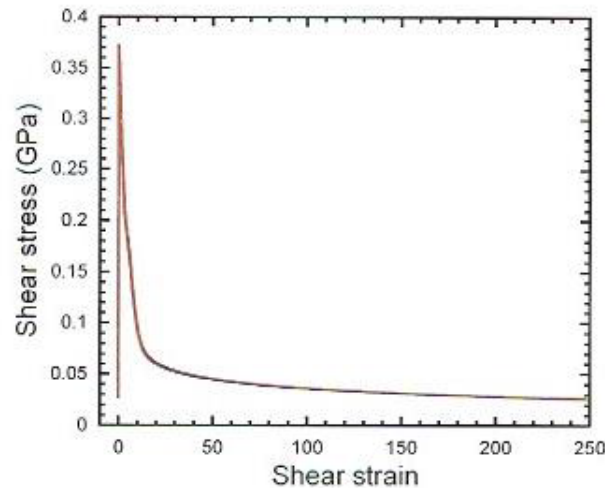


Figure 4. Shear stress as function of shear strain in the shear band for the reference computation

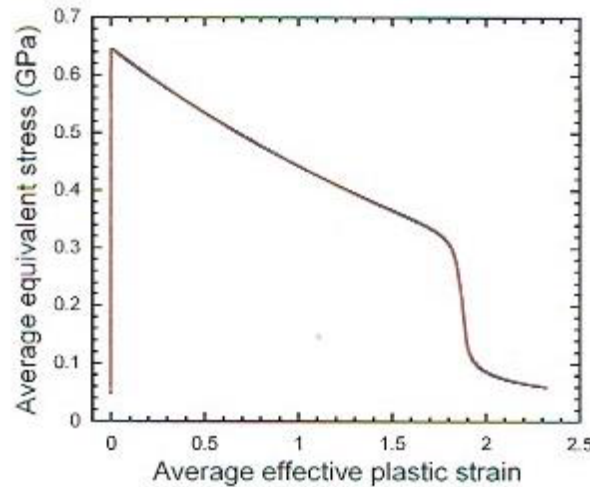


Figure 5. Average equivalent shear stress as function of the average effective plastic strain in the tube

From Fig. 2 we see that: 1) the accumulated strain in the shear band is quite large; 2) the strain shear band thickness is 0.02mm (or 20 μ), 10 times smaller than the temperature shear band thickness.

From Fig. 3 we see that the shear band develops slowly for about 65 μ s. Then it develops fast and gets close to the melting temperature, but does not quite reach it.

From Fig. 4 we see that: 1) the shear stress in the shear band rises initially to a value somewhat higher than the yield stress. It is then able to increase a little more before thermal softening pushes it back down; 2) the shear stress drops to quite low values after a strain of 10. Then it continues slowly down towards zero, but never gets there.

We see from Fig. 5 that: 1) the effective plastic strain along the tube is much smaller than the strain in the shear band. This is of importance when translating the results of a quasi-

mesoscopic computation (like the one presented here) to macroscopic computations; 2) when the shear band forms after $65\mu\text{s}$ (see Fig. 3), the average stress decreases fast but doesn't go down to zero, and beyond that it decreases slowly with time and with the average strain.

From the results of Figs. 1-5 we see that a dynamic shear failure (decrease of the shear stress to zero) is not caused by heat alone, as metals generally fail under dynamic shear at plastic strain much smaller than that needed for failure from heat alone. We therefore assume in what follows that at a certain stage of the shear band development, mechanical damage starts to develop there, and this mechanical damage brings the material to failure before the shear band temperature reaches melting.

MECHANICAL DAMAGE AT THE SHEAR BAND

As seen above, the material in the shear band undergoes fast and large strains, and it's plausible to assume that such a process would lead to significant mechanical damage. We find in the literature photos of shear bands with highly damaged material [7]. But generally, it's hard or impossible to represent what we see in those photos as an ordered mechanical process that can be reproduced by simulations. It therefore seems to us that generally it's rather difficult to realistically simulate the development of damage in a shear band. It's plausible to assume that damage develops on the mesoscale, and that it's specific to the type of material considered and to its structure. We assume generally that the creation and accumulation of such damage is related to the creation and development of voids and cracks, and to the breaking of large material crystals into small ones. We assume next that at the end of such processes, a highly broken and crushed material is obtained, which has no strength, and responds much like a granular material. In a granular material (like sand or sugar) under shear stress, the grains move with respect to each other without resistance, but if in addition pressure is applied, there is friction between the grains, which creates shear resistance. As the process of damage accumulation in a shear band is quite complex and may vary with the material, we see a way to represent it only by using a schematic model. What we use, which is quite common in the literature is the following. We define a damage parameter (expressing the relative amount of damage) that varies between 0 and 1. Damage accumulation takes place according to some criterion, which is expressed in terms of the effective plastic strain. We denote the amount of damage by D , and the damage accumulation threshold by γ_D . From lack of any specific knowledge on damage accumulation we assume the following simple relation:

