

Machine Learning Solutions of Stress Concentration Factors in Perforated Plate With Single Circular Hole

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ABSTRACT

The discontinuities such as holes, grooves, notches and fillets in the structural geometry in structures would lead to significant increase of stress level, which can be represented by stress concentration factor (SCF), and further influence the strength of structures. Therefore, SCF plays a vital role in quantitatively understanding the influence of discontinuities on the peak stress. However, analytical or empirical solutions for predicting localized high-stress around the discontinuities are usually difficult to be derived, owing to the complex interaction of the specific boundary conditions and the discontinuities. In this work, machine learning (ML) solutions of SCF of circular holes in a finite perforated plate under tension are modeled using finite element analysis and back propagation neural network (BPNN) technique. The locations and sizes of circular holes are input as input variables, and the SCFs are target variable. The feasibility and accuracy of the model are demonstrated through numerical examples. It's found that the developed ML approach base on BPNN technique can provide accurate prediction of SCF value for each set of structural parameters, and vice versa.

Keywords: Machine learning, neural network, stress concentration factor, circular hole.

INTRODUCTION

Considering the fact that almost all engineering structures involve interruptions in section and/or shape, i.e. cracks on aircraft wings, grooves or shoulders on shafts, oil holes, screw threads, shape reentrant corners of auxetics, etc. Typically, these interruptions arise from discontinuities of geometry and may cause a local increase in stress in the engineering structure, where a high stress gradient exists [1-3]. Such stress rise is known as stress concentration normally. Obviously, the stress concentration may result in the part failure in practice, and further affects the material or structure designing. Therefore, stress concentration analysis plays an important role, especially in designing the structures under alternating loads.

For engineers, a simple and reliable solution can bring great convenience to engineering applications. However, so far, most of the existing estimators for stress concentration factor (SCF) near a circular hole in a finite plate are some empirical formulas, which are provided by curve fitting technology based on the experimental measurement (usually using photoelasticity) involving many contributors through several years [4]. Moreover, these empirical solutions are usually not reliable for the extreme cases that the circular hole diameter is large compared to the dimensions of finite plate. As an alternative to the empirical solutions, the finite element solutions are popularly resorted, due to the advantages of easy implementation of programing and strong applicability to complex problems [5, 6]. However, the

efficiency of numerical solutions is doubtable and you have to conduct simulation for each structural configuration and each loading condition. Even today, it is difficult to determine the exact value of SCF for most extreme cases. Therefore, a practical requirement seeking reliable solutions is naturally raised.

Machine learning (ML) solutions are a promising way to achieve this purpose [7-11]. It has been reported that the ML solutions behave not only ease of implementation but also low computational cost for some complex engineering problems, i.e. fracture mechanics problems of the pentagonal cross-section microcantilever at nanoscale [9], stress hotspots in polycrystalline materials under uniaxial tensile deformation [12], stress concentration depiction at the low points of rough surfaces by interpretable convolutional neural network [13]. More recently, Ozkan and Erdemir developed nonlinear ML models based on neural network (also known as artificial neural network, ANN) for predicting SCF for circular/elliptical holes in an infinite plate [11]. Interestingly, in these successful applications, the ML process thoroughly avoids the complex mathematical formulas related to the controlling parameters and the desired mechanical response.

Taking inspiration from these successful applications of ML, in this context, the stress concentration in a finite plate perforated with a circular hole is investigated by using BPNN based ML technology. The size and position of the circular hole can be arbitrary. Thus, from the view of data analysis, the input in the BPNN framework is the geometrical information (e.g., radius of hole, location coordinates) and the output is the results of stress concentration factor. To drive the BPNN algorithm, a dataset including the geometric parameters of the problem and the corresponding stress response is synthetically generated from finite element simulation. After that, we used genetic algorithm to continuously optimize and find the maximum stress from all the predicted data.

PROBLEM DESCRIPTION

The peak stress in a finite plate perforated by a circular hole located at an arbitrary position is investigated in this study, as shown in Fig. 1. The plate is subject to a uniform tension with a specific tension force σ_0 along the x direction. The solid material is isotropic and elastic.

