

Analysis of Correct Use of Time Transformation in Physics

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ABSTRACT

The paper deals with the analysis of mathematically correct use of time derivative in coordinate systems with time transformation. It combines the use of time derivative in different areas of physics with the results of this analysis. The results are used to verify the most well-known theories using time transformation. A general limit of the use of time transformation in physics is formulated and proved.

The analysis is supplemented by specific examples which illustrate the consequences of using a nonlinear (curved) time transformation. They show the formation of a resistor in the LC circuit as a result of time transformation, energy generation or loss, tilting of high rock walls, change in distance between the Earth's centre and the Earth's surface, twisting of Earth's axis, astronomical paradoxes, unknown acceleration properties. The internal consistency of both theories of relativity is also analysed, including the equivalence principle.

The result of the analysis shows that the use of a nonlinear (curved) time transformation is a dangerous tool of contemporary physics that decomposes its internal consistency. This also applies to the time transformations used in both theories of relativity.

Keywords: Time transformation; Time derivative; Coordinate system; Coordinate transformation; Special and General theory of relativity; Space-time.

1 Introduction

Physics often works with a time derivative. The use of time derivative is not limited to one area of physics. It is a common mathematical apparatus of experimental and theoretical physics.

Theoretical physics works with hypotheses or theories based on the transformation of time between different coordinate systems.

The text deals with the general analysis of the impact of the time transformation to time derivative of general physical quantities.

The results of the general analysis are used to analyse selected time transformations, namely the time transformations included in Special theory of relativity (STR) [1] and General theory of relativity (GTR) [2].

A general limit of the use of time transformation in physics is formulated and proved.

2 General Analysis of Time Transformation

Consider two coordinate systems with generally different time. The relationship between time in one system and another system is described by a time transformation.

Time t is used in the reference system, time τ is used in the second system. Time transformation is described by general dependence $\tau = \tau(t)$ or $t = t(\tau)$ where $\tau(t)$ and $t(\tau)$ are the time transformation functions.

We have a general physical time dependence f of a physical quantity (e.g. spatial coordinate, voltage, electrical current, energy, momentum, temperature) such that in both coordinate systems the time dependence is identical for the corresponding pair of times t and τ .

Writing: $\hat{f}(\tau) = f(t) = \hat{f}(\tau(t)) = f(t(\tau))$. This means that the measured values of the physical quantity are the same at the same place and moment in both coordinate systems. Emphasize that this is of such physical quantities which can be measured objectively.

2.1 Types of Time Transformations

Let's divide time transformations into two types:

- Linear time transformation
- Non-linear (curved) time transformation

2.1.1 Linear Time Transformation

Let us define a linear time transformation with a condition $\frac{d^2 t(\tau)}{d\tau^2} = 0 = \frac{d^2 \tau(t)}{dt^2}$ valid throughout the scope of the time transformation validity. This means that the second derivative of the time-to-time transformation is zero at all points in space and for all times.

From this condition we can calculate (by double integration) a general description of linear time transformation

$$\tau = \tau(t) = A \cdot t + B \text{ or } t = t(\tau) = \hat{A} \cdot \tau + \hat{B} \quad (1)$$

where $A, B, \hat{A}$ and $\hat{B}$ are constants. Physical meaning of linear time transformation is a change of units of time and/or the time offset. Such a transformation of time is independent of the position in the space and hence the time changes equally in all movements in space.

An important feature of the linear time transformation is that it is stable over time. It is the same at any moment.

2.1.2 Non-linear Time Transformation

Let us define a non-linear (curved) time transformation as a complement to a linear time transformation, thus a condition $\frac{d^2 t}{d\tau^2} \neq 0$ or $\frac{d\tau^2 t}{dt^2} \neq 0$ is valid in at least one point in the domain of validity of the time transformation. This means any other time transformation that cannot be described as a linear time transformation in at least one point of validity scope (at least one time and/or in any area of space). Physically it is a case of time-varying time transformation, signifying the curvature of the time transformation. It is irrelevant for what reason the time transformation is changed, whether it is directly included in the analytical notation of the time transformation, or whether it is the result of another physical phenomenon, e.g. the displacement of a body between places with different time transformation.

Generally, objects can move in space between arbitrary places at any time. If the time transformation were different in different places of space, then there would be a time change of the time transformation if the body was moved between places with different time transformation. Therefore, invariance of time transformation in space is a necessary condition for invariance of time

transformation in time. Thus, a sufficient condition for the non-linearity of the time transformation is the dependence of the time transformation on the position in space.

2.2 Transformation of Time Derivative

Now we analyse the behaviour of the first and second time derivative of the general function f in coordinate systems with a general time transformation. In the second coordinate system, the function $\hat{f}$ is a composite function and therefore we calculate the derivative according to the derivative rules of the composite function. When using Leibniz notation,

$$\text{we get: } \frac{d\hat{f}(\tau)}{d\tau} = \frac{df(t)}{dt} \cdot \frac{dt(\tau)}{d\tau} \quad (2)$$

$$\frac{d^2\hat{f}(\tau)}{d\tau^2} = \frac{d^2f(t)}{dt^2} \cdot \left(\frac{dt(\tau)}{d\tau}\right)^2 + \frac{df(t)}{dt} \cdot \frac{d^2t(\tau)}{d\tau^2} \quad (3)$$

Let us pay attention to the second derivative (3). The second derivative contains the sum of two components, where the first component is the second time derivative of the analysed physical function f multiplied by the square of the first time derivative of the transformation function.

The second component is the product of the first time derivative of the analysed function f and the second time derivative of the transformation function. This means that this second term will exist or disappear depending on the type of time transformation.

For a linear time transformation, i.e. the time transformation described by (1), this term disappears and has no physical meaning. For non-linear time transformation, i.e. for all other time transformations, the term is non-zero. Let us analyse the physical impacts of this option in the following text.

2.3 Example LC Circuit

For further analysis we will use an example of LC circuit. The LC circuit is described in the reference coordinate system using the generalized coordinate $Q(t)$ (capacitor charge) and the generalized velocity $I(t) = dQ(t)/dt$ (capacitor current).

$$\ddot{Q} + \frac{1}{LC}Q = 0 \quad \text{thus} \quad \frac{d^2Q(t)}{dt^2} + \frac{1}{LC}Q(t) = 0 \quad (4)$$

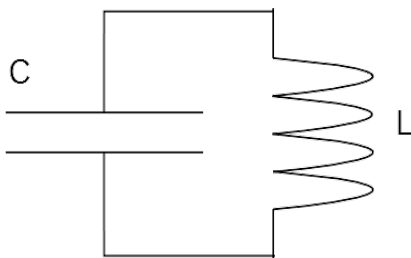


Figure 1: LC circuit in reference coordinate system

In the second coordinate system using time transformation $\tau = \tau(t)$, the LC circuit is described by a transformed second time derivative according to (3).

We substitute the transformed second derivative of the function $\hat{Q}(\tau)$

$$\begin{aligned} \frac{d^2\hat{Q}(\tau)}{d\tau^2} \cdot \left(\frac{d\tau(t)}{dt}\right)^2 + \frac{d\hat{Q}(\tau)}{d\tau} \cdot \frac{d^2\tau(t)}{dt^2} + \frac{1}{LC}Q(t(\tau)) &= 0 \text{ and adjust to the standard shape} \\ \frac{d^2\hat{Q}(\tau)}{d\tau^2} + \frac{\frac{d^2\tau(t)}{dt^2}}{\left(\frac{d\tau(t)}{dt}\right)^2} \cdot \frac{d\hat{Q}(\tau)}{d\tau} + \frac{1}{LC \cdot \left(\frac{d\tau(t)}{dt}\right)^2} Q(t(\tau)) &= 0 \end{aligned} \quad (5)$$

Before we analyse the equation (5) in the transformed coordinate system, we need to complete the transformation. The capacitance C and the inductance L appear to be constants that the time transformation should not concern. But it is not.

