ISSN: 2634-9221
European Journal of Applied Sciences; Vol. 14 No. 06 (2026) (78-94)
https://doi.org/10.14738/ejas.1406.12480

Energy Levels and Quantum Defect of Ions Silicon Si^{2+} and Germanium Ge^{2+} Calculated via Screening Constant by Unit Nuclear Charge Method

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Abstract: Resonance energies associated with the spectral energy levels of Rydberg atoms are of great practical importance for understanding the formation of heavy atoms in the galaxy, the opacity of emitted lines across all spectral regions, fusion research, stellar nucleosynthesis, environmental studies based on absorption spectroscopy, and the study of the chemical properties of astrophysical objects. In this paper, we calculate the energy of doubly excited states $3s^2ns\ ^2S$, $3s^2nd\ ^2D$, $3s^2np\ ^2P$, $3s^2nf\ ^2F$ et $3s^2ng\ ^2G$ from Si^{2+} et Ge^{2+}. Using the screen constant by unit nuclear charge (SCUNC) method as part of a semi-empirical procedure. The resonance energies, expressed in cm^{-1}, are listed in tables for $4 \leq n \leq 40$, and the results obtained are compared with those found in the NIST database as well as those from the "Weakest Bound Electron Potential Model Theory" (WBEPMT), and the results of W. C. Martin and Romuald Zalubas, (1983) and the works of Jack Sugar and Arlene Musgrove, (1993)

Keyword: Rydberg series, resonance energies, Quantum defects, screen constant by unit nuclear charge (SCUNC),

INTRODUCTION

Photoionization plays an important role in understanding photon-plasma interactions within many astrophysical objects, such as stars and nebulae [1]. In recent years, intensive research has been conducted on the photoionization of elements resulting from neutron capture ("n-capture" elements) [2], produced by nucleosynthesis via slow or fast neutron capture. This research has shed light on stellar nucleosynthesis processes as well as the chemical evolution of astrophysical objects such as galaxies. The 2009 work by Bautista and al. showed that silicon, once ionized as Si II, is predominant in the ultraviolet (UV) and optical spectra of various astrophysical plasmas [3]. With regard to absorption spectra, in the spectral region beyond 912 Å, Si II exhibits eight absorption line complexes associated with the fundamental term $3s^23p^2P^\circ$ [4]. In photoionized plasmas characterized by electron temperatures (Te) on the order of 10^4 K and electron densities [3] (ne) $\leq 10^4\ cm^{-3}$, the relative optical depths of the absorption troughs arising from the two ground-state levels depend on ne [12]. Consequently, this dependence can serve as a diagnostic tool for. In this regard, the lines centered at 1814 Å ($3s^23p\ ^2P^\circ$ – $3s3p^2\ ^2D$) are of particular interest due to their unusually weak oscillator forces; among all Si II lines, these are the ones most likely to fall within the linear portion of the growth curve, thus allowing for a precise

measurement of their column densities. However, determining precise and reliable oscillator forces for these transitions is, unfortunately, particularly difficult[13]. Like the other members of its group—carbon (lighter) as well as germanium, tin, and lead (heavier) it has as many valence electrons as it has valence orbitals. Synchrotron radiation was used to measure the absolute single-photoionization cross sections [16]. These experiments made it possible to precisely determine the positions of the resonances and quantum defects associated with the various series of silicon Si^{2+} synchrotron. Here we present additional data from laboratory experiments on the absorption of X-rays by the K-shell of Si^{+}, Si^{2+}, and Si^{3+} ions, which enable the study of the gaseous component of the interstellar medium. Like the other members of its group carbon (lighter) as well as germanium, tin, and lead (heavier) it has as many valence electrons as it has valence orbitals. According to the IUPAC (International Union of Pure and Applied Chemistry) in 2004, global production was estimated at 40 metric tons. In 2006, China was the leading producer, accounting for 79% of the global supply of germanium. To obtain ultra-high-purity germanium, germanium after being refined to remove copper, zinc, and lead is often used as a semiconductor in alloys containing small amounts of phosphorus, arsenic, gallium, and antimony. As such, it is used in transistors and optoelectronics. **Andersen and Lindgard** performed semi-empirical calculations to determine the lifetimes of the excited levels of the isoelectronic sequences of Ga I and In I, including Ge II [10]. When alloyed with silicon, germanium can be used to manufacture high-performance integrated circuits. Germanium can also be used in glass due to its infrared transparency. Generally speaking, as of 2007, germanium is primarily used in: the manufacture of optical fibers (35%), infrared optics (20%), catalysts (20%), and electronics (15%). Germanium minerals are extremely rare, but the element can be recovered from coal ash [7]. Kessler et al. (2007) studied multistage laser ionization via Rydberg states for Si and Ge. They examined the Rydberg series in the range $20 \leq n \leq 80$ and determined the ionization potential by fitting the series and quantum defects [8]. From a theoretical standpoint, few studies have been conducted to compare these results with previous measurements of single-photon photoionization cross sections [9]. In this context, the approximations by Jack Sugar and Arlene Musgrove were used to determine the first theoretical values of the positions of the resonances and quantum defects for the dominant states. Preliminary studies have shown that the SCUNC formalism has been used to determine the precise resonance energies of double-excited states for systems with different numbers of valence electrons. Section 2 provides a brief summary of the SCUNC formalism. Section 3 presents an analysis of the results obtained by comparing them with data from W. C. Martin and Romuald Zalubas available in the literature [17]. Finally, Section 4 summarizes and concludes this study.

THEORY

Brief Description of the SCUNC Formalism

In the framework of the Screening Constant per Unit Nuclear Charge formalism, the total energy of the $(Nl, nl', {}^{2S+1}L_j^{\pi})$ excited states is expressed in the form (in Rydberg) [9] [10]:

$$E\left(Nl, nl', {}^{2S+1}L_j^{\pi}\right) = -Z^2\left(\frac{1}{N^2} + \frac{1}{n^2}\left[1 - \beta\left(Nl, nl'; {}^{2S+1}L_j^{\pi}; Z\right)\right]^2\right). \quad (1)$$

In this equation, the principal quantum numbers N and n are, respectively, for the inner and the outer electron of the two-electron atomic systems. The ß-parameters are screening constants per unit nuclear charge expanded in inverse powers of the nuclear charge Z and given by:

$$\beta\left(Nl, nl'; {}^{2S+1}L_j^{\pi}; Z\right) = \sum_{k=1}^{q} f_k \left(\frac{1}{Z}\right)^k . \quad (2)$$

where $f_k = f_k\left(Nl, nl'; {}^{2S+1}L_j^{\pi}\right)$ are screening constants to be evaluated empirically.