$$\begin{aligned}\dot{D} &> 0 \quad \text{for} \quad \gamma_{\text{eff}}^p > \gamma_D \\ \dot{D} &= 0 \quad \text{for} \quad \gamma_{\text{eff}}^p \leq \gamma_D \\ \gamma_D \quad \text{and} \quad \dot{D} \quad \text{are given}\end{aligned}\tag{7}$$

As stated in Eq. (7), the rate of damage accumulation is given. It may depend on state variables (like pressure and temperature), but in what follows we assume the rate of damage accumulation to be constant. We decrease the yield stress Y as a result of damage accumulation by:

$$\begin{aligned}f_D &= 1 - D \\ Y &= Y_0 f_T f_D\end{aligned}\tag{8}$$

In Fig. 6 we show the influence of damage accumulation on decrease of the average equivalent stress down to zero.

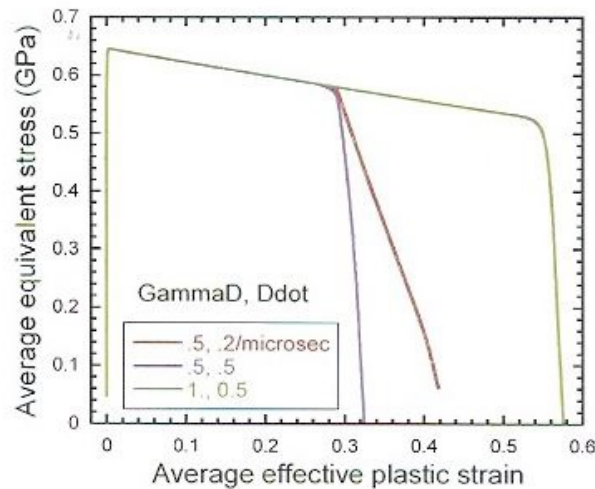


Figure 6. Average equivalent shear stress in the tube as function of the average effective plastic strain in the tube, resulting from thermal softening plus mechanical damage. Red: $\gamma_D=0.5$, $\dot{D}=0.2/\mu\text{s}$, Blue: $\gamma_D=0.5$, $\dot{D}=0.5/\mu\text{s}$, Green: $\gamma_D=1$, $\dot{D}=0.5/\mu\text{s}$

From Fig. 6 we see that: 1) the stress decline curve includes two sections. The first is shallow and results from thermal softening, and the second is steep and results from mechanical damage accumulation; 2) when the threshold of damage accumulation is higher, yield stress reduction and its average equivalent stress reduction start later; 3) when the rate of damage accumulation is higher, the rate of average equivalent stress reduction is faster.

STRAIN HARDENING

In the figures above we see that stress decrease in the shear band starts right when the shear band starts to develop. But in tests [2] we see that stress starts to decrease later, when thermal softening overcomes strain hardening. To reproduce this behavior, we add strain hardening to our model. Strain hardening is given by:

$$f_\gamma = f_{\text{inf}} + (1 - f_{\text{inf}}) \exp\left(-\frac{\gamma_{\text{eff}}^p}{\gamma_{\text{ref}}}\right)$$

$$Y = Y_0 f_1 f_D f_\gamma \quad (9)$$

where: $Y_{\text{inf}}=f_{\text{inf}}Y_0$ =maximal yield stress, γ_{ref} =parameter determining the rate of yield stress increase as function of the effective plastic strain.

In Fig. 7 we show average stress-strain curves demonstrating the effect of strain hardening.