Time derivative is part of the definition of capacitance and inductance. Therefore, it is also necessary to transform C and L . Capacitance and inductance are defined as the ratio between one physical quantity and the time derivative of another physical quantity

$$C = i_c(t) / \frac{du_c(t)}{dt} \quad L = -u_l(t) / \frac{di_l(t)}{dt} \quad (6)$$

From the definition of capacitance $\hat{C} = \hat{i}_c(\tau) / \frac{d\hat{u}_c(\tau)}{d\tau}$ and inductance $\hat{L} = -\hat{u}_l(\tau) / \frac{d\hat{i}_l(\tau)}{d\tau}$ and assuming that measurable currents and voltages do not differ at the respective moments and place in the two coordinate systems (they are measured by the same voltmeter in the same moment), and using the rule of transformation of time derivative (2) follows

$$\begin{aligned} \frac{d\hat{u}_c(\tau)}{d\tau} &= \frac{dt}{d\tau} \cdot \frac{du_c(t)}{dt} & \hat{C} &= \hat{i}_c(\tau) / \frac{d\hat{u}_c(\tau)}{d\tau} = i_c(t) / \left(\frac{dt}{d\tau} \cdot \frac{du_c(t)}{dt} \right) = \frac{1}{\frac{dt}{d\tau}} \cdot C \\ \frac{d\hat{i}_l(\tau)}{d\tau} &= \frac{dt}{d\tau} \cdot \frac{di_l(t)}{dt} & \hat{L} &= -\hat{u}_l(\tau) / \frac{d\hat{i}_l(\tau)}{d\tau} = -u_l(t) / \left(\frac{dt}{d\tau} \cdot \frac{di_l(t)}{dt} \right) = \frac{1}{\frac{dt}{d\tau}} \cdot L \end{aligned}$$

$$\text{Consequently } C = \frac{dt}{d\tau} \cdot \hat{C} \quad L = \frac{dt}{d\tau} \cdot \hat{L} \quad (7)$$

Thus, the equation describing the LC circuit (5) can be modified by substituting to

$$\begin{aligned} \frac{d^2 Q(\tau)}{d\tau^2} + \frac{\frac{d^2 \tau}{dt^2}}{\left(\frac{d\tau}{dt}\right)^2} \cdot \frac{dQ(\tau)}{d\tau} + \frac{1}{\hat{L}\hat{C} \cdot \left(\frac{d\tau}{dt}\right)^2} Q(t(\tau)) &= 0 \text{ consequently} \\ \frac{d^2 Q(\tau)}{d\tau^2} + \frac{\frac{d^2 \tau}{dt^2}}{\left(\frac{d\tau}{dt}\right)^2} \cdot \frac{dQ(\tau)}{d\tau} + \frac{1}{\hat{L}\hat{C}} Q(t(\tau)) &= 0 \end{aligned} \quad (8)$$

The resulting equation in the second coordinate system differs from the original equation. The additional member is dependent on the rate of charge change, i.e. it is current dependent.

Comparing this equation to the equation describing the RLC circuit

$$\frac{d^2 Q(t)}{d\tau^2} + \frac{R}{L} \cdot \frac{dQ(t)}{d\tau} + \frac{1}{LC} Q(t) = 0 \quad (9)$$

we find that the new term corresponds to the ratio of resistance and inductance.

$$\hat{R}(\tau) = \frac{\frac{d^2 \tau}{dt^2}}{\left(\frac{d\tau}{dt}\right)^2} \cdot \hat{L} \quad (10)$$

Thus, the time transformation can create a new physical element. This is a surprising conclusion.

In this case, it is generally the time-varying resistance $\hat{R}$ (10). It has a zero value only if the second derivative of the time transformation is zero, i.e. in terms of the linear time transformation.

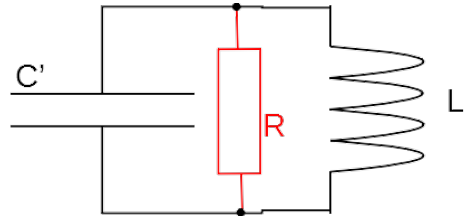


Figure 2: LC circuit in the second coordinate system with non-linear time transformation

Let us also analyse the solution of the original equation (4) and the transformed equation (8). In the reference system, the solution of the equation (4) is a harmonic oscillation $a \sin(\omega t)$ with angular velocity $\omega = \sqrt{\frac{1}{LC}}$.

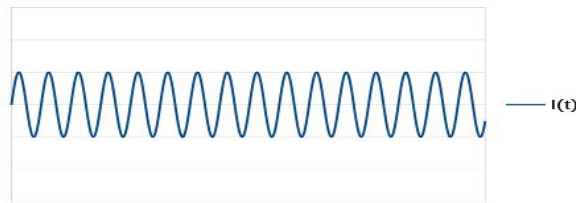


Figure 3: solution of LC circuit in reference coordinate system

In a transformed system, the approximate solution of equation (8) is increasing or damped harmonic oscillation $a \sin(\hat{\omega}\tau) e^{b\tau}$ where $\hat{\omega} = \sqrt{\frac{1}{\hat{L}\hat{C}}}$.

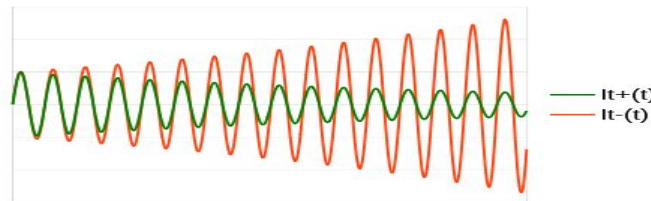


Figure 4: solution of LC circuit in second coordinate system with non-linear time transformation

Physically, the angular velocity is the same, it is only expressed by a different number in other time units.

Notable is the effect of attenuation or amplitude growth

$$b = -\frac{\frac{d^2\tau}{dt^2}}{2\left(\frac{d\tau}{dt}\right)^2} \quad (11)$$

It has significant physical consequences. If the second derivative of the time transformation is positive, the amplitude decreases and thus the energy is lost and vice versa if the second derivative of the time transformation is negative, the amplitude increases and energy is generated.

2.3.1 Conclusion from the Example

Analysis of the LC circuit shows that in the case of using non-linear time transformation may occur physically surprising effects. These effects should be measurable. The measurement can be used to

verify the time transformation hypothesis. The physical impact of linear time transformation is significantly different from the physical impact of non-linear (curved) time transformation.

3 STR Time Transformation

Let us analyse the Special relativity theory (SRT) time transformation [1]. Is the time transformation linear or non-linear?

The transformation according to SRT (time dilation) is described $\tau = \gamma(t - vx(t)/c^2)$ where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ and where v is the velocity of the second coordinate system relative to the reference coordinate system.

Derivative of time transformation is $\frac{d\tau}{dt} = \gamma(1 - v \frac{dx(t)}{dt} / c^2) = \gamma(1 - \frac{v}{c^2} \cdot \frac{dx(t)}{dt}) = \gamma - \frac{\gamma v}{c^2} \cdot \frac{dx(t)}{dt}$ (12)

Note that the first derivative of the SRT time transformation is not constant. It is not just time dilation. The derivative of time transformation contains a component dependent on speed of movement in the reference coordinate system.