For a given Rydberg series originating from a ${}^{2S+1}L_j^{\pi}$ state, we obtain [6]:

$$E_n = E_\infty - \frac{Z^2}{n^2}\left[1 - \beta(nl\,; s, \mu, \nu, {}^{2S+1}L_j^{\pi}; Z)\right]^2 . \quad (3)$$

In Equation (3), ν and μ (μ > ν) denote the principal quantum numbers of the $\left({}^{2S+1}L_j^{\pi}\right)nl$ Rydberg series used in the empirical determination of the f_i screening constants, s represents the spin of the nl-electron $\left(s = \frac{1}{2}\right)$, E_∞ is the energy value of the series limit, E_n denotes the resonance energy and Z_0 stands for the atomic number. The ß-parameters are screening constants per unit nuclear charge expanded in inverse powers of Z_0 and given by [6]:

$$\beta(Z, {}^{2S+1}L_j^{\pi}, n, s, \mu, \nu) = \sum_{k=1}^{q} f_k \left(\frac{1}{Z}\right)^k . \quad (4)$$

where $f_k = f_k\left({}^{2S+1}L_j^{\pi}, n, s, \mu, \nu\right)$ are screening constants to be evaluated empirically.

For a given Rydberg series originating from a ${}^{2S+1}L_j^{\pi}$ state, resonance energy is the in the form [6]:

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1\left({}^{2S+1}L_j^{\pi}\right)}{Z(n-1)} - \frac{f_2\left({}^{2S+1}L_j^{\pi}\right)}{Z} \pm \sum_{k=1}^{q}\sum_{k'=1}^{q'} f_1^{k'} F(n, \mu, \nu, s) \times \left(\frac{1}{Z}\right)^k\right\}^2 . \quad (5)$$

In Eq. (5), $f_1\left({}^{2S+1}L_j^{\pi}\right)$ and $f_2\left({}^{2S+1}L_j^{\pi}\right)$ and are screening constants to be evaluated empirically.

$\pm\sum_{k=1}^{q}\sum_{k'=1}^{q'} f_1^{k'} F(n, \mu, \nu, s) \times \left(\frac{1}{Z}\right)^k$ is a corrective term introduced to stabilize the resonance energies with increasing the principal quantum number n. The correct expression of this term is determined iteratively by imposing general Eq. (5) to give accurate data with a constant quantum defect values along all the considered series.

Besides, resonance energies are analyzed from the well-known standard quantum-defect expansion formula:

$$E_n = E_\infty - \frac{RZ_{core}^2}{(n-\delta)^2} . \quad (6)$$

In this equation, R, E_∞, Z_{core} and δ denote respectively the Rydberg constant, the converging limit, the electric charge of the core ion the quantum defect. In the framework

of the SCUNC procedure, theoretical and measured energy positions can be analyzed by calculating the Z^* effective charge given by:

$$Z^* = Z\left\{1 - \frac{f_1\left(^{2S+1}L_j^{\pi}\right)}{Z(n-1)} - \frac{f_2\left(^{2S+1}L_j^{\pi}\right)}{Z} \pm \sum_{k=1}^{q}\sum_{k'=1}^{q'} f_1^{k'}\, F(n,\mu,\nu,s) \times \left(\frac{1}{Z}\right)^k\right\}. \quad (7)$$

Besides, the f_2-parameter in Eq. (5) can be theoretically determined from Eq. (6) via the condition:

$$\lim_{n\to\infty} Z^* = Z\left\{1 - \frac{f_2\left(^{2S+1}L_j^{\pi}\right)}{Z}\right\} = Z_{core}\,. \quad (8)$$

So, we get then:

$$f_2 = Z - Z_{core}\,. \quad (9)$$

The single photoionization process from an atomic X^{p+}system is given by

$$X^{p+} + h\nu \rightarrow X^{(p+1)+} + e^-\,. \quad (10)$$

Using Eq. (10), we find $Z_{core} = p + 1$.

Besides, the relationship between the Z^*-effective charge and the δ-quantum defect is in the shape

$$Z^* = \frac{Z_{core}}{\left(1 - \frac{\delta}{n}\right)}\,. \quad (11)$$

Using Eq. (11), one gets the SCUNC procedure for the analysis of resonance energies

$$\begin{cases} Z^* \geq Z_{core}\ if\ \delta \geq 0 \\ Z^* \leq Z_{core}\ if\ \delta \leq 0 \\ \lim\limits_{n\to\infty} Z^* = 0 \end{cases}. \quad (12)$$

Analysis of Resonance Energies Using the Quantum Defect Model

For the Si^{2+} and Ge^{2+} ions considered in this work, Eq. (10) gives:

$$S_i{}^{2+} + h\nu \rightarrow S_i{}^{3+} + e^-\,. \quad (13)$$

$$G_e{}^{2+} + h\nu \rightarrow G_e{}^{3+} + e^-\,. \quad (14)$$

- For the Si^{2+} ion, we have Z_{core} = 3 and f_2 = (14-3) =11.0.
- For the Ge^{2+} ion we have Z_{core} = 3 and f_2 = (32-3) =29.0.

Resonance Energy Si^{2+} of 3s□ nl Transitions (l=s, p, d, f, g)

Using Eq. (5), we find the semi-empirical formulas used in the present work. (in Cm^{-1}).