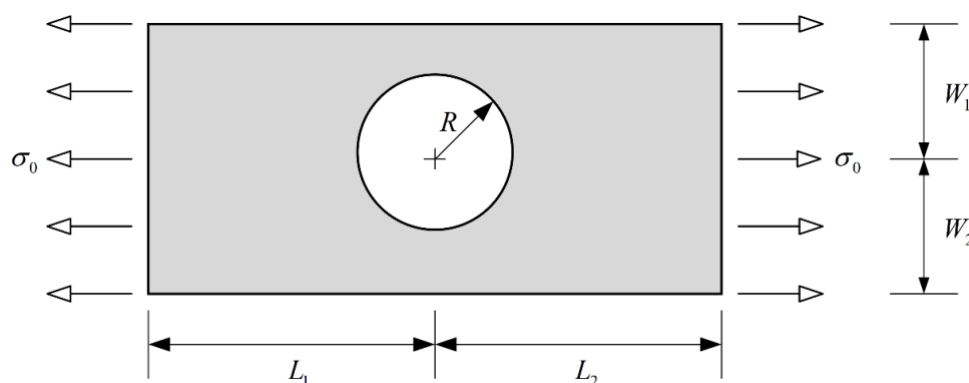


Fig 1. Schematic diagram of finite plate with circular perforation subject to unidirectional tension

To characterize the peak stress better, the SCF concept is normally introduced to perform both design and analysis of the loaded members with geometric discontinuities. Here the SCF for the problem under consideration is defined as the ratio of the peak stress to the normal stress at the circular hole,

$$K_s = \frac{\sigma_{max}}{\sigma_{nom} \frac{\sigma_{max}}{\sigma_0}} \quad (1)$$

where K_s denotes the SCF, σ_{max} is the unknown peak stress, and σ_{nom} is the reference nominal stress, which usually corresponds to the case without hole.

Obviously, the value of SCF for isotropic material is independent of material properties and is strongly relevant to the geometries of the structure, including the size of circular hole, the location of circular hole in the plate. Thus, K_s is a function of L_1, L_2, W_1, W_2 and R

$$K_s = K_s(L_1, L_2, W_1, W_2, R) \quad (2)$$

Traditional theoretical methods to compute the SCF of the structure involving multiple parameters are usually very complex. In order to solve this problem and further improve the predictive efficiency, the nonlinear machine learning model with BPNN algorithm is established to predict the SCF.

BACK-PROPAGATION NEURAL NETWORK

Generally, a typical BPNN model composes of three layers [14, 15]. The first layer is input layer containing inputs (also known as predictors), and the last layer is output layer containing target outputs. Between them are one or more hidden layers which are interconnected between neuron nodes which simulate the biological nervous system of human brain to process information, as shown in Fig. 2. The input layer takes in input information and then transfer them throughout the middle neuron network layer in which nonlinear activation functions are employed to capture the nonlinear relationship in the data set. Theoretically, the more the hidden layers and hidden neurons, the more capabilities a network can model a complex and nonlinear system.

