

Unification of Strong-Weak Interactions and Possible Unified Scheme of Four-Interactions

Yi-Fang Chang

Department of Physics, Yunnan University, Kunming 650091, China

yifangch@sina.com

ABSTRACT

First, various known unified theories of interactions in particle physics are reviewed. Next, strong and weak interactions are all short-range, which should more be unified. Except different action ranges their main character is: strong interactions are attraction each other, and weak interactions are mutual repulsion and derive decay. We propose a possible method on their unification, whose coupling constants are negative and positive, respectively. Further, we propose a figure on the unification of the four basic interactions in three-dimensional space, and search some possible tests and predictions, for example, strong-weak interactions transform each other, some waves may be produced. Finally, based on the simplest unified gauge group $GL(6, \mathbb{C})$ of four-interactions, a possible form of Lagrangian is researched. Some relations and equations of different interactions are discussed.

Key words: interaction; unification; short-range force; coupling constant; Lagrangian; equation

DOI ·10.13140/RG.2.2.18002.71368

1 Introduction

In nature have four different strong, electromagnetic, weak and gravitational interactions. Their distinction justified by relative strengths. The strength of weak interaction is very small, but it involves all observed particles (hadrons and leptons) except photon. Only proton, electron, neutrino and photon are stable, which are just the smallest particles with four interactions Other particles all decay.

Another important property is the range of interactions. Electromagnetic and gravitational interactions have infinite range. Strong and weak interactions have short range, and weak interaction possesses the shortest range of all interactions [1].

It is known that the coupling constant of strong interaction is $g^2 / 4\pi \cong 15$, the coupling constant of weak interaction is $\cong 10^{-6}$. The range of strong interaction is $m_{\pi}^{-1} \cong 1.4 \times 10^{-13}$ cm [1], the range of weak interaction should be $m_{WZ}^{-1} \approx 2 \times 10^{-16}$ cm. In this paper, we research unification of strong and weak interactions as short-range, and some results of the simplest unified gauge group $GL(6, \mathbb{C})$ of four-interactions.

2 Various Known Unifications on Interactions

The unification of various interactions is always an important question in physics. The early unifications are mainly some theories on the gravitational and electromagnetic fields, which are all long-range [2-4]. We proposed the gravitational field and the source-free electromagnetic field can be unified preliminarily by the equations in the Riemannian geometry [5,6]:

$$R_{klm}^i = \kappa T_{klm}^i \quad (1)$$

Both are contractions of im and ik , respectively. If $R_{klm}^i = \kappa T_{klm}^i = \text{constant}$, so it will be equivalent to the Yang's gravitational equations [7]:

$$R_{km;l} - R_{kl;m} = 0, \quad (2)$$

which include $R_{lm} = 0$. From $R_{lm} = 0$ we can obtain the Lorentz equations of motion, the first system and second source-free system of Maxwell field equations. This unification can be included in the gauge theory, and the unified gauge group is $SL(2, \mathbb{C}) \times U(1) = GL(2, \mathbb{C})$, which is just the same as the gauge group of the Riemannian manifold. Another unification on the general nonsymmetric metric field with high-dimensional space-time is analyzed mathematically, and we proposed an imaginative representation on the ten dimensional space-time [6].

The mathematical basis in unified theories of particle physics is the gauge groups. Weinberg and Salam proposed a well-known electroweak theory unified the weak and electromagnetic interactions, whose unified gauge group is $SU(2) \times U(1) = U(2)$ [8,9]. Further, various grand unified theories (GUT) of the strong, weak and electromagnetic interactions are researched [10-16], whose pioneer is Bars-Halpern-Yoshimura model [10,11]. Pati and Salam proposed the unified lepton-hadron symmetry and a gauge theory $SU(2) \times U(1) \times SU(3')$ of the basic interaction [12]. There is the same gauge group in Itoh-Minamikawa-Miura-Watanabe model [13]. A famous theory is the Georgi-Glashow $SU(5)$ theory [14], which is only a non-Abel field in which proton will decay to electron. Moreover, there are Fritzsche-Minkowski $SU(n) \times SU(n)$ ($n=8,12,16$) and $SO(n)$ ($n=10,14$) unified interactions theories of leptons and hadrons [15], and a universal gauge theory model based on $E(6)$ [16]. Calmet, et al., showed grand unification and some enhanced quantum gravitational effects [17]. Blumenhagen investigated gauge coupling unification for F-theory $SU(5)$ GUT with gauge symmetry breaking via nontrivial hypercharge flux [18].

Einstein gravitational Lagrangian possesses two invariances: the $GL(4, \mathbb{R})$ invariance of Einstein under coordinate transformations, and the $SL(2, \mathbb{C})$ gauge invariant of Weyl. The strong interaction of quarks possesses internal $SU(3)$ symmetry. From these symmetries Isham, Salam and Strathdee proposed the unified scheme on gravitational and strong interactions, whose gauge group is $SU(3) \times SL(2, \mathbb{C}) = SL(6, \mathbb{C})$ [19-21].

The supersymmetry theory describes a basic symmetry between bosons and fermions, and arouses the supergravity in the gravitational theory, and derives a superstring combining a string model. This is related with the unified theory of interactions. Barr and Raby proved minimal $SO(10)$ unification in the supersymmetric grand unified theory [22]. Kakushadze and Tye researched the classification of three-family grand unification of $SO(10)$, $E(6)$, $SU(5)$ and $SU(6)$ models in string theory [23]. Das and Jain discussed dynamical gauge symmetry breaking in an $SU(3) \times U(1)$ extension of the standard model [24]. Albright and Barr discussed explicit $SO(10)$ and $U(1) \times Z(2) \times Z(2)$ supersymmetric grand unified model for the Higgs and Yukawa sectors [25]. Moreover, the unified forms of supersymmetry are also connected with the statistics unifying BE and FD statistics, and with the possible violation of Pauli exclusion principle [26,5].

A possible development is the higher dimensional complex space [27]. Triantaphyllou and Zoupanos searched strongly interacting fermions from higher (4-12) dimensional $E(8) \times E(8)'$ unified gauge theory [28].

3 Unification of Strong-Weak Interactions with Short-Range

3.1 Some Possible Unified Methods

It is known that the strong and weak interactions are all the short-range, which should more be unified. But, so far their unification is almost neglected.

Recently, some confusion exists on weak interactions. How do weak interactions may overcome strong interactions leading to particle decay? This should be for the shorter distance, and smaller distance corresponds to higher energy. The weak interactions include two types: one determines particle decay, and another exists for interaction of all particles except photon, especially leptons, which are mainly electron-neutrino ($e-\nu_e$) and $\nu-\nu$ interactions, and which exchange large mass W and Z, respectively. But, neutrino ν is introduced to ensure energy conservation, and it can run through the Earth without hindrance, so $\nu-\nu$ is very small interaction. Strong interactions between nucleons (p, n) pass through π^+, π^0 and gluon. Weak interaction among all baryons and leptons pass through W^+, Z^0 ($m=80, 91\text{GeV}$).

