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Mathematics to Physics --- Mathematically Deriving Novel Field Equations of Electromagnetics and Gravity with Sources Moving with Velocity, Acceleration, and Spin, Respectively

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Abstract: In this article, we show that the mathematical vector calculus identities are powerful tool in both rederiving the existing physical laws, such as Maxwell equations, and discovering new physical laws. First, we show that the following physical laws can be rederived from the vector calculus identities: (1) Poynting's Theorem; (2) Gauss's Law for Magnetism; (3) Charge density Continuity equation; (4) Wave equation of potential A, electric field E, and magnetic field B; (5) Faraday's Law of Induction; (6) energy density of the magnetic field; (7) time change of energy density of the electric field; (8) time change of energy density of the magnetic field. Rederiving the above physical laws (1) to (8) shows the validation and power of the vector calculus identities in discovering physical laws. Then, applying the vector calculus identities, we derive NEW physical field equations: (1) for electric charges moving with either uniform velocity v, or constant acceleration a, or continuous spin S; (2) for gravitational charges (mass) moving with either uniform velocity v, or constant acceleration a, or continuous spin S. Novel Electromagnetics and gravitoelectromagnetics are derived from the vector calculus identities and have the same form, and thus can be unified. The new physical field equations need to be tested experimentally.

Keyword: vector calculus identities, field equation, electric field, magnetic field, electromagnetics, gravitoelectric field, gravitomagnetic field, gravitoelectromagnetics, electric field source, magnetic field source, gravitation source, velocity, acceleration, spin, unification of electromagnetics and gravity

INTRODUCTION

Historically, some of the physical laws were discovered based on experimental data,

Experiment → physics law

In this article, we show a different approaching: (1) starting with mathematical vector calculus identities; (2) substituting the parameters of physical fields into the mathematical vector calculus identities to obtain the physical laws; (3) the so-derived laws need to be tested by experiments:

Mathematical equations → physics laws → Experimental testing

In this article, we start with the vector calculus identities which are a branch of mathematics concerned with differentiation of vector fields (gradient, divergence, curl, and combination of above) [1]. We show that the mathematical vector calculus identities can be used to analyze how physical fields, such as electric fields, magnetic fields, and gravitation fields, change in magnitude and direction across space. By substituting the parameters of physical fields into vector calculus identities, we derive the new physical field equations to describe physical fields, such as electric field, magnetic field, and gravity. Some of so-derived physical field equations can be confirmed by existing experiments. New derived physical field equations need to be confirmed experimentally.

The purpose of this article is to derive new physical laws due to the source moving with different motion status. The magnetic field B is created by current density $\mathbf{j} = \rho\mathbf{v}$, where the velocity v in vacuum is treated as a constant. By analogy, we list three moving statuses of charges.

Moving statuses of source	**Current Density**
Stationary	$\mathbf{j} = 0$
Uniform velocity: **v**	$\mathbf{j} \propto \mathbf{v}$
Constant acceleration: **a**	$\mathbf{j_a} \propto \mathbf{a}$
Constant spin: **S**	$\mathbf{j_S} \propto \mathbf{S}$

Note: For the purpose of deriving field equation, we treat the Spin as one of motion status.

Since the velocity **v** of electric charge creates a magnetic field, we propose the following:

1. The acceleration **a** of electric charge creates magnetic-type fields;
2. The Spin **S** of electric charge creates magnetic-type fields;
3. The velocity **v** of gravitational charge creates gravitomagnetic fields;
4. The acceleration **a** of gravitational charges creates gravitomagnetic-type fields;
5. The Spin **S** of gravitational charge creates gravitomagnetic-type fields.

In Section 2, we: (1) list the existing vector calculus identities; (2) propose new vector calculus identities.

In Section 3, we apply vector calculus identities to electromagnetics: (1) rederive the Poynting's theorem, and derive new physical laws; (2) derive new extended electric field equations and magnetic field equations due to the ***velocity***, due to the ***acceleration***, and due to the ***Spin***, respectively.

In Section 4, following the same approach of Section 3, we utilize the concept of the gravitoelectromagnetic field, then apply vector calculus identities to gravitoelectromagnetic fields, and derive new field equation for them.

VECTOR CALCULUS IDENTITIES

In Section 2, we list 11 vector calculus identities ([Wikipedia] [1]), and propose 9 new vector calculus identities. Then in Section 3, we utilized them to derive existing and new field equations of Electromagnetics. In Section 4, we utilized them to derive new field equations of Gravity, Gravitoelectromagnetics.

1. Divergence of cross product of two vector fields: $\nabla \cdot (\mathrm{Y} \times \mathrm{Z})$

$$\nabla \cdot (\mathrm{Y} \times \mathrm{Z}) = (\nabla \times \mathrm{Y}) \cdot \mathrm{Z} - \mathrm{Y} \cdot (\nabla \times \mathrm{Z}) \tag{1}$$

2. Curl of cross product of two vector fields: $\nabla \times (\mathrm{Y} \times \mathrm{Z})$

$$\nabla \times (\mathrm{Y} \times \mathrm{Z}) = \mathrm{Y}(\nabla \cdot \mathrm{Z}) - \mathrm{Z}(\nabla \cdot \mathrm{Y}) + (\mathrm{Z} \cdot \nabla)\mathrm{Y} - (\mathrm{Y} \cdot \nabla)\mathrm{Z} \tag{2}$$

3. Divergence of Curl of one vector field: $\nabla \cdot (\nabla \times \mathrm{Y})$

$$\nabla \cdot (\nabla \times \mathrm{Y}) = 0 \tag{3}$$

4. Vector Laplacian of one vector field: $\nabla \times (\nabla \times \mathrm{Y})$

$$\nabla \times (\nabla \times \mathrm{Y}) = \nabla(\nabla \cdot \mathrm{Y}) - \nabla^2 \mathrm{Y} \tag{4}$$

5. Scalar Laplacian of a scalar potential: $\nabla \cdot (\nabla \varphi)$

$$\nabla \cdot (\nabla \varphi) = \nabla^2 \varphi \tag{5}$$

6. Curl of Gradient of one scalar potential: $\nabla \times (\nabla \varphi)$

$$\nabla \times (\nabla \varphi) = 0 \tag{6}$$

7. Divergence of product of one scalar potential and one vector field: $\nabla \cdot (\varphi \mathrm{Y})$

$$\nabla \cdot (\varphi \mathrm{Y}) = \varphi(\nabla \cdot \mathrm{Y}) + \mathrm{Y} \cdot (\nabla \varphi) \tag{7}$$

8. Curl of product of one scalar potential and one vector field: $\nabla \times (\varphi \mathrm{Y})$

$$\nabla \times (\varphi \mathrm{Y}) = \varphi(\nabla \times \mathrm{Y}) + (\nabla \varphi) \times \mathrm{Y} \tag{8}$$

9. Gradient of dot product of two vector fields: $\nabla(\mathrm{Y} \cdot \mathrm{Z})$.

$$\nabla(\mathrm{Y} \cdot \mathrm{Z}) = (\mathrm{Y} \cdot \nabla)\mathrm{Z} + (\mathrm{Z} \cdot \nabla)\mathrm{Y} + \mathrm{Y} \times (\nabla \times \mathrm{Z}) + \mathrm{Z} \times (\nabla \times \mathrm{Y}) \tag{9}$$

10. Vector-Cross-Del operator of two vector fields: $(\mathrm{Y} \times \nabla) \cdot \mathrm{Z}$

$$(\mathrm{Y} \times \nabla) \cdot \mathrm{Z} = \mathrm{Y} \cdot (\nabla \times \mathrm{Z}) \tag{10}$$

11. Vector-Cross-Del operator of two vector fields: $(\mathrm{Y} \times \nabla) \times \mathrm{Z}$

$$(\mathrm{Y} \times \nabla) \times \mathrm{Z} = \mathrm{Y} \times (\nabla \times \mathrm{Z}) + (\mathrm{Y} \cdot \nabla)\mathrm{Z} - \mathrm{Y}(\nabla \cdot \mathrm{Z}) \tag{11}$$