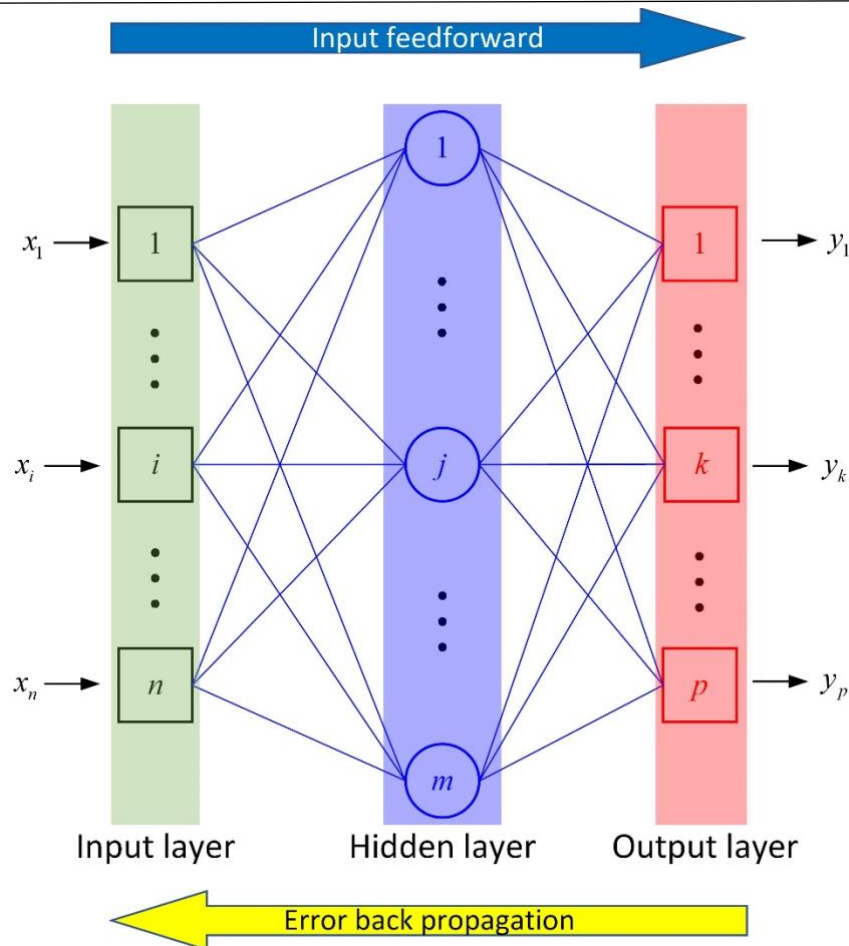


Fig 2. Diagram of typical structure of three-layer back-Propagation neural network

To understand the details of how the ANN works, a neuron is considered, as given in Fig. 3. When the input signal passes through the network, the neuron will give an initial prediction by multiplying it by the respective weights

$$g_j = \left(\sum_{i=1}^n w_{ij}^{(1)} x_i \right) + b_j^{(1)} \quad (3)$$

where $w_{ij}^{(1)}$ is the weight connecting the input i and the neuron j , and $b_j^{(1)}$ is the related bias.

Further, the summation in Eq. **Error! Reference source not found.** is modified by a nonlinear activation (transfer) function to generate an output of each neuron,

$$h_j = f(g_j) = f \left[\left(\sum_{i=1}^n w_{ij}^{(1)} x_i \right) + b_j^{(1)} \right] \quad (4)$$

where f is the activation function which is continuous and differentiable. Subsequently, the output of neuron can be mapped into an output by

$$y_k = \phi(h_j) = \phi \left(w_{jk}^{(2)} h_j + b_k^{(2)} \right) \quad (5)$$

where $w_{jk}^{(2)}$ is the weight connecting the neuron j and the output variable k , and $b_k^{(2)}$ is the related bias. Usually, the transfer function ϕ is linear.

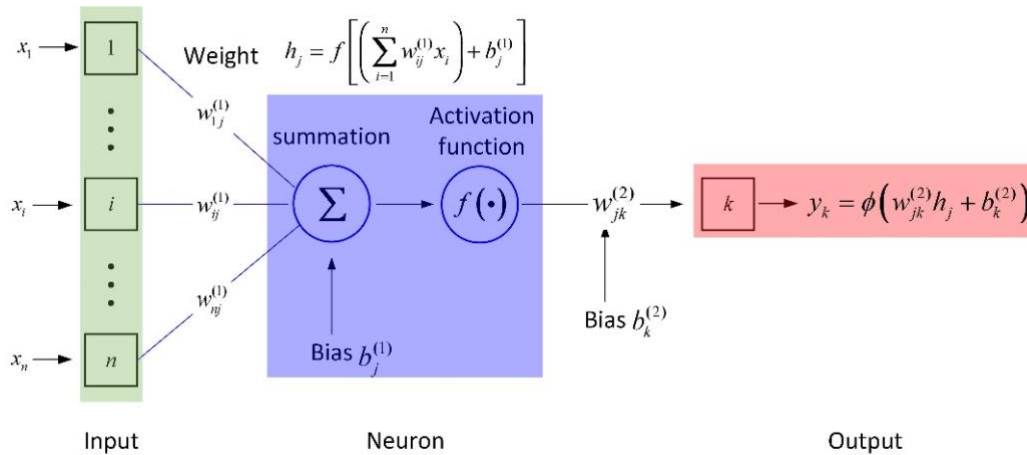


Fig. 3 Details of input feedforward process for an artificial neuron

During the ANN training process, the actual output y_k generated by Eq. **Error! Reference source not found.** may be different to the target output $\tilde{y}_k$, then a back propagation of error is needed to increase predicting accuracy and to make the model reliable. Generally, if the error between the actual output and the target output is

