

Solution Concepts in Multiobjective Location Problems with Variable Domination Structure

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ABSTRACT

We study multiobjective location problems with respect to a variable domination structure. To solve such problems, we apply minimal and nondominated solutions from vector optimization problems with respect to a variable domination structure. We investigate these solution concepts to be used in multiobjective location problems. These solution concepts are proposed for multiobjective location problems where variable preferences of objective functions at each location are assumed; our results are applied for selecting a location to establish a new branch.

Keywords: Multiobjective location problems, Variable domination structure, Minimal solution, Nondominated solution

INTRODUCTION

In many real-life problems, we have to deal with multiobjective decision making problems. In these problems, one would like to find the best alternative among a given set of feasible alternatives in the condition of considered objective function or criteria, see [12, 2]. Moreover, in location problems, a decision maker would like to select the best location among a set of candidate locations with respect to a set of given criteria or objective functions. The location problem is utilized for finding an optimal location in different categories of problems. In general, minimizing the distance is an essential objective in many location problems. Up to the real-world applications of the problem, other objectives may affect the decision, including cost (e.g., land cost, labor cost, production cost, material cost, logistic cost, and the like), time, financial risks, laws and regulations, quality, economic factors, etc. In some location problems, the decision maker seeks an optimal location, while in some others, the decision maker wants to rank the candidate locations. In other vital problems in logistics, the goal is to find a route by minimizing the given objectives. Such problems consist of finding the best solution to visit a set of locations considering constraints for better efficiency and cost/time effectiveness.

Farahani et al. [18] investigated different categories of location problems and their solutions. Kirschstein and Bierwirth [10] introduced selective traveling salesman problems with emission allocation rules.

The aim was to choose a subset of transport requests and select the corresponding route by minimizing the transport emissions that can be allocated to a particular request based on the emission reporting standard. In [10], a mixed-integer linear optimization problem is considered, and a corresponding solution procedure is derived.

In multiobjective decision making problems, a fixed domination structure is generally considered, while in many problems, there exists variable domination structure related to different alternatives/locations (in multicriteria location problems) [6]. In multiobjective decision-making problems, the decision objectives could be minimizing the time, cost, proximity to the suppliers, transportation costs, and distances, on the other hand maximizing proximity to the competitors, customer satisfaction, quality, and other objectives. In many multiobjective location problems, it is clear that the priority of these objectives could be different at each location. For instance, up to the condition of a problem, minimizing the time in some candidate locations is much more preferred than minimizing the cost, while it is another way around in some other candidate locations. Variable domination structure represents these varieties.

Some articles presented an application of variable domination structure in psychology, medical engineering, and radiotherapy, see [3, 4, 7, 11]. We introduced an inverse variational inequality to investigate variable domination structure in multiobjective location problems [16]. In some multiobjective decision making problems, the importance of objective functions (the weights) is different, as Bertsch studied [5]. Zargini investigated the multicriteria decision making problems where different criteria weights with respect to the alternatives are assumed [17]. In this study, we investigate multiobjective location problems with different preferences of objective functions; These preferences are related to the candidate locations or some restrictions related to the objective functions. These conditions are presented in Sections 5 and 4, respectively. Section 2 refers to the fundamental concepts of vector optimization problems with respect to variable domination structure. Section 3 presents the concept of solutions in multiobjective location problems with respect to variable domination structure. This solution concept leads to two types of solutions, namely minimal and nondominated. The minimal solution is derived by comparing alternatives in terms of their preferences; the nondominated solution is obtained by comparing the alternatives with the preferences of the other alternatives.

Moreover, Section 4 presents an application to study multiobjective problems in terms of fixed ordering cone and variable domination structure and how minimal and nondominated solutions are utilized. In Section 5, we apply the result of Section 3 to select a location for establishing a new branch of a market.

PRELIMINARIES

In this part, we recall some basic concepts on multiobjective decision making problems. Let $\mathbb{R}^n$ and $\mathbb{R}^p$ be preimage space or decision space and image space or objective space, respectively. A decision making process contains the following steps:

1. Defining the goal of the decision making process;
2. Collecting the set of candidate **alternatives**, possible actions, or candidate locations belong to the decision space, which is known as the **feasible set**;
3. Determining the set of criteria or **objective functions** given by $f: \mathbb{R}^n \rightarrow \mathbb{R}^p, i = \overline{1, p}$;
4. Defining a cone in order to compare objective function values;
5. Assigning a weight for each criterion to represent their importance;
6. Selecting a method to solve such problems.

For more detailed results, we refer the reader to [1, 12].