Let us calculate the second derivative $\frac{d^2\tau}{dt^2} = -\frac{\gamma v}{c^2} \cdot \frac{d^2x(t)}{dt^2}$ (13)

The second derivative of the STR time transformation is directly proportional to the acceleration of motion in the reference coordinate system. Thus, the STR time transformation depends on the movement of the body in the reference coordinate system.

It follows that the STR time transformation is non-linear.

Another question is the relationship between acceleration and STR. This question is answered in a specialized chapter 5.5 STR and Acceleration.

3.1 LC Circuit and SRT

Apply the STR time transformation for example LC circuit. Calculate the amplitude attenuation of the accelerating LC circuit by substituting (12) and (13) to (11)

$$b = -\frac{\frac{d^2\tau}{dt^2}}{2(\frac{d\tau}{dt})^2} = -\frac{\frac{\gamma v}{c^2} \frac{d^2x(t)}{dt^2}}{2(\gamma - \frac{\gamma v}{c^2} \frac{dx(t)}{dt})^2} = \frac{\frac{v}{c^2} \frac{d^2x(t)}{dt^2}}{2(1 - \frac{v}{c^2} \frac{dx(t)}{dt})^2} \quad (14)$$

The LC circuit oscillation attenuation is an easily measurable quantity.

It follows from equation (14) that the attenuation is proportional to the product of the velocity of the second coordinate system and the acceleration of the LC circuit. Acceleration is the same in both coordinate systems.

Remarkably, using attenuation of the LC circuit should be able to measure the velocity of the coordinate system - see chapter 5 Discussion.

4 GTR Time Transformation

Let us analyse the General relativity theory (GTR) time transformation [2] using free fall analysis. The free fall is described by the Newton equation in the reference coordinate system

$$F_i = m_i \cdot \frac{d^2r(t)}{dt^2} = m_g \cdot g = F_g \quad (15)$$

where F_i is the inertia force, F_g is the gravitational force, m_i is the inertia mass, m_g is the gravitational mass, g is the gravitational acceleration and r is the distance between the bodies.

If the gravitational mass and the inertial mass are the same $m_i = m_g$, the gravitational and inertial accelerations are the same $\frac{d^2 r(t)}{dt^2} = g$. Thus, by measuring inertial acceleration (free fall acceleration), gravitational acceleration is also measured.

After transformation (15) into the second coordinate system using (3) we get

$$\hat{F}_i = \hat{m}_i \cdot \frac{d^2 r(\tau)}{d\tau^2} = \hat{m}_i \cdot \frac{d^2 \hat{r}(t)}{dt^2} \cdot \left(\frac{dt}{d\tau}\right)^2 + \hat{m}_i \cdot \frac{d\hat{r}(t)}{dt} \cdot \frac{d^2 t}{d\tau^2} = \hat{m}_g \cdot \hat{g} = \hat{F}_g \quad (16)$$

In the second coordinate system it is different. If the free fall velocity is non-zero $\frac{d\hat{r}(t)}{dt} \neq 0$ and the time transformation is non-linear (i.e. $\frac{d^2 t}{d\tau^2} \neq 0$ in at least one point of the free fall path), then it cannot be valid at the same time $\hat{m}_i = \hat{m}_g$ and $\frac{d^2 \hat{r}(t)}{dt^2} = \hat{g}$.

Either one or the other can apply.

$$\text{If } \hat{m}_i = \hat{m}_g \text{ we get from (16) } \frac{d^2 \hat{r}(t)}{dt^2} \cdot \left(\frac{dt}{d\tau}\right)^2 + \frac{d\hat{r}(t)}{dt} \cdot \frac{d^2 t}{d\tau^2} = \hat{g} \text{ then } \frac{d^2 \hat{r}(t)}{dt^2} = \frac{\hat{g} - \frac{d\hat{r}(t)}{dt} \cdot \frac{d^2 t}{d\tau^2}}{\left(\frac{dt}{d\tau}\right)^2} \quad (17)$$

$$\text{If } \frac{d^2 \hat{r}(t)}{dt^2} = \hat{g} \text{ we get from (16) } \hat{m}_i \cdot \hat{g} \cdot \left(\frac{dt}{d\tau}\right)^2 + \hat{m}_i \cdot \frac{d\hat{r}(t)}{dt} \cdot \frac{d^2 t}{d\tau^2} = \hat{m}_g \cdot \hat{g} \text{ then}$$

$$\hat{m}_g = \hat{m}_i \cdot \left(\left(\frac{dt}{d\tau}\right)^2 + \frac{1}{\hat{g}} \cdot \frac{d\hat{r}(t)}{dt} \cdot \frac{d^2 t}{d\tau^2}\right) \quad (18)$$

However, the simultaneous validity of both conditions is the basic assumption on which the GTR is based (equivalence principle) [2, 3].

Is GTR time transformation linear or non-linear? Is it guaranteed $\frac{d^2 t}{d\tau^2} = 0$ to be valid at all points of space and time in free fall? Or not?

The basic difference of GTR from Newtonian gravity is the mismatch of clock speed in places with different gravity [2]. There are experiments that are interpreted as evidence of the dependence of local clock speed on gravitational acceleration [4].

In a free fall of a body in a gravitational field where gravitational acceleration depends on the distance from the material body. The body gets over time from point A , where gravitational acceleration is g_A to point B , where gravitational acceleration is g_B , and this is different from g_A .

If the time transformation is dependent on gravitational acceleration, it follows that it is a non-linear (curved) transformation of time.

According to GTR, the speed of the clock placed on the falling object will change over time. Therefore, the GTR time transformation is non-linear.

It follows from the previous analysis (16, 17, 18) that for non-linear time transformation the assumptions of the equivalence principle as formulated in GTR cannot be fulfilled.

This means that the GTR time transformation excludes the validity of the basic GTR principle (unless gravity is constant in space and time).

5 Discussion

5.1 Relativistic Compass

Place the LC circuit on a centrifuge arm. The acceleration will be constant in absolute terms, but the x-axis projection (as indicated by STR) will be a sine wave with one maximum and one minimum per revolution.

According to (14), the attenuation of the electrical current of the LC circuit is proportional to the product of the velocity of the second coordinate system and the projection of the acceleration of the LC circuit in the direction of relative motion of the coordinate systems. At one revolution of the centrifuge, the acceleration of the LC circuit will have the same direction as speed v at one point, and will be reversed at the point on the opposite side. Therefore, the current attenuation of the LC circuit will be non-zero and opposite at both points. An experiment can have two different results:

- o Confirm the validity of the relation (14), thus increasing the amplitude of oscillations with the maximum in the direction of movement of the coordinate systems and decreasing with the maximum in the opposite direction. In doing so, the growth rate will be proportional to the square of the centrifuge speed (acceleration).
- o The relationship will not be valid, the measured effect will be different or zero.

In the first case, the centrifuge with the LC circuit will act as a compass and speedometer that will allow you to measure the direction and speed of movement between coordinate systems. This confirms the validity of the SRT time transformation while refuting the validity of the STR postulate (relativity, equivalence of physical processes in all coordinate systems) and hence the validity of STR.

In the latter case, the validity of the time transformation will be experimentally refuted. This means refuting STR.

If the experiment does not have enough sensitivity to distinguish between these two variants, it will not be known whether STR is invalid due to invalid postulate or due to invalid time transformation. Even in this case, there is no way to validate STR.

5.2 Equivalence Principle

The analysis shows (16, 17, 18) that the dependence of the time transformation on the gravitational acceleration is in contradiction with the equivalence principle if the gravitational acceleration is not constant in the whole space and in time.