- For the Rydberg serie [$3s^2$ ($^2S_{1/2}$)] ns

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1\left(^2S^o_{1/2}\right)}{Z(n-1)} - \frac{12.0}{Z} - \frac{(n-\mu)^2 f_1\left(^2S^o_{1/2}\right)}{Z^2(n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2S^o_{1/2}\right)}{Z\ (n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2S^o_{1/2}\right)}{Z\ (n-\nu+s)^3}\right\}^2 \quad (15)$$

- For the Rydberg serie [$3s^2(^2D_{3/2})$]nd

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1\left(^2D^o_{3/2}\right)}{Z(n-1)} - \frac{12.0}{Z} - \frac{(n-\mu)^2 f_1\left(^2D^o_{3/2}\right)}{Z^2(n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2D^o_{3/2}\right)}{Z\ (n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2D^o_{3/2}\right)}{Z\ (n-\nu+s)^3}\right\}^2 \quad (16)$$

- For the Rydberg serie [$4s(^2F_{1,2,3})$]nf

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1\left(^2F^o_{4/2}\right)}{Z(n-1)} - \frac{12.0}{Z} - \frac{(n-\mu)^2 f_1\left(^2F^o_{4/2}\right)}{Z^2(n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2F^o_{4/2}\right)}{Z\ (n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2F^o_{4/2}\right)}{Z\ (n-\nu+s)^3}\right\}^2 \quad (17)$$

- For the Rydberg serie [$4s\ (^2G_{5/2,7/2})$]ng pour

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1\left(^2G^o_{5/2}\right)}{Z(n-1)} - \frac{12.0}{Z} - \frac{(n-\mu)^2 f_1\left(^2G^o_{5/2}\right)}{Z^2(n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2G^o_{5/2}\right)}{Z\ (n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2G^o_{5/2}\right)}{Z\ (n-\nu+s)^3}\right\}^2 \quad (18)$$

Resonance Energy Ge^{2+} of 4s□ nl Transitions (l=s, p, d, f, g)

- For the Rydberg serie *[$3s^2\ (^2P_{1/2,3/2})$]np*

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1\left(^2P^o_{1/2}\right)}{Z(n-1)} - \frac{30.0}{Z} - \frac{(n-\mu)^2 f_1\left(\left(^2P^o_{1/2}\right)\right)}{Z^2(n-\nu+s/2)^3} + \frac{(n-\mu)^2 f_1\left(^2P^o_{1/2}\right)}{Z^2(n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2P^o_{1/2}\right)}{Z^2(n-\nu+4s)7}\right\}^2 \quad (19)$$

- For the Rydberg serie *[$4s^2(^2G_{7/2})$]ng*

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1\left(^2G^o_{7/2}\right)}{Z(n-1)} - \frac{12.0}{Z} - \frac{(n-\mu)^2 f_1\left(\left(^2G^o_{7/2}\right)\right)}{Z^2(n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2G^o_{7/2}\right)}{Z\ (n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2G^o_{7/2}\right)}{Z\ (n-\nu+s)^3}\right\}^2 \quad (20)$$

- For the Rydberg serie *[$4s^2(^2D_{3/2,7/2})$]nd*

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1\left(^2D^o_{3/2}\right)}{Z(n-1)} - \frac{12.0}{Z} - \frac{(n-\mu)^2 f_1\left(^2D^o_{3/2}\right)}{Z^2(n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2D^o_{3/2}\right)}{Z\ (n-\nu+s)^3} + \frac{(n-\mu)^2 f_1\left(^2D^o_{3/2}\right)}{Z\ (n-\nu+s)^3}\right\}^2 \quad (21)$$

- For the Rydberg serie *[$4s^2(^2F_{7/2})$]nf*

$$E_n = E_\infty - \frac{Z^2}{n^2}\left\{1 - \frac{f_1\left(^2F^o_{7/2}\right)}{Z(n-1)} - \frac{30.0}{Z} - \frac{(n-\mu)^2 f_1\left(^2F^o_{7/2}\right)}{Z^2(n-\nu+4s)^3} + \frac{(n-\mu)^2 f_1\left(^2F^o_{7/2}\right)}{Z\ (n-\nu+5s)^3} + \frac{(n-\mu)^2 f_1\left(^2F^o_{7/2}\right)}{Z\ (n-\nu+10s)^3}\right\}^2 \quad (22)$$

RESULTS AND DISCUSSION

The energy levels of the Rydberg series and the quantum defects of the second ionic state of silicon and germanium were calculated using experimental data from the National Institute of Standards and Technology (NIST) [15].

The doubly excited states with excitation energies from the Rydberg series and the quantum defects are calculated and then classified in Tables 2 through 9. For the doubly excited states of the silicon ion Si^{2+} $3s^2ns\ ^2S_{1/2,}$ $3s^2ns\ ^2D_{1/2,}$ $3s^2nf\ ^2F_{1/2,}$ $3s^2ng\ ^2G_{7/2,\ 9/2}$ as well as for the germanium ion Ge^{2+}, we calculated the following series: $4s^2np\ ^2P_{3/2,1/2,}$ $4s^2ng\ ^2G_{7/2,9/2}$, $4s^2nf\ ^2F_{5/2,7/2,}$ $4s^2nd\ ^2D_{3/2\ ,5/2}$. This second table presents the results for the resonance energies expressed En cm^{-1}, the quantum defect δ, and the effective charge Z* of the $3s^2ns\ ^2S_{1/2}$ Rydberg series of Si^{2+}. The results obtained using the SCUNC method are compared to

the values found in the NIST database by Martin and Romuald Zalubas [19], as well as to the results of theoretical calculations using WBEPMT. So far, the values obtained using the SCUNC method all converge toward the same energy limit, E = 128521.3 cm^{-1}. Thus, the adjustment of the SCUNC method's formula yields a fairly precise value of E_6 = 111150.534 cm^{-1} very close to the value in the NIST database, E_6 = 111184. 46 cm^{-1} with a difference ΔE = 33 cm^{-1} ≈ 0.0040 eV. It is observed that the results of the method are consistent from n = 4 to 9 with a quantum defect that remains constant and varies slightly in a decreasing manner.