Consider the objective function $f: \mathbb{R}^n \rightarrow \mathbb{R}^p, i = \overline{1, p}$ given by

$$f(x) := \begin{pmatrix} f_1(x) \\ \cdots \\ f_p(x) \end{pmatrix}, \quad x \in X,$$

where, $X \subseteq \mathbb{R}^n$.

Let $X := \bigcup_{k=1}^m X_k$, be the feasible set where $X_k \subset \mathbb{R}^n$ for $k = \overline{1, m}$, and $X_s \cap X_r = \emptyset$ whenever $s \neq r$, and $f(X) := \bigcup_{x \in X} f(x)$.

Assume that $C: \mathbb{R}^n \rightrightarrows \mathbb{R}^p$ is a set-valued mapping which is called domination map. The corresponding multiobjective decision making problem with respect to the variable domination structure generated by C is given by

$$\text{Min}(f(X), C(\cdot)). \quad (PC(\cdot))$$

In the following, we present optimality concepts for multiobjective decision making problems with respect to a variable domination structure.

Definition 2.1 (Eichfelder, 2012, [8]). Let $x^0 \in X$. Corresponding to the problem $(PC(\cdot))$; we say that:

$f(x^0)$ is a **minimal element** of $f(X)$ w.r.t. the domination map C if

$$(f(x^0) - C(x^0) \setminus \{0\}) \cap f(X) = \emptyset.$$

The set of all minimal elements is denoted by $M(f(X), C(\cdot))$.

Figure 1 shows that $f(x^0)$ is a minimal element of $f(X)$ with respect to the domination structure C .

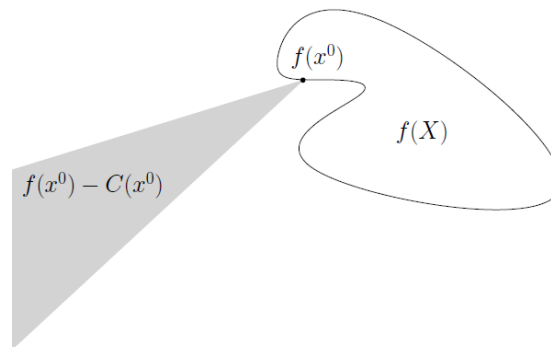


Figure 1: $f(x^0)$ is a minimal element of $f(X)$ w.r.t. C

Definition 2.2 (Yu, 1974, [15]). $f(x^0)$ is a **nondominated element** of $f(X)$ w.r.t. the domination map C if

$$\forall x \in X: (f(x^0) - C(x) \setminus \{0\}) \cap \{f(x)\} = \emptyset.$$

or equivalently, if no $x \in f(X)$ exists s.t

$$f(x^0) \in f(x) + C(x)$$

The set of all nondominated elements is denoted by $N(f(X), C(\cdot))$.

Figure 2 shows that $f(\bar{x})$ is not a nondominated element of $f(x)$ with respect to the domination structure C .

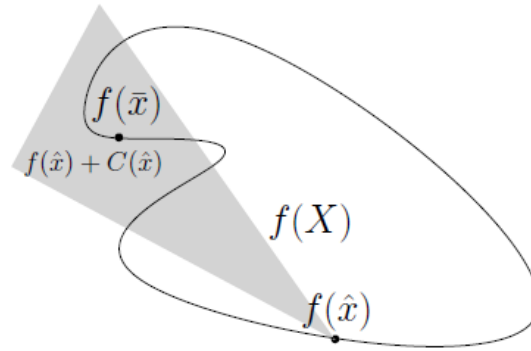


Figure 2: $f(\bar{x})$ is not a nondominated element of $f(x) + \text{w.r.t. } C$

Definition 2.3 (Eichfelder, 2012, [8]). $f(x_0)$ is a weakly nondominated element of $f(x)$ w.r.t. the domination map C if

$$\forall x \in X: (f(x_0) - \text{int } C(x) \setminus \{0\}) \cap \{f(x)\} = \emptyset.$$

The set of all weakly nondominated solutions is denoted by $\text{WN}(f(X), C(\cdot))$.

MULTIOBJECTIVE LOCATION PROBLEMS WITH RESPECT TO VARIABLE DOMINATION STRUCTURE

A multiobjective location problem considers a set of known points and determines the number of alternative points to investigate in order to optimize a distance function between candidate alternatives and the existing points.

Definition 3.1. A function $\|\cdot\|: \mathbb{R}^n \rightarrow \mathbb{R}_+$ is called norm on $\mathbb{R}^p$ if the following conditions are held for all $x, \hat{x} \in X$ and for all $\lambda \in \mathbb{R}$

- (i) $\|x\| = 0 \Leftrightarrow x = 0_X$;
- (ii) $\|\lambda x\| = |\lambda| \|x\|$;
- (iii) $\|x + \hat{x}\| \leq \|x\| + \|\hat{x}\|$.

