

Research on Basic Scientific Characters of Complexity, and Their Mathematics and Applications

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ABSTRACT

Based on the brief introduction of complexity, first we propose four basic scientific characters of complexity: much elements, various interactions, hierarchies, evolutions. Next, some corresponding mathematical methods of complexity are researched. Third, we discuss complexity in physics, and entropy is an important quantity. Fourth, nonlinearity of complexity is searched. Fifth, we research complexity in biology and neural networks. Finally, we study general sciences of complexity from chemistry, social sciences, to various applications. In a word, complexity is a complex science. Its investigations, developments and applications must be simplification and quantification.

Key words: complexity, elements, interaction, hierarchy, evolution, mathematics, physics, nonlinearity, biology, social science.

INTRODUCTION

At present the study of complexity is very important, and is also very complex, and the complexity may be shown in any field [1]. Even the hypercomplexity is researched.

In 1973 E. Morin proposed the thinking and paradigm of complexity, which include three rules: dialogic, cycle and hologram. Prigogine, et al., explored complexity science [2,3]. Santa Fe Institute studies the science of complexity, and published a series of lectures from 1989, and becomes the center of world complexity research.

Complexity can be defined in various ways [4] from information, computation [5,6], network, statistics [7,8], thermodynamic depth [9], entropy [10,11] to effective complexity [12,13] and philosophy [1], etc. Rescher proposed that the complexity of systems has some problems [1]: First, it is related to the number of system components and diversity. Second, the ontological complexity has three main aspects: the compositional complexity, the structural complexity, the functional complexity, and list many models of complexity. They are more complex complexity. Only for the mechanism of complexity, Rescher discussed four theories: intelligent theory, inherent teleology theory, chance-plus-self-perpetuation theory, and automatic self-potential theory [1]. Mainzer research thinking in complexity, and try to construct an interdisciplinary general methodology by the computational dynamics of matter, mind and mankind [14].

We discussed generally the four variables and the eight aspects in social physics, and searched social thermodynamics and the five fundamental laws of social complex systems, and proposed the nonlinear whole sociology and its four basic laws [15]. Further, we researched the evolutionary equation of system, the educational equation, the nonlinear theory of economic growth and its three laws: Economic

takeoff-growth-stagnancy law, social conservation and economic decay law, and economic growth mode transition and new developed period law. The social open-reform is a necessary and sufficient condition for further economic development [15]. We proposed the multiply connected topological economics, and four theorems on knowledge economic theory, etc [16]. In this paper we propose four basic scientific characters of complexity and research corresponding mathematics and applications.

BASIC SCIENTIFIC CHARACTERS OF COMPLEXITY

Any systems are determined by their elements and interactions each other. Therefore, first the complexity has two basic characters:

- I. Much elements. They may be the same or be different (diversity). Single element is simple system theory.
- II. Various interactions. This includes internal and external interactions. Nonlinearity is an important mathematical method, which can also produce chaos, fractal and so on. Synergetics describes various phase transitions of nonlinear complex systems by the order parameters and the slaving principle, etc [17-19].
- III. Only I exist, and interaction may be neglected, it corresponds to statistics. If interactions are simple, it may be crystals, rigid body, etc. Different interactions may form solids, liquids, Brown movement and so on.
- IV. I and II are two necessary bases of complexity. Two correspond to element and relation in mathematics. Further, we can derive two important emerged results:
- V. Hierarchies of complexity. A combination between I and II can produce hierarchies and different structures. Such they will be more complex, and to complicate (i.e. complexification) [20], and may show emergence, hypercycle and so on.
- VI. Simon defined complexity by the degree of hierarchy [21], whose important character is emergence [22]. McShea discussed the hierarchical structure of organisms as a scale and documentation of a trend in the maximum [23]. In *The Emergence of Complexity*, Fromm proposed some outstanding problems, self-organization, evolution, cooperation, differentiation and so on [24]. Rescher's first problem is basis on element and interactions, and corresponds to two basic characters. Second problem is some results [1].
- VII. Complexity with time. If the system is dynamic, it will be from being to becoming [25]. This not only produces difficulty of prediction, but also more complex. Bonner searched the evolution of complexity [26].