The third table presents the results for the resonance energies (expressed in cm^{-1}), the quantum defect δ, and the effective charge Z* of the $3s^2ns\ ^2D_{1/2}$ Rydberg series of Si^{2+}. The results obtained using the SCUNC method are compared to the values found in the NIST database by Martin and Romuald Zalubas as well as to the results of theoretical calculations using the WBEPMT method. It is observed that the results of the method agree for n = 4 to 9 with a quantum defect of 0.21, compared to that of Martin and Romuald Zalubas and the WBEPMT results of Ahmed et al. [1], which is 0.22 a value that remains constant and varies slightly in a decreasing trend. In addition, the fourth table lists the results for the resonance energies (expressed in cm^{-1}), the quantum defect δ, and the effective charge Z^* of the $3s^2nf\ ^2F_{1/2}$ Rydberg series of Si^{2+}. The results obtained using the SCUNC method are compared with the values found in the NIST database by Martin and Romuald Zalubas as well as with the results of theoretical calculations using the WBEPMT method. It can be seen that the results from the SCUNC method are in agreement for n = 4 to 9, with energies equal to E_5 = 103556. 160 cm^{-1} and E_9 = 126356.669 cm^{-1}, respectively, with a quantum defect of 0.06 compared to that of Martin and Romuald Zalubas and the WBEPMT method by Ahmed et al. [1], which is also 0.06 a value that remains constant and varies slightly in a decreasing trend.

Table 5 lists the results obtained using the SCUNC method, which are compared to the NIST database as well as to the results of theoretical calculations using WBEPMT [1]. For the SCUNC resonance energies n = 4 to 9 associated with a constant quantum defect of 0.015, the SCUNC formulas once again exactly reproduce the data found in the NIST database.

Table 6 lists the resonance energies, the quantum defect (δ), and the effective charge (Z*) of the $4s^2np\ ^2P_{3/2,1/2}$ Rydberg series of Ge^{2+}. The results obtained by the SCUNC method are compared with NIST measurements from [3]. The energy limit E_∞ = 128521.3 cm^{-1} is taken from the NIS database.

Table 7 lists the resonance energies, quantum defect (δ), and effective charge (Z*) of the Ge^{2+}, $4s^2ng\ ^2G_{7/2,9/2}$ Rydberg series. The results obtained using the SCUNC method are compared with NIST measurements from [3].

Table 8 presents the results for the resonance energies, the quantum defect (δ), and the effective charge (Z^*) of the $4s^2nf\ ^2F_{5/2,7/2}$ Rydberg series of Ge^{2+}. The results obtained are consistent with those of Jack Sugar and Arlene Musgrove [3]. It should be noted that, within the SCUNC formalism, the parameter f1 is evaluated based on a single experimental data point for a given Rydberg series. The first experimental data point corresponding to the lowest energy value of the principal quantum number n associated with a specific quantum defect is the one taken into account, as explained in our various published articles since 2013; thus, several states are not used to adjust the value of f1. Based on this single data point, the SCUNC formulas generally reproduce other experimental data corresponding

to a constant quantum defect as n increases with excellent accuracy, as demonstrated in Tables 1 through 8 of this article.

Table 1: Rydberg series and screen constants f_1 and f_2 calculated using equation (6). The resonance energies (En cm^{-1}) and the energy at infinity are taken from the NIST database [19] and from [3].

Element	Spectroscopic notation		ν	E_∞ (Cm^{-1})	E_n(Cm^{-1})	f_1	f_2
Si^{2+}	$3s^2ns$	$(^2S^{\circ}_{1/2})ns$	4	128521.3	65500.47	-3.32994	12
	$3s^2ns$	$(^2D_{1/2})nd$	4	128521.3	101023.08	-0.35888	12
	$3s^2nf$	$(^2F_{1/2})nd$	4	128521.3	101023.05	-0.09191	12
	$3s^2ng$	$(^2G^2_{7/2,9/2})nf$	5	128521.3	114177.1	-0.02335	12
Ge^{2+}	$4s^2np$	$(^2P_{3/2,1/2})np$	5	131838.14	179366.53	-5.38538	30
	$4s^2ng$	$(^2G_{7/2,9/2})ng$	5	131838.14	110868.45	-0.02149	30
	$4s^2nf$	$(^2F_{5/2,7/2})nf$	4	131838.14	100317.28	-0.08351	30
	$4s^2nd$	$(^2D_{3/2,5/2})nd$	5	131838.14	100089.97	-2.17996	30

Table 2: Lists the resonance energies (in cm^{-1}), the quantum defect (δ), and the effective charge (Z*) of the $3s^2ns\ ^2S_{1/2}$ Rydberg series of Si^{2+}. The results obtained using the SCUNC method are compared with NIST measurements [19]as well as with WBEPMT calculations [1]. The energy limit (E_∞ = 131,838.14 cm^{-1}) is taken from the NIST database.

	SCUNC				WBEPMT		
	$3s^2ns\ ^2S_{1/2}$				$3s^2ns\ ^2S_{1/2}$		
n	SCUNC	NIST	SCUNC		WBEPMT		NIST
	E (Cm^{-1})	E (Cm^{-1})	δ	Z^*	E (Cm^{-1})	δ	E (Cm^{-1})
4	65500.470	65500.47	1.38	2.81	65500.47	1.42	65500.47
5	98351.222	97972.09	1.39	2.83	97972.09	1.38	97972.09
6	111150.534	111184.46	1.41	2.67	111184.46	1.38	111184.46
7	117783.962	117914.8	1.43	2.55	117914.80	1.38	117914.8
8	121680.206	121814.38	1.44	2.48	121814.38	1.38	121814.38
9	124160.696	124276.7	1.45	2.42	124276.68	1.38	124276.7
10	125835.313		1.46	2.37	125930.97	1.37	
11	127018.000		1.46	2.33	127095.96	1.37	
12	127883.705		1.47	2.30	127947.31	1.37	
13	128536.117		1.48	2.28	128588.33	1.37	
14	129039.818		1.48	2.26	129083.02	1.37	
15	129436.716		1.48	2.24	129472.76	1.37	
16	129754.951		1.49	2.22	129785.28	1.37	
17	130013.984		1.49	2.21	130039.71	1.37	
18	130227.620		1.49	2.20	130249.61	1.37	
19	130405.863		1.50	2.18	130424.79	1.37	
20	130556.113		1.50	2.18	130572.51	1.37	
21	130683.931		1.50	2.17	130698.23	1.37	
22	130793.566		1.50	2.16	130806.10	1.37	
23	130888.307		1.50	2.15	130899.36	1.37	
24	130970.732		1.51	2.14	130980.52	1.37	