A norm always induces a distance function by

$$d(x, \hat{x}) := \|x - \hat{x}\|.$$

We investigate multiobjective location problems with respect to a variable domination structure. The following assumption will be needed throughout this section.

Assumption 3.1. Let $X := \bigcup_{j=1}^m X_j$, $m \in \mathbb{N}$ be the feasible set where $X_j \subseteq \mathbb{R}^n$ for $j = \overline{1, m}$, and $X_s \cap X_r = \emptyset$ whenever $s \neq r$, and $f(X) := \bigcup_{x \in X} f(x)$. Assume that $a^i \in \mathbb{R}^n$ are presented existing facilities.

Let $\|\cdot\|$ be an arbitrary norm (usually Manhattan norm), the objective function $f: \mathbb{R}^n \rightarrow \mathbb{R}^p$ is given by

$$f(x) := \begin{pmatrix} f_1(x) \\ \vdots \\ f_p(x) \end{pmatrix}. \quad (2)$$

In our setting, each X_j has its own preferences with respect to the location a^i , for, $i = \overline{1, p}$ and $j = \overline{1, m}$. Moreover, the domination map $C: \mathbb{R}^n \rightrightarrows \mathbb{R}^p$ is given by

$$C(x) = \left\{ (y_1, y_2, \dots, y_p)^T \in \mathbb{R}^p \mid \alpha_1^j y_1 + \alpha_2^j y_2 + \dots + \alpha_p^j y_p \geq 0 \right\}, x \in X_j, \quad (3)$$

where $\alpha^j := (\alpha_1^j, \dots, \alpha_p^j)^T \in \mathbb{R}^p$ for $j = \overline{1, m}$.

The corresponding multiobjective location problem with respect to the variable domination structure defined in (3) is given by

$$\text{Min}(f(X), C(\cdot)). \quad (P_L)$$

In the following, we recall a theorem representing sufficient conditions of the minimal and weakly nondominated solutions of (P_L) .

Theorem 3.1 (Zargini, Doagooei, 2018 [16]). *Consider the problem (P_L) under Assumption 3.1. Assume that $x^j \in X_j$. Consider an element $x^0 \in X$ such that following inequality holds for all $x^j \in X, j = \overline{1, m}$.*

$$\sum_{i=1}^p \alpha_i^j \|x^0 - a^i\| \leq \sum_{i=1}^p \alpha_i^j \|x^j - a^i\|.$$

Then, $f(x^0) \in WN(f(X), C(\cdot))$.

- For $x^0 \in X$, if the following assertion holds true

$$\sum_{i=1}^p \alpha_i^0 \|x^0 - a^i\| \leq \sum_{i=1}^p \alpha_i^0 \|x^j - a^i\|.$$

for $x^j \in X$ then, $f(x^0) \in M(f(X), C(\cdot))$.

SELECTING A LOCATION TO CONSTRUCT AN INDUSTRIAL PLANT COMPARING THE TERM OF FIXED ORDERING AND VARIABLE DOMINATION STRUCTURE

The goal is to find an optimal location to construct an industrial plant by minimizing the distance and cost.

Consider $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by

$$f(x) := \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix},$$

- where $f_1(x) := d(x, a)$, $d(x, a) := \|x - a\|$, $x, a \in \mathbb{R}^2$, see Definition 3.1 and $f_2(x)$ represents costs. Suppose that $f(X) := \bigcup_{x \in X} f(x)$, $X \subseteq \mathbb{R}^2$. $f_1(x)$ represents the distance to the city center, and $f_2(x)$ refers to the final production costs, including land, human

resource, production, material, and transport costs. The goal is to minimize both these objective functions.

- (ii) Let $C = \mathbb{R}_+^2$. The corresponding bi-criteria location problem with respect to the fixed domination structure $\mathbb{R}_+^2$ is given by

Determine (Pareto) efficient elements of f w.r.t. C . (6)

The candidate locations are compared up to the same states, and Figure 3 shows that $f(\bar{x})$ belongs to the Pareto efficient solutions of (6).

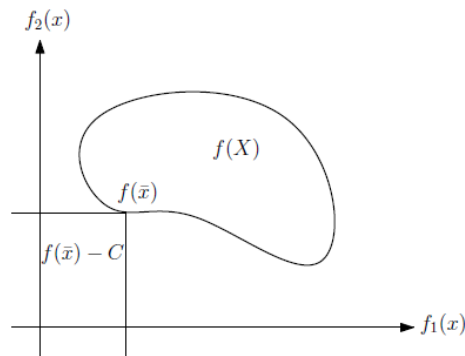


Figure 3: $f(\bar{x})$ is a Pareto efficient solution of (6)

- (iii) However, in many real-life problems, there are different characterizations and preferences related to the given locations. Now we propose the multiobjective optimization problem with respect to the variable domination structure. Suppose that in area $\|x - a\| < \|\hat{x} - a\|$ (very close to the city center), the total production cost is more than others and at area $\|x - a\| < \|\hat{x} - a\|$, up to the customer satisfactions minimizing the total cost is more preferred. It means that the locations with a distance to the city center are more than $\|\hat{x} - a\|$, the decision maker defines the domination structure up to these preferences.