Flood, et al., discussed complexity [27,28], in which people with power, and system are all elements in I; nonlinear, non-symmetry, non-holistic constraints, etc., are all interactions in II; and values, beliefs, and interests are the result of comparison.

This mode agrees with the Occam's Razor. Four aspects can produce all kinds of contingency, these are the results. The self-organization is not a general result of complexity, and it is opposite with chaos.

MATHEMATICS OF COMPLEXITY

From I to IV we can define different complexities. Elements in I may have k types. Interactions in II may have j types. Hierarchies in III may have i layers. Therefore, we try to define a mathematical function of complexity $C_{oml}(e_k, g_j, x_i, t)$, in which e_k, g_j, x_i represent different elements, interactions, and hierarchies of space, respectively.

The four quantitative methods contribute to the computation of complexity.

Based on I we may use system theory, and set theory and union $\cup$, intersection $\cap$, subser $A \subseteq B$, difference $B - A$, etc. Discrete elements can be described by matrix and matrix mechanics. If they are invariant, there is a corresponding conservation law. General form of the generalized conservation law may be equation [29]:

$$\frac{dG}{dq} = \frac{\partial G}{\partial q} + [G, W] = 0. \quad (1)$$

It represents the quantity G is conservation for generalized coordinate q . These generalized conservation laws may be represented by four dimensional divergence forms:

$$\frac{\partial S_i}{\partial x_i} = 0. \quad (2)$$

Based on II we may use graph theory, network [30], topology, etc.

Based on III we may use the fractal dimension D [10], which may be extended to complex dimension in both aspects of mathematics and physics [31]:

$$D_z = D + iT. \quad (3)$$

When the complex dimension is combined with relativity, whose dimensions are three real spaces and one imaginary time, it expresses a change of the fractal dimension with time or energy, etc., and exists in the fractal's description of meteorology, seismology, medicine and the structure of particle, etc. [32-35].

Based on IV we may discuss dynamics [36], arrows of time and semi-group [37], etc. Any life system must be related to evolution.

Symmetry principle and its violation are the first basic principles of physics [29]. Symmetries and groups exist probably for elements (I) and interactions (II), for example, in four basic interactions in physics and their possible unification [38]. But, they must be violations of symmetries [39,40] for hierarchies (III) and time, evolution (IV). Symmetry is uniform and beautiful, but symmetry breaking can produce complexity and evolution.

We may use various corresponding partial differentials:

$$\frac{\partial C_{oml}}{\partial e_k}, \frac{\partial C_{oml}}{\partial g_j}, \frac{\partial C_{oml}}{\partial x_i}, \frac{\partial C_{oml}}{\partial t}, \quad (4)$$

and $\frac{\partial C_{oml}}{\partial e_k} \geq 0$ is monotone increasing function, $\frac{\partial C_{oml}}{\partial e_k} \leq 0$ is monotone decreasing function.

Further, we introduce

$$\frac{\partial^2 C_{oml}}{\partial e_k \partial e_{k'}}, \frac{\partial^2 C_{oml}}{\partial e_k \partial g_j}, \text{ etc.} \quad (5)$$

From this the points $C_{oml}(e_{k0}, g_j, x_i, t)$ of $\frac{\partial C_{oml}}{\partial e_k} = 0$ are the stationary points, and it is maximum when

$$\frac{\partial^2 C_{oml}(e_k, g_j, x_i, t)}{\partial e_k^2} \Big|_{e_k=e_{k0}} < 0. \quad (6)$$

It is minimum when

$$\frac{\partial^2 C_{oml}(e_k, g_j, x_i, t)}{\partial e_k^2} \Big|_{e_k=e_{k0}} > 0. \quad (7)$$