A base of electroweak unification is that $Z^0 - \gamma$ are neutral particles and neutral currents. Base of the unification of weak and strong interactions is that $W^\pm - Z^0$ and $\pi^\pm - \pi^0$ (for u and d, $m=140, 135\text{MeV}$) are symmetry, and the difference of both interaction scales is $90/0.14=64.286$. This is strong interaction as 10^{-13} cm , so weak interaction scale is $10^{-15}-10^{-16}\text{ cm}$. Further, $K^\pm - K^0$ (add s, $m=494, 497\text{MeV}$), $D^\pm - D^0$ (add c, $m=1870, 1865\text{MeV}$), $B^\pm - B^0$ (add b, $m=5279\text{MeV}$) all are completely the systems.

We propose the possible theoretical approach on unification of strong and weak interactions can be:

- (1) The removal of electromagnetic fields from the grand unified theory (GUT) seems to be the easiest way.
- (2) In the electroweak unified theory the electromagnetic field is transformed into a strong interaction. If the gluon is simplified to one type, it is similar to a photon, and can be obtained as a similar electroweak unified theory, in which positive and negative charges correspond to quarks u and d. It has neutral charged (pn, nn) and (pp) strong interaction, but p-p has weaker charge interactions. In experiments pp, pn (elastic and total), and $\pi^+ p, \pi^- p$ and $K^+ p, K^+ n, K^- p, K^- n$ are all similar, but they are not the same, so there have symmetry violations.
- (3) The juxtaposed strong and weak interactions and their interactions each other, whose Lagrangian is:

$$L = L_s + L_w + L_{in} . \quad (3)$$

We may establish a simplified theory of strong and weak unity: the first generation quark-lepton has basic symmetry, and are all SU(2). The second and third generations are some excited states, and are heavy electrons and heavy quarks. SU(3) is originally three kinds of quark u, d, s, and now become three kinds of color, and total gauge group should be $SU(6)_q \times SU(3)_c$. Leptons with weak interaction are three generations SU(2).

- (4) The best way seems to transform each other with distance, energy-momentum, and action strength, etc., so the interaction direction and coupling constants are opposite.

Except different action ranges their main character is: strong interactions are attraction each other, and weak interactions are mutual repulsion and derive decay. Now we propose a new possible method on their unification, whose coupling constants are negative and positive, respectively. Any attract forces cannot obtain decay of particles. In electromagnetic field attraction or repulsion is determined by opposite charges. In particle physics strong and weak interactions should be determined by opposite coupling constants.

3.2 Change and Unification of Coupling Constants

It is known that the strong interaction is a big attraction, which corresponds to the quarks confinement. When the distance decreases, it is asymptotic freedom, i.e., no interaction. They correspond to that strong interaction, SU(3) and QCD first become zero along with distance decrease. Then, the distance becomes smaller and the corresponding energy becomes larger, it becomes weak interaction, SU(2) and QWD (quantum weak dynamics), and derive decay. It has a critical point of transformation between strong and weak interactions. Further, probably it may include electromagnetic interactions.

The asymptotic freedom is the biggest characteristic of the QCD. The formula is:

$$\alpha_s(Q^2) = \frac{g_s^2}{4\pi} = \frac{4\pi}{\beta_0 \ln(Q^2 / \Lambda^2)}. \quad (4)$$

When energy $Q^2 \rightarrow \infty$, the coupling constant of strong interaction $\alpha_s(Q^2) \rightarrow 0$. Further, reduction of distance should be weak interaction with repulsive force. It can be inaccessibility and exclusion, are repulsion. This can be the unity of strong and weak interactions with short-range.

Defining a new kind of renormalized coupling constant $g(\mu)$ depends on a sliding energy scale μ [29]. The 't Hooft-Weinberg renormalization group equations are [30-32]:

$$[M^2 \frac{\partial}{\partial M^2} + \beta \frac{\partial}{\partial \lambda} + \gamma_\theta m^2 \frac{\partial}{\partial m^2} - n\gamma] \Gamma(p_1, \dots, p_n, \lambda, M^2, m^2) = 0, \quad (5)$$

where $\beta(\lambda) = M^2 \frac{d\lambda}{dM^2}$, etc.

The coupling satisfies the renormalization group equations [33-36]:

$$\mu_R^2 \frac{d\alpha_s}{d\mu_R^2} = \beta(\alpha_s) = -(b_0 \alpha_s^2 + b_1 \alpha_s^3 + b_2 \alpha_s^4 + \dots), \quad (6)$$

where $b_0 = (33 - 2n_f)/(12\pi)$ is referred to as the l-loop beta function coefficient, etc.

Some possible forms of the function $\beta(g_l)$ depend on the running coupling g_l . $\beta(g_l)$ may be positive or negative or 0. Fig.1 is Fig.18.5b [37] and Fig.18.4c (but, $\beta(g) \rightarrow -\beta(g_l)$ and $g_* \rightarrow q$) [29] and Fig.3.3.2b [38], $\beta(g_l) < 0$ (for $0 < g_l < q$), $\beta(g_l) = 0$ (when $g_l = q$), and $\beta(g_l) > 0$ (for $g_l > q$).

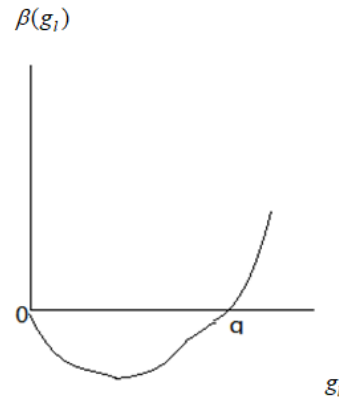


Fig.1 $g_i - \beta(g_i)$

In Fig.1 we propose that $\beta(g_i) > 0$ is strong interaction with big g_i and attraction, $\beta(g_i) < 0$ is weak interaction with small g_i and opposite repulsion, and $\beta(g_i) = 0$ with $g_i = q$ is the asymptotic freedom. The theory of strong interactions based on the gauge group SU(3) is asymptotically free, which first is discovered by Gross and Wilczek [39] and Politzer [40] in 1973.

The solution of Eq.(6) may obtain [29]:

$$\alpha_s(Q^2) = \frac{\alpha}{[1 + b(n-1)\alpha^{n-1} \ln(Q^2 / \Lambda^2)]^{1/(n-1)}}. \quad (7)$$

The effective 'running' coupling constant with SU(N) is [36-38]:

$$\alpha_s(Q^2) = \frac{g^2}{4\pi} = \frac{\alpha}{1 + \frac{\alpha}{12\pi} [11C_2(G) - 4n_f T(R)] \ln(Q^2 / \Lambda^2)}, \quad (8)$$

where n_f is the number of quark flavors participating in the interaction at this Q. It is determined by momentum and energy. When $Q^2 \rightarrow \infty$ (higher energy and shorter range), $\alpha_s = 0$, i.e., the asymptotically freedom. The experiments shown $\alpha_s = 0.1184$ decrease, but it is not $\alpha_s \rightarrow 0$.