We derive several new vector calculus identities:

Equations (12) and (13) have two parameters of a field: $(\varphi \text{ and } \mathrm{Y})$

12. Divergence of Curl of product of one scalar potential and one vector field: $\nabla \cdot \big(\nabla \times (\varphi \mathrm{Y})\big)$

$$\nabla \cdot \big(\nabla \times (\varphi \mathrm{Y})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{Y}) + (\nabla \varphi) \times \mathrm{Y}] = 0 \tag{12}$$

13. Curl of curl of product of one scalar potential and one vector field: $\nabla \times \big(\nabla \times (\varphi \mathrm{Y})\big)$

$$\nabla \times \big(\nabla \times (\varphi \mathrm{Y})\big) = \nabla[\varphi(\nabla \cdot \mathrm{Y}) + \mathrm{Y} \cdot (\nabla \varphi)] - \nabla^2(\varphi \mathrm{Y}) \tag{13}$$

Equation (14) to Equation (20) have three parameters: $(\varphi, \mathrm{Y}, \text{and } \mathrm{Z})$

14. Divergence of product of one scalar potential and two vector fields: $\nabla \cdot \big((\varphi \mathrm{Y}) \times \mathrm{Z}\big)$

$$\nabla \cdot \big((\varphi \mathrm{Y}) \times \mathrm{Z}\big) = \varphi(\nabla \times \mathrm{Y}) \cdot \mathrm{Z} - \varphi(\nabla \times \mathrm{Z}) \cdot \mathrm{Y} + [(\nabla \varphi) \times \mathrm{Y}] \cdot \mathrm{Z} \tag{14}$$

15. Curl of cross product of one scalar potential and two vector fields: $\nabla \times \big((\varphi \mathrm{Y}) \times \mathrm{Z}\big)$

$$\nabla \times \big((\varphi \mathrm{Y}) \times \mathrm{Z}\big) = \varphi \mathrm{Y}(\nabla \cdot \mathrm{Z}) - \varphi \mathrm{Z}(\nabla \cdot \mathrm{Y}) - \mathrm{Z}[\mathrm{Y} \cdot (\nabla \varphi)] + (\mathrm{Z} \cdot \nabla)(\varphi \mathrm{Y}) - \varphi(\mathrm{Y} \cdot \nabla)\mathrm{Z} \tag{15}$$

16. Gradient of dot product of one scalar potential and two vector fields: $\nabla\big((\varphi \mathrm{Y}) \cdot \mathrm{Z}\big)$

$$\nabla\big((\varphi Y)\cdot Z\big) = \big((\varphi Y)\cdot\nabla\big)Z + (Z\cdot\nabla)(\varphi Y) + (\varphi Y)\times(\nabla\times Z) + Z\times\big(\nabla\times(\varphi Y)\big) \quad (16)$$

17. Vector-Cross-Del operator of one scalar potential and two vector fields: $((\varphi Y)\times\nabla)\cdot Z$

$$\big((\varphi Y)\times\nabla\big)\cdot Z = \varphi Y\cdot(\nabla\times Z). \quad (17)$$

18. Vector-Cross-Del operator of one scalar potential and two vector fields: $(Y\times\nabla)\cdot(\varphi Z)$

$$(Y\times\nabla)\cdot(\varphi Z) = Y\cdot(\nabla\times(\varphi Z)) \quad (18)$$

19. Vector-Cross-Del operator of one scalar potential and two vector fields: $((\varphi Y)\times\nabla)\times Z$

$$((\varphi Y)\times\nabla)\times Z = \varphi Y\times(\nabla\times Z) + \varphi(Y\cdot\nabla)Z - \varphi(\nabla\cdot Z)Y. \quad (19)$$

20. Vector-Cross-Del operator of one scalar potential and two vector fields: $(Y\times\nabla)\times(\varphi Z)$

$$(Y\times\nabla)\times(\varphi Z) = Y\times\big[\big(\nabla\times(\varphi Z)\big)\big] + Y\times[(\nabla\varphi)\times Z] +$$
$$(Y\cdot\nabla)(\varphi Z) - Y[\varphi(\nabla\cdot Z) + Z\cdot(\nabla\varphi)] \quad (20)$$

Note: the physical meaning of an equation containing scalar potential and two vector fields is unclear.

MATHEMATICALLY DERIVING CLASSICAL ELECTROMAGNETICS AND NOVEL ELECTROMAGNETICS

Classical Electromagnetics

In Section 3, let Y and Z represent either electric field E, or magnetic field B, or vector potential A, respectively, and φ is scalar potential in Equations (5) to Equation (8), and in Equation (12) to Equation (20). Then substituting E, B, A, and φ into vector calculus identities, we obtain the field equations, respectively.

Divergence of Cross Product of Vector Fields: $\boldsymbol{\nabla}\cdot(\boldsymbol{E}\times\boldsymbol{B})$, $\boldsymbol{\nabla}\cdot(\boldsymbol{A}\times\boldsymbol{B})$, and $\boldsymbol{\nabla}\cdot(\boldsymbol{A}\times\boldsymbol{E})$

$$\nabla\cdot(Y\times Z) = (\nabla\times Y)\cdot Z - Y\cdot(\nabla\times Z) \quad (1)$$

Poynting's Theorem: $\boldsymbol{\nabla}\cdot(\boldsymbol{E}\times\boldsymbol{B})$:

Substituting Y = E and Z = B, Equation (1) gives Poynting's Theorem:

$$\nabla\cdot(E\times B) = (\nabla\times E)\cdot B - E\cdot(\nabla\times B) = -\tfrac{1}{2}\partial(E^2+B^2)/\partial t - E\cdot j \quad (3.1)$$

Where $\nabla\times B = j + (\partial E/\partial t)$ and $\nabla\times E = -(\partial B/\partial t)$ are used. The term $E\times B$ is the Poynting vector representing the energy flux (power per unit area) of the electromagnetic field. The term, $\partial(E^2+B^2)/\partial t$, is the rate of change of the stored electromagnetic energy density. The term $E\cdot J$ represents the rate at which the electromagnetic field does work on the charge carriers, and $J = \rho v$.

New Field Equation: $\nabla \cdot (A \times B)$ and $\nabla \cdot (A \times E)$:

For completeness, let consider vector potential A as well. We obtain new field equations.

Substituting (Y = A, Z = B) and (Y = A, Z = E), respectively, Equation (1) gives NEW field equation:

$$\nabla \cdot (\mathrm{A} \times \mathrm{B}) = (\nabla \times \mathrm{A}) \cdot \mathrm{B} - \mathrm{A} \cdot (\nabla \times \mathrm{B}) = \mathrm{B} \cdot \mathrm{B} - \mathrm{A} \cdot \mathrm{j} - \mathrm{A} \cdot (\partial \mathrm{E}/\partial t). \quad (3.2)$$

$$\nabla \cdot (\mathrm{A} \times \mathrm{E}) = (\nabla \times \mathrm{A}) \cdot \mathrm{E} - \mathrm{A} \cdot (\nabla \times \mathrm{E}) = \mathrm{E} \cdot \mathrm{B} + \mathrm{A} \cdot (\partial \mathrm{B}/\partial t).. \quad (3.3)$$

Equation (3.2) represent the energy of magnetic field.