Moreover, we may introduce different nonlinear functions:

$$C_{oml} = a_0 + a_1 e_k + a_2 e_k^2 + a'_2 e_k e_{k'} + a_3 e_k^3 + \dots, \quad (8)$$

$$C_{oml} = b_0 + b_1 g_j + b_2 g_j^2 + b'_2 g_j g_{j'} + b_3 g_j^3 + \dots, \text{ etc.} \quad (9)$$

From this we may determine Lagrange theorem, Taylor theorem and Taylor series, Cauchy theorem, etc.

If element e_k is invariant, the corresponding conservation law will be:

$$\frac{\partial e_k}{\partial t} + \nabla(e_k \vec{v}) = \frac{\partial}{\partial q_\mu}(e_k v_\mu) = 0. \quad (10)$$

Here q_μ are four time-spaces. If complexity C_{oml} of a system is stable, it will be:

$$\frac{\partial C_{oml}}{\partial t} = 0. \quad (11)$$

Integrals are:

$$\int C_{oml} de_k, \int C_{oml} dg_j, \int C_{oml} dx_i, \int C_{oml} dt. \quad (12)$$

$$\text{And } \iiint \dots \int f(x_i) dx_1 dx_2 \dots dx_n = F(x_i, C_n), \text{ etc.} \quad (13)$$

Assume that C_{oml} is scalar field, there will have gradient:

$$\text{grad} C_{oml} = \nabla C_{oml} = \sum_{k=1}^n \frac{\partial C_{oml}}{\partial e_k} \vec{e}_k, \text{ etc.} \quad (14)$$

It is a vector field.

If C_{oml} is vector field, there will have divergence:

$$\text{div} \vec{C}_{oml} = \nabla \cdot C_{oml} = \sum_{k=1}^n \frac{\partial \vec{C}_{oml}}{\partial e_k}, \text{ etc.} \quad (15)$$

and curl. Further, C_{oml} may be tensor, spinor, torsion, twistor and so on.

COMPLEXITY OF PHYSICS, AND ENTROPY

In physics from particle mechanics, rigid body mechanics, ideal fluid mechanics to friction, elasticity and viscosity mechanics, it is becoming more and more complex. The complex turbulence originates from interactions.

Thermodynamics develops from the entropy increasing principle of closed system to the theory of dissipative structure of open system. We researched possible entropy decrease in isolated system with various complex internal interactions [41-57], and proposed a universal formula for any isolated system [42]:

$$dS = dS^a + dS^i. \quad (16)$$

It is symmetry with the formula:

$$dS = d_i S + d_e S, \quad (17)$$

In the theory of dissipative structure. From this we derived a complete symmetrical structure on change of entropy [45,56]:

$$Entropy \rightarrow \begin{cases} \text{increase.} \\ \text{decrease} \rightarrow \begin{cases} dS = d_i S + d_e S. \\ dS = dS^a + dS^i. \end{cases} \end{cases} \quad (18)$$

General correlation, entanglement and coherence are all internal interactions in systems, which will increase complexity.

In 1990 Zuzek edited book *Complexity, Entropy and the Physics of Information* [9], in which complexity and entropy are related each other. For IV change is related to entropy. Any complex system can be as an information processing system [14], the increase of complexity should be the increase information. This is the entropy change system, in which increasing entropy always is too simple for complexity. Complexity is the unity of unification and diversity, the unity of order and disorder, that is, the unity of entropy increase and entropy decrease.

Entropy of degree distribution is defined by formula:

$$H = -\sum_k P(k) \lg P(k). \quad (19)$$

The maximum entropy corresponds to the uniform random networks, and the minimum entropy corresponds to the regular networks. It shows that degree distribution of small-world network method is analogy to degree distribution of random figure [58]. Further, it is known that there is structural entropy, and we should be extended to hierarchical entropy.