For SU(N) $C_2(G) = N$. For SU(3) and QCD, $C_2(G) = 3$ and $T(R) = 1/2$,

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - 2n_f/3) \ln(Q^2 / \Lambda^2)}. \quad (9)$$

General $n_f < 33/2$, $\alpha_s > 0$. Strong-weak interactions and QED may be unified by $g_0^2 \delta$, in which $\delta > 0$ for QCD, $\delta < 0$ for SU(2), and $\delta = 0$ for QED [37]. Let $n_f = 33/2$, $\alpha_s = \infty$. Let $n_f = 6$ and $\beta = 0$, $\alpha_f = 12.726$ is an inflection point, from infrared attraction to ultra-violet repulsion [38].

In each energy range there have a different scale Λ , for example, $\Lambda = 253 \text{ MeV}$ [29] or $\Lambda = 500 \text{ MeV}$ [41]. For unified α_s in Eq.(8), if $n_f < 33/2$, $\alpha_s > 0$ for $Q > \Lambda$; when $Q \rightarrow \infty$, $\alpha_s \rightarrow 0$ is the asymptotic freedom; when $Q = \Lambda$, $\alpha_s \rightarrow \infty$ is an impossible state; $\alpha_s < 0$ for $Q < \Lambda$.

For SU(2) $C_2(G) = 2$ and $T(R) = 1/2$,

$$\alpha_w(Q^2) = \frac{4\pi}{(11-n_f)\ln(Q^2/\Lambda^2)}. \quad (10)$$

For weak interaction and QWD, the coupling constant should be the same change. The known Fermi coupling constant $G_F/(\hbar c)^3 = 1.16637 \times 10^{-5} \text{ GeV}^{-2} < 0$. For Eq.(10) if $n_f > 11$, $\alpha_w < 0$; or $\alpha_w < 0$ for $Q < \Lambda$. Or Eq.(8) becomes:

$$\alpha_w(Q^2) = \frac{g^2}{4\pi} = \frac{\alpha}{1 + \frac{\alpha}{6\pi}(11-n_f)\ln(Q^2/\Lambda^2)}. \quad (11)$$

For α_w in Eq.(11), if $n_f < 11$, α_w decrease for $Q > \Lambda$; when $Q \rightarrow \infty$, $\alpha_w \rightarrow 0$ is the asymptotic freedom; when $Q = \Lambda$, α_w is an inflection point; α_w increase for $Q < \Lambda$. Probably, it corresponds to the “strong decay”. Assume that $n_f = 2$, if $\ln(Q/\Lambda) < -(\pi/3\alpha)$, it will be $\alpha_w < 0$. If $\alpha = 1$, $Q < \Lambda e^{-1.05}$. If $\alpha = 15$ is strong coupling constant, $Q < \Lambda e^{-0.07}$.

For QED and U(1), $C_2(G) = 0$ and $T(R) = 1$,

$$\alpha_{em}(Q^2) = \frac{\alpha}{1 - (\alpha/3\pi)n_f \ln(Q^2/\Lambda^2)}. \quad (12)$$

For α_{em} in Eq.(12), if $n_f = 2$, α_{em} increase for $Q > \Lambda$; when $Q \rightarrow \infty$, $\alpha_{em} \rightarrow 0$ is the asymptotic freedom; when $Q = \Lambda$, α_{em} is an inflection point; α decrease for $Q < \Lambda$. If $\ln(Q/\Lambda) > (3\pi/4\alpha)$, it will be $\alpha_{em} < 0$. If $\alpha = 1$, $Q > \Lambda e^{2.355}$. If $\alpha = 15$ is strong coupling constant, $Q > \Lambda e^{0.157}$.

For QED, β function is positive. But, for Yang-Mills (YM) field and strong interaction, β function is negative. For weak interaction, β function should be positive. So β function is related with QCD and QWD, and unified strong and weak interactions.

Assume that α all hold in general cases. For the extensive air shower (EAS) $n_f > 33/2$, so $\alpha < 0$, and corresponds to repulsion. Further, when strong interaction becomes weak interaction and electromagnetic interaction, the scaling parameter Λ as the earliest cutoff factors linked to the renormalization process should be changeable, for example, $\Lambda = m_e$ for QED [38].

Assume that the coupling constant $g < 0$ corresponds to repulsion, $\varepsilon = -g > 0$ in QCD. Fig.1 may describe unification of strong and weak interactions.

It is known that the short-range potential $V(r) = g_0 e^{-mr}/r$, in which the coupling constant $g_0 < 0$ is attraction and strong interaction, and $g_0 > 0$ is repulsion and weak interaction, and $g_0 = 0$ corresponds to the asymptotic freedom. $\varepsilon = g_0$ in QCD. When the combination of positive and negative quark forms the color singlet state, whose interaction $\varepsilon = -8/3 < 0$ is attraction.

Quarks confinement corresponds to a large scale of strong interactions. Strong interactions exist outside the hadron, and within the hadron QCD tends to asymptotically freedom and to mutually repulsive weak interactions. Thus two short-range interactions are unified. Gluon (meson) and W-Z boson are unified. This no interaction must go through inevitable transition from strong interaction to weak interaction with the smaller scale of mutual repulsion.

Experiments of asymptotic freedom are based on the deep inelastic electron-nucleon scattering, and from this obtain parton-quark model which should have the property that the interaction between partons-quarks becomes weaker at shorter distance. Bjorken scaling implies that the constituents of the nucleon look almost free and point-like, and suggests that the quark dynamics (QCD) must have the property of asymptotic freedom [37], and Callan-Gross relation is $F_1(\omega) = (\omega/2m_N)F_2(\omega)$,

$$\text{where } \omega \equiv 2m_N \nu / q^2 \text{ [29].}$$

The strong interactions are attraction each other, whose scale are about $10^{-13} \sim 10^{-15}$ cm in atomic nucleus and nucleon. The weak interactions are mutual repulsion and derive decay, whose scale are about $10^{-15} \sim 10^{-17}$ cm, since mass 80.4 (91.2)GeV of $W^\pm$ (Z) of 'Yukawa-type' exchange particle between weak interactions are 100s times of mass 137 (496)MeV of π (K) of exchange particle between strong interaction. We calculated the weak interaction range about 1.604×10^{-16} cm [42]. From strong interaction to weak interaction there is a process along with range decrease and energy increase, and must be the existence of zero interaction, i.e., the asymptotic freedom.

The amplitudes of boson and fermion are:

$$J_{\mu\nu}^{(B)} = \frac{i}{16\pi^2} (g_b \mu^{-2\rho\varepsilon})^2 \delta_{ab} [-\frac{5}{3} C_2(G)] (k_\mu k_\nu - g_{\mu\nu} k^2) \frac{1}{\varepsilon}, \quad (13)$$

$$J_{\mu\nu}^{(F)} = \frac{i}{16\pi^2} (g_b \mu^{-2\rho\varepsilon})^2 \delta_{ab} [\frac{4}{3} n_f T(R)] (k_\mu k_\nu - g_{\mu\nu} k^2) \frac{1}{\varepsilon}. \quad (14)$$

Both are negative and positive, respectively.

Gauge group $SU(3)_c \times SU(2)_w$ of unified strong and weak interactions has two coupling constants g_s and g_w . This can seem to be reduced to SU(4), where the fourth row corresponds to W-Z and weak interaction. It is the first generation quark-lepton u, d; e, ν_e , or d_i (i is 3 colors); e, ν_e . It is a 3 rank group with 15 generating elements.