Curl of Cross Product of Two Vector Fields: $\nabla \times (E \times B)$, $\nabla \times (A \times B)$, and $\nabla \times (A \times E)$

$$\nabla \times (\mathrm{Y} \times \mathrm{Z}) = \mathrm{Y}(\nabla \cdot \mathrm{Z}) - \mathrm{Z}(\nabla \cdot \mathrm{Y}) + (\mathrm{Z} \cdot \nabla)\mathrm{Y} - (\mathrm{Y} \cdot \nabla)\mathrm{Z} \quad (2)$$

Substituting (Y = E, Z = B), (Y = A, Z = B), and (Y = A, Z = E), respectively, Equation (2) gives NEW field equations:

$$\begin{aligned}\nabla \times (\mathrm{E} \times \mathrm{B}) &= \mathrm{E}(\nabla \cdot \mathrm{B}) - \mathrm{B}(\nabla \cdot \mathrm{E}) + (\mathrm{B} \cdot \nabla)\mathrm{E} - (\mathrm{E} \cdot \nabla)\mathrm{B} \\ &= -\rho\mathrm{B} + (\mathrm{B} \cdot \nabla)\mathrm{E} - (\mathrm{E} \cdot \nabla)\mathrm{B} \quad (3.4)\end{aligned}$$

$$\begin{aligned}\nabla \times (\mathrm{A} \times \mathrm{B}) &= \mathrm{A}(\nabla \cdot \mathrm{B}) - \mathrm{B}(\nabla \cdot \mathrm{A}) + (\mathrm{B} \cdot \nabla)\mathrm{A} - (\mathrm{A} \cdot \nabla)\mathrm{B} \\ &= -\mathrm{B}(\nabla \cdot \mathrm{A}) + (\mathrm{B} \cdot \nabla)\mathrm{A} - (\mathrm{A} \cdot \nabla)\mathrm{B} \quad (3.5)\end{aligned}$$

$$\begin{aligned}\nabla \times (\mathrm{A} \times \mathrm{E}) &= \mathrm{A}(\nabla \cdot \mathrm{E}) - \mathrm{E}(\nabla \cdot \mathrm{A}) + (\mathrm{E} \cdot \nabla)\mathrm{A} - (\mathrm{A} \cdot \nabla)\mathrm{E} \\ &= \rho\mathrm{A} - \mathrm{E}(\nabla \cdot \mathrm{A}) + (\mathrm{E} \cdot \nabla)\mathrm{A} - (\mathrm{A} \cdot \nabla)\mathrm{E} \ . \quad (3.6)\end{aligned}$$

Where $\nabla \cdot \mathrm{E} = \rho$.

Divergence of Curl of Vector Field: $\nabla \cdot (\nabla \times A)$, $\nabla \cdot (\nabla \times E)$, and $\nabla \cdot (\nabla \times B)$

$$\nabla \cdot (\nabla \times \mathrm{Y}) = 0 \quad (3)$$

Gauss's Law for Magnetism (1): $\nabla \cdot (\nabla \times A)$:

Substituting Y = A and the definition of the magnetic field, $\mathrm{B} = \nabla \times \mathrm{A}$, Equation (3) gives Gauss's Law for Magnetism:

$$\nabla \cdot (\nabla \times \mathrm{A}) = \nabla \cdot \mathrm{B} = 0 \quad (3.7)$$

Equation (3.7) mathematically indicates no magnetic monopoles.

Gauss's Law for Magnetism (2): $\nabla \cdot (\nabla \times E)$:

Substituting Y = E, Equation (3) gives:

$$\nabla \cdot (\nabla \times \mathrm{E}) = 0 \quad (3.8)$$

Substituting the Faraday's Law of Induction into Equation (3.8), we obtain:

$$\nabla \cdot (\nabla \times \mathrm{E}) = -\nabla \cdot (\partial \mathrm{B}/\partial t) = -\frac{\partial}{\partial t}(\nabla \cdot \mathrm{B}) = 0. \tag{3.9a}$$

Equation (3.9a) has two solutions:

1. The magnetic field B is a constant field:

$$\partial \mathrm{B}/\partial t = 0. \tag{3.9b}$$

2. Gauss's Law for Magnetism

$$\nabla \cdot \mathrm{B(t)} = 0. \tag{3.9c}$$

Equation (3.9a) is valid for both situations:

1. constant magnetic field, Equation (3.9b); and
2. time-varying magnetic field Equation (3.9c).

Charge Density Continuity Equation: $\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \boldsymbol{B})$:

Substituting Y = B, Equation (3) gives:

$$\nabla \cdot (\nabla \times \mathrm{B}) = 0 \tag{3.10}$$

Substituting the Ampere-Maxwell equation into Equation (3.10), we obtain Charge density Continuity equation,

$$\nabla \cdot (\nabla \times \mathrm{B}) = \nabla \cdot \mathrm{j} + \nabla \cdot \frac{\partial \mathrm{E}}{\partial t} = \nabla \cdot \mathrm{j} + \frac{\partial}{\partial t}\rho = 0 \tag{3.11}$$

Curl of Curl of Vector Field: Wave Equation: $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{A})$, $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{E})$, and $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{B})$

$$\nabla \times (\nabla \times \mathrm{Y}) = \nabla(\nabla \cdot \mathrm{Y}) - \nabla^2 \mathrm{Y} \tag{4}$$

Wave Equation of Vector Potential A: $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{A})$:

Substituting Y = A, Equation (4) gives:

$$\nabla \times (\nabla \times \mathrm{A}) = \nabla \times \mathrm{B} = \mathrm{j} + \frac{\partial}{\partial t}\mathrm{E} \tag{3.12a}$$

and

$$\nabla(\nabla \cdot \mathrm{A}) - \nabla^2 \mathrm{A} = -\frac{\partial}{\partial t}(\nabla \varphi) - \nabla^2 \mathrm{A} = \frac{\partial}{\partial t}\mathrm{E} + \frac{\partial^2}{\partial t^2}\mathrm{A} - \nabla^2 \mathrm{A} \tag{3.12b}$$

Combining Equation (3.12a) and Equation (3.12b), we obtain wave equation of vector potential A:

$$\frac{\partial^2}{\partial t^2}\mathrm{A} - \nabla^2 \mathrm{A} = \mathrm{j} \tag{3.12c}$$

Where the definition of E field is applied: $\mathrm{E} = -\nabla \varphi - \frac{\partial}{\partial t}\mathrm{A}$.

Wave Equation of Electric Field E: $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{E})$:

Substituting Y = E, Equation (4) gives

$$\nabla \times (\nabla \times \mathrm{E}) = -\nabla \times \frac{\partial}{\partial t}\mathrm{B} = -\frac{\partial}{\partial t}\mathrm{j} - \frac{\partial^2}{\partial t^2}\mathrm{E} \tag{3.13a}$$

And

$$\nabla(\nabla \cdot \mathrm{E}) - \nabla^2\mathrm{E} = \nabla\rho - \nabla^2\mathrm{E} \tag{3.13b}$$

Combining Equation (3.13a) and Equation (3.13b), we obtain wave equation of electric field E,

$$\frac{\partial^2}{\partial t^2}\mathrm{E} - \nabla^2\mathrm{E} = -\frac{\partial}{\partial t}\mathrm{j} - \nabla\rho \tag{3.13c}$$

Wave Equation of Magnetic Field B: $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{B})$:

Substituting Y = B, Equation (4) gives

$$\nabla \times (\nabla \times \mathrm{B}) = \nabla \times \mathrm{j} + \nabla \times \frac{\partial}{\partial t}\mathrm{E} = \nabla \times \mathrm{j} - \frac{\partial^2}{\partial t^2}\mathrm{B} \tag{3.14a}$$

And

$$\nabla(\nabla \cdot \mathrm{B}) - \nabla^2\mathrm{B} = -\nabla^2\mathrm{B} \tag{3.14b}$$

Combining Equation (3.14a) and Equation (3.14b), we obtain wave equation of B,

$$\frac{\partial^2}{\partial t^2}\mathrm{B} - \nabla^2\mathrm{B} = \nabla \times \mathrm{j} \tag{3.14c}$$

Divergence of Gradient of Scalar Field: Wave Equation of Scalar Potential $\boldsymbol{\varphi}$: $(\boldsymbol{\nabla\varphi})$

$$\nabla \cdot (\nabla\varphi) = \nabla^2\varphi \tag{5}$$

Substituting the definition of electric field, $\mathrm{E} = -\nabla\varphi - \frac{\partial}{\partial t}\mathrm{A}$ into Equation (5). We obtain wave equation of potential φ:

$$\nabla \cdot (\nabla\varphi) = \nabla \cdot \left(-\mathrm{E} - \frac{\partial}{\partial t}\mathrm{A}\right) = -\rho + \frac{\partial^2}{\partial t^2}\varphi \tag{3.15a}$$

i.e.,

$$\frac{\partial^2}{\partial t^2}\varphi - \nabla^2\varphi = \rho. \tag{3.15b}$$

Curl of Gradient of Scalar Field: Faraday's Law of Induction: $\boldsymbol{\nabla} \times (\boldsymbol{\nabla\varphi})$

$$\nabla \times (\nabla\varphi) = 0 \tag{6}$$

Substituting the definition of the electric field, $\mathrm{E} = -\nabla\varphi - \partial\mathrm{A}/\partial\mathrm{t}$, into Equation (6).