Anderson proposed: more is different (MID) [59,60]. He indicated that the theory of symmetry breaking in multi-body physics shows the failure of constructivism in reduction theory. Different levels of multi-body can have symmetry breaking. This similar gas can be condensed into different liquids, solids (crystals, supercrystals). MID means that the whole is not only not, and but may be very different from the sum of the parts that make up the whole.

From simple to complex include: Atoms from hydrogen to period eight, more complex general atoms, molecules, and organisms, etc.

Complex nano-systems [61] include $10^3 - 10^9$ atoms-molecules, which may produce self-replication, self-assembled, self-organization, etc. From this simple molecules may form dendritic macromolecules [62], and great balls of carbon as buckminsterfullerene [63]. We believe self-organization, etc., must be entropy decrease [46,56]. Kauffman searched laws of self-organization and complexity [64].

Electromagnetic interaction in microscopic physics is simple Abel group $U(1)$, then weak and strong interactions are non-Abel gauge fields $SU(2)$ and $SU(3)$, etc. Corresponding particles are also more and more complex.

COMPLEXITY, AND NONLINEARITY, STOCHASTIC RESONANCE

The specific methods of complexity science include: general system theory, cybernetics, artificial intelligence, and theory of dissipative structure, synergetics [17-19], hypercycle [65], catastrophe theory, chaos, fractals and the cellular automata theory. Mainzer researched complex systems and various evolutions from matter, life, mind-brain to computation, economy, artificial intelligence, human society-culture [14].

An important mathematical characteristic of complexity is nonlinearity. Mainzer was special attention nonlinearity in complexity [14]. We have explored the nonlinear whole biology [66], the nonlinear whole medicine [67], the nonlinear whole ecology, the nonlinear whole sociology [15].

The stochastic resonance proposed first by Benzi, et al. [68,69], which not only is observed in the experiments, and should derive the stochastic complexity. Fauve and Helslot found stochastic resonance in Schmitt bistable system [70]. McNamara, et al., in a bistable ring laser found stochastic resonance [71]. Kaufman and McClintock in a superconducting quantum interference of Josephson junction [72], Qian, et al., in the pumped biochemical reactions [73], Longtin in neuron models [74] found stochastic resonance. It may be caused by the combined action of periodic signal and noise, or the coupled effect without periodic driving [75].

COMPLEXITY IN BIOLOGY AND NEURAL NETWORKS

Complexity is common in biology [76,14]. There are different plants and animals in nature, and have different relations each other [77]. The ant and bee populations are very much and complex behaviors emerge to form an orderly society [10]. A combination of four characters must obtain complexity in biological systems, in which a typical example is many hierarchies from molecules, synapses, neurons, networks to nervous system, central nervous system (CNS) [14].

This complexity of biology includes the metabolic scaling theory with $3/4$ power law [78]. It is related to fractal dimension [79]. Ravasz, et al., discussed the statistical description of complex network [80,81]. In phase transition, a phenomenological rule of order parameter is:

$$\xi \propto |t - t_c|^\beta. \quad (20)$$

It is a nonlinear formula, and β is fractal dimension. The scale-free network has stronger robustness, and obeys also the power law [82]:

$$p(k) \sim k^{-n}. \quad (21)$$

This is the degree distribution formula with a fractal dimension $D = -n$. The scale-free and clustering in bionetworks coexist, and emerge the new hierarchical networks, whose formations [80] are some classical fractals, for example, Eq.(21). Primary model of neural transference may apply hydrodynamics and its three-dimensional shape. Assume $A \propto C^{D/(D+1)}$ [83]. For $D=3$, so $A \propto C^{3/4}$.