The two ways may be field theories and QCD methods that describe different interactions.

The long-range color force is screened by the plasma and becomes short-range, and is manifested by the quark-gluon plasma whose interaction strength α_s becomes smaller [41].

3.3 Some Problems of Weak Interaction

Weak interactions are universal decay of all particles, and give rise to the creation of the neutrino. Further, decay includes: 1. 'Strong' decays without violating any conservation of quantum numbers are rapid, $t_s \sim 10^{-22}$ s. 2. Electromagnetic decays involve the emission of photon, etc., and $t_e \sim 10^{-15} - 10^{-20}$ s. 3. Weak decays with some violations of quantum numbers (S, I and so on) are $t_w \sim 10^{-13} - 10^{-14}$ s, and the selection rules are $\Delta S = \Delta Q = \pm 1$ for hadrons, etc [43]. They include leptonic decay $\mu^\pm \rightarrow e^\pm \nu_e \bar{\nu}_\mu$, semileptonic decay $A \rightarrow B e^\pm \bar{\nu}_e$, $\pi^\pm \rightarrow l \nu$, $K^\pm \rightarrow \pi^0 e^\pm \nu_e$ and nonleptonic decay $H \rightarrow B \pi$, etc.

Various decays from weak to strong decay increase strengths, and time becomes shorter, corresponding to energy becomes larger, and corresponds to shorter distance. The general weak interaction is exchange W-Z bosons. But, various decays do not seem to require exchange of particles.

Moreover, we proposed that hadrons are similar to the limit cycles, whose exterior is an attractive strong interaction, and interior is a repulsive weak interaction which derives decays, and both combination obtains hadron [42]. Strong interaction with longer-range force; and interior of attractor all is repulsive each other, which is weak interaction with shorter-range force. A zero dimensional strange attractor becomes a point charge, and corresponds to electron. Electron e in electromagnetic interaction corresponds to stable focal point, and strong interaction is the limit cycle. At certain degree GUT is namely unification of the limit cycles and singular attractor. Limit of strong and weak interactions is namely electromagnetic interaction of positive and negative charges, whose mass $m=0$ corresponds to zero dimension. For different interactions the equations of quantum mechanics are different. Electromagnetic interaction is Abel group $U(1)$, and strong and weak interactions are non-Abel groups $SU(2)$ and $SU(3)$, and are Yang-Mills (YM) field. Further, the cycle may extend to high dimensions. The non-Abel group equations of interactions with $SU(N)$ symmetry obtain the period solution, which corresponds to the limit cycle.

The decay modes and fractions in particle physics are some quantitative and very complex questions. We discussed various decays of particles and some known decay formulas. Many important decays of particles (0^{--} mesons, $(1/2)^+$ baryons and $\mu^\pm$) and some known decays of resonances can be generally described by the square of some types of the associated Legendre functions P_l^l and P_{l+1}^l . Its universal formula is:

$$\Gamma = \frac{G^2}{4\pi N} m_i^\alpha |P_l^{|m|}(x)|^2, \quad (15)$$

in which x is the ratio of the mass-sum of the final state and the mass of the initial state of particle. It is combined with the $\Delta I=1/2$ rule, then the agreements are very good. The decay formulas of the similar decay modes are the same. For the same types of particles, the coupling constants are kept to be invariant, and only six constants are independent. The simplest dynamical basis is probably that the decay formulas are the square of the solution of free field equation of the decay particles in the momentum coordinate system [5,44]. Further, we discuss some general decays and their rules, and apply the decay formulas to some massive hadrons [44]. Various baryons pass through weak interaction to become proton. Various mesons pass through weak interaction to become electron or photon.

3.4 Unified Figures on Interactions and Possible Tests

Based on Dirac's negative energy state, we developed to the negative matter as the simplest candidate of dark matter and dark energy [45-51], and proposed some tests and a judgment test [52]. Further, we proposed the most perfect symmetrical world on the four types of positive, opposite, and negative, negative-opposite matters (Fig.2) [51,53].

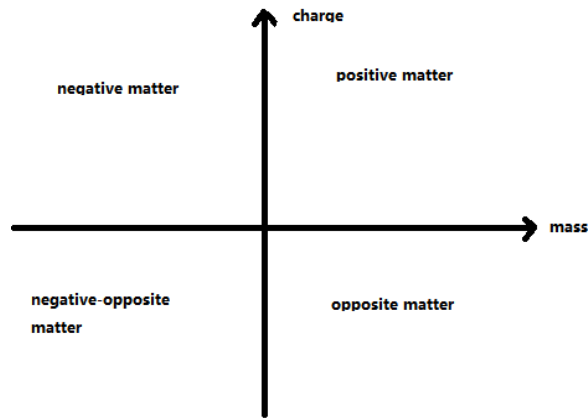


Fig. 2. A new most perfect symmetrical world

Fig. 2 as a two-dimensional plane also corresponds to gravitational and electromagnetic fields determined by mass and charge, respectively. We may combine the four known fundamental interactions to develop into the four-dimensional space. But in some respect, this can be simplified to three-dimensional space, where the third dimension is assumed to be the strong and weak interactions of the short-range. It is known that the strong interactions are attraction each other, and the weak interactions are mutual repulsion and derive decay, both correspond to the coupling constant $G = -g_0 > 0$ of the upper sides, and $G < 0$ of the lower side, respectively. The two aspects are QCD with SU (3) and QWD with SU (2), and may be unified by the YM gauge field, and between them is asymptotically free $G=0$, so that Fig. 3 can uniformly describe the four basic interactions.

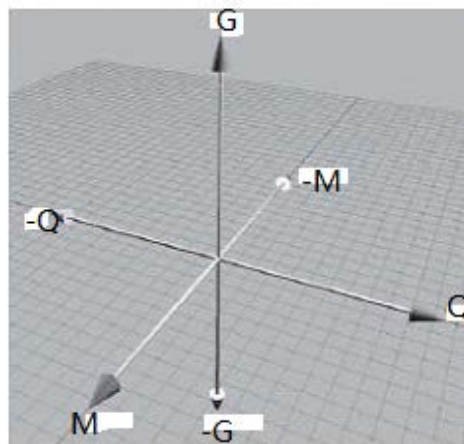


Fig. 3 The unification on the four basic interactions in three-dimensional space

We research some predictions of these unifications, and known experiments and possible tests. It shows clearly that weak interaction appears for higher energy and shorter range. It is consistent completely with those experiments show the scattering in the quark-parton model, they indicate a strong repulsion at a smaller distance. It also seems to correspond to various resonances that are produced by strong interactions, and then are strong decay.

At high energy scattering the cross sections, which connect the structure functions, usually agree with Pomeranchuk theorem that all quark-quark scattering amplitudes are asymptotically equal, and the differences between some scattering particles tend toward zero. This is related to the scaling. In some

aspects it seems to correspond to the asymptotic freedom. But, at higher energy the cross sections gradual increase.