$$\delta_k = y_k - \tilde{y}_k \quad (6)$$

an error (loss) function, i.e. mean square error (MSE),

$$\text{MSE} = \frac{1}{p} \sum_{k=1}^p \delta_k^2 = \frac{1}{p} \sum_{k=1}^p (y_k - \tilde{y}_k)^2 \quad (7)$$

can be defined to iteratively tune the weights and biases of a neural net to minimize the MSE. The basic process of back propagation can be found in Fig. 4.

To implement the BPNN modeling, the Neural Network Toolbox of MATLAB is used, in which the popular back propagation learning algorithm Bayesian regularization (TRAINBR) is employed to perform the back propagation of error, and the inbuilt transfer functions TANSIG (hyperbolic tangent sigmoid function) in the second layer and PURELIN (the linear function) in the third layer are used for the feedforward process.

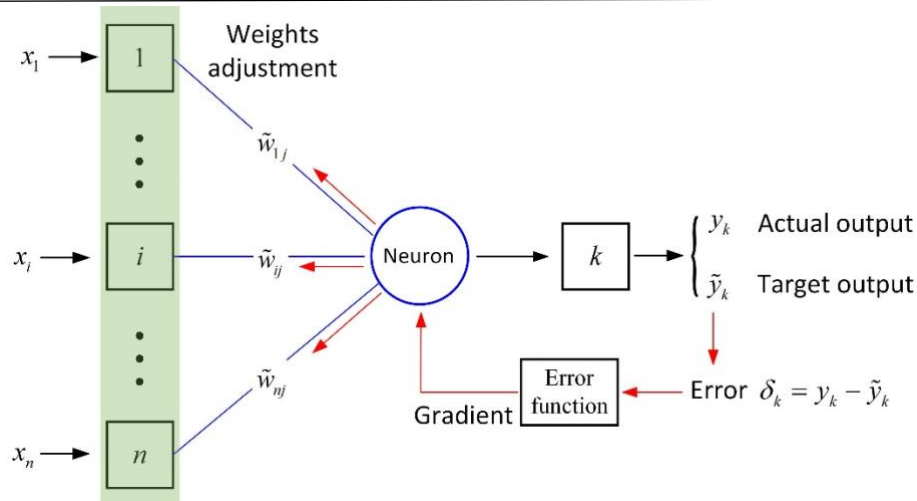


Fig4. Illustration of back propagation process for an artificial neuron

RESULTS AND DISCUSSION

Machine learning methods often require a data set, which contains a large amount of required input and output data [12]. These data can be used to train and test ML models to find accurate ML solutions. Therefore, it is very important to establish an appropriate data set for each problem. In this work, the SFC of the structure is independent of the elastic properties of the solid material and is only related to the five structural dimensions, so the ML model can be depicted by,

$$\begin{cases} \text{Input variables: } x = (x_1, x_2, x_3, x_4, x_5) = (L_1, L_2, W_1, W_2, R) \\ \text{Target variable: } y = K_s \end{cases} \quad (8)$$

To form the required data set of the ML model, the structural dimensions are generated randomly in the space of input variables, and each element of the data set corresponds to one structural configuration, which is solved by finite element analysis to provide the mapping relation of the target variable and the input variables. The mesh density for each FEM simulation is set to be extra fine to achieve mesh-independent solutions. To obtain a satisfying ML model, the elements in the data set are randomly divided into three portions: 70% samples for training, 15% samples for validation, and 15% for test. Additionally, to assess the prediction accuracy of the ML model, the following error R^2 is defined as,

$$R^2 = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (9)$$

where y is the value to be fitted, $\bar{y}$ is its average value, and the fitted value is $\hat{y}$. Obviously, the closer it is to 1, the higher the accuracy of the prediction results.