We obtain Faraday's Law of Induction:

$$\nabla \times (\nabla\varphi) = \nabla \times (-\mathrm{E} - \partial\mathrm{A}/\partial\mathrm{t}) = -\nabla \times \mathrm{E} - \partial\mathrm{B}/\partial\mathrm{t} = 0. \tag{3.16a}$$

i.e.,

$$\nabla \times \mathrm{E} = -\partial\mathrm{B}/\partial\mathrm{t}. \tag{3.16b}$$

Divergence of Product of Scalar Potential φ with Vector Potential A, Electric Field E, Magnetic Field B:

$$\nabla \cdot (\varphi Y) = \varphi(\nabla \cdot Y) + Y \cdot (\nabla\varphi) \tag{7}$$

Substituting Y = A, Y = E, and Y = B into Equation (7), respectively, we obtain NEW field equations:

$$\nabla \cdot (\varphi A) = \varphi(\nabla \cdot A) + A \cdot (\nabla\varphi) \tag{3.17}$$

$$\nabla \cdot (\varphi E) = \varphi(\nabla \cdot E) + E \cdot (\nabla\varphi) = \rho\varphi + E \cdot (\nabla\varphi) \tag{3.18}$$

$$\nabla \cdot (\varphi B) = \varphi(\nabla \cdot B) + B \cdot (\nabla\varphi) = B \cdot (\nabla\varphi) \tag{3.19}$$

Curl of Product of Scalar Potential φ and Vector Field: $\boldsymbol{\nabla} \times (\boldsymbol{\varphi A})$, $\boldsymbol{\nabla} \times (\boldsymbol{\varphi E})$, and $\boldsymbol{\nabla} \times (\boldsymbol{\varphi B})$

$$\nabla \times (\varphi Y) = \varphi(\nabla \times Y) + (\nabla\varphi) \times Y. \tag{8}$$

Substituting Y = A, Y = E, and Y = B into Equation (8), respectively, we obtain NEW field equations:

$$\nabla \times (\varphi A) = \varphi(\nabla \times A) + (\nabla\varphi) \times A = \varphi B + (\nabla\varphi) \times A \tag{3.20}$$

$$\nabla \times (\varphi E) = \varphi(\nabla \times E) + (\nabla\varphi) \times E = -\varphi\partial B/\partial t + (\nabla\varphi) \times E \tag{3.21}$$

$$\nabla \times (\varphi B) = \varphi(\nabla \times B) + (\nabla\varphi) \times B = j\varphi + \varphi\partial E/\partial t + (\nabla\varphi) \times B \tag{3.22}$$

Gradient of Scalar Product of Two Vector Fields: $\boldsymbol{\nabla}(\boldsymbol{E} \cdot \boldsymbol{B})$, $\boldsymbol{\nabla}(\boldsymbol{A} \cdot \boldsymbol{B})$, $\boldsymbol{\nabla}(\boldsymbol{A} \cdot \boldsymbol{E})$, $\boldsymbol{\nabla}(\boldsymbol{A} \cdot \boldsymbol{A})$, $\boldsymbol{\nabla}(\boldsymbol{E} \cdot \boldsymbol{E})$, $\boldsymbol{\nabla}(\boldsymbol{B} \cdot \boldsymbol{B})$

$$\nabla(Y \cdot Z) = (Y \cdot \nabla)Z + (Z \cdot \nabla)Y + Y \times (\nabla \times Z) + Z \times (\nabla \times Y). \tag{9}$$

Substituting (Y = E, Z = B), (Y = A, Z = B), (Y= A, Z = E), (Y = Z = B), (Y = Z = A), $Y = Z = E$), into Equation (9), respectively, we obtain NEW field equations:

$$\begin{aligned}\nabla(E \cdot B) &= (E \cdot \nabla)B + (B \cdot \nabla)E + E \times (\nabla \times B) + B \times (\nabla \times E)\\ &= (E \cdot \nabla)B + (B \cdot \nabla)E + E \times j\end{aligned} \tag{3.23}$$

$$\begin{aligned}\nabla(A \cdot B) &= (A \cdot \nabla)B + (B \cdot \nabla)A + A \times (\nabla \times B) + B \times (\nabla \times A)\\ &= (A \cdot \nabla)B + (B \cdot \nabla)A + A \times j + A \times \partial E/\partial t\end{aligned} \tag{3.24}$$

$$\begin{aligned}\nabla(A \cdot E) &= (A \cdot \nabla)E + (E \cdot \nabla)A + A \times (\nabla \times E) + E \times (\nabla \times A)\\ &= (A \cdot \nabla)E + (E \cdot \nabla)A - A \times \partial B/\partial t + E \times B\end{aligned} \tag{3.25}$$

$$\begin{aligned}\nabla(A \cdot A) &= 2(A \cdot \nabla)A + 2A \times (\nabla \times A)\\ &= 2(A \cdot \nabla)A + 2A \times B\end{aligned} \tag{3.26}$$

$$\begin{aligned}\nabla(E \cdot E) &= (E \cdot \nabla)E + (E \cdot \nabla)E + E \times (\nabla \times E) + E \times (\nabla \times E)\\ &= 2(E \cdot \nabla)E - 2E \times (\partial B/\partial t)\end{aligned} \tag{3.27}$$

$$\nabla(B \cdot B) = (B \cdot \nabla)B + (B \cdot \nabla)B + B \times (\nabla \times B) + B \times (\nabla \times B)$$

$$= 2(\mathrm{B}\cdot\nabla)\mathrm{B} + 2\mathrm{B}\times(\nabla\times\mathrm{B}). \tag{3.28}$$

Cross Product: $(\boldsymbol{E}\times\boldsymbol{\nabla})\cdot\boldsymbol{B}$, $(\boldsymbol{B}\times\boldsymbol{\nabla})\cdot\boldsymbol{E}$, $(\boldsymbol{A}\times\boldsymbol{\nabla})\cdot\boldsymbol{B}$, $(\boldsymbol{B}\times\boldsymbol{\nabla})\cdot\boldsymbol{A}$, $(\boldsymbol{E}\times\boldsymbol{\nabla})\cdot\boldsymbol{A}$, ***and*** $(\boldsymbol{A}\times\boldsymbol{\nabla})\cdot\boldsymbol{E}$

Equation (10),

$$(\mathrm{Y}\times\nabla)\cdot\mathrm{Z} = \mathrm{Y}\cdot(\nabla\times\mathrm{Z}) \tag{10}$$