Note that, in this case, the scales of two objective functions are not the same; therefore, the given data should be normalized.

In $f_1(x) > f_1(\hat{x})$ Minimizing the cost five times is more preferred than minimizing the distance; therefore, we formulate the problem as a multiobjective location problem with respect to a variable domination structure. The variable domination structure is given by the domination map $C: \mathbb{R}^2 \rightrightarrows \mathbb{R}^2$ is defined by

$$C(x) := \begin{cases} \{(y_1, y_2)^T \in \mathbb{R}^2 \mid y_1 \leq 5y_2\} & \text{if } f_1(x) > f_1(\hat{x}), \\ \mathbb{R}_+^2 & \text{if } f_1(x) \leq f_1(\hat{x}). \end{cases} \quad (7)$$

Then, the corresponding problem is defined by

Determine nondominated (or minimal) solutions of f w.r.t. $C(\cdot)$. (8)

Figure 4 illustrates the nondominated and minimal solutions of problem (8) (see Definition 2.1, 2.2).

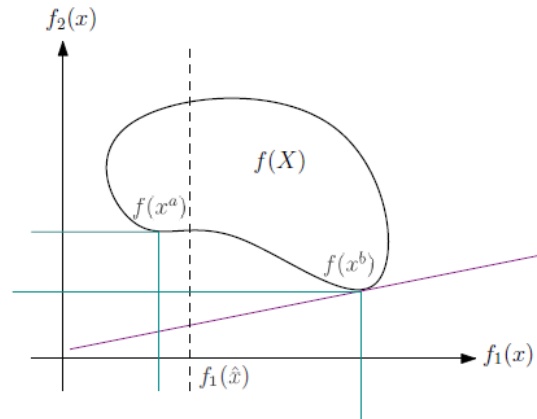


Figure 4: Nondominated and minimal solution of (8)

$f(x^a)$ and $f(x^b)$ are minimal solutions of $f(X)$ with respect to the domination map $C(\cdot)$, while $f(x^b)$ is also a nondominated solution of $f(X)$ with respect to the domination map $C(\cdot)$. In the current situation, all minimal solutions can be a proper location to construct an industrial plant. However, the nondominated solution x^b is preferred while the minimal solutions apply for the corresponding preferences of the alternatives (i.e., the candidate locations). In contrast, it is also true that the nondominated solutions are based on the preferences of the other alternatives. For instance, x^b is the proper location to construct a plant considering the current restrictions, even for the situation without restrictions. On the other hand, x^a is a proper location to construct a plant due to the current preferences of this location, and if there are restrictions in the future, the location will not be optimal for that condition.

Remark 4.1. Note that, up to the condition of a problem, the domination structure could be defined from image space to image space or from preimage space to image space. In this section, we defined domination structure $C: \mathbb{R}^2 \rightrightarrows \mathbb{R}^2$.

OPENING A NEW BRANCH OF BUSINESS

Selecting a business location is a significant part of establishing a shopping or business market (i.e., trade). Many attributes play an essential role in selecting the proper location.

Some of these attributes may include operating costs, production costs, land costs, logistic costs, manpower costs, proximity to the supplier, proximity to the competitors, proximity to the customers, laws and regulations, environmental risks, and accessibility.

In the previous models, the decision maker collected the set of candidate locations as the alternative and compared them based on the considered criteria. To solve such problems, the decision maker assigns a weight to each criterion and selects a proper location using multicriteria decision making methods.

In a multiobjective location problem with a continuous feasible set, one can find the optimal solution set using Pareto optimality. In many multiobjective location problems, each location has its own preferences. We apply the multiobjective location problem with respect to the variable domination structure to compare the candidate locations with their corresponding preferences. These variable structures represent the performance of each criterion in the corresponding location. At each location, minimizing or maximizing some criteria will be preferred to others. These preferences may vary based on customer satisfaction in the given location. In the case of the alternative locations, comparing the facilities located in different countries or states, economic factors, the business climate, laws and regulations, financial risks, and other attributes can affect the preferences of the location.

For the practical application, we assume that a factory owner wants to establish a new market branch by minimizing the distance to twelve given suppliers. Suppose that $X = \mathbb{R}^2$, and $i = \overline{1,12}$ under Assumption 3.1.