Thus, the gravitational acceleration described by Newton's law (15) directly excludes either the validity of the equivalence between the inertia and the gravitational mass or any non-linear time transformation.

If the time transformation changes during the free fall (due to the displacement of the body between places with different gravity), the coincidence of inertial acceleration with gravity acceleration and the coincidence of inertial and gravitational mass cannot be valid simultaneously.

This match can only be true if the time transformation does not change with any movement of the body, i.e. either gravity is constant or the time transformation does not depend on gravity.

According to Newton's law of gravity, gravitational acceleration decreases with distance.

Thus, **Newton's law of gravity excludes either the basic postulate of GTR** and thus the principle of GTR **or the result of GTR**, i.e. the relativistic dependence of time on gravity, i.e. the relativistic curvature of space-time.

5.3 Measurement of Gravitational Dependence of Local Time

There are several published results interpreted as experimental verification of the dependence of local time on gravity e.g. [4]. These results are interpreted as experimental proof of GTR.

According to the analysis described above, the following options are possible:

- o The local time has been measured correctly, the speed of the local time depends on the position in space. Thus, linear time transformation cannot be used to describe physical phenomena. It is necessary to use non-linear time transformation. Thus, the equivalence principle cannot be valid. This means that the basic principle of GTR is excluded.
- o Measurement results are affected by a measurement error that is interpreted as a measurement result. This can be a technical feature of a particular measuring device. Analogous to e.g. the dependence of oscillation frequency on temperature. Watches are delayed or preceded in real life. In practice it is common clock is behind or precede right time. This tells nothing about the time at the measurement site. Clock does not control time. A clock is measurement apparatus.

This option means that the GTR time transformation is not experimentally confirmed.

5.4 Astronomical and Geological Observations

There are empirical observations that are very difficult to interpret to confirm the GTR time transformation. Let us analyse following examples.

5.4.1 The Distance between the Pole and the Centre of the Earth

The centre of the Earth is a zero-gravity site, at the Earth's poles is the standard gravitational acceleration of the Earth. In both places there are no centrifugal accelerations caused by the Earth's rotation.

According to the GTR time transformation, the time at the centre of the Earth must be 0.0219 s per year faster than at the pole [5]. This means a 21.9 s time difference over a thousand years.

The Earth orbits the Sun at 29.78 km/s, the Solar System including the Earth orbits the centre of the Milky Way at 210 km/s. The measurement of the red and blue shifts of the CMB (Cosmic Microwave Background) shows the speed of the Earth's movement relative to the CMB background of 552 km/s.

The distance travelled by a body s over a given time t is generally described by the time integral $s = \int v(t)dt$. For constant speed it is $s = v \cdot t$. At the pole according to GTR time is running differently than in the centre of the Earth. We analyse the time dependence of the distance between the pole and the centre of the Earth for all three velocities described above.

We have two options:

- A. The distance s between the pole and the centre of the Earth is constant
- B. The speed of motion v of the centre of the Earth and the pole is the same

Both conditions cannot apply simultaneously because the time in the centre of the Earth is different from the time on the surface and $s = v \cdot t$.

Option A: the velocity of the pole movement in the orbit of the Sun, in the orbit of the Milky Way or towards the CMB is different than the velocity of the Earth's centre. According to (2), the relationship

between velocities is dependent on the derivation of the time transformation. In this case, the time transformation is dependent on gravity (gravitational acceleration).

The relationship between speed v and acceleration a is $v = \int a(t)dt$.

If a body moves between the pole and the centre of the Earth, it will have to change the velocity, i.e. it will be subjected to additional acceleration due to the time transformation, i.e. the change in the velocity of time (3). Such acceleration would have to have interesting physical properties:

- o It must be in the direction of the respective speed (the individual speeds do not have the same direction and their directions change with time).
- o It must be dependent on the time of movement of the body, i.e. the speed of movement in the appropriate direction, so that the time integral of the acceleration is always equal to the difference in velocity of the pole and the centre of the Earth.
- o This is an inertial acceleration that depends on gravity in a direction other than the direction of the gravitational field gradient.
- o Since the additional acceleration is non-zero from (16, 17, 18) it follows that the principle of equivalence does not apply.

Experimental verification of such acceleration can only result in a single value. Thus, for at least two speeds (from three ones described above), the measured value must be mismatched. So, we must rule out this option.

Note that GTR does not describe such a dependence of speed nor acceleration.

Option B assumes that due to the different time in the centre of the Earth and at the pole, the relative position and therefore the distance between the pole and the centre of the Earth is changing ($s = v \cdot t$). Quantification is 652 m/year, 4.6 km/year and 12.1 km/year.

If we calculate the distance from Earth's origin (about 3.5 billion years ago), the total time difference between the pole and the Earth's centre is 2.43 years (the Earth's centre is 2.43 years "older" than the Earth's pole). This means a distance difference of 15.3 AU, 108 AU and 283 AU.

Note, the distance from Earth to the Sun is 1 AU, large half-axis of Neptune orbit is 30 AU.

No such phenomenon was observed.

5.4.2 High Rock Walls

There are undoubtedly high rock walls on Earth, their dimensions and geological parameters are known. As an example:

- o North face of Eiger (Switzerland), height 1800m, at least 35 million years old
- o El Capitan (California), 900m high, 103 million years old

According to the GTR time transformation, the time at the top of the wall runs faster than at the base of the wall, where gravitational acceleration is greater.

The same applies to the top of the wall and the base of the wall as described in the previous analysis, except that they are angular distances φ and angular velocities ω due to the Earth's rotation where $\varphi = \int \omega(t)dt$. So, the options are:

- A. the angular velocity ω of rotation of the top wall and the base of the wall is different, exactly so that there is no change in the angle φ between the line connecting the top wall and the earth's axis relative to line connecting the base of the wall and the earth's axis.
- B. the angular velocity ω of the top of the wall and the base of the wall is identical.

Thus, in **option A**, there is a necessary horizontal additional acceleration which, moreover, needs to be dependent on the latitude (due to the geometry of rotation and gravity). Also, in this case it is an additional horizontal inertial acceleration dependent on gravity. Also, in this example of the (16, 17, 18) indicates an invalid equivalence principle. Even such acceleration is not posted in GTR.

According to **option B**, the distance between the top of the wall and the base changes over time $\Delta\varphi = \omega \Delta t$. For the wall of Eiger, the top of the wall had to be 70 km further west than the base of the wall 35 million years ago. For the wall El Capitan comes 117.3 km at the time of creation the wall. Both are geological nonsense.

5.4.3 Twisting the Earth's Axis

With the Earth's age of about 3.5 billion years, the total time difference in the centre of the Earth and the pole is 77 million seconds, i.e. approximately 887 days.

In the case of **option A**, the angular velocity of the Earth's rotation ω depends on the depth, that is, the centre of the Earth rotates at a different angular velocity than the Earth's surface.

For **option B**, i.e. the same angular speed of rotation ω of the entire Earth, it follows that the Earth's axis at the centre of the Earth has rotated 887 more revolutions since the Earth's origin than the Earth's axis at the poles.

5.4.4 Gyroscopes

We have a pair of lossless gyroscopes equipped with flywheel speed sensors (e.g. flywheel mark). Both gyroscopes will have the same flywheel rotation speed ω and the same flywheel orientation at the beginning of the experiment. The reference gyroscope will remain on the surface of the Earth, the other will be dropped into the well toward the centre of the Earth.