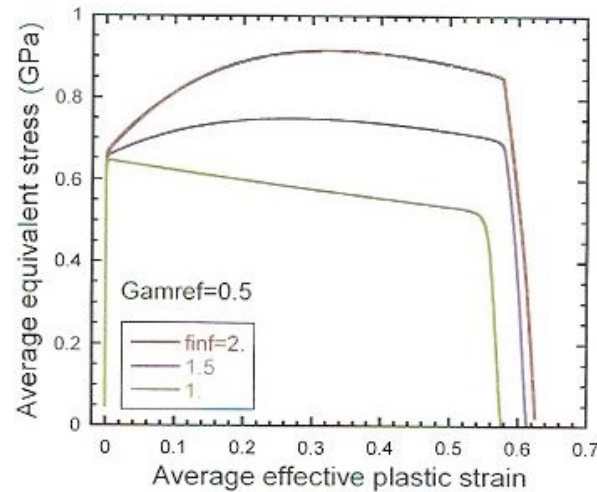


Figure 7. Average equivalent shear stress in the tube as function of the average plastic strain resulting from thermal softening, mechanical damage, and strain hardening

We see from Fig. 7 that: 1) upon adding strain hardening, stress rises after crossing the initial yield stress point; 2) as the rate of hardening decreases, at some point thermal softening overcomes strain hardening, and beyond that point stress decreases at a rate determined by thermal softening; 3) when mechanical damage softening comes into play, a fast stress decline phase takes over.

PRESSURE HARDENING

As is well known [], pressure increases the yield stress, and below about 10GPa we may assume for that a linear dependence:

$$f_p = 1 + h_p P$$

$$Y = Y_0 f_T f_D f_\gamma f_p \quad (10)$$

As seen above, for shear band development in torsion, the isotropic stress (or pressure) is not in the equations, and we apply it from the outset just for the purpose of including the influence of pressure hardening. In Fig. 8 we show simulation results with the same data as in Fig 7, except that we added pressure hardening. The hardening coefficient in these computations is: $h_p=0, 0.2, 0.5/\text{GPa}$, and the pressure applied from the outset is 10GPa.

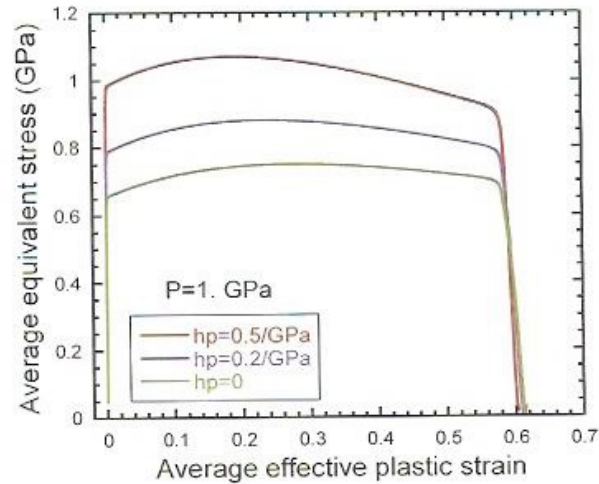


Figure 8. Average equivalent shear stress in the tube as function of the average effective plastic strain, resulting from all four softening/hardening effects: thermal softening, mechanical damage softening, strain hardening and pressure hardening

We see from Fig. 8 that: 1) for higher pressure levels the stress-strain curve (average along the whole tube) goes up, and the point of maximum stress moves to lower values of strain; 2) in addition, it's plausible to assume that for a fast-increasing pressure, thermal softening will not be able to overcome pressure hardening, a shear band will not form, and shear failure will not happen.

APPLICATION TO MACROSCALE COMPUTATIONS

Macroscale computations (using a hydrocode) are usually without heat conduction and with computational cells of about 0.1mm or more. This is why it's not possible to reproduce shear band formation on the macroscale. To predict shear failure with a macroscale computation we therefore need to represent the evolution of shear bands all the way to failure in terms of macroscopic quantities. The boundary velocities of the tube shear band computations discussed above are represented in a cell of a macroscopic computation by the effective strain rate of the cell. This quantity follows from the particle velocity deformation gradient tensor (or deformation rate tensor) of the cell. We therefore check next the influence of the tube boundary velocities on the shear band formation process.

In Fig. 9 we show the average equivalent stress in the tube as function of the average effective strain in the tube for three values of the boundary velocities.

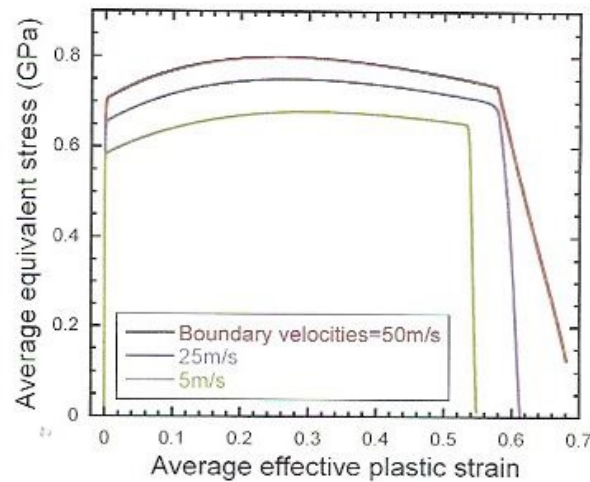


Figure 9. The influence of boundary velocities

We see from Fig. 9 that for higher boundary velocities the maximal stress is higher and the strain to failure is somewhat higher. The reason is that for higher boundary velocities the stress increase is faster, but the rate of damage accumulation stays as before. It follows that the path to failure and the strain to failure depend on the boundary conditions.