The distances from the new market to the suppliers are calculated by Manhattan norm $\|\cdot\|_1 : \mathbb{R}^n \rightarrow \mathbb{R}_+$ with

$$\|x - a^i\|_1 := \sum_{j=1}^p |x^j - a_j^i|, \quad i = \overline{1,p}.$$

The objective function $f: \mathbb{R}^2 \rightarrow \mathbb{R}^p$ for the corresponding location problem is given by

$$f(x) = \begin{pmatrix} \|x - a^1\|_1 \\ \dots \\ \|x - a^{12}\|_1 \end{pmatrix},$$

the corresponding multiobjective location problem with respect to the fixed cone $\mathbb{R}^2$ is given by

$$\text{Min}(f(X), \mathbb{R}^2). \quad (P_C)$$

Note that $f(X)$ can include the distance functions and other objective functions to be minimized (i.e., cost, quality, time, and so on).

If there are no other preferences or restrictions, Figure 5 illustrates the Pareto efficient solution set of (P_C) using FLO software.

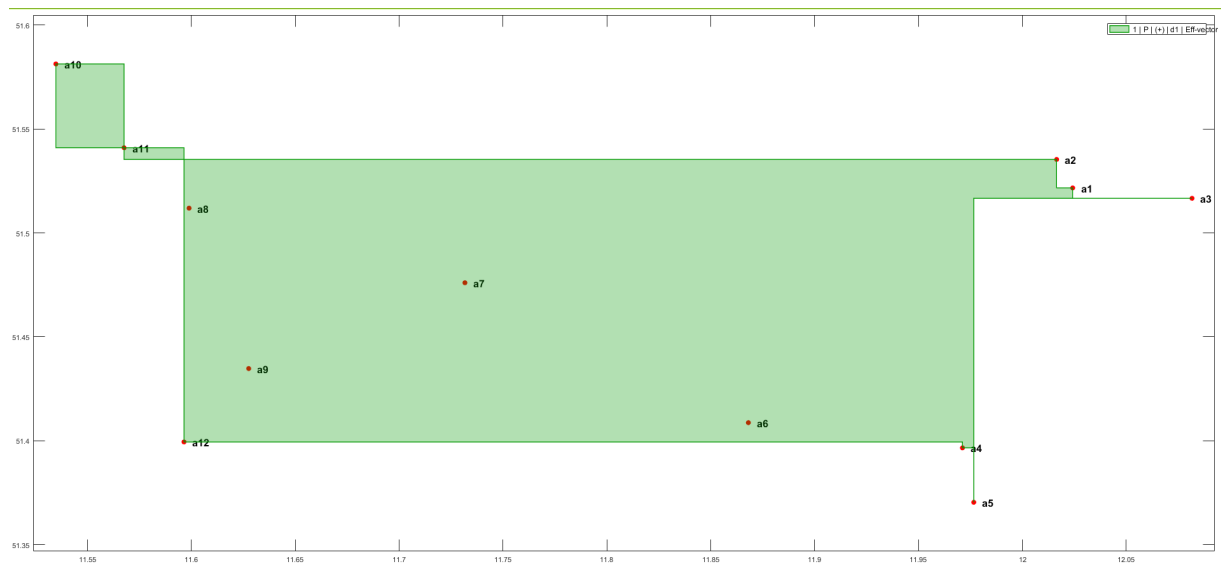


Figure 5: Pareto efficient solutions of (P_C) using Manhattan norm

Note that the decision maker selects an appropriate norm regarding the urban structure of a city. Figure 6 compares the Pareto efficient solutions of (P_C) applying Manhattan and infinity norm.