25	131042.885				1.51		2.14		131051.59		1.37		
26	131106.403				1.51		2.13		131114.18		1.37		
27	131162.611				1.51		2.13		131169.59		1.37		
28	131212.587				1.51		2.12		131218.87		1.37		
29	131257.219				1.51		2.12		131262.90		1.37		
30	131297.242				1.51		2.11		131302.39		1.37		
31	131333.270				1.51		2.11		131337.95		1.37		
32	131365.817				1.52		2.11		131370.08		1.37		
33	131395.317				1.52		2.10		131399.22		1.37		
34	131422.138				1.52		2.10		131425.71		1.37		
35	131446.596				1.52		2.10		131449.88		1.37		
36	131468.959				1.52		2.10		131471.98		1.37		
37	131489.460				1.52		2.09		131492.25		1.37		
38	131508.301				1.52		2.09		131510.88		1.37		
39	131525.656				1.52		2.09		131528.05		1.37		
40	131541.676				1.43		2.09		131543.90		1.37		
…	…												
∞	131838.14												

Table 3: Lists the resonance energies (in cm^{-1}), the quantum defect (δ), and the effective charge (Z*) of the $3s^2ns\ ^2D_{1/2}$ Rydberg series of Si^{2+}. The results obtained using the SCUNC method are compared to NIST measurements [19] as well as to WBEPMT calculations [1]. The energy limit (E_∞ = 131,838.14 cm^{-1}) is taken from the NIST database

	SCUNC						WBEPMT			
	$3s^2nd\ ^2D_{1/2}$						$3s^2nd\ ^2D_{1/2}$			
n	SCUNC		NIST		SCUNC		WBEPMT			NIST
	E (Cm^{-1})		E (Cm^{-1})		δ	Z^*	E (Cm^{-1})		δ	E (Cm^{-1})
4	101023.080		101023.05		0.21	2.12	101023.05		0.10	101023,05
5	112696.612		112394.56		0.21	2.09	112394.56		0.22	112394,56
6	118754.996		118522.86		0.20	2.07	118522.86		0.24	118522,86
7	122336.056		122163.48		0.20	2.06	122163.14		0.25	122163,48
8	124623.276		124495.7		0.20	2.05	124495.21		0.26	124495,7
9	126173.060				0.20	2.04	126076.98		0.26	
10	127271.766				0.19	2.04	127198.41		0.27	
11	128079.017				0.19	2.04	128022.00		0.27	
12	128689.548				0.19	2.03	128644.48		0.27	
13	129162.488				0.19	2.03	129126.32		0.27	
14	129536.299				0.19	2.03	129506.87		0.27	
15	129836.879				0.19	2.03	129812.64		0.27	
16	130082.186				0.19	2.02	130062.00		0.27	
17	130284.990				0.19	2.02	130268.01		0.27	
18	130454.576				0.19	2.02	130440.16		0.28	
19	130597.822				0.19	2.02	130585.49		0.27	
20	130719.916				0.19	2.02	130709.28		0.27	
21	130824.826				0.19	2.02	130815.59		0.27	
22	130915.634				0.19	2.02	130907.57		0.27	
23	130994.758				0.19	2.02	130987.67		0.27	

24	131064.120				0.19		2.02		131057.86		0.27		
25	131125.264				0.19		2.01		131057.86		0.27		
26	131179.437				0.19		2.01		131119.71		0.27		
27	131227.661				0.18		2.01		131174.49		0.27		
28	131270.776				0.18		2.01		131223.23		0.27		
29	131309.479				0.18		2.01		131266.80		0.27		
30	131344.353				0.18		2.01		131341.11		0.27		
31	131375.886				0.18		2.01		131372.94		0.27		
32	131404.492				0.18		2.01		131401.81		0.27		
33	131430.522				0.18		2.01		131428.07		0.27		
34	131454.277				0.18		2.01		131452.03		0.27		
35	131476.014				0.18		2.01		131473.95		0.27		
36	131495.955				0.18		2.01		131494.06		0.27		
37	131514.293				0.18		2.01		131512.55		0.27		
38	131531.196				0.18		2.01		131529.58		0.27		
39	131546.809				0.18		2.01		131545.31		0.27		
40	131561.260				-1.82		2.01		131559.87		0.27		
…	…								…				
∞	131838.14								131838.14				

Table 4: Lists the resonance energies (in cm^{-1}), the quantum defect (δ), and the effective charge (Z*) of the $3s^2nf\ ^2F_{1/2}$ Rydberg series of Si^{2+}. The results obtained using the SCUNC method are compared to NIST measurements [19] as well as to WBEPMT calculations [1]. The energy limit (E_∞ = 131838.14 cm^{-1}) is taken from the NIST database.

	SCUNC						WBEPMT			
	$3s^2nf\ ^2F_{1/2}$						$3s^2nf\ ^2F_{1/2}$			
N	SCUNC		NIST		SCUNC		WBEPMT			NIST
	E (Cm^{-1})		E (Cm^{-1})		δ	Z^*	E (Cm^{-1})		δ	E (Cm^{-1})
4	103556.160		103556.16		0.06	2.00	103556.16		0.06	103556.16
5	113912.158		113760.32		0.05	2.00	113760.32		0.07	113760.32
6	119425.244		119311.34		0.05	2.00	119311.34		0.08	119311.34
7	122743.494		122655.37		0.05	2.00	122655.37		0.08	122655.37
8	124889.566		124822.14		0.05	2.00	124822.21		0.09	124822.14
9	126356.669		126304.82		0.05	2.00	126305.00		0.09	126304.82
10	127403.711				0.05	2.00	127363.64		0.09	
11	128177.014				0.05	2.00	128145.52		0.09	
12	128764.320				0.05	2.00	128739.22		0.09	
13	129220.835				0.05	2.00	129200.56		0.09	
14	129582.702				0.05	2.00	129566.13		0.10	
15	129874.389				0.05	2.00	129860.69		0.10	
16	130112.938				0.05	2.00	130101.49		0.10	
17	130310.516				0.05	2.00	130300.86		0.10	
18	130475.995				0.05	2.00	130467.79		0.10	
19	130615.971				0.05	2.00	130608.93		0.10	
20	130735.428				0.05	2.00	130729.35		0.10	
21	130838.189				0.05	2.00	130832.91		0.10	
22	130927.226				0.05	2.00	130922.61		0.10	