The cross section for the annihilation of quark pair through gluon is [54,55]:

$$\frac{d\sigma}{d\Omega}(q\bar{q} \rightarrow q'\bar{q}') \approx \frac{\alpha_s^2}{9s}. \quad (16)$$

Here s is energy squared at c.m. For hadroproduction of heavy quarks $Q=c, b, t$, the formula is:

$$\frac{d\sigma}{d\Omega}(q\bar{q} \rightarrow Q\bar{Q}) \approx \frac{\alpha_s^2}{9s^3}. \quad (17)$$

Eq.(4) replaces into Eq.(16), we may obtain a minimum solution: When $s = Q_0^2 = \Lambda^2 e^{-2\Lambda^2}$, the minimum value is:

$$\left. \frac{d\sigma}{d\Omega} \right|_{s=Q_0^2} = \frac{64\pi^2}{9\beta_0^2} \Lambda^2 e^{2\Lambda^2}. \quad (18)$$

From $\beta(g_l)$ squared in Fig.1 we can also derive the similar result, a minimum value exists at a point $g_l=q$. It seems to be a universal rule that the cross sections of scattering tend toward equal from big, and then gradual increase.

From the minimum value Q_0^2 , we may derive $Q_0 = \Lambda e^{\pm i\sqrt{2}\Lambda}$. It shows the periodicity. In the standard model three generations of quark-lepton possess some periodicities, and they are also very clear for two systems $\eta_c = c\bar{c}$ and $Y = b\bar{b}$.

The electromagnetic fields produce each other and derive electromagnetic wave, whose equation is:

$$\frac{\partial^2 A}{\partial x_\alpha^2} - \frac{\partial^2 A}{c^2 \partial t^2} = 0. \quad (19)$$

Usual gravitational fields change to produce gravitational wave, in which $m=0$ derives $v=c$.

When general strong and weak interactions are changed, strong and weak waves should also be produced, in particular, those particle waves produced by strong interactions and weak interactions decay. Different interactions produce different particles, which have wave property. It possesses the uncertainty relation [5,56]. According to the de Broglie relation $v\bar{v} = c^2$, the particle velocity v is related to the phase velocity $\bar{v} > c$ that obeys the generalized Lorentz transformation (GLT) [5,57], and corresponds to the quantum entangled state [57].

Klein-Gordon equation becomes to a wave form:

$$\frac{\partial^2 \varphi}{\partial x_\alpha^2} - \frac{\partial^2 \varphi}{c^2 \partial t^2} - \frac{m^2 c^2}{\hbar^2} \varphi = \frac{\partial^2 \varphi}{\partial x_\alpha^2} - \frac{\partial^2 \varphi}{v^2 \partial t^2} = 0. \quad (20)$$

Here the wave velocity $v < c$. Assume that $v = \frac{c}{1 + amt}$ or $v = ce^{-(mc/\hbar)t}$, so the bigger m , v the smaller.

When $t \rightarrow \infty$, $v \rightarrow 0$.

The strong and weak interactions produce each other, which should obtain the particle waves whose m is not 0, and $v < c$. It is related to duality. They are probably resonances and resonance waves. It can

be divided into boson waves and fermion waves, generally is short-range waves. These waves correspond to not-Abel gauge fields and nonlinear waves and soliton waves.

$e^+e^- \rightarrow 2\gamma$ is particle with mass and gravitational fields-electromagnetic fields changes to light waves. GRB is huge mass change into light wave.

Huge strong interaction changes exist in the Sun and the nuclear bomb. Standard solar model $4^1H \rightarrow ^4He + 2e^+ + 2\nu_e$ is strong interaction change to light wave and neutrino flow. Huge gravitational field changes are black holes. The known gravitational waves are obtained by huge mass conversion [51].

$n \rightarrow pe\nu$ is n-p strong and weak interactions transform each other to produce neutrino waves with weak interacting. $\pi^0 \rightarrow 2\gamma$ is hadron weak decay into electromagnetic interactions and light waves.

An important example of weak interaction waves is the neutrino flow. If weak interacting wave is W-Z flow, the wave ranges are extremely short $\Gamma_W = 2.14\text{GeV}$ and $\Gamma_Z = 2.50\text{GeV}$. Particle waves constantly collide, transform, and decay to form propagating waves, such as $\pi^\pm \rightarrow \mu\nu$, $\mu \rightarrow e\bar{\nu}\nu_\mu$ form the neutrino waves. High energy particles may form the wave surface of extensive air showers (EAS) [58]. So are the nuclear and atomic waves.

The general strong and weak interactions transform each other into weak interactions often appear neutrino flow-wave $v=c$. They transform to strong interactions, and appear p flow and the unstable π flow. Eventually, they decay into stable minimal elements with four basic interactions p, e, ν_e , and photons.

$K^\pm \rightarrow \mu\nu_\mu$ (63.4%), $K^\pm \rightarrow \pi^\pm\pi^0$ (20.9%), $K^\pm \rightarrow \pi^\pm\pi^+\pi^-$ (5.6%), $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$ (1.76%). $K^\pm \rightarrow \pi^0e\nu$ (4.98%), $K_L^0 \rightarrow \pi^\pm e\nu$ (40.5%), $K_L^0 \rightarrow \pi^\pm l\nu$ (67.55%) are the branch ratio of weak semileptonic decay of hadrons.

$K_L^0 \rightarrow 3\pi^0$ (19.56%), $K_L^0 \rightarrow \pi^+\pi^-\pi^0$ (12.56%) both sum 32.12% is about 1/3. $K_S^0 \rightarrow \pi^+\pi^-$ (69.2%), $K_S^0 \rightarrow \pi^0\pi^0$ (30.7%). They show CP violation.

$\Lambda \rightarrow p\pi^-, \rightarrow n\pi^0$; $\Sigma^+ \rightarrow p\pi^0, \rightarrow n\pi^+$; $\Sigma^- \rightarrow n\pi^-$; $\Xi^0 \rightarrow \Lambda\pi^0$, $\Xi^- \rightarrow \Lambda\pi^-$ are all 100%, and are that hadrons pass through weak decay to transform to the branch ratio of nonleptonic decay. For this if the energy is equal before and after decay, it is π wave; if it is not equal, another new wave will exist.

$\Sigma^0 \rightarrow \Lambda\gamma$ (100%) is that hadron passes through strong-electromagnetic interaction to transform to produce light waves.

Various radiative decays $\Lambda \rightarrow n\gamma$, $\Sigma^+ \rightarrow p\gamma$, $\Xi^0 \rightarrow \Lambda\gamma, \Sigma^0\gamma$ and $\Xi^- \rightarrow \Sigma^-\gamma$, etc., are all transform to light waves.

$\Omega^- \rightarrow \Xi \rightarrow \Sigma^\pm(\Lambda) \rightarrow p(n)$ are the most obvious the quantum number S layer by layer decay. $\Sigma^0 \rightarrow \Lambda\gamma$ is electromagnetic decay at the same layer.