Substituting $(\mathrm{Y}=\mathrm{E}$ and $\mathrm{Z}=\mathrm{B})$, $(\mathrm{Y}=\mathrm{B}$ and $\mathrm{Z}=\mathrm{E})$, $(\mathrm{Y}=\mathrm{A}$ and $\mathrm{Z}=\mathrm{B})$, $(\mathrm{Y}=\mathrm{B}$ and $\mathrm{Z}=\mathrm{A})$, $(\mathrm{Y}=\mathrm{E}$ and $\mathrm{Z}=\mathrm{A})$, and $(\mathrm{Y}=\mathrm{A}$ and $\mathrm{Z}=\mathrm{E})$, respectively, Equation (10) gives the following equations,

$$(\mathrm{E}\times\nabla)\cdot\mathrm{B} = \mathrm{E}\cdot(\nabla\times\mathrm{B}) = \mathrm{E}\cdot\mathrm{j} + \mathrm{E}\cdot\partial\mathrm{E}/\partial\mathrm{t} \tag{3.29}$$

$$(\mathrm{B}\times\nabla)\cdot\mathrm{E} = \mathrm{B}\cdot(\nabla\times\mathrm{E}) = -\mathrm{B}\cdot\partial\mathrm{B}/\partial\mathrm{t}. \tag{3.30}$$

$$(\mathrm{A}\times\nabla)\cdot\mathrm{B} = \mathrm{A}\cdot(\nabla\times\mathrm{B}) = \mathrm{A}\cdot\mathrm{j} + \mathrm{A}\cdot\partial\mathrm{E}/\partial\mathrm{t} \tag{3.31}$$

$$(\mathrm{B}\times\nabla)\cdot\mathrm{A} = \mathrm{B}\cdot(\nabla\times\mathrm{A}) = \mathrm{B}^2 \tag{3.32}$$

$$(\mathrm{E}\times\nabla)\cdot\mathrm{A} = \mathrm{E}\cdot(\nabla\times\mathrm{A}) = \mathrm{E}\cdot\mathrm{B} \tag{3.33}$$

$$(\mathrm{A}\times\nabla)\cdot\mathrm{E} = \mathrm{A}\cdot(\nabla\times\mathrm{E}) = -\,\mathrm{A}\cdot\partial\mathrm{B}/\partial\mathrm{t} \tag{3.34}$$

Cross Product: $(\boldsymbol{E}\times\boldsymbol{\nabla})\times\boldsymbol{B}$, $(\boldsymbol{B}\times\boldsymbol{\nabla})\times\boldsymbol{E}$, $(\boldsymbol{A}\times\boldsymbol{\nabla})\times\boldsymbol{B}$, $(\boldsymbol{B}\times\boldsymbol{\nabla})\times\boldsymbol{A}$, $(\boldsymbol{A}\times\boldsymbol{\nabla})\times\boldsymbol{E}$, ***and*** $(\boldsymbol{E}\times\boldsymbol{\nabla})\times\boldsymbol{A}$

$$(\mathrm{Y}\times\nabla)\times\mathrm{Z} = \mathrm{Y}\times(\nabla\times\mathrm{Z}) + (\mathrm{Y}\cdot\nabla)\mathrm{Z} - \mathrm{Y}(\nabla\cdot\mathrm{Z}) \tag{11}$$

Substituting $(\mathrm{Y}=\mathrm{E}$ and $\mathrm{Z}=\mathrm{B})$, $(\mathrm{Y}=\mathrm{B}$ and $\mathrm{Z}=\mathrm{E})$, $(\mathrm{Y}=\mathrm{A}$ and $\mathrm{Z}=\mathrm{B})$, $(\mathrm{Y}=\mathrm{B}$ and $\mathrm{Z}=\mathrm{A})$, $(\mathrm{Y}=\mathrm{A}$ and $\mathrm{Z}=\mathrm{E})$, and $(\mathrm{Y}=\mathrm{E}$ and $\mathrm{Z}=\mathrm{A})$, Equation (11) gives new field equations:

$$(\mathrm{E}\times\nabla)\times\mathrm{B} = \mathrm{E}\times(\nabla\times\mathrm{B}) + (\mathrm{E}\cdot\nabla)\mathrm{B} - \mathrm{E}(\nabla\cdot\mathrm{B}) = \mathrm{E}\times\mathrm{j} + \mathrm{E}\times\partial\mathrm{E}/\partial\mathrm{t} + (\mathrm{E}\cdot\nabla)\mathrm{B}$$

$$= \mathrm{E}\times\mathrm{j} + (\mathrm{E}\cdot\nabla)\mathrm{B} \tag{3.35}$$

$$(\mathrm{B}\times\nabla)\times\mathrm{E} = \mathrm{B}\times(\nabla\times\mathrm{E}) + (\mathrm{B}\cdot\nabla)\mathrm{E} - \mathrm{B}(\nabla\cdot\mathrm{E}) = -\mathrm{B}\times\partial\mathrm{B}/\partial\mathrm{t} + (\mathrm{B}\cdot\nabla)\mathrm{E} - \rho\mathrm{B}$$

$$= -\rho\mathrm{B} + (\mathrm{B}\cdot\nabla)\mathrm{E} \tag{3.36}$$

$$(\mathrm{A}\times\nabla)\times\mathrm{B} = \mathrm{A}\times(\nabla\times\mathrm{B}) + (\mathrm{A}\cdot\nabla)\mathrm{B} - \mathrm{A}(\nabla\cdot\mathrm{B})$$

$$= \mathrm{A}\times\mathrm{j} + \mathrm{A}\times\partial\mathrm{E}/\partial\mathrm{t} + (\mathrm{A}\cdot\nabla)\mathrm{B} \tag{3.37}$$

$$(\mathrm{B}\times\nabla)\times\mathrm{A} = \mathrm{B}\times(\nabla\times\mathrm{A}) + (\mathrm{B}\cdot\nabla)\mathrm{A} - \mathrm{B}(\nabla\cdot\mathrm{A})$$

$$= (\mathrm{B}\cdot\nabla)\mathrm{A} + \mathrm{B}(\partial\varphi/\partial\mathrm{t}) \tag{3.38}$$

$$(\mathrm{A}\times\nabla)\times\mathrm{E} = \mathrm{A}\times(\nabla\times\mathrm{E}) + (\mathrm{A}\cdot\nabla)\mathrm{E} - \mathrm{A}(\nabla\cdot\mathrm{E})$$

$$= -\mathrm{A}\times\partial\mathrm{B}/\partial\mathrm{t} + (\mathrm{A}\cdot\nabla)\mathrm{E} - \rho\mathrm{A} \tag{3.39}$$

$$(\mathrm{E}\times\nabla)\times\mathrm{A} = \mathrm{E}\times\mathrm{B} + (\mathrm{E}\cdot\nabla)\mathrm{A} - \mathrm{E}(\nabla\cdot\mathrm{A})$$

$$= \mathrm{E}\times\mathrm{B} + (\mathrm{B}\cdot\nabla)\mathrm{A} + \mathrm{E}(\partial\varphi/\partial\mathrm{t}) \tag{3.40}$$

Next let us derive several new physics laws from new vector calculus identities:

Divergence of Curl of Vector Field: $\boldsymbol{\nabla} \times (\boldsymbol{\varphi Y})$

$$\nabla \cdot \big(\nabla \times (\varphi \mathrm{Y})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{Y}) + (\nabla\varphi) \times \mathrm{Y}] = 0 \tag{12}$$

Substituting (Y = A), (Y = E), and (Y = B), respectively, Equation (12) gives NEW field equation:

$$\nabla \cdot \big(\nabla \times (\varphi \mathrm{A})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{A}) + (\nabla\varphi) \times \mathrm{A}] = 0, \tag{3.41}$$

$$\nabla \cdot \big(\nabla \times (\varphi \mathrm{E})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{E}) + (\nabla\varphi) \times \mathrm{E}] = 0, \tag{3.42}$$

$$\nabla \cdot \big(\nabla \times (\varphi \mathrm{B})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{B}) + (\nabla\varphi) \times \mathrm{B}] = 0. \tag{3.43}$$