23	131004.879				0.05		2.00		131000.82		0.10		
24	131073.010				0.05		2.00		131069.42		0.10		
25	131133.113				0.05		2.00		131129.93		0.10		
26	131186.403				0.05		2.00		131183.56		0.10		
27	131233.871				0.05		2.00		131231.33		0.10		
28	131276.335				0.05		2.00		131274.05		0.10		
29	131314.476				0.05		2.00		131312.42		0.10		
30	131348.861				0.05		2.00		131347.00		0.10		
31	131379.966				0.05		2.00		131378.27		0.10		
32	131408.197				0.05		2.00		131406.65		0.10		
33	131433.897				0.05		2.00		131432.49		0.10		
34	131457.359				0.05		2.00		131456.07		0.10		
35	131478.837				0.05		2.00		131477.65		0.10		
36	131498.547				0.05		2.00		131497.46		0.10		
37	131516.679				0.05		2.00		131515.67		0.10		
38	131533.396				0.05		2.00		131532.47		0.10		
39	131548.842				0.05		2.00		131547.98		0.10		
40	131563.143				0.05		2.00		131562.35		0.10		
…	…								…				
∞	131838.14								131838.14				

Table 5: Lists the resonance energies (in cm^{-1}), the quantum defect (δ), and the effective charge (Z^*) of the $3s^2ng\ ^2G_{7/2,\ 9/2}$ Rydberg series of Si^{2+}. The results obtained using the SCUNC method are compared to NIST measurements [19] as well as to WBEPMT calculations [1]. The energy limit (E_∞ = 131,838.14 cm^{-1}) is taken from the NIS database.

	SCUNC					WBEPMT		
	$3s^2ng\ (^2G_{7/2,\ 9/2})$					$3s^2ng\ (^2G_{7/2,\ 9/2})$		
n	SCUNC	NIST	SCUNC		WBEPMT		NIST
	E (Cm^{-1})	E (Cm^{-1})	δ	Z^*	E (Cm^{-1})	δ	E (Cm^{-1})
5	114177.1	114177.1	0.015	2.00	114177.10	0.014	114177.1
6	119594.484	119578.23	0.012	2.00	119578.23	0.016	119578.23
7	122845.921	122835.13	0.013	2.00	122835.13	0.017	122835.13
8	124956.783	124948.4	0.013	2.00	124948.40	0.017	124948.4
9	126403.162	126396.47	0.013	2.00	126396.65	0.017	126396.47
10	127437.195	127432.02	0.013	2.00	127432.11	0.017	
11	128201.923		0.013	2.00	128197.92	0.018	
12	128783.348		0.013	2.00	128780.16	0.018	
13	129235.697		0.013	2.00	129233.12	0.018	
14	129594.531		0.013	2.00	129592.43	0.018	
15	129883.957		0.013	2.00	129882.22	0.018	
16	130120.786		0.012	2.00	130119.34	0.018	
17	130317.033		0.012	2.00	130315.81	0.018	
18	130481.466		0.012	2.00	130480.43	0.018	
19	130620.609		0.012	2.00	130619.72	0.018	
20	130739.393		0.012	2.00	130738.63	0.018	
21	130841.605		0.012	2.00	130840.94	0.018	
22	130930.191		0.012	2.00	130929.61	0.018	
23	131007.468		0.012	2.00	131006.96	0.018	
24	131075.284		0.012	2.00	131074.84	0.018	
25	131135.122		0.012	2.00	131134.72	0.018	
26	131188.185		0.012	2.00	131187.83	0.018	

27	131235.46		0.012	2.00	131235.14	0.018			
28	131277.759		0.012	2.00	131277.47	0.018			
29	131315.755		0.012	2.00	131315.50	0.018			
30	131350.015		0.012	2.00	131349.78	0.018			
31	131381.011		0.012	2.00	131380.80	0.018			
32	131409.146		0.012	2.00	131408.95	0.018			
33	131434.761		0.012	2.00	131434.59	0.018			
34	131458.149		0.012	2.00	131457.99	0.018			
35	131479.56		0.012	2.00	131479.41	0.018			
36	13149.211		0.012	2.00	131499.08	0.018			
37	131517.29		0.012	2.00	131517.17	0.018			
38	131533.96		0.012	2.00	131533.85	0.018			
39	131549.363		0.012	2.00	131549.26	0.018			
40	131563.626		0.012	1.98	131563.53	0.018			
…	…				…				
∞	131838,14				131838,14				

Table 6: Lists the resonance energies (in cm^{-1}), the quantum defect (δ), and the effective charge (Z*) of the $4s^2np\ ^2P_{3/2\ 1/2}$ Rydberg series of Ge^{2+}. The results obtained using the SCUNC method are compared with NIST measurements [3]. The energy limit (E_∞ = 128,521.3 cm^{-1}) is taken from the NIS database.

	SCUNC				SCUNC		
	$4s^2np\ ^2P_{3/2}$				$4s^2np\ ^2P_{1/2}$		
n	SCUNC	NIST	SCUNC		SCUNC		NIST
	E (Cm^{-1})	E (Cm^{-1})	δ	Z^*	E (Cm^{-1})	δ	E (Cm^{-1})
5	79366.530	79366.494	2.01	2.00	79007.315	2.02	79366.494
6	101187.415	101244.63	1.99	2.00	101125.772	2.00	101244.63
7	110477.215	111091.55	2.07	2.00	111007.073	1.99	111091.55
8	115786.660	116406.61	2.13	2.00	116430.171	1.97	116406.61
9	119092.415	119607.77	2.18	2.00	119707.571	1.94	119607.77
10	121277.043	121685.74	2.22	2.00	121829.257	1.90	121685.74
11	122790.524		2.25	2.00	123276.637	1.85	
12	123879.664		2.28	2.00	124305.650	1.80	
13	124688.227		2.30	2.00	125062.068	1.74	
14	125304.253		2.32	2.00	125633.636	1.67	
15	125783.969		2.34	2.00	126075.620	1.60	
16	126164.582		2.35	2.00	126424.181	1.53	
17	126471.481		2.37	2.00	126703.751	1.46	
18	126722.448		2.38	2.00	126931.305	1.38	
19	126930.232		2.39	2.00	127118.923	1.31	
20	127104.159		2.40	2.00	127275.385	1.23	
21	127251.176		2.41	2.00	127407.195	1.15	
22	127376.541		2.42	2.00	127519.250	1.07	
23	127484.292		2.43	2.00	127615.293	0.99	
24	127577.570		2.43	2.00	127698.224	0.91	
25	127658.848		2.44	2.00	127770.317	0.82	
26	127730.093		2.45	2.00	127833.374	0.74	
27	127792.887		2.45	2.00	127888.840	0.66	
28	127848.513		2.46	2.00	127937.882	0.57	
29	127898.018		2.46	2.00	127981.453	0.48	