For hadrons of $C=1$, $\Lambda_c^+ \rightarrow p\bar{K}^0$, $\Lambda_c^+ \rightarrow pK^-\pi^+$, $\Lambda_c^+ \rightarrow p\bar{K}^0\pi^0$, $\Lambda_c^+ \rightarrow p\bar{K}^0\eta$ and $D^0 \rightarrow K^-e^+\nu_e$, $D^0 \rightarrow K^-\mu^+\nu_\mu$, $D^0 \rightarrow \bar{K}^*(892)^-e^+\nu_e$, $D^0 \rightarrow \bar{K}^*(892)^-\mu^+\nu_\mu$. $D^+ \rightarrow \bar{K}^0e^+\nu_e$, $D^+ \rightarrow \bar{K}^0\mu^+\nu_\mu$, $D^+ \rightarrow \bar{K}^*(892)^0e^+\nu_e$, $D^+ \rightarrow \bar{K}^*(892)^0\mu^+\nu_\mu$.

Strong and weak interaction waves derive cosmic rays, most of which are protons, next are electrons at high energy [59,60].

More generally, when various interactions change, and they transform each other, different waves may be produced. Positive and negative matters [45-53] produced each other should obtain the material waves.

4 A Possible Unified Scheme of Four-Interactions

4.1 Unified Gauge Group

Based on the Weinberg-Salam unified electroweak theory [8,9] and Isham-Salam-Strathdee strong-gravitational interactions unified scheme [19-21], in 1974 we proposed that the simplest unified gauge group of four-interactions must be $GL(6,C)$ or its extension [61,5]. In 1977 Terazawa proposed also a unified gauge symmetry of $GL(32N,C)$ or $GL(12+2n,C)$ for combining the $SU(16N)$ or $SU(6+n)$ group of lepton-quark internal symmetry and the $SL(2,C)$ Lorentz group of space-time symmetry (where $N=1,2,3,\dots$ and $n=0,1,2,3,\dots,N$) [62].

$GL(6,C)$ group is the six-dimensional general complex linear group. For the general complex number, it can be a 12-dimensional space-time. $GL(6,C)$ contains $GL(6,R)$ and $GL(4,R)$, that is the coordinate transformations group $GL(4,R)$ in general relativity is embedded in $GL(6,C)$. $GL(6,C)$ contains the six-dimensional special complex linear group $SL(6,C)$, which is the simplest relativistic generalization of $SU(6)$, and the gauge group of Isham-Salam-Strathdee strong and gravitational unification, and contains the gauge invariant group $SL(2,C)$ in the gravitational field.

$GL(6,C)$ also directly contains various simple symmetry groups $SU(6)$, $SU(3)$, $SU(2)$ and $U(6)$, $U(2)$ on different interactions, and the gauge group $U(1)$ of the electromagnetic field. The symmetry group of weak interactions is $SU(2)$, then the symmetry is exactly the same as that of the initial YM field. $SU(3)$ contains $SU(2)$, then the leptons are embedded in the strong interaction $SU(3)$, which is some extension to the unified strong and weak interactions. It is similar to the Bars-Halpern-Yoshimura approach, where leptons and weak interactions are embedded in hadrons and strong interactions. Meanwhile, the strong and electromagnetic interactions should be initially unified as $U(3)$. The gauge group of unified gravitational and electromagnetic interactions should be $SL(2,C) \times U(1) = GL(2,C)$.

$GL(6,C)$ should also be generalized if the strong and weak interactions are separated as $SU(3) \times SU(2)$. If various interactions correspond to different groups, the unified group also varies. When there is no gravitational interaction, it becomes GUT. If electromagnetic interaction is also omitted, it will be a combination of short-range interactions, further, i.e., individual strong and weak interactions.

Of course, this unified group is a non-compact group, and seems to construct a 'no-go' law. But, it is not only reasonable but also necessary, because the classical gravitational interaction corresponds to a non-compact group $SL(2,C)$. Therefore, it should be that the gravitational theory is developed to the quantum gravity theory, for example, the supergravity and the loop quantum gravity theory [63-65] and so on. Further, $GL(6,C)$ is related to four- or six-dimensional Riemann geometry and nonsymmetric field [6].

4.2 Some Unified Lagrangian Forms

Further, we researched a possible form of Lagrangian on group GL(6,C) of unified four interactions [66], and discussed some relations among these results and other unified theories and the equations of different interactions [27].

The present interaction unifications are mainly based on the YM field and its generalized gauge theory. For SU(2) gauge group, Lagrangian of YM field is:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}\gamma_{\mu}(\partial_{\mu} - i\varepsilon\mathbf{t}b_{\mu})\psi - m\bar{\psi}\psi, \quad (21)$$

where $F_{\mu\nu} = \partial_{\nu}b_{\mu} - \partial_{\mu}b_{\nu} - 2\varepsilon b_{\mu} \times b_{\nu}$. It is also Lagrangian of weak interaction [15].

QCD is the modern theory of strong interactions, and is a non-Abelian theory based on the gauge group SU(3) [29]. The most general renormalizable Lagrangian for QCD is [29]:

$$L = -\frac{1}{4}F_{\alpha}^{\mu\nu}F_{\alpha\mu\nu} - \sum_n \bar{\psi}_n[\partial - igA_{\alpha}t_{\alpha} + m_n]\psi_n. \quad (22)$$

Here A_{α} is the color gauge vector potential, and g is the strong coupling constant.

In Weinberg-Salam electroweak unified theory, Lagrangian is [8]:

$$L_W = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \bar{R}\gamma^{\mu}D_{\mu}R - \bar{L}\gamma^{\mu}D'_{\mu}L - \frac{1}{2}|D'_{\mu}\phi|^2 - G_e(\bar{L}\phi R + \bar{R}\phi^{\dagger}L) - P(\phi), \quad (23)$$

where $F_{\mu\nu} = \partial_{\mu}\vec{A}_{\nu} - \partial_{\nu}\vec{A}_{\mu} + g\vec{A}_{\mu} \times \vec{A}_{\nu}$, $B_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}$ and $D_{\mu} = \partial_{\mu} - ig'B_{\mu}$,

$D'_{\mu} = \partial_{\mu} - igt\vec{A}_{\mu} - \frac{i}{2}g'B_{\mu}$, $P(\phi) = M^2\phi^{\dagger}\phi - h(\phi^{\dagger}\phi)^2$. In Eq.(23), first term corresponds to the

weak interaction of YM field with SU(2) symmetry, second term corresponds to the electromagnetic field, third and fourth terms correspond to the lepton and its interaction with the electroweak field, and fifth and sixth terms correspond to the interaction of the scalar field with other fields and the lepton, and seventh term is the self-interaction of the scalar field. Lagrangian can also add a mass term $-\bar{\psi}m_0\psi$.