Curl of Curl of Vector Field: $(\boldsymbol{\varphi Y})$

$$\nabla \times \big(\nabla \times (\varphi \mathrm{Y})\big) = \nabla[\varphi(\nabla \cdot \mathrm{Y}) + \mathrm{Y} \cdot (\nabla\varphi)] - \nabla^2(\varphi \mathrm{Y}) \tag{13}$$

Substituting (Y = A), (Y = E), and (Y = B), respectively, Equation (13) gives NEW field equation:

$$\nabla \times \big(\nabla \times (\varphi \mathrm{A})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{A}) + (\nabla\varphi) \times \mathrm{A}] = 0, \tag{3.44}$$

$$\nabla \times \big(\nabla \times (\varphi \mathrm{E})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{E}) + (\nabla\varphi) \times \mathrm{E}] = 0, \tag{3.45}$$

$$\nabla \times \big(\nabla \times (\varphi \mathrm{B})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{B}) + (\nabla\varphi) \times \mathrm{B}] = 0. \tag{3.46}$$

Divergence of Cross Product of Two Vector Fields: $(\varphi \mathrm{Y}) \times \mathrm{Z}$

$$\nabla \cdot \big((\varphi \mathrm{Y}) \times \mathrm{Z}\big) = \varphi(\nabla \times \mathrm{Y}) \cdot \mathrm{Z} - \varphi(\nabla \times \mathrm{Z}) \cdot \mathrm{Y} + [(\nabla\varphi) \times \mathrm{Y}] \cdot \mathrm{Z} \tag{14}$$

Substituting (Y = A, Z = B), (Y = A, Z = E), and (Y = E, Z = B), respectively, Equation (14) gives NEW field equation:

$$\nabla \cdot \big((\varphi \mathrm{A}) \times \mathrm{B}\big) = \big(\nabla \times (\varphi \mathrm{A})\big) \cdot \mathrm{B} - (\varphi \mathrm{A}) \cdot (\nabla \times \mathrm{B})$$
$$= [\varphi \mathrm{B} + (\nabla\varphi) \times \mathrm{A}] \cdot \mathrm{B} - (\varphi \mathrm{A}) \cdot \mathrm{j} - (\varphi \mathrm{A}) \cdot (\partial \mathrm{E}/\partial t). \tag{3.47}$$

$$\nabla \cdot \big((\varphi \mathrm{A}) \times \mathrm{E}\big) = \big(\nabla \times (\varphi \mathrm{A})\big) \cdot \mathrm{E} - (\varphi \mathrm{A}) \cdot (\nabla \times \mathrm{E})$$
$$= [\varphi \mathrm{B} + (\nabla\varphi) \times \mathrm{A}] \cdot \mathrm{E} + (\varphi \mathrm{A}) \cdot (\partial \mathrm{B}/\partial t). \tag{3.48}$$

$$\nabla \cdot \big((\varphi \mathrm{E}) \times \mathrm{B}\big) = [\varphi(\nabla \times \mathrm{E})] \cdot \mathrm{B} - \varphi(\nabla \times \mathrm{B}) \cdot \mathrm{E} + [(\nabla\varphi) \times \mathrm{E}] \cdot \mathrm{B}$$
$$= -\varphi \mathrm{B} \cdot \left(\frac{\partial \mathrm{B}}{\partial t}\right) - \varphi \mathrm{E} \cdot \mathrm{j} - \varphi \mathrm{E} \cdot (\partial \mathrm{E}/\partial t) - \left[\left(\frac{\partial \mathrm{A}}{\partial t}\right) \times \mathrm{E}\right] \cdot \mathrm{B}. \tag{3.49}$$

Curl of Cross Product of Two Vector Fields: $(\boldsymbol{\varphi Y}) \times \boldsymbol{Z}$

$$\nabla \times \big((\varphi \mathrm{Y}) \times \mathrm{Z}\big) = \varphi \mathrm{Y}(\nabla \cdot \mathrm{Z}) - \varphi \mathrm{Z}(\nabla \cdot \mathrm{Y}) - \mathrm{Z}[\mathrm{Y} \cdot (\nabla\varphi)] + (\mathrm{Z} \cdot \nabla)(\varphi \mathrm{Y}) - \varphi(\mathrm{Y} \cdot \nabla)\mathrm{Z} \tag{15}$$

Substituting (Y = A, Z = B), (Y = A, Z = E), and (Y = E, Z = B), respectively, Equation (15) gives NEW field equation:

$$\nabla \times \big((\varphi \mathrm{A}) \times \mathrm{B}\big) = -\varphi \mathrm{B}(\nabla \cdot \mathrm{A}) - \mathrm{B}[\mathrm{A} \cdot (\nabla\varphi)] + (\mathrm{B} \cdot \nabla)(\varphi \mathrm{A}) - \varphi(\mathrm{A} \cdot \nabla)\mathrm{B} \tag{3.50}$$

$$\nabla \times \big((\varphi A) \times E\big) = \varphi A(\nabla \cdot E) - \varphi E(\nabla \cdot A) - E[A \cdot (\nabla\varphi)] + (E \cdot \nabla)(\varphi A) - \varphi(A \cdot \nabla)E \quad (3.51)$$

$$\nabla \times \big((\varphi E) \times B\big) = -\varphi B(\nabla \cdot E) - B[E \cdot (\nabla\varphi)] + (B \cdot \nabla)(\varphi E) - \varphi(E \cdot \nabla)B \quad (3.52)$$

Gradient of Dot Product of Two Vector Fields: $(\boldsymbol{\varphi Y} \cdot \boldsymbol{Z})$.

$$\nabla\big((\varphi Y) \cdot Z\big) = \big((\varphi Y) \cdot \nabla\big)Z + (Z \cdot \nabla)(\varphi Y) + (\varphi Y) \times (\nabla \times Z) + Z \times \big(\nabla \times (\varphi Y)\big) \quad (16)$$

Substituting (Y = A, Z = B), (Y = A, Z = E), and (Y = E, Z = B), respectively, Equation (16) gives NEW field equation:

$$\nabla\big((\varphi A) \cdot B\big) = \big((\varphi A) \cdot \nabla\big)B + (B \cdot \nabla)(\varphi A) + (\varphi A) \times (\nabla \times B) + B \times \big(\nabla \times (\varphi A)\big) \quad (3.53)$$

$$\nabla\big((\varphi A) \cdot E\big) = \big((\varphi A) \cdot \nabla\big)E + (E \cdot \nabla)(\varphi A) + (\varphi A) \times (\nabla \times E) + E \times \big(\nabla \times (\varphi A)\big) \quad (3.54)$$

$$\nabla\big((\varphi E) \cdot B\big) = \big((\varphi E) \cdot \nabla\big)B + (B \cdot \nabla)(\varphi E) + (\varphi E) \times (\nabla \times B) + B \times \big(\nabla \times (\varphi E)\big) \quad (3.55)$$

Vector-Cross-Del Operator: $((\boldsymbol{\varphi Y}) \times \boldsymbol{\nabla}) \cdot \boldsymbol{Z}$

$$\big((\varphi Y) \times \nabla\big) \cdot Z = \varphi Y \cdot (\nabla \times Z). \quad (17)$$

Substituting $(Y = E$ and $Z = B)$, $(Y = E$ and $Z = A)$, $(Y = B$ and $Z = E)$, $(Y = B$ and $Z = A)$, $(Y = A$ and $Z = B)$, $(Y = A$ and $Z = E)$, respectively, Equation (17) gives new field equations:

$$((\varphi E) \times \nabla) \cdot B = \varphi E \cdot (\nabla \times B) \quad (3.56)$$

$$\big((\varphi E) \times \nabla\big) \cdot A = \varphi E \cdot (\nabla \times A). \quad (3.57)$$

$$\big((\varphi B) \times \nabla\big) \cdot E = \varphi B \cdot (\nabla \times E). \quad (3.58)$$

$$((\varphi B) \times \nabla) \cdot A = \varphi B \cdot (\nabla \times A) \quad (3.59)$$

$$\big((\varphi A) \times \nabla\big) \cdot B = \varphi A \cdot (\nabla \times B). \quad (3.60)$$

$$\big((\varphi A) \times \nabla\big) \cdot E = \varphi A \cdot (\nabla \times E). \quad (3.61)$$