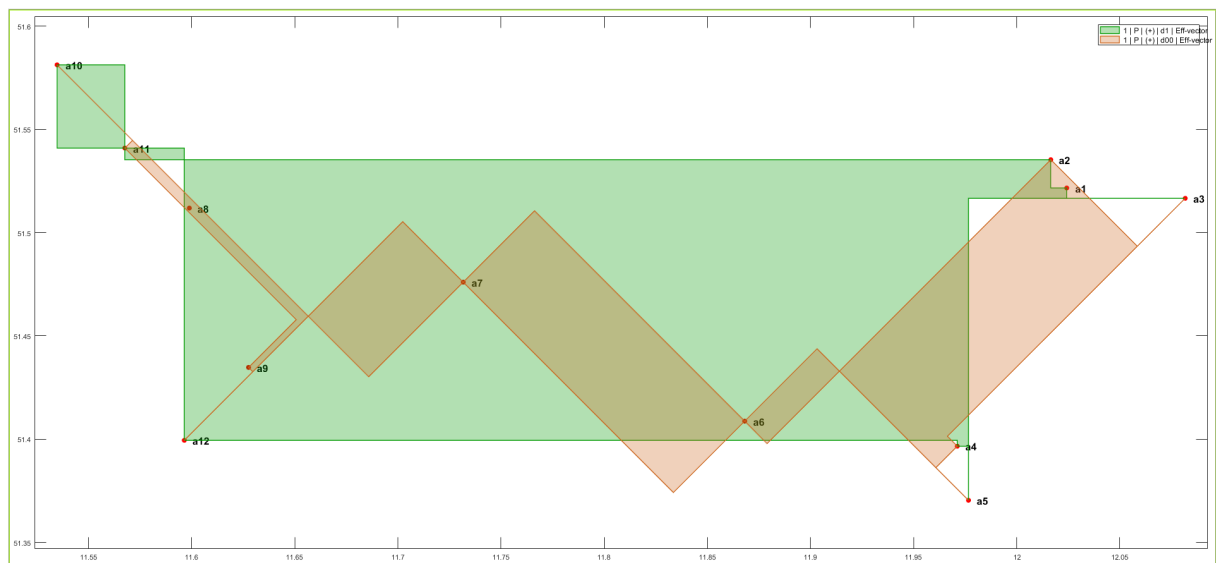


Figure 6: Pareto efficient solutions of (P_C) using Manhattan and infinity norms

Now, we assume a finite set of alternatives; we are looking for a new facility location for a new branch in one of five candidate locations by minimizing the distance to twelve given suppliers. Suppose that $X_j = \{x^j\}$, $j = \overline{1,5}$, and $i = \overline{1,12}$.

Figure 7 illustrates the given locations on the map, where the asterisks represent the locations of the suppliers, and the flags denote the five feasible locations. Assuming the same condition for other aspects, the decision goal involves minimizing the proximity to all given suppliers.

If there are no other preferences involved, Figure 8 shows that x^3 and x^4 are not the optimal solutions of (P_C) .

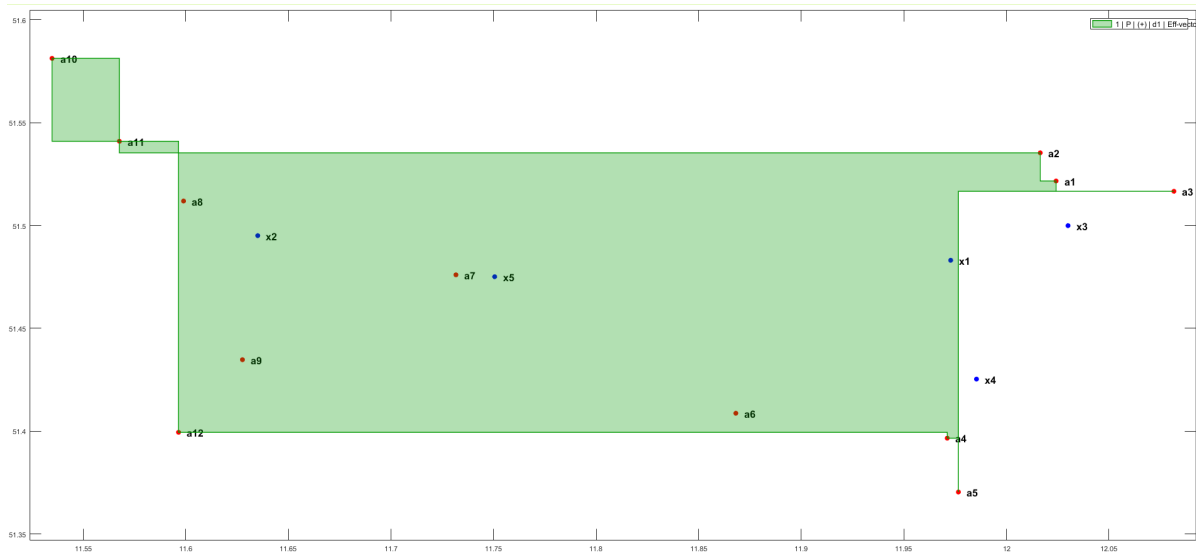


Figure 8: Pareto efficient solutions of (P_C) with a finite feasible set

If we consider the problem using the fixed domination structure, the decision maker can assign a weight to each given criteria (i.e., proximity to each supplier) and find the solution accordingly.