The observer will simultaneously observe the rotation of the flywheels of both gyroscopes from the Earth's surface.

According to the GTR time transformation, the time of the second gyroscope runs differently than the reference because it is at a location with different gravity.

Again, we have two options:

- A. the angular velocity ω of each flywheel of the second gyroscope varies, the total angle of rotation of the flywheel φ is the same as the reference flywheel on the Earth's surface,
- B. the angular velocity ω of the flywheel is constant and the same as the flywheel on the Earth's surface.

In **option A**, there must be an additional angular acceleration such that the change in angular velocity corresponds to the change in the velocity of time, i.e. gravity at the appropriate location. This means not only the dependence of the angular acceleration on the speed of movement of the gyroscope in the respective direction (which is generally different from the direction of the axis of rotation). In addition, the angular acceleration must be dependent on the orientation of the flywheel axis and be directly proportional to the angular velocity of the flywheel.

Also, in this case, the GTR does not describe such an acceleration and by (16, 17, 18) the existence of such accelerations is contrary to the principle of equivalence according to the GTR.

In **option B**, the gyro flywheel angular velocity ω is constant. Because the time velocity varies according to the GTR time transformation, and the total rotational angle φ of the flywheel is the product of the angular velocity and relevant time, the total rotational angle of the second gyroscope is different from the total rotational angle of the reference gyroscope.

Both gyroscopes are equipped with the same rotation sensors, so the Earth observer can compare information from both sensors. That is, the observer measures a different number of revolutions of the reference gyroscope than the second gyroscope at a given duration of the experiment.

This means that the flywheel speed of both gyroscopes varies over the same time period. But on the surface of the Earth before the experiment began, the flywheel speed of both gyroscopes was the same. And the assumption of this option was that the angular velocity had not changed and no additional angular acceleration was effective. How is it possible?

5.4.5 Astronomical Paradoxes

At the top of Mauna Kea (Hawaii) is an observatory. Suppose the second observatory is at the base of Mauna Kea at sea level.

According to **option A**, the angular speed of Earth's rotation ω is different at the top of Mauna Kea than at the base. This means that the astronomer on top will be watching stars moving in the sky at a different angular velocity than the astronomer at the base of Mauna Kea. And their distance to each other will be constant.

An even more interesting image will be seen by an astronomer watching the Earth from space. The angular velocity of planets' movement across the sky is the same (except for the direction) of the Earth's angular velocity. So, the astronomer in the space should see different angular speeds of Earth's rotation, another for the top of Mauna Kea and another for the base of Mauna Kea. And yet the distance between the two observatories does not change. How is it possible?

According to **option B**, the angular velocity ω at the top of Mauna Kea is the same as at the base. Thus, the overall angle of movement of the observed planets is different because time is different. Thus, both astronomers will observe the same angular velocity of planetary orbit, but each will see the planet in a different direction. The angle between the position of the same planet observed by the astronomer at the top of Mauna Kea and the position of the same planet observed by the astronomer at the base of Mauna Kea will gradually increase.

And what will an astronomer observe from space? Will he see Earth in two different directions, will he have to look in a different direction to see the astronomer at the top of Mauna Kea and in another direction to see an astronomer at the base of Mauna Kea? Will the direction difference increase over time? And the distance between the observatories will still be the same. How is it possible?

There is also an astronomical definition of time units based on the geometry of motion of astronomical objects. E.g. galactic year, which is defined as the time of the Sun's orbit around the centre of the Milky Way. It is currently estimated 225 to 250 million terrestrial years.

This means that, according to the GTR, the galactic year is about 1 hour longer on the top of Mauna Kea than on the coast below Mauna Kea.

The observer may also be in other places with significantly different gravity. The Sun's orbit around the centre of the galaxy is about 47.5 days longer for the Moon's surface observer than for the Earth's surface observer. On the other hand, an observer on the surface of the Sun would measure 1 year and 109 days less time. During that time, the Sun travels 59.4 AU. This is comparable to twice the length of the large half-axis of Neptune orbit.

Can the Sun orbit the centre of the galaxy at several different moments, depending on different gravity at the observer's site?

If the definition of time is based on the dynamics of distant objects, such as the galactic year or the period of a stable pulsar, the results of the analyses will be completely different.

The deviations of the atomic clock measurements will have to be evaluated as a measurement error, because the time of a distant object cannot be dependent on the gravitational field at the locations of various observers.

5.5 STR and Acceleration

Often, STR is interpreted as a special case of GTR limited to inertial systems. That is, systems where there is no acceleration. In this case, if the objects move without acceleration, there are no forces and Newton's laws cannot be applied. Is it possible or impossible to consider acceleration within STR?

A. Einstein gives a consistent answer to this question in several places. Let's choose some:

[2] p.120: *"Thus, according to the general theory of relativity, gravitation occupies an exceptional position with regard to the other forces, especially the electromagnetic forces, since the ten functions representing the gravitational field at the same time define the metrical properties of the space measured."*

[2] p.143: *"We now make the assumption, which readily suggests itself, that this covariant system of equations also defines the motion of the point in the gravitational field in the case when there is no system of reference, with respect to which the special theory of relativity holds good in finite region."*

[2] p.157 - § 21 Newton's Theory as a First Approximation: *"As has already been mentioned more than once, the special theory of relativity as a special case of the general theory is characterized by the having constant values (4). From that has already been said, this means complete neglect of the effects of gravitation."*

[1] p.61 - "§10. Dynamic of the Slowly Accelerated Electron"

The answer to the question is clear. The acceleration of bodies is part of STR with one exception, and that is gravitational acceleration. This describes GTR.

Let's compare the acceleration transformation used in STR [1] p.61 - the chapter "§10. Dynamic of the Slowly Accelerated Electron" with a mathematically correct time transformation.

Substitute the time transformation STR (12), (13) into a general relation for the calculation of the second time derivative (3) applied to the distance s .

$$\frac{d^2s(\tau)}{d\tau^2} = \gamma^2 \frac{d^2s'(t)}{dt^2} \cdot \left(1 - 2 \frac{v}{c^2} \cdot \frac{dx(t)}{dt} + \frac{v^2}{c^4} \cdot \frac{dx(t)^2}{dt^2}\right) - \gamma \frac{ds'(t)}{dt} \cdot \frac{v}{c^2} \cdot \frac{d^2x(t)}{dt^2} \quad (19)$$

According to STR, the contraction of length occurs only in the direction of the x-axis and the distance in the direction of the y-axis and the z-axis is the same in both coordinate systems. We get a "Transverse" result.

$$\frac{d^2y(\tau)}{d\tau^2} = \frac{d^2y(t)}{dt^2} \cdot \gamma^2 \left(1 - 2 \frac{v}{c^2} \cdot \frac{dx(t)}{dt} + \frac{v^2}{c^4} \cdot \frac{dx(t)^2}{dt^2} \right) - \frac{d^2x(t)}{dt^2} \cdot \gamma \frac{dy(t)}{dt} \cdot \frac{v}{c^2} \quad (20)$$

and

$$\frac{d^2z(\tau)}{d\tau^2} = \frac{d^2z(t)}{dt^2} \cdot \gamma^2 \left(1 - 2 \frac{v}{c^2} \cdot \frac{dx(t)}{dt} + \frac{v^2}{c^4} \cdot \frac{dx(t)^2}{dt^2} \right) - \gamma \cdot \frac{v}{c^2} \cdot \frac{d^2x(t)}{dt^2} \cdot \frac{dz(t)}{dt} \quad (21)$$

For movement in the direction of the x-axis described according to STR $x' = \gamma(x - vt)$ therefore $\frac{dx'(t)}{dt} = \gamma \left(\frac{dx(t)}{dt} - v \right)$ and $\frac{d^2x'(t)}{dt^2} = \gamma \frac{d^2x(t)}{dt^2}$. After substituting into (19) we get

$$\frac{d^2x(\tau)}{d\tau^2} = \gamma^3 \cdot \frac{d^2x(t)}{dt^2} \cdot \left(1 - 2 \frac{v}{c^2} \cdot \frac{dx(t)}{dt} + \frac{v^2}{c^4} \cdot \frac{dx(t)^2}{dt^2} \right) - \gamma^2 \cdot \frac{d^2x(t)}{dt^2} \cdot \left(\frac{dx(t)}{dt} - v \right) \cdot \frac{v}{c^2} \quad (22)$$

We compare with the content of the STR [1].