Vector-Cross-Del Operator: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \cdot (\boldsymbol{\varphi Z})$

$$(Y \times \nabla) \cdot (\varphi Z) = Y \cdot (\nabla \times (\varphi Z)) \quad (18)$$

Substituting $(Y = E$ and $Z = B)$, $(Y = E$ and $Z = A)$, $(Y = B$ and $Z = E)$, $(Y = B$ and $Z = A)$, $(Y = A$ and $Z = B)$, $(Y = A$ and $Z = E)$, respectively, Equation (18) gives new field equations:

$$(E \times \nabla) \cdot (\varphi B) = E \cdot \big(\nabla \times (\varphi B)\big) \quad (3.62)$$

$$(E \times \nabla) \cdot (\varphi A) = E \cdot \big(\nabla \times (\varphi A)\big) \quad (3.63)$$

$$(B \times \nabla) \cdot (\varphi E) = B \cdot \big(\nabla \times (\varphi E)\big) \quad (3.64)$$

$$(B \times \nabla) \cdot (\varphi A) = B \cdot \big(\nabla \times (\varphi A)\big) \quad (3.65)$$

$$(A \times \nabla) \cdot (\varphi B) = A \cdot \big(\nabla \times (\varphi B)\big) \quad (3.66)$$

$$(\mathrm{A} \times \nabla) \cdot (\varphi\mathrm{E}) = \mathrm{A} \cdot \big(\nabla \times (\varphi\mathrm{E})\big) \tag{3.67}$$

Vector-Cross-Del Operator: $((\boldsymbol{\varphi Y}) \times \boldsymbol{\nabla}) \times \boldsymbol{Z}$

$$((\varphi\mathrm{Y}) \times \nabla) \times \mathrm{Z} = \varphi\mathrm{Y} \times (\nabla \times \mathrm{Z}) + \varphi(\mathrm{Y} \cdot \nabla)\mathrm{Z} - \varphi(\nabla \cdot \mathrm{Z})\mathrm{Y}. \tag{19}$$

Substituting ($\mathrm{Y} = \mathrm{E}$ and $\mathrm{Z} = \mathrm{B}$), ($\mathrm{Y} = \mathrm{E}$ and $\mathrm{Z} = \mathrm{A}$), ($\mathrm{Y} = \mathrm{B}$ and $\mathrm{Z} = \mathrm{E}$), ($\mathrm{Y} = \mathrm{B}$ and $\mathrm{Z} = \mathrm{A}$), ($\mathrm{Y} = \mathrm{A}$ and $\mathrm{Z} = \mathrm{B}$), ($\mathrm{Y} = \mathrm{A}$ and $\mathrm{Z} = \mathrm{E}$), respectively, Equation (19) gives new field equations:

$$((\varphi\mathrm{E}) \times \nabla) \times \mathrm{B} = \varphi\mathrm{E} \times (\nabla \times \mathrm{B}) + \varphi(\mathrm{E} \cdot \nabla)\mathrm{B} \tag{3.68}$$

$$\big((\varphi\mathrm{E}) \times \nabla\big) \times \mathrm{A} = \varphi\mathrm{E} \times (\nabla \times \mathrm{A}) + \varphi(\mathrm{E} \cdot \nabla)\mathrm{A} - \varphi(\nabla \cdot \mathrm{A})\mathrm{E} \tag{3.69}$$

$$\big((\varphi\mathrm{B}) \times \nabla\big) \times \mathrm{E} = \varphi\mathrm{B} \times (\nabla \times \mathrm{E}) + \varphi(\mathrm{B} \cdot \nabla)\mathrm{E} - \varphi(\nabla \cdot \mathrm{E})\mathrm{B} \tag{3.70}$$

$$((\varphi\mathrm{B}) \times \nabla) \times \mathrm{A} = \varphi\mathrm{B} \times (\nabla \times \mathrm{A}) + \varphi(\mathrm{B} \cdot \nabla)\mathrm{A} - \varphi(\nabla \cdot \mathrm{A})\mathrm{B} \tag{3.71}$$

$$\big((\varphi\mathrm{A}) \times \nabla\big) \times \mathrm{B} = \varphi\mathrm{A} \times (\nabla \times \mathrm{B}) + \varphi(\mathrm{A} \cdot \nabla)\mathrm{B} \tag{3.72}$$

$$\big((\varphi\mathrm{A}) \times \nabla\big) \times \mathrm{E} = \varphi\mathrm{A} \times (\nabla \times \mathrm{E}) + \varphi(\mathrm{A} \cdot \nabla)\mathrm{E} - \varphi(\nabla \cdot \mathrm{E})\mathrm{A} \tag{3.73}$$

Vector-Cross-Del Operator: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \times (\boldsymbol{\varphi Z})$

$$(\mathrm{Y} \times \nabla) \times (\varphi\mathrm{Z}) = \mathrm{Y} \times \big[\big(\nabla \times (\varphi\mathrm{Z})\big)\big] + \mathrm{Y} \times [(\nabla\varphi) \times \mathrm{Z}] + (\mathrm{Y} \cdot \nabla)(\varphi\mathrm{Z}) - \mathrm{Y}[\varphi(\nabla \cdot \mathrm{Z}) + \mathrm{Z} \cdot (\nabla\varphi)] \tag{20}$$

Substituting ($\mathrm{Y} = \mathrm{E}$ and $\mathrm{Z} = \mathrm{B}$), ($\mathrm{Y} = \mathrm{E}$ and $\mathrm{Z} = \mathrm{A}$), ($\mathrm{Y} = \mathrm{B}$ and $\mathrm{Z} = \mathrm{E}$), ($\mathrm{Y} = \mathrm{B}$ and $\mathrm{Z} = \mathrm{A}$), ($\mathrm{Y} = \mathrm{A}$ and $\mathrm{Z} = \mathrm{B}$), ($\mathrm{Y} = \mathrm{A}$ and $\mathrm{Z} = \mathrm{E}$), respectively, Equation (19) gives new field equations:

$$(\mathrm{E} \times \nabla) \times (\varphi\mathrm{B}) = \mathrm{E} \times \big[\big(\nabla \times (\varphi\mathrm{B})\big)\big] + \mathrm{E} \times [(\nabla\varphi) \times \mathrm{B}] + (\mathrm{E} \cdot \nabla)(\varphi\mathrm{B}) - \mathrm{E}[\mathrm{B} \cdot (\nabla\varphi)] \tag{3.74}$$

$$(\mathrm{E} \times \nabla) \times (\varphi\mathrm{A}) = \mathrm{E} \times \big[\big(\nabla \times (\varphi\mathrm{A})\big)\big] + \mathrm{E} \times [(\nabla\varphi) \times \mathrm{A}] + (\mathrm{E} \cdot \nabla)(\varphi\mathrm{A}) - \mathrm{E}[\varphi(\nabla \cdot \mathrm{A}) + \mathrm{A} \cdot (\nabla\varphi)] \tag{3.75}$$

$$(\mathrm{B} \times \nabla) \times (\varphi\mathrm{E}) = \mathrm{B} \times \big[\big(\nabla \times (\varphi\mathrm{E})\big)\big] + \mathrm{B} \times [(\nabla\varphi) \times \mathrm{E}] + (\mathrm{B} \cdot \nabla)(\varphi\mathrm{E}) - \mathrm{B}[\varphi(\nabla \cdot \mathrm{E}) + \mathrm{E} \cdot (\nabla\varphi)] \tag{3.76}$$