In many real-world problems, like this one, up to the customer requirement and customer satisfaction, there are different preferences at each candidate location up to the customer demand. In one location, minimizing the time is preferred to minimizing the cost, while in the other, it is the other way around. While the competitors and other aspects can affect such decisions, we consider that the other conditions for all feasible locations are equal. We compare these alternatives subject to minimizing the distance to twelve given suppliers. Since the customers at each location have different requirements, these requirements should be met by different suppliers. Therefore, minimizing the distances to some suppliers is more preferred to some others at each feasible location. Up to each location's characterization, the importance of minimizing the distance to different suppliers varies at different candidate locations. Therefore, to solve such problem, the multiobjective location problem with respect to a variable domination structure is proposed.

Consider multiobjective location problem (P_L) with respect to the variable domination structure defined in (3).

The decision-maker determines the domination structure, defined in (3), according to the customer requirements at each location. We collect all these coefficients α^j in Table 1.

Furthermore, we calculate the distance between candidate locations and suppliers using their geographical coordination and Manhattan norm.

For each x^j , at each period, the customer demands should be met by different suppliers. The variable domination structure defined in (3) represents these preferences.

$C(x^1)$

$$= \{(y_1, \dots, y_{12})^T \in \mathbb{R}^p | y_1 + y_2 + 2y_3 + 4y_4 + 3y_5 + 3y_6 + 2y_7 + y_8 + y_9 + 3y_{10} + 3y_{11} + 3y_{12} \geq 0\},$$

as the same way $C(x^2)$, $C(x^3)$, $C(x^4)$, and $C(x^5)$ are defined. We normalize the data to convert them to comparable data, see [14]. Table 1 depicts preferences, where each row of Table 1 as a vector of $\mathbb{R}^{12}$ and denote the i^{th} component of the j^{th} row by a_{ij} , for $j = \overline{1,5}$, and $i = \overline{1,12}$.

Table 1: The preference matrix

$a1$	$a2$	$a3$	$a4$	$a5$	$a6$	$a7$	$a8$	$a9$	$a10$	$a11$	$a12$
x^1	0.037	0.074	0,148	0.111	0.111	0.074	0.037	0.037	0.111	0.111	0.111
37	0.0										
x^2	0.105	0.158	0.052	0.105	0.052	0.105	0.052	0.052	0.052	0.105	0.052
05	0.1										
x^3	0.045	0.182	0.045	0.136	0.09	0.045	0.09	0.045	0.136	0.09	0.045
05	0.0										
x^4	0.055	0.055	0.111	0.111	0.055	0.055	0.055	0.111	0.167	0.055	0.055
45	0.1										
x^5	0.074	0.037	0.074	0.111	0.148	0.148	0.111	0.148	0.037	0.037	0.037
11	0.0										
37											

The following data are at hand from the Google map, illustrated in Figure 7.