The procedure for calculating the second derivative in the second coordinate system is not described in STR. We can only speculate about whether the procedure was or was not mathematically correct. We can compare the results.

STR [1] p. 62 shows (in the original unmarked equation - the original notation is used, which is different from the notation used today):

$$m \frac{d^2\xi}{d\tau^2} = \epsilon X' \quad m \frac{d^2\eta}{d\tau^2} = \epsilon Y' \quad m \frac{d^2\zeta}{d\tau^2} = \epsilon Z'$$

Transcription of equations (A) [1] p. 62:

$$m\beta^3 \frac{d^2x}{dt^2} = \epsilon X = \epsilon X' \quad m\beta^2 \frac{d^2y}{dt^2} = \epsilon \beta \left(Y - \frac{v}{c} N \right) = \epsilon Y' \quad m\beta^2 \frac{d^2z}{dt^2} = \epsilon \beta \left(Z + \frac{v}{c} M \right) = \epsilon Z'$$

From the comparison of these triplets, the relation for Longitudinal mass and Transverse mass is developed at [1] - p. 63:

$$Longitudinalmass = \frac{m}{\left(\sqrt{1 - \frac{v^2}{c^2}} \right)^3} \quad Transversemass = \frac{m}{1 - \frac{v^2}{c^2}}$$

Let's compare this with (20), (21) and (22).

The equations are similar, but not the same.

(22) contains the term γ^3 (corresponds to β^3 according to the STR notation) as well as the first of the triplets in STR from which the relation for Longitudinal mass is derived.

(22) further comprises speed-dependent members and a speed square dependence in the x-axis direction, and also a member including γ^2 .

(20, 21) contain a member γ^2 (corresponds to β^2 according to the STR notation) as well as the second and third of the triplets in STR from which the relation for the Transverse mass is derived.

(20, 21) additionally comprise members depending on the velocity, the square of the velocity and also on the acceleration in the direction of the x-axis, i.e. in the direction perpendicular to the direction of the transformed acceleration.

The comparison also reveals serious physical consequences:

- o Additive members of the mathematically correct acceleration transformation in STR are missing. There is no comment in STR to justify their possible intentional neglect. We will no longer find out why the author did not mention them or comment on their possible intentional neglect.

- o The physical interpretation of the relationship between force and acceleration in both coordinate systems as a change in mass is defective. The accelerations in both systems (while respecting all additive members) are the same, i.e. the same force causes the same acceleration and therefore the mass does not change. It is another mathematical notation of the same acceleration based on a mathematically correct transformation of the time derivative.

Recall that this is the effect of the non-linearity of the STR time transformation, i.e. the dependence of the time change on the position in the x-direction. This is reflected in the mathematical description of acceleration due to the mathematical definition of derivation as the ratio of the change in functional value to the change in independent variable. And in this case, the independent variable is the time that changes due to the movement of the body in the x-axis direction.

Therefore, in the relations (20) and (21), the members are dependent on the movement in the perpendicular direction.

The complete relations (20), (21) and (22) thus describe a physically identical acceleration in both coordinate systems with a STR time transformation. Thus, the consideration of the dependence of matter on the transformation of time is not physically substantiated.

Because the relationships used in the derivation of Longitudinal mass and Transverse mass are incomplete, there is a hypothetical variant that the missing members describe the change in mass. That is, the change in mass is given by the difference between the mathematically correct description of the transformed acceleration and the acceleration given in STR on page 62. However, this is contrary to the equations for Longitudinal mass and Transverse mass given in STR [1] p. 63.

5.5.1 Linear Time Transformation and Acceleration

For comparison, let's analyse the same problem that STR [1] solves in §10 but in coordinate systems with linear time transformation.

Let's analyse the slow acceleration of an electron observed by two observers *A* and *B*. Each of them is equipped with a very accurate atomic clock with a digital output. Observer *A* has the clock calibrated in the time unit $[a]$ and observer *B* in the time unit $[b]$. For simplicity, both clocks have the same beginning, i.e. the moment of start.

The time transformation is given by a linear equation $\tau = \frac{1}{\alpha} t$ where α is the constant of the ratio of the units of time $[a]$ and $[b]$.

The distances are the same in both coordinate systems, so the situation is the same as in the case of the y-axis and the z-axis according to STR.

Substitute into the general equation for the transformation of the second time derivative (3) and we get

$$\frac{d^2s(\tau)}{d\tau^2} = \alpha^2 \frac{d^2s(t)}{dt^2} \quad (23)$$

In our example, observer *A* measures the time it takes for an electron to travel a given distance and calculates the acceleration from it.

As in STR, the observer knows the electric field and the charge of the electron and calculates the applied force. He gets a relationship $m \frac{d^2s}{d\tau^2} = \epsilon X^A$. Observer *B* will do the same $m \frac{d^2s}{dt^2} = \epsilon X^B$.

Since the electron in both cases was identical and therefore the electric field was identical, the question is how it is possible that the acceleration is numerically different and the relation (23) holds?

Is it because the mass of the electron is different for observer *A* than for observer *B*, as interpreted by STR? Or is it different?

Is it possible that the mass of the body depends on how the clock is calibrated? The following specific numerical example should explain it.

Time τ is measured by observer *A* in minutes and time t is measured by observer *B* in seconds. Then it is $\alpha = 60$.

The electron travels a distance of 1m in 1s, so observer *A* measures the value 0.016667 on its clock and observer *B* measures the value 1.000000.

Both observers calculate the acceleration according to the same relation $a = \frac{2s}{time^2}$. Therefore, observer *A* calculates acceleration 7200 and observer *B* calculates acceleration 2.

Since the electric field and the electron charge is the same for both observers, there are two possible options for interpretation of the results:

- o STR
- o Classical physics

The interpretation **according to STR** is described in the already cited [1] chapter §10 in relation to Transverse mass. It is enough to replace the coefficient or by using the coefficient of linear transformation of time. This means that the electron mass is 3600 times smaller for observer *A* than for observer *B*, in our example due to the differently set clock used by the observers.

The interpretation according to **classical physics** is that the mass of an electron is the same, also the acceleration is the same, it is only measured in other time units and therefore a different numerical value is obtained. Acceleration 2 [**m/s²**] is exactly the same as acceleration 7200 [**m/min²**]!

5.6 Internal Consistency of Relativity Theories and Black Hole

According to the citations given in the previous chapter, STR is a special case of GTR. Is that really so? Are the two theories consistent or contradictory?