A typical example of complexity is brain and neurobiology. I are a huge number 10^{11} of neurons, II are complex relations in brain, III are brain regions, IV corresponds to evolution, and even the formation of new synapses. The brain model is a complex cellular system with nonlinear dynamics [14]. Churchland discussed the matrix equation of neurons [84]:

$$(x, y, z) = (a, b, c, d) \begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \\ w_{41} & w_{42} & w_{43} \end{pmatrix}. \quad (22)$$

The brain may form the neural networks [14]. Hopfield [85,86] proposed a type of neural networks model, whose basic nonlinear equations are:

$$C_i \frac{du_i(t)}{dt} = -\frac{u_i(t)}{R_i} + \sum_{j=1}^N T_{ij} g_j(u_j(t)) + I_i. \quad (23)$$

Christopher discussed bishop, neural networks for pattern recognition [87]. Wolfram found that cellular automata have four different results: stable state, period states, chaos and complex state [88]. Further, there are also quasiperiod and random states. We discussed the quantum neutral network, which may be related to biological loop quantum theory [66].

Network includes random network and scale-free network [89]. General bionetwork is not random networks, and form often network clusters, which are described by the clustering coefficient, and form “small-world” networks with the shortest path length [90]. Bionetworks may construct network motifs with feed-forward, bi-fan, bi-parallel and three chains forms [91,92], and modularity [93], etc.

It is well-known that the neurobiological system is a very complex nonlinear system. Much neuron constructs the neural networks (NN), which are origin of neural system. Their models may be continuous or discrete. The pan-brain network combines the pan-brain level and the pan-brain relation. It includes complex macrocircuit and microcircuit. These may be applied to neuron, neurobiology, and constitute their theoretical models, and research their functions and mechanism.

Based on quantum biology and biological gauge field theory, we proposed the biological lattice gauge theory as modeling of quantum neural networks [94]. This

method applies completely the same lattice theory in quantum field, but, whose two anomaly problems may just describe the double helical structure of DNA and violated chiral symmetry in biology. Further, we discussed the NN model and the quantum neutral networks, which are related to biological loop quantum theory. Moreover, we researched some possible developments on described methods of networks by the extensive graph theory and their new mathematical forms.

There is the perception of neural networks. They may be self-adapt and self-organization, which corresponds to learning. Information may be storage and memory, may be control and learning. A three-layer connected model of NN is:

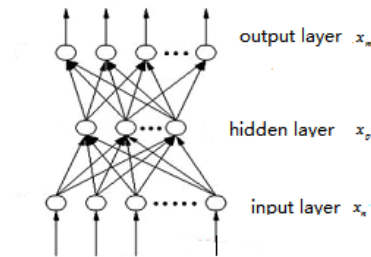


Fig. 1. A three-layer connected model of NN

In this model input layer x_n has n neurons, hidden layer x_p has p neurons and output layer x_m has m neurons. While x and y are not direct connection.

Barabasi and Albert [95] proposed the average field equations of evolutionary model on scale-free network:

$$\frac{\partial k_i}{\partial t} = mk_i / \sum_{j=1}^{N-1} k_j. \quad (24)$$

When time t is enough long, a solution is $k_i = m(t/t_i)^\beta$. When network increases with complete random, the average field equations are:

$$\frac{\partial k_i}{\partial t} = \frac{m}{m_0 + t - 1}. \quad (25)$$

Their solution is $p(k) \propto e^{-k/m}$, which is the same with Eq.(21).

Jeong, et al., [96] and Tanaka [97] researched metabolic networks, for example, amino acid biosynthesis, carbohydrate metabolism, lipid metabolism, nucleotide metabolism, etc. Interacting networks of protein are also scale-free [98,99]. They and the channels in traditional Chinese medicine possess fractal structures. Holme-Kim model of scale-free network has tunable clustering [100,101]. Networks can show dynamics with scale-free structures and nodes of networks [102-104], and form the coevolution network model [105,106], the adaptive network model [107], the self-organized network model [104], etc.