30	127942.267			2.47	2.00	128020.334	0.40		
31	127981.976			2.47	2.00	128055.174	0.31		
32	128017.746			2.48	2.00	128086.512	0.23		
33	128050.079			2.48	2.00	128114.802	0.14		
34	128079.401			2.48	2.00	128140.425	0.05		
35	128106.074			2.49	2.00	128163.706	-0.04		
36	128130.407			2.49	2.00	128184.920	-0.12		
37	128152.666			2.49	2.00	128204.305	-0.21		
38	128173.080			2.50	2.00	128222.066	-0.30		
39	128191.846			2.50	2.00	128238.377	-0.39		
40	128209.138			2.50	1.98	128253.393	-0.48		
…	…					….			
∞	128521.3					128521.3			

Table 7: Lists the resonance energies (in cm^{-1}), the quantum defect (δ), and the effective charge (Z*) of the Ge^{2+} $3s^2$ ng $^2G_{7/2,9/2}$ Rydberg series. The results obtained using the SCUNC method are compared with NIST measurements [3]. The energy limit (E_∞ = 128521.3 cm^{-1}) is taken from the NIS database.

	SCUNC				WBEPMT		
	$3s^2$ ($^2G_{7/2}$)ng				$3s^2$ ($^2G_{9/2}$)ng		
n	SCUNC	NIST	SCUNC		WBEPMT		NIST
	E (Cm^{-1})	E (Cm^{-1})	δ	Z*	E (Cm^{-1})	δ	E (Cm^{-1})
5	110868.45	110868.45	0.013	2.36	110868.45	0.013	110868.45
6	116275.51	116266.75	0.013	2.09	116266.75	0.015	116266.75
7	119530.83	119521.83	0.013	2.91	119521.84	0.016	119521.83
8	121641.49	121633.87	0.012	2.78	121633.87	0.016	121633.87
9	123087.48	123081.34	0.012	2.68	123081.33	0.017	123081.34
10	124121.22		0.012	2.60	124116.30	0.017	110868.45
11	124885.74		0.012	2.54	124881.79	0.017	
12	125467.01		0.012	2.49	125463.82	0.018	
13	125919.25		0.012	2.45	125916.65	0.018	
14	126278.01		0.012	2.42	126275.86	0.018	
15	126567.37		0.012	2.39	126565.59	0.018	
16	126804.16		0.011	2.36	126802.66	0.018	
17	127000.37		0.011	2.34	126999.10	0.018	
18	127164.77		0.011	2.32	127163.69	0.018	
19	127303.89		0.011	2.30	127302.97	0.018	
20	127422.66		0.011	2.29	127421.86	0.018	
21	127524.86		0.011	2.27	127524.16	0.018	
22	127613.43		0.011	2.26	127612.82	0.018	
23	127690.70		0.011	2.25	127690.16	0.018	
24	127758.51		0.011	2.24	127758.03	0.018	
25	127818.34		0.011	2.23	127817.91	0.018	
26	127871.39		0.011	2.22	127871.02	0.018	
27	127918.66		0.011	2.21	127918.33	0.018	
28	127960.96		0.011	2.20	127960.65	0.018	
29	127998.95		0.011	2.19	127960.65	0.018	
30	128033.21		0.011	2.19	128032.96	0.018	

31	128064.20				0.011		2.18		128063.97		0.018		
32	128092.33				0.011		2.18		128092.13		0.018		
33	128117.94				0.011		2.17		128117.76		0.018		
34	128141.33				0.011		2.16		128141.16		0.018		
35	128162.74				0.011		2.16		128162.58		0.018		
36	128182.39				0.011		2.16		128182.24		0.018		
37	128200.47				0.011		2.15		128200.33		0.018		
38	128217.14				0.011		2.15		128217.01		0.018		
39	128232.54				0.011		2.14		128232.42		0.018		
40	128246.80				0.011		2.14		128246.69		0.018		
…	…								…				
∞	128521.3								128521.3				

Table 8: Lists the resonance energies (in cm^{-1}), the quantum defect (δ), and the effective charge (Z*) of the $4s^2nf\ ^2F_{5/2\ 7/2}$ Rydberg series of Ge^{2+}. The results obtained using the SCUNC method are compared with NIST measurements [3]. The energy limit (E_∞ = 128521.3 cm^{-1}) is taken from the NIS database.

	SCUNC					SCUNC		
	$4s^2nf\ ^2F_{5/2}$					$4s^2nf\ ^2F_{7/2}$		
n	SCUNC	NIST	SCUNC		SCUNC		NIST	
	E (Cm^{-1})	E (Cm^{-1})	δ	Z^*	E (Cm^{-1})	δ	E (Cm^{-1})	
4	100317.283	100317.283	0.05	2.02	100317.976	0.05	100317.976	
5	110594.422	110504.58	0.05	2.02	110594.748	0.05	110505.22	
6	116123.476	116041.231	0.05	2.02	116123.654	0.05	116041.81	
7	119437.826	119521.835	0.05	2.01	119437.932	0.05	119521.837	
8	121580.493	121532.26	0.05	2.01	121580.561	0.05	121532.56	
9	123045.333	123008.34	0.05	2.01	123045.379	0.05	123008.96	
10	124090.884	124062.8	0.05	2.01	124090.916	0.05	124062.8	
11	124863.18	124841.2	0.05	2.01	124863.204	0.05	124841.2	
12	125449.787		0.05	2.01	125449.805	0.05		
13	125905.802		0.05	2.01	125905.816	0.05		
14	126267.304		0.04	2.01	126267.314	0.04		
15	126558.717		0.04	2.01	126558.725	0.04		
16	126797.057		0.04	2.01	126797.064	0.04		
17	126994.473		0.04	2.01	126994.478	0.04		
18	127159.825		0.04	2.00	127159.829	0.04		
19	127299.699		0.04	2.00	127299.703	0.04		
20	127419.074		0.04	2.00	127419.077	0.04		
21	127521.768		0.04	2.00	127521.77	0.04		
22	127610.75		0.04	2.00	127610.752	0.04		
23	127688.357		0.04	2.00	127688.359	0.04		
24	127756.449		0.04	2.00	127756.451	0.04		
25	127816.52		0.04	2.00	127816.521	0.04		
26	127869.782		0.04	2.00	127869.783	0.04		
27	127917.226		0.04	2.00	127917.227	0.04		
28	127959.67		0.04	2.00	127959.671	0.04		
29	127997.793		0.04	2.00	127997.794	0.04		
30	128032.162		0.04	2.00	128032.163	0.04		
31	128063.255		0.04	2.00	128063.255	0.04		
32	128091.474		0.04	2.00	128091.474	0.04		
33	128117.163		0.04	2.00	128117.164	0.04		
34	128140.616		0.04	2.00	128140.617	0.04		
35	128162.085		0.04	2.00	128162.086	0.04		