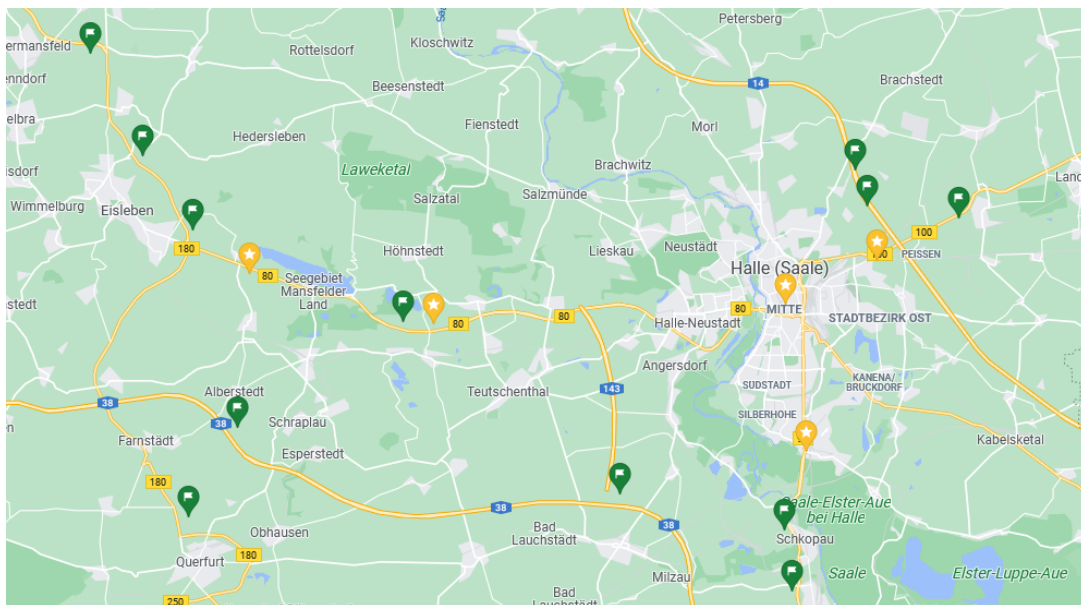


Figure 7: Alternatives and suppliers locations

Table 2: The decision matrix

$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$	$f_5(x)$	$f_6(x)$	$f_7(x)$	$f_8(x)$	$f_9(x)$	$f_{10}(x)$	$f_{11}(x)$	$f_{12}(x)$
x^1	0.1	0.14	0, 09	0.112	0.18	0.25	0.4	0.4	0.54	0.47	0.46
9	0.0										
x^2	0.42	0.47	0.43	0.47	0.32	0.12	0.05	0.07	0.19	0.11	0.13
2	0.4										
x^3	0.05	0.07	0.16	0.18	0.25	0.32	0.44	0.47	0.58	0.5	0.53
3	0.0										
x^4	0.14	0.19	0.04	0.06	0.13	0.03	0.47	0.37	0.61	0.53	0.41
4	0.1										
x^5	0.33	0.33	0.3	0.33	0.18	0.02	0.19	0.16	0.32	0.25	0.23
2	0.3										

The geographical coordinates for suppliers:

$\alpha^1 = (51.521651, 12.024222)$, $\alpha^2 = (51.535372, 12.016491)$, $\alpha^3 = (51.516655, 12.081668)$,
 $\alpha^4 = (51.396550, 11.971150)$, $\alpha^5 = (51.370349, 11.976604)$, $\alpha^6 = (51.408680, 11.868088)$,
 $\alpha^7 = (51.476018, 11.731622)$, $\alpha^8 = (51.511917, 11.598831)$, $\alpha^9 = (51.434721, 11.627542)$,
 $\alpha^{10} = (51.581313, 11.534716)$, $\alpha^{11} = (51.540977, 11.567546)$, $\alpha^{12} = (51.399403, 11.596385)$.

The geographical coordinates for feasible locations:

$x^1 = (51.483101, 11.972841)$, $x^2 = (51.495076, 11.635011)$, $x^3 = (51.499950, 12.030045)$,
 $x^4 = (51.425299, 11.985413)$, $x^5 = (51.475089, 11.750506)$.

Consider the set of objective functions given in 9; the Manhattan norm calculates the distances from the new market to the suppliers. Table 2 collected these objective functions values for each feasible location; for instance, the first component represents that $\|x^1 - a^1\|_1 = 0.09$.

For this problem, we are seeking minimal solution/solutions at the first step, while it means that this location is a proper location by considering the current preferences. If the minimal solution is also a nondominated solution, it means that this location is a proper location even by considering the customer requirements in the other feasible locations.

$$\sum_{i=1}^{12} \alpha_i^1 \|x^5 - a^i\| \leq \sum_{i=1}^{12} \alpha_i^1 \|x^1 - a^i\|;$$

$$\sum_{i=1}^{12} \alpha_i^2 \|x^5 - a^i\| \leq \sum_{i=1}^{12} \alpha_i^2 \|x^2 - a^i\|;$$

$$\sum_{i=1}^{12} \alpha_i^3 \|x^5 - a^i\| \leq \sum_{i=1}^{12} \alpha_i^3 \|x^3 - a^i\|;$$

$$\sum_{i=1}^{12} \alpha_i^4 \|x^5 - a^i\| \leq \sum_{i=1}^{12} \alpha_i^4 \|x^4 - a^i\|.$$

Moreover,

$$\begin{aligned} \sum_{i=1}^{12} \alpha_i^5 \|x^5 - a^i\| &\leq \sum_{i=1}^{12} \alpha_i^5 \|x^1 - a^i\|; \\ \sum_{i=1}^{12} \alpha_i^5 \|x^5 - a^i\| &\leq \sum_{i=1}^{12} \alpha_i^5 \|x^2 - a^i\|; \\ \sum_{i=1}^{12} \alpha_i^5 \|x^5 - a^i\| &\leq \sum_{i=1}^{12} \alpha_i^5 \|x^3 - a^i\|; \\ \sum_{i=1}^{12} \alpha_i^5 \|x^5 - a^i\| &\leq \sum_{i=1}^{12} \alpha_i^5 \|x^4 - a^i\|. \end{aligned}$$

Applying Theorem 3.1, the following data shows that x^5 is a minimal and nondominated solution with respect to the variable domination structure generated by the preferences.

CONCLUSION

In many multiobjective location problems, each location has its restrictions or preferences; consequently, it is proposed to apply multiobjective location problems with respect to variable domination structure. Utilizing such problems leads us to two kinds of solutions. The minimal solution is derived where alternatives are compared in terms of their preferences or restrictions. The nondominated solution is obtained where alternatives are compared in terms of the preferences or restrictions of the other alternatives. This study investigated the application for both types of solutions.

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