For structures of functional networks [108] the memory function is defined by the edge weight function $f(e)$ in networks flow. When $f(e) \geq f(e_c)$, threshold value is

namely long memory. It is related to initial conditions. Clauset, et al., researched hierarchical structure and the prediction of missing links in networks [109]. Newman and Clauset discussed structure and inference in annotated networks [110]. Network dynamics researches the networks whose status changes in time [111-113]. It may be based on the principle of least action, and obtain network structures from selection principles [114-117].

In a word, the quantum neutral networks and general bionetworks may combine various theories and mathematical methods. It is possibly a direction of biology and neurobiology, and may be perfected and developed continually [94].

GENERAL SCIENCES OF COMPLEXITY AND APPLICATIONS

Scientific developments are from simple to complex. Geometry is from one-dimensional straight line and two-dimensional plane to n-dimensional and Hilbert space, and curved space.

Hydrogen nucleus (proton) and hydrogen atom are simple, but with the increasing number of nucleons and electrons, various nuclei and atoms begin to some layers and become complex. The periodic table of elements is also relatively simple at the beginning, and gradually complex later [118]. Atoms, molecules to macromolecules are also increasingly complex. Lehn discussed the self-organization in supermolecular chemistry [119].

The diversity of elements leads to chemistry-biology. From inorganic chemistry to organic chemistry, from single cell to multicellular biology are all constantly increasing complexity. A typical example of the hierarchies (III) is ecological systems.

In astronomy there is complexity with self-organization that the celestial bodies from simple nebulae to planets, various stars, and galaxies, etc. The Galaxy includes 2×10^{12} stars, among which various interactions exist widely.

Geoscience forms different geological structures and different strata. It is known that earthquakes are very complex nonlinear phenomena. Based on the nonlinear equations of fluid dynamics and so on, we derive a simplified nonlinear solution of momentum, which correspond to the accumulation of the energy. Their values are excess a faulting threshold, earthquake will take place. From this a chaos equation is obtained, in which chaos corresponds to the earthquake, which shows complexity on seismology, and impossibility of exact prediction of earthquakes. But, combining the Carlson-Langer model and the Gutenberg-Richter relation, we may obtain approximately the magnitude-period formula of the earthquakes proposed in 1989 [120,121]:

$$T = T_0 10^{-b(M_0 - M)}. \quad (26)$$

From this we may predict quantitatively earthquakes. For California that is a multi-earthquake area, but bigger ones are still not more, according to the formula (26) we predicted future earthquakes will take place in 2009, 2014 and 2019 from 2004. So far earthquakes (M=6.9) occurred on 3 August of 2009, and M=6 on 24 August of

2014. Further, we take the period $T=33$, a large earthquake of 2019 will be more possible [122-124]. In 2019 California occurred two bigger earthquakes on 4 July $M=6.4$ and 5 July $M=7.1$. This shows that earthquakes may be predicted initially, and be calculated simply. Further, earthquake predictions are combined different methods: various modern scientific instruments, different scientific theories, and some paranormal ways, which include special functions and animals [125] and plants, etc., they will be able to increase the accuracy of prediction.

The general social science is more complex. Human society has different social levels, different religious and cultures. The complexity of a highly industrialized society in the social sciences lies mainly in its numerous citizens and their relationships [126,14]. Moreover, complex systems include also psychology, economics, sociology, etc.

Usual economic theories are some idealized models from Mill's homo oeconomicus on the definition of political economy to the econophysics [127,128]. Economies based on human are more complex. In the microeconomic theory of consumer behavior either a utility function or a binary relation can describe the preferences of an individual. The strict equivalence of these two primitive concepts, ordinal utility functions and preference relations, was first axiomatized by Debreu [129,130].

Using the similar formulas of the preference relation and the utility function, we proposed the confidence relations and the corresponding influence functions that represent various interacting strengths of different families, cliques and systems of organization. Since they can affect products, profit, prices, and so on in an economic system, and are usually independent of economic results, therefore, the system can produce a multiply connected topological economics. If the political economy is an economy chaperoned polity, it will produce consequentially a binary economy. When the changes of the product and the influence are independent one another, they may be a node or saddle point. When the influence function large enough achieves a certain threshold value, it will form a wormhole with loss of capital. Various powers produce usually the economic wormhole and various corruptions [16,131].