Strong interaction should also add Lagrangian

$$L_s = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \bar{\psi}\gamma^{\mu}(\partial_{\mu} + ig_B B_{\mu})\psi - \frac{1}{2}\mu_B^2 B_{\mu}B^{\mu}. \quad (24)$$

In Isham-Salam-Strathdee gravitational and strong unified theory, Lagrangian with SL(2,C) invariance is [20]:

$$L_{ISS} = \frac{1}{8}\text{Tr}\left[-\frac{1}{k^2}(\nabla_{\mu}L_{\nu}\nabla_{\nu}L_{\mu} - \nabla_{\mu}L_{\mu}\nabla_{\nu}L_{\nu}) + \alpha L_{\mu}(S\nabla_{\nu}S^{-1})L_{\mu}(S\nabla_{\nu}S^{-1}) + \beta_1 L_{\mu}L_{\mu} + \beta_2 L_{\mu}L_{\mu}L_{\nu}L_{\nu} + \beta_3 L_{\mu}L_{\nu}L_{\mu}L_{\nu}\right], \quad (25)$$

where L is Einstein-Weyl Lagrangian. It may become approximately the Pauli-Fierz form, which describes particles with arbitrary spins. For a certain extent, it is also unification. In this case, the forms

of gravitational interaction and strong interaction are symmetrical. When $\alpha = \beta = 0$, it is Lagrangian of gravitational interaction [21]:

$$L_g = \frac{i}{8k^2} \sqrt{g} \text{Tr}([L^\mu, L^\nu](\partial_\mu B_\nu - \partial_\nu B_\mu + i[B_\mu, B_\nu])). \quad (26)$$

Quark Lagrangian with $SL(6, C)$ gauge invariance is [20]:

$$L_q = \frac{1}{2} i \bar{\psi} L_\mu \nabla_\mu \psi + H.c. - m \bar{\psi} \psi. \quad (27)$$

In gravitational field, Lagrangian of classical lepton field is [21]:

$$L_{kp} = \sqrt{g} \left[\frac{1}{2} i (\bar{\psi} L^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} L^\mu \psi) - m \bar{\psi} \psi \right]. \quad (28)$$

For the same case Lagrangian of strong interaction is [21]:

$$L_s = \frac{i}{16k^2} \sqrt{f} \text{Tr}([L^\mu, L^\nu] B_{\mu\nu}). \quad (29)$$

According to the three steps of gauge field unification theory: apply the appropriate group; write gauge invariant Lagrangian; symmetry spontaneously breaks. This relates to the Higgs mechanism by introducing a scalar field that produces the mass of lepton, baryon and other fermions (except neutrinos), and obtain also the mass of meson as the field quantum and of boson $W^\pm, Z^0$. In the various unified theories, Lagrangian is only partially identical because of the different groups and starting points.

Combining various known results on unified interactions, we may obtain the simplest Lagrangian of the unified four-interactions:

$$\begin{aligned} L_{tot} = & \frac{\sqrt{g}}{8k^2} \text{Tr}([L^\mu, L^\nu] B_{\mu\nu}) - \frac{\sqrt{g}}{2} F_{\mu\nu} F^{\mu\nu} - \frac{\sqrt{g}}{4} E_{\alpha\mu\nu} E_\alpha^{\mu\nu} - \\ & - \sqrt{g} (\bar{\psi} L^\mu D_\mu \psi + m \bar{\psi} \psi) - \frac{\sqrt{g}}{2} \mu_B^2 A_\mu A^\mu - P(\varphi) + L_i, \end{aligned} \quad (30)$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + i[B_\mu, B_\nu]$ is the connected covariant curl. In Lagrangian (30), first term is caused by gravitational field of $SL(2, C)$ (whose spin is 2); second and third terms are caused by $U(1)$ of electromagnetic field and $SU(3)$ of the vector field (both spin is 1); fourth term is caused by the spin field of quark (whose spin is 1/2), and D_μ is its covariant differential; fifth and sixth terms are mass; seventh term is the polynomials of the scalar field (whose spin is 0); and eighth term describes other various interactions. In gravitational interaction, various terms in Lagrangian are covariant in generalized coordinates. At this state, strong interaction contains weak interaction, i.e., leptons embedded hadrons. If the symmetry group is $SU(3) \times SU(2)$, etc., the corresponding group will be larger, and similar form terms appear in Lagrangian.

Based on Lagrangian of GUT [10,11,13], the Lagrangian must add one of gravitational field, and should be covariant (these are namely the interactions between gravity and electromagnetic, strong and weak fields). It can also obtain four-interactions unified Lagrangian. This Lagrangian, under certain conditions, is transformed into Weinberg-Salam electroweak unification and Isham-Salam-Strathdee gravitation-strong unified Lagrangian, respectively. When the symmetry breaks spontaneously, it can

be further transformed into quantized strong, weak interactions and geometrical gravitational interaction. From this we can derive Lagrangian of QCD and QED. Some Lagrangian is very detailed and complex in the known unified theories.

4.3 Various Filed Equations

Based on the equation:

$$\Delta\varphi - m_0^2\varphi = 4\pi Gm, \quad (31)$$

its solution is:

$$\varphi(r) = -\frac{Gm}{r} e^{-m_0 r}. \quad (32)$$

From this may derive the short-range force:

$$F = -\frac{Gm_1 m_2}{r^2} e^{-m_0 r}. \quad (33)$$

Lagrangian of unified four-interactions includes gravitational field, quark field, non-Abel gluon field of strong interaction (lepton field and weak interaction field embedded therein), electromagnetic field, Higgs scalar field and interactions between these five fields. It can contain the mass term, or the mass is caused by Higgs field. The equations obtained from Eq.(30) can uniformly describe the four-interactions. First term derives the Riemann-Christoffel curvature tensor $R_{\lambda\mu\nu}^\alpha$, and in Einstein gravitational field equations $T_{\mu\nu}$ are determined by total mass. In this case, every term has the bending effect. If there is no first term, and when the gravitational effect is omitted, it is GUT, which can have various schemes. When strong and weak interactions are separated, it is extended $E_{\alpha\mu\nu} E_\alpha^{\mu\nu} \rightarrow F_{\alpha\mu\nu} F_\alpha^{\mu\nu} + G_{\mu\nu} G^{\mu\nu}$, which is namely Lagrangian of Weinberg unified electroweak-strong interactions, and corresponds to quark-lepton field separated. Fourth term differential for $\bar{\psi}$ obtain:

$$\sqrt{g}(L^\mu D_\mu + m)\psi = J. \quad (34)$$

It is similar to Dirac equation, but considering the gravitational bending effect, J is the interaction term.

If consider only two quarks (u,d), $\psi = \begin{bmatrix} u \\ d \end{bmatrix}$. It is Dirac equation for mass is not zero, and mass of

electron is smaller. For $m=0$, it has isospin SU(2) and chiral symmetry. In QCD for Goldstone particle, SU(2) corresponds to π meson, whose mass approximate 0. But, mass is not zero, it corresponds to the partial conservation of axial-vector current (PCAC).

Eq.(30) may derive the general Proca equation of vector field on strong and weak interactions:

$$\sqrt{g}(\partial_\nu E^{\mu\nu} + \mu_B^2 A^\mu) = J^\mu, \quad (35)$$

and special Maxwell equation of electromagnetic field with mass of zero. It can also derive Klein-Gordon equation of scalar field:

$$\sqrt{g}(D^\mu D_\mu - m_0^2)\varphi = J_0. \quad (36)$$

In a word, the research of various unified theories is very meaningful, and very difficult. It includes that strong and weak unified theory, the four-interaction unified theory are now unified form. Further, we must investigate their predictions and more tests. We believe that it should be different stages as a goal that scientists will always pursue.