On this basis we propose to apply our failure model described above to computations on the macroscale in the following way

We assume that in each computational cell on the macroscale, at least one shear band may form. We further assume that the macroscopic variables in the macroscale cell represent in some way the variables of the shear band. From the hydrocode computation we get for the considered cell the values of the following variables: total effective strain rate, pressure, temperature, equivalent stress, effective strain, level of mechanical damage and yield stress. From these variables we compute the: effective plastic strain rate, effective elastic strain rate, rate of increase of the equivalent stress, rate of hardening/softening of the yield stress. From the temperature increase we then compute the rate of thermal softening. From the rate of change of pressure (resulting from the hydrodynamics) we compute the rate of pressure hardening/softening, and from the last three rates we then compute the rate of change of the yield stress. When the rate of softening of the yield stress becomes higher than its rate of hardening, this is the threshold of localization, and from then on, we start checking whether the conditions for mechanical damaging are being fulfilled. These conditions are given in terms of the effective plastic strain and the rate of change of the effective plastic strain.

When crossing the threshold of mechanical softening, we add to the above three rates the rate of mechanical softening, which may decrease with pressure. Usually, the rate of mechanical softening is the dominant one on the way to failure. Failure is reached when the yield stress reaches zero, and when this happens, we also put to zero the equivalent stress and the shear modulus. Finally, we compute the shear stress components from the Prandtl-Reuss equations [8]. In the future we intend to incorporate the scheme described above into our hydrocode and use it to compute examples of shear failure in a macroscopic single cell.

SUMMARY

We're dealing here with failure of viscoplastic materials through the formation of shear bands. Shear bands may form in a viscoplastic material when loaded dynamically in a way that leads to localization of the plastic flow. Such a localization happens from strength reduction which results from plastic heating. Dynamic loading is essential for such a localization, as it prevents the heat created to escape. Localization of plastic flow may start at an initial or subsequent strength reduction (perturbation). The perturbation may then strengthen by plastic heating and weaken by pressure hardening and strain hardening. If the net result is that strength continues to decrease, an instability is formed, and it leads to localization of the viscoplastic flow and to formation of a shear band. The shear band then heats up and becomes localized. At some point the material in the shear band undergoes significant mechanical damage (voids and cracks), which then leads quickly to failure. We model shear band formation followed by shear failure with a 1D computation of a thin-walled tube loaded in torsion. We start with the viscoplastic response influenced only by thermal softening, and then add in turn three other mechanisms: mechanical damage, strain hardening and pressure hardening. We don't need to add strain rate hardening, as this feature is intrinsically included by the overstress approach to viscoplasticity that we use. Finally, we lay down schematically the way to apply our shear band shear failure model to a computational cell of a hydrocode on the macroscale.

References

- [1]. Y. Bai and T. Wierzbicki, A new model of metal plasticity and fracture with pressure and Lode angle dependence, *Int. J. Plast.* 24, 1071-1096 (2008).
- [2]. A. Marchand and J. Duffy, An experimental study of the formation process of adiabatic shear bands in a structural steel, *J. Mech. Phys. Solids*, 36, 251-283 (1987).
- [3]. Y. Partom, Modeling of shear band interaction in 1D torsion, *Shock compression of condensed matter conference* (2015).
- [4]. Y. Partom and A. Lindenfeld, Computer modeling of dynamic necking in bars, *Shock compression of condensed matter conference* (2015).
- [5]. Y. Partom, Overstress and flowstress approaches to dynamic plasticity, *DYMAT conference*, Lugano Switzerland (2015).
- [6]. D.E. Grady, Structured Shock Waves and the 4th Power Law, *J. Appl. Phys.* 107, 013506 (2010).
- [7]. A. Molinari, S. Mercier and N. Jacques, Dynamic failure of ductile materials, 23rd *Int. Con. of Theor. and Appl. Mech.*, IUTAM 10 201-220 (2014).
- [8]. O.T. Bruhns, The Prandtl-Reuss equations revisited, *Z. Angew Math. Mech*, 94, No 3, 187-202 (2014).