STR is based on two basic principles - relativity and the absolute speed of light. GTR is based on the principle of equivalence. At the same time, let us assume the validity of mass on the speed of the coordinate system dependence as described STR [1] chapter §10:

$$Longitudinalmass = \frac{m}{(\sqrt{1 - \frac{v^2}{c^2}})^3} \quad Transversemass = \frac{m}{1 - \frac{v^2}{c^2}}$$

According to these relations, the inertial mass increases with increasing velocity v , i.e. with the speed of movement of the coordinate system. According to the principle of relativity, the physical effects are the same in all coordinate systems. Imagine a light source in which there is an electron, which is the subject of chap. §10 STR [1]. Thus, in the coordinate system of the emitted light, the electron of the light source has an infinite inertial mass.

According to the GTR principle of equivalence, an electron also has the same gravitational mass, i.e. an infinite gravitational mass. This means that it will become a super Black Hole.

Recall that the same dependence of the change of gravitational mass as the inertial mass is explicitly described in [3] p. 103 (equation in the original marked 1b). In addition, [3] implies the formation of infinite energy.

With its infinite gravitational force, this Black Hole will also act on the emitted photon, which means that the light does not leave the light source.

According to [3] p. 107 (equation in the original marked 3), the speed of light in such a gravitational field will be negative and infinite.

But according to STR, the speed of light is constant everywhere and always. This is one of the two basic postulates of STR [1] p. 38 ", *namely that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of emitting body.*"

At the same time, this super Black Hole will gravitationally act not only on this selected photon, but on all other electrons in the entire Universe, and therefore the gravitational collapse of the Universe will occur.

But nothing like that was observed.

5.7 Relationship between Time Transformations in Relativity Theories

According to the above citations, STR is a special case of GTR.

STR and GTR use time transformation. Verify what is the relationship between the two time transformations, whether or not it is true that the STR time transformation is a special case of the GTR time transformation.

The time transformation STR depends on the speed of the coordinate systems and on the position in the direction of the x-axis - STR [1] p. 48.

The time transformation of GTR depends only on the gravitational potential. See GTR [2] pp. 159-162 (equations in the original labelled 68a, 69, and 72). This is also in line with the description of time slowdown due to gravity in [3] p. 106.

In addition, there are other physical relationships:

- o Due to the STR time transformation, the inertial mass changes differently in the longitudinal direction and differently in the transverse direction.
- o The dependence of the mass according to STR is on the speed of the coordinate systems, not on the position in the direction of the x-axis or other axis. This is a different dependency than the STR time transformation dependency.
- o According to the principle of equivalence, which is the essence of GTR, the gravitational mass changes in the same way as inertial mass.
- o The gravitational potential is directly proportional to the gravitational mass.

If we include these gravitational physical consequences, the GTR time transformation should directly or indirectly include all of these effects:

- o dependence on the speed of the coordinate system,
- o dependence on the distance from the origin of the coordinate system in the direction of movement of the coordinate system,
- o different dependence of the gravitational potential in different directions depending on the speed and direction of movement of the coordinate system.

The GTR time transformation does not depend on the speed of coordinate systems. It also does not depend on the position in the direction of the x-axis. This alone suffices as proof of the claim that the STR time transformation cannot be a special case of the GTR time transformation.

By simplifying (excluding gravitational effects) from the GTR time transformation, it is not possible to derive:

- o nor the relation for the dependence of gravitational mass on the speed of the coordinate system,
- o neither the time transformation STR, i.e. its dependence on the speed of the coordinate system nor the dependence on the position in the direction of the x-axis.

This contradicts the claim that STR is a special case of GTR.

The analysis shows that GTR and STR are mutually exclusive theories.

It is left to the reader what conclusion he will draw from it, which theory he will consider invalid or whether it will be both theories.

6 Generalization

Let us have two functions $\alpha(\vec{x}, t)$ and $\beta(\vec{x}, t)$ in reference coordinate system $\vec{x}, t$, where $\vec{x}$ is an n-dimensional vector and t is the n+1 coordinate of the space. Consider the second coordinate system $\vec{X}, T$ and the coordinate transformation γ where $\vec{X}, T = \gamma(\vec{x}, t)$.

Suppose that functions α and β are invariant under coordinate transformations γ . So $\alpha(\vec{x}, t) = \hat{\alpha}(\vec{X}, T)$ applies and also $\beta(\vec{x}, t) = \hat{\beta}(\vec{X}, T)$ where $\vec{X}, T = \gamma(\vec{x}, t)$.

At the same time, the function β is a derivative of the function α according to the variable t (it can be understood that it is a time derivative), i.e. $\beta(\vec{x}, t) = \frac{d\alpha(\vec{x}, t)}{dt}$, also $\hat{\beta}(\vec{X}, T) = \frac{d\hat{\alpha}(\vec{X}, T)}{dT}$.

It follows that a coordinate transformation γ cannot be a general time transformation, but only a transformation where $T = t + k$ holds, where k is any constant. It can be understood that the time must be the same for both coordinate systems.

6.1 Proof

Derivative definition is: $f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$ e.g [6-8]. Because $\beta(\vec{x}, t) = \frac{d\alpha(\vec{x}, t)}{dt}$ holds and also

$$\hat{\beta}(\vec{X}, T) = \frac{d\hat{\alpha}(\vec{X}, T)}{dT}, \text{ we substitute in the definition of derivation in the original coordinate system}$$

$$\beta(\vec{x}, t) = \frac{d\alpha(\vec{x}, t)}{dt} = \lim_{t \rightarrow t_0} \frac{\alpha(\vec{x}, t) - \alpha(\vec{x}, t_0)}{t - t_0} \text{ and the same in transformed coordinate system}$$

$$\hat{\beta}(\vec{X}, T) = \frac{d\hat{\alpha}(\vec{X}, T)}{dT} = \lim_{T \rightarrow T_0} \frac{\hat{\alpha}(\vec{X}, T) - \hat{\alpha}(\vec{X}, T_0)}{T - T_0}.$$

Because, as we assume $\beta(\vec{x}, t) = \hat{\beta}(\vec{X}, T)$, we substitute the definition of derivative on both sides

$$\beta(\vec{x}, t) = \frac{d\alpha(\vec{x}, t)}{dt} = \lim_{t \rightarrow t_0} \frac{\alpha(\vec{x}, t) - \alpha(\vec{x}, t_0)}{t - t_0} = \hat{\beta}(\vec{X}, T) = \frac{d\hat{\alpha}(\vec{X}, T)}{dT} = \lim_{T \rightarrow T_0} \frac{\hat{\alpha}(\vec{X}, T) - \hat{\alpha}(\vec{X}, T_0)}{T - T_0} \quad \text{therefore}$$

$$\lim_{t \rightarrow t_0} \frac{\alpha(\vec{x}, t) - \alpha(\vec{x}, t_0)}{t - t_0} = \lim_{T \rightarrow T_0} \frac{\hat{\alpha}(\vec{X}, T) - \hat{\alpha}(\vec{X}, T_0)}{T - T_0}. \text{ Under the assumption } \alpha(\vec{x}, t) = \hat{\alpha}(\vec{X}, T) \text{ is also true. We}$$

substitute on the left side $\lim_{t \rightarrow t_0} \frac{\alpha(\vec{x}, t) - \alpha(\vec{x}, t_0)}{t - t_0} = \lim_{t \rightarrow t_0} \frac{\hat{\alpha}(\vec{X}, T) - \hat{\alpha}(\vec{X}, T_0)}{t - t_0}$. By comparison with the previous

$$\text{relationship we get } \lim_{t \rightarrow t_0} \frac{\hat{\alpha}(\vec{X}, T) - \hat{\alpha}(\vec{X}, T_0)}{t - t_0} = \lim_{T \rightarrow T_0} \frac{\hat{\alpha}(\vec{X}, T) - \hat{\alpha}(\vec{X}, T_0)}{T - T_0}.$$

Because the numerator in the fraction is the same, $\lim_{t \rightarrow t_0} (t - t_0) = \lim_{T \rightarrow T_0} (T - T_0)$ must hold where $\vec{X}, T = \gamma(\vec{x}, t)$ for all $\vec{x}$, all t and all t_0 .