36	128181.788			0.04	2.00	128181.789	0.04		
37	128199.914			0.04	2.00	128199.914	0.04		
38	128216.625			0.04	2.00	128216.626	0.04		
39	128232.066			0.04	2.00	128232.067	0.04		
40	128246.362			0.04	2.00	128246.363	0.04		
…									
∞									

Table 9: Lists the resonance energies (in cm^{-1}), the quantum defect (δ), and the effective charge (Z*) of the Ge^{2+} Rydberg series $4s^2nd\ ^2D_{5/2,3/2s}$. The results obtained using the SCUNC method are compared with NIST measurements [3]. The energy limit (E_∞ = 128521.3 cm^{-1}) is taken from the NIS database.

	SCUNC					SCUNC		
	$4s^2nd\ ^2D_{3/2}$					$4s^2nd\ ^2D_{5/2}$		
n	SCUNC	NIST	SCUNC			SCUNC		NIST
	E (Cm^{-1})	E (Cm^{-1})	δ		Z^*	E (Cm^{-1})	δ	E (Cm^{-1})
5	100089.97	100089.7	2.15		2.54	100130.812	1.07	100130.812
6	110432.32	110378.07	1.07		2.44	110454.041	1.07	110397.63
7	116012.49	115977.8	1.07		2.36	116025.388	1.07	115989.77
8	119360.29	119339.11	1.08		2.31	119368.57	1.07	119345.75
9	121524.73	121511.36	1.08		2.27	121530.356	1.08	121515.82
10	123004.08		1.08		2.24	123008.083	1.08	
11	124059.60		1.08		2.22	124062.541	1.08	
12	124838.93		1.08		2.20	124841.156	1.08	
13	125430.63		1.08		2.18	125432.354	1.08	
14	125890.41		1.08		2.17	125891.778	1.08	
15	126254.76		1.08		2.16	126255.861	1.08	
16	126548.37		1.08		2.15	126549.262	1.08	
17	126788.42		1.08		2.14	126789.157	1.08	
18	126987.19		1.08		2.13	126987.807	1.08	
19	127153.62		1.08		2.12	127154.149	1.08	
20	127294.38		1.08		2.11	127294.827	1.08	
21	127414.48		1.09		2.11	127414.861	1.08	
22	127517.77		1.09		2.10	127518.101	1.08	
23	127607.25		1.09		2.10	127607.539	1.08	
24	127685.28		1.09		2.09	127685.529	1.08	
25	127753.72		1.09		2.09	127753.946	1.08	
26	127814.10		1.09		2.09	127814.294	1.08	
27	127867.62		1.09		2.08	127867.793	1.08	
28	127915.28		1.09		2.08	127915.443	1.08	
29	127957.92		1.09		2.08	127958.065	1.08	
30	127996.22		1.09		2.08	127996.343	1.08	
31	128030.73		1.09		2.07	128030.847	1.08	
32	128061.95		1.09		2.07	128062.059	1.08	
33	128090.29		1.09		2.07	128090.383	1.08	
34	128116.08		1.09		2.07	128116.166	1.08	
35	128139.62		1.09		2.06	128139.702	1.08	
36	128161.17		1.09		2.06	128161.245	1.08	
37	128180.95		1.09		2.06	128181.015	1.08	
38	128199.14		1.09		2.06	128199.199	1.08	
39	128215.91		1.09		2.06	128215.964	1.08	
…	128231.40		1.09		2.06	100130.812	1.07	
	….		…		…			
∞	128521.3					128521.3		

CONCLUSION

In this article, we used the screen constant per unit nuclear charge method to calculate the resonance energies of the Rydberg series of the doubly excited states of Si^{2+} and Ge^{2+} ions. We calculated the following series: $3s^2ns\ ^2S_{1/2,}\ 3s^2ns\ ^2D_{1/2,}\ 3s^2nf\ ^2F_{1/2,}\ 3s^2ng\ ^2G_{7/2,\ 9/2}$ for the Si^{2+} ion, and for the Ge^{2+} ion, we calculated the following series: $4s^2np\ ^2P_{3/2,1/2,}\ 4s^2ng\ ^2G_{7/2,9/2}$, $4s^2nf\ ^2F_{5/2,7/2,}\ 4s^2nd\ ^2D_{3/2\ ,5/2}$. In this work, the Rydberg energy levels and quantum defects of the first ionic states of silicon II and germanium II are calculated [1] theoretically using the screen constant per unit nuclear charge (SCUNC) method [3]. The experimental data were obtained from NIST (National Institute of Standards and Technology) [16]. Based on the experimental data, quantum defects and Rydberg energy states were calculated; principal quantum numbers were assigned to each resonance and confirmed by correlation with known low-energy levels. In the case of nickel Si^{2+} and Ge^{2+}, spectroscopic studies aimed at identifying an intense autoionization resonance suitable for titanium-sapphire laser systems will be conducted shortly.

Acknowledment:

Authors are grateful to the Orsay Institute of Molecular Sciences (OIMS), Orasy, Paris, France, and the Abdus Salam International Center for Theoretical Physic (ICTP), Trieste, Italy.

Funding:

No external funding received

Competing interests:

Authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported to this paper.

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