Next, it is assumed that the parameters L_1 , L_2 , W_1 , W_2 are changed between 15-30mm, and the radius of R varies between 1-10mm. The applied tensile stress is $\sigma_0 = 1000\text{MPa}$. A total of 1792 samples are prepared, in which 1770 samples are used to form the data set and 22 samples are used to test the accuracy of the present BPNN model.

Fig. 5 gives the peak stress values predicted by the BPNN model for the prepared 22 samples, and the R^2 error between the ML solutions and the finite element results is 0.98716. Thus, the present BPNN model is accurate enough to predict the maximum stress along the circular hole boundary induced by the stress concentration.

Besides, benefiting from the function of error back propagation, the BPNN model can also be used to perform the inverse prediction with the cooperation of genetic algorithm, that is, if the desired peak stress value is specific, the BPNN model can give the approximate predictions of structural parameters. To illustrate this capability, the structural parameters $(L_1, L_2, W_1, W_2, R) = (15, 15, 30, 15, 10)$ mm is used to produce an eccentrically placed circular hole in a rectangular plate and the corresponding SCF is evaluated to be 7.7508 by finite element simulation. During the inverse prediction of the BPNN model, the desired value of SCF 7.7508 is used as the target variable. Through the error back propagation iteration, the approximated structural parameters are found as,

$$(L_1, L_2, W_1, W_2, R) = (15, 15.0029, 29.2512, 15.0003, 9.9867) \text{ mm}$$

which induce the SCF of 7.6596. From this procedure it is observed that the predicted SCF is very close to the accurate value 7.7508 and the relative error between them is only 1.18%. Therefore, the present BPNN model is capable to simulate the stress concentration phenomenon in the perforated plate, and give the accurate prediction of SCF for each set of structural parameters, and vice versa.

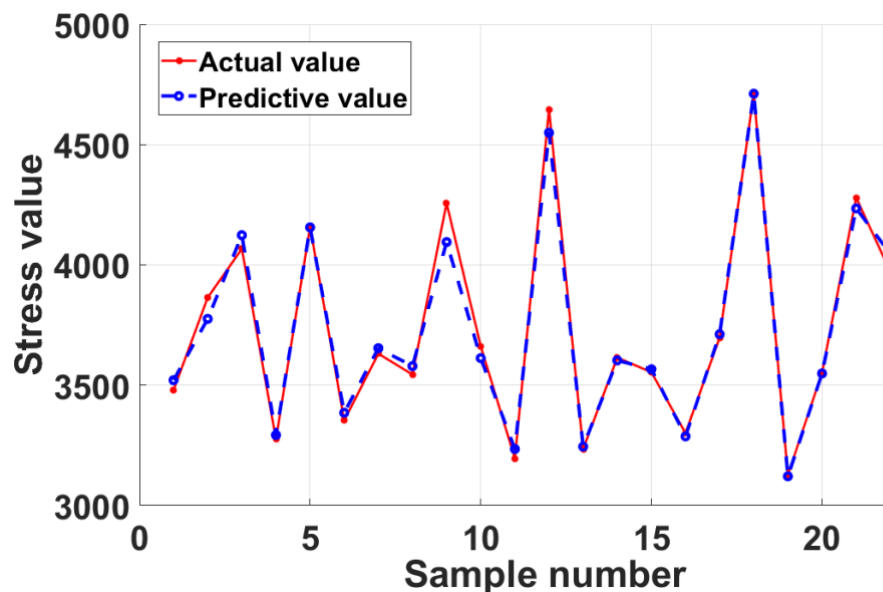


Fig5. BPNN predictions for the selected 22 samples with accuracy $R^2=0.98716$

CONCLUSIONS

This paper concerns the prediction of stress concentration factor for perforated solid plate under tension using the machine learning method implemented by the BPNN algorithm. The position and size of circular hole are represented by the five structural parameters. Based on this knowledge, the BPNN model is developed including five input variables and one target variable. The finite element simulation

is employed to prepare the data set for training the present BPNN model and also provides the reference values to assess the accuracy of the present BPNN model. Using the BPNN model, an easy-to-use and accurate solution of SCF is presented for each set of structural parameters and also the powerful capability of inverse prediction of the structural parameters is demonstrated if the desired value of SCF is specific.

Declaration of competing interest

The authors declare that there is no conflict of interest.

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