This is fulfilled only if $T = t + k$ where k is any constant.

6.2 Physical consequences

According to the previous chapter from a pair of assumptions follows that time must be the same everywhere and must not change during the transformation between different coordinate systems.

The assumptions are:

1. invariance of at least two time-dependent physical quantities
2. the existence of a link between these quantities consisting in the fact that one quantity is a time derivative of the other quantity

The invariance condition is physically equivalent to the condition of objectivity of the measurement of the relevant quantity, i.e. that the measurement result of the given quantity is independent of the coordinate system in which the analysis is performed (of course with respect to possible conversions between physical units).

There are a number of such pairs of functions in physics, where one physical quantity is a time derivative of another physical quantity. These are distance and velocity, velocity and acceleration, electromagnetic induction (magnetic field and induced voltage - Faraday's law of induction), displacement current (Ampere's law with Maxwell's addition), charge and electric current, etc.

Therefore, there are two theoretical options:

- o There is at least one pair of physical quantities satisfying the pair of assumptions in the real world. Thus, there is at least one pair of objectively measurable physical quantities, one of which is the time derivative of the other quantity. The consequence of the existence of such a pair is the coincidence of time in all physically usable coordinate systems. Thus, the exclusion of any non-linear time transformation for any physical quantity.
- o Negation of the previous condition. There is not even one such pair of physical quantities in the real world. Thus, either physics cannot use the mathematical apparatus of differential calculus for all such pairs, or at least one quantity from each existing pair of physical quantities is not invariant. In such a case, the possibility of using a non-linear time transformation in physics can be considered. In both sub-variant we can get on thin ice outside the boundaries of serious science.

Let us illustrate the physical consequences of using three examples.

The first example - discharging or charging a capacitor. One physical quantity is the charge of the capacitor, the other is the electric current of charging or discharging, i.e. the time derivative of the charge.

It would be strange to assume that the charge of a capacitor, i.e. the number of electrons in a capacitor, is not an objective quantity, that the number of electrons depends on the coordinate system or even on the speed or acceleration of the observer or on gravity. It is also unbelievable that the charge of an electron changes depending on the coordinate system or the motion of the observer. Maxwell's equations or other known laws of physics do not describe anything like this.

The current is measured with an ammeter located at the capacitor. Let's imagine that this is a very high-quality digital ammeter. It is not expected that the ammeter will display different values, different

digits depending on the coordinate system, or that the observer will see different numbers on the ammeter display depending on the speed or acceleration of its movement or gravity. Such effects have not yet been experimentally verified.

Let's formulate two hypotheses:

- e1-A. Electric charge and electric current are invariant in coordinate system transformation,
- e1-B. Either the electric charge and/or the electric current varies depending on the coordinate system

The first hypothesis (e1-A) guarantees the same time in all coordinate systems and excludes the use of non-linear time transformation in physics.

The second hypothesis (e1-B) formulates indirectly necessary (not sufficient) conditions for a physically acceptable non-linear time transformation.

The second example - discharging a capacitor through a resistor. This is a typical task of the basic course of Circuit Theory.

Recall that Circuit Theory works with a special case of Maxwell's equations applied to discrete elements. At the same time, let us recall that this is the use of the original (extended) set of Maxwell's equations, which also includes Ohm's law.

Recall that the capacitance of a capacitor is defined as the ratio of the current and the time derivative of the voltage (6).

The definition of capacity $C = i_c(t) / \frac{du_c(t)}{dt}$ can be rewritten into the form $i_c(t) = C \frac{du_c(t)}{dt}$.

When discharging a charged capacitor through a resistor $i_c(t) = C \frac{du_c(t)}{dt}$ is valid where C is the capacitance of the capacitor, i_c the current through the capacitor and u_c the voltage across the capacitor. And at the same time, it applies $u_r(t) = Ri_r(t)$ (Ohm's law) where R is the resistance of the resistor, i_r the current through the resistor and u_r the voltage across the resistor. Because $i_c(t) = -i_r(t)$ and $u_c(t) = u_r(t)$, we can substitute $i_c(t) = C \frac{du_c(t)}{dt} = -i_r(t) = -\frac{1}{R}u_r(t) = -\frac{1}{R}u_c(t)$ and adjust $C \frac{du_c(t)}{dt} = -\frac{1}{R}u_c(t)$ to $u_c(t) = -RC \frac{du_c(t)}{dt}$.

This means that if the product RC is constant, all the conditions defined above are met and therefore the consequence holds.

It is known and experimentally verified that the capacity of a capacitor is given by the physical parameters of the mass from which the capacitor is made. Specifically, for example, the capacity of a plate capacitor depends on the area of the electrodes, the distance of the electrodes and the material of the dielectric.

The same applies to electrical resistance. This is determined by the mechanical dimensions, the shape of the resistor and the material of the resistor.

So, we have two possible hypotheses:

- e2-A. matter has physical properties influencing capacitance and resistance independent of the coordinate system
- e2-B. the properties of matter change depending on the coordinate system so that for all the different design solutions and materials of the capacitor and resistor, the changes in

properties correspond to the time transformation and the definition of capacitance and resistance.

The first hypothesis (e2-A) guarantees the same time in all coordinate systems and excludes the use of non-linear time transformation in physics.

The second hypothesis (e2-B) formulates indirectly necessary (not sufficient) conditions for a physically acceptable non-linear time transformation.

The third example - rotary motion. We know that there are planets and moons in the universe that rotate around their axis at a constant speed. A well-known example is the Earth.

Let us describe the rotation of the body of the functions $\varphi(t) = \omega \cdot t + \varphi_0 = \alpha(t)$ and $\omega(t) = \omega_0 = \beta(t)$ where φ_0 and ω_0 are constants. The general relationship between angle and angular velocity is $\omega(t) = \frac{d\varphi(t)}{dt}$.

If the rotation of the body is invariant with respect to the coordinate transformation, the pair of above-defined assumptions are fulfilled.

Thus, the geometric properties of rotary motion with a constant speed of rotation are not compatible with any non-linear time transformation.

Also, in this case we have two options:

- e3-A. the first option guarantees the same time in all coordinate systems and excludes the use of non-linear time transformation in physics,
- e3-B. the second option requires a change in the geometry of rotation, the creation of a new hitherto unknown geometry of rotation corresponding to a non-linear time transformation (or use of another mathematical apparatus of differential calculus) .

Note that physics has inner integrity. Time is the same for all laws of physics. The options from the previous examples cannot be freely combined. Either e1-A applies together with e2-A and e2-A or e1-B applies together with e2-B and e3-B.

It is left to the reader what conclusion he will draw from it, which hypothesis he will consider invalid.

7 Motivation

The analysis was not performed by accident. The analysis was processed as part of analytical work associated with experiments with unknown gravitational interactions and their surprising results [9-13].

8 Conclusion

The linear time transformation generally described by equation (1) is equivalent to conversion between units of time measurement and/or time offset. It can be used without restrictions and without the risk of unexpected physical impacts.

Non-linear (curved) time transformation is a very dangerous tool in theoretical physics. It's at first sight unseen consequences may be contrary to the internal consistency of physics and thus destroy a theory or hypothesis built on it.

This is illustrated by the example of the time transformations used in STR and GTR.

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