The self-reinforcing mechanism of positive feedback is a typical feature of nonlinear complex systems in economy and society [132,14]. Bechtel and Richardson proposed discovering complexity [133].

McIntyre studied complexity and social scientific laws [134]. Weidlich discussed sociodynamics as a systematic approach to mathematical modeling in the social sciences [135]. Complexity is applied to management and organizational sociology [136]. Sterman searched business dynamics as systems thinking and modeling for a complex world [137]. Godfrey-Smith searched complexity and the function of mind in nature [138].

Prediction is always infatuated, although it difficult. For general futurology as predictions of various future natural and social phenomena, we researched its six

methods: natural scientific methods, social scientific methods, computer simulation methods, cultural methods (including Chinese traditional Yi), religious methods, and mystical methods (including parapsychology). Then mathematical and physical methods are discussed. Further, we searched Back Map and its some results [139]. Social dynamics may pass through nonlinear mechanism to derive social chaos, which obtains social storm. From the maximum (6) and minimum (7) of complexity we can predict the critical value of the phase transformation point. The dramatic changes in Eastern European Countries, the Arab Spring and some are waves on freedom-democracy due to the butterfly effect caused by accidental factors. For the future and crisis usual analysis is divided into pessimism or optimism. But, we believe in the real strength and the progress of the society for our world.

DISCUSSION

Complexity is a complex science. Its investigations, developments and applications must be simplification and quantification.

Nature and human society are all complex. The simplifications of elements (I) may include: the Yin-Yang (Eight Diagrams) and the Five-Elements of Chinese traditional culture, and Four Elements of India. In ancient Greece everything originated from water of Thales of Miletus, or fire of Heraclitus of Ephesus, or atoms of Democritus. Dualism of matter and mind corresponds to Yang and Yin. Further, it may be mass-force and atom-molecule of classical physics. Quantum mechanics is that all particles are divided into bosons and fermions, which correspond to Yin and Yang, respectively. In ancient Greece Empedocles assumed that world only has two relations: Attraction (love) and repulsion (hate). It is a simplification of interactions (II), and is consistent with promotion and restraint in the Five-Elements of Chinese traditional culture.

The simplest is the binary Tai-Ji Figure, which can be developed to the many layers Tai-Ji Figure [140]. Each element in Five Elements can also have Yin and Yang. Five elements can also be developed into multiple layers. This can be related to hypercycles [65].

A way of simplification is study of single element or few elements, and of single level or few levels. For example, in astronomy a single mass increase also creates different neutron stars and black holes, etc. If there are only two bodies, the simplest relations will be to attract or repel each other, which are described completely in Newtonian mechanics. The three bodies may be chaotic. But, chaos is only a nonlinear descriptive result. Such we may let $e_k \geq 3$ in element (I).

In economics Cobb-Douglas production function is [141]:

$$Y = AK^\alpha L^\beta, \quad (27)$$

in which the two essential factors: labor L is elements, and capital K is interaction. For the epoch of knowledge economy, knowledge is first in various bases, talented person is first in various resources, innovation is first in various developments, and cooperation is first in various managements. Its precondition is a right decision-making. Based on these characters we proposed the four theorems on the

knowledge economic theory, and expounded the production function and basic equations. Some possible directions of the development on the knowledge economy and a sustainable development theory of new economics are discussed [16,142].

Recently, we proposed three dimensional body-mind-spirit worlds on human society and parapsychology. Some observed results imply “ghosts” are probably the existences of some biological or non-biological objects. It will be the discovery of a new class of substances. Any observations and detections on mind and spirit worlds and on relations and entanglements between both and matter are all valuable [143,144]. This will derive a more complex world.

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