REFERENCES

- [1] R.E. Marshak, Riazuddin and C.P. Ryan, *Theory of Weak Interactions in Particle Physics*. New York: A Division of John Wiley & Sons, Inc. **1969**.
- [2] A. Einstein, *The Meaning of Relativity* (Fifth edition). Princeton. **1955**.
- [3] V. Hlavaty, *Geometry of Einstein's Unified Field Theory*. P.Noordhoff Ltd. **1957**.
- [4] V.D. Sabbata and E. Schmutzer, *Unified Field Theories of More Than 4 Dimensions Including Exact Solutions*. World Scientific. **1983**.
- [5] Yi-Fang Chang, *New Research of Particle Physics and Relativity*. Yunnan Science and Technology Press. **1989**. p184-216. Phys.Abst.93,1371(**1990**).
- [6] Yi-Fang Chang, *International Review of Physics*. 8(**2014**):132.
- [7] C.N. Yang, *Phys.Rev.Lett.* 33(**1974**):445.
- [8] S. Weinberg, *Phys.Rev.Lett.* 19(**1967**):1264.
- [9] S. Weinberg, *Phys.Rev.Lett.* 27(**1971**):1688.
- [10] I. Bars, M.B. Halpern and M. Yoshimura, *Phys.Rev.Lett.* 29(**1972**):969.
- [11] I. Bars, M.B. Halpern and M. Yoshimura, *Phys.Rev.* D7(**1973**):1233.
- [12] J.C. Pati and A. Salam, *Phys.Rev.* D8(**1973**):1240.
- [13] C. Itoh, T. Minamikawa, K. Miura and T. Watanabe, *Prog.Theor.Phys.* 51(**1974**):1575.
- [14] H. Georgi and S.L. Glashow, *Phys.Rev.Lett.* 32(**1974**):438.
- [15] H. Fritzsch and P. Minkowski, *Ann.Phys.* 93(**1975**):193.
- [16] F. Gursey, P. Ramond and P. Sikivie, *Phys.Lett.* 60B(**1976**):177.
- [17] X. Calmet, S.D. Hsu and D. Reeb, *Phys.Rev.Lett.* 101(**2008**):171802.
- [18] R. Blumenhagen, *Phys.Rev.Lett.* 102(**2009**):071601.
- [19] C.J. Isham, A. Salam and J. Strathdee, *Lett.Nuov.Cim.* 5(**1972**):969.
- [20] C.J. Isham, A. Salam and J. Strathdee, *Phys.Rev.* D8(**1973**):2600.

- [21] C.J. Isham, A. Salam and J. Strathdee, *Phys.Rev.* D9(**1975**):1702.
- [22] S.M. Barr and S. Raby, *Phys.Rev.Lett.* 79(**1997**):4748.
- [23] Z. Kakushadze and S.H.H. Tye, *Phys.Rev.* D55(**1997**):7878.
- [24] P. Das and P. Jain, *Phys.Rev.* D62(**2000**):075001.
- [25] C.H. Albright and S.M. Barr, *Phys.Rev.Lett.* 85(**2000**):244.
- [26] Yi-Fang Chang, *Hadronic J.* 7(**1984**):1118.
- [27] Yi-Fang Chang, *International Journal of Modern Mathematical Sciences.* 10(**2014**):75.
- [28] G. Triantaphyllou and G. Zoupanos, *Phys.Lett.* B489(**2000**):420.
- [29] S. Weinberg, *The Quantum Theory of Fields II. Modern Applications*. Cambridge University Press. **1995**.
- [30] G. 'tHooft, *Nucl.Phys.* B61(**1973**):455.
- [31] S. Weinberg, *Phys.Rev.* D8(**1973**):5497.
- [32] C. Nash, *Relativistic Quantum Fields*. Academic Press. **1978**.
- [33] T. van Ritbergen, J.A.M. Vermaseren and S.A. Larin, *Phys.Lett.* B400(**1997**):379.
- [34] K.G. Chetyrkin, B.A. Kniehl and M. Steinhauser, *Nucl.Phys.* B510(**1998**):61.
- [35] M. Czakon, *Nucl.Phys.* B710(**2005**):485.
- [36] J.C. Collins and A.J. Macfarlane, *Phys.Rev.* D10(**1974**):1201.
- [37] T.D. Lee, *Particle Physics and Introduction to Quantum Field Theories*. Amsterdam: OPA. **1981**.
- [38] Bing-Lin Yong, *Introduction to Field Theory*. Science Press. **1987**.
- [39] D.J. Gross and F. Wilczek, *Phys.Rev.Lett.* 30(**1973**):1343.
- [40] H.D. Politzer, *Phys.Rev.Lett.* 30(**1973**):1346.
- [41] K. Yagi, T. Hatsuda and Y. Miake, *Quark-Gluon Plasma: From Big Bang to Little Bang*. Cambridge University Press. **2005**.
- [42] Yi-Fang Chang, *International Journal of Physics and Applications.* 1(**2019**):30.
- [43] B.T. Feld, *Models of Elementary Particles*. Blaisdell Publishing Company. **1969**.

- [44] Yi-Fang Chang, *International Journal of Theoretical Physics*. 2(**2013**):184.
- [45] Yi-Fang Chang, *J. Yunnan University*. 30(**2008**):41.
- [46] Yi-Fang Chang, *J. Shangqiu Teachers College*. 24(**2008**):57.
- [47] Yi-Fang Chang, *Internal Review of Physics*. 5(**2011**):340.
- [48] Yi-Fang Chang, *International Journal of Theoretical Physics*. 2(**2013**):100.
- [49] Yi-Fang Chang, *International Journal of Modern Applied Physics*. 4(**2013**):69.
- [50] Yi-Fang Chang, *International Journal of Advanced Research in Physical Science*. 1(**2014**):11.
- [51] Yi-Fang Chang, *International Journal of Modern Theoretical Physics*. 5(**2016**): 22.
- [52] Yi-Fang Chang, *International Journal of Modern Applied Physics*. 9(**2019**):1.
- [53] Yi-Fang Chang, *Hadronic J*. 40(**2017**):291.
- [54] J.F. Owens, et al., *Phys.Rev.* D18(**1978**):1501.
- [55] H. Baer and R.N. Cahn, *J.Phys.G*. 37(**2010**)7A:376.
- [56] Yi-Fang Chang, *International Journal of Theoretical Physics*. 7(**2018**):16.
- [57] Yi-Fang Chang, *International Journal of Theoretical Physics*. 2(**2013**):53.
- [58] P.K.F. Grieder, *Extensive Air Showers*. Heidelberg, Springer-Verlag. **2010**.
- [59] T.K. Gaisser, *Cosmic Rays and Particle Physics*. Cambridge University Press. **1990**.
- [60] T. Stanev, *High Energy Cosmic Rays* (second edition). Springer-Praxis. **2010**.
- [61] Yi-Fang Chang, A preliminary scheme on a unification of four interactions. **1974**.
- [62] H. Terazawa, *Prog.Theor.Phys*. 58(**1977**):1276.
- [63] A. Ashtekar, *Phys.Rev.Lett*. 57(**1986**):2244.
- [64] A. Ashtekar, *Phys.Rev.* D36(**1987**):1587.
- [65] T. Jacobsoni and L. Smolin, *Nucl.Phys.* B299(**1988**):295.
- [66] Yi-Fang Chang, *J.Xinyang Normal University*. 17(**2004**)1:30.