$$(\mathrm{B} \times \nabla) \times (\varphi\mathrm{A}) = \mathrm{B} \times \big[\big(\nabla \times (\varphi\mathrm{A})\big)\big] + \mathrm{B} \times [(\nabla\varphi) \times \mathrm{A}] + (\mathrm{B} \cdot \nabla)(\varphi\mathrm{A}) - \mathrm{B}[\varphi(\nabla \cdot \mathrm{A}) + \mathrm{A} \cdot (\nabla\varphi)] \tag{3.77}$$

$$(\mathrm{A} \times \nabla) \times (\varphi\mathrm{B}) = \mathrm{A} \times \big[\big(\nabla \times (\varphi\mathrm{B})\big)\big] + \mathrm{A} \times [(\nabla\varphi) \times \mathrm{B}] + (\mathrm{A} \cdot \nabla)(\varphi\mathrm{B}) - \mathrm{A}[\mathrm{B} \cdot (\nabla\varphi)] \tag{3.78}$$

$$(\mathrm{A} \times \nabla) \times (\varphi\mathrm{E}) = \mathrm{A} \times \big[\big(\nabla \times (\varphi\mathrm{E})\big)\big] + \mathrm{A} \times [(\nabla\varphi) \times \mathrm{E}] + (\mathrm{A} \cdot \nabla)(\varphi\mathrm{E}) - \mathrm{A}[\varphi(\nabla \cdot \mathrm{E}) + \mathrm{E} \cdot (\nabla\varphi)] \tag{3.79}$$

Electric Field and Magnetic Field Created by Electric Charge Moving with Velocity v

Divergence of Cross Product of Velocity v and Vector Fields: $\boldsymbol{\nabla}\cdot(\boldsymbol{v}\times\boldsymbol{A})$, $\boldsymbol{\nabla}\cdot(\boldsymbol{v}\times\boldsymbol{E})$, ***and*** $\boldsymbol{\nabla}\cdot(\boldsymbol{v}\times\boldsymbol{B})$

$$\nabla\cdot(\mathrm{Y}\times\mathrm{Z})=(\nabla\times\mathrm{Y})\cdot\mathrm{Z}-\mathrm{Y}\cdot(\nabla\times\mathrm{Z}) \tag{1}$$

Substituting (Y = v, Z = A), (Y = v, Z = E), and (Y = v, Z = B), respectively, Equation (1) gives New field equations:

$$\nabla\cdot(\mathrm{v}\times\mathrm{A})=(\nabla\times\mathrm{v})\cdot\mathrm{A}-\mathrm{v}\cdot(\nabla\times\mathrm{A})=-\mathrm{v}\cdot\mathrm{B}. \tag{3.80}$$

$$\nabla\cdot(\mathrm{v}\times\mathrm{E})=(\nabla\times\mathrm{v})\cdot\mathrm{E}-\mathrm{v}\cdot(\nabla\times\mathrm{E})=\mathrm{v}\cdot\partial(\mathrm{B})/\,\partial\mathrm{t}. \tag{3.81}$$

$$\nabla\cdot(\mathrm{v}\times\mathrm{B})=(\nabla\times\mathrm{v})\cdot\mathrm{B}-\mathrm{v}\cdot(\nabla\times\mathrm{B})=-\mathrm{v}\cdot\mathrm{j}-\mathrm{v}\cdot\partial(\mathrm{E})/\,\partial\mathrm{t} \tag{3.82}$$

Where v is particles' average drift velocity.

Curl of Cross Product of Velocity v and Vector Fields: $\boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{A})$, $\boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{E})$, $\boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B})$

$$\nabla\times(\mathrm{Y}\times\mathrm{Z})=\mathrm{Y}(\nabla\cdot\mathrm{Z})-\mathrm{Z}(\nabla\cdot\mathrm{Y})+(\mathrm{Z}\cdot\nabla)\mathrm{Y}-(\mathrm{Y}\cdot\nabla)\mathrm{Z} \tag{2}$$

Substituting (Y = v, Z = A), (Y = v, Z = E), and (Y = v, Z = B), respectively, Equation (2) gives NEW field equations:

$$\nabla\times(\mathrm{v}\times\mathrm{A})=\mathrm{v}(\nabla\cdot\mathrm{A})-\mathrm{A}(\nabla\cdot\mathrm{v})+(\mathrm{A}\cdot\nabla)\mathrm{v}-(\mathrm{v}\cdot\nabla)\mathrm{A}=\mathrm{v}(\nabla\cdot\mathrm{A})-\partial(\mathrm{A})/\,\partial\mathrm{t}. \tag{3.83}$$

$$\nabla\times(\mathrm{v}\times\mathrm{E})=\mathrm{v}(\nabla\cdot\mathrm{E})-\mathrm{E}(\nabla\cdot\mathrm{v})+(\mathrm{E}\cdot\nabla)\mathrm{v}-(\mathrm{v}\cdot\nabla)\mathrm{E}=\rho\mathrm{v}-\partial(\mathrm{E})/\,\partial\mathrm{t} \tag{3.84}$$

$$\nabla\times(\mathrm{v}\times\mathrm{B})=\mathrm{v}(\nabla\cdot\mathrm{B})-\mathrm{B}(\nabla\cdot\mathrm{v})+(\mathrm{B}\cdot\nabla)\mathrm{v}-(\mathrm{v}\cdot\nabla)\mathrm{B}=-\,\partial(\mathrm{B})/\,\partial\mathrm{t}. \tag{3.85}$$

Divergence of Product of Scalar Potential $\boldsymbol{\varphi}$ ***and Velocity v:*** $\boldsymbol{\nabla}\cdot(\boldsymbol{\varphi v})$

$$\nabla\cdot(\varphi\mathrm{Y})=\varphi(\nabla\cdot\mathrm{Y})+\mathrm{Y}\cdot(\nabla\varphi) \tag{7}$$

Substituting Y = v into Equation (7), we obtain NEW field equation

$$\nabla\cdot(\varphi\mathrm{v})=\varphi(\nabla\cdot\mathrm{v})+\mathrm{v}\cdot(\nabla\varphi)=\mathrm{v}\cdot(\nabla\varphi) \tag{3.86}$$

Curl of Product of Scalar Field $\boldsymbol{\varphi}$ ***and Velocity v:*** $\boldsymbol{\nabla}\times(\boldsymbol{\varphi v})$

$$\nabla\times(\varphi\mathrm{Y})=\varphi(\nabla\times\mathrm{Y})+(\nabla\varphi)\times\mathrm{Y}. \tag{8}$$

Substituting Y = v into Equation (8), we obtain NEW field equation:

$$\nabla\times(\varphi\mathrm{v})=\varphi(\nabla\times\mathrm{v})+(\nabla\varphi)\times\mathrm{v}=(\nabla\varphi)\times\mathrm{v} \tag{3.87}$$

Gradient of Scalar Product of Velocity v and Vector Fields: $\boldsymbol{\nabla}(\boldsymbol{v}\cdot\boldsymbol{A})$, $\boldsymbol{\nabla}(\boldsymbol{v}\cdot\boldsymbol{E})$, ***and*** $\boldsymbol{\nabla}(\boldsymbol{v}\cdot\boldsymbol{B})$

$$\nabla(\mathrm{Y}\cdot\mathrm{Z})=(\mathrm{Y}\cdot\nabla)\mathrm{Z}+(\mathrm{Z}\cdot\nabla)\mathrm{Y}+\mathrm{Y}\times(\nabla\times\mathrm{Z})+\mathrm{Z}\times(\nabla\times\mathrm{Y}). \tag{9}$$

Substituting (Y = v, $\mathrm{Z}=\mathrm{A}$), or (Y = v, $\mathrm{Z}=\mathrm{E}$), or (Y = v, $\mathrm{Z}=\mathrm{B}$), into Equation (9), respectively, we obtain NEW field equations:

$$\nabla(\mathrm{v}\cdot\mathrm{A})=(\mathrm{v}\cdot\nabla)\mathrm{A}+(\mathrm{A}\cdot\nabla)\mathrm{v}+\mathrm{v}\times(\nabla\times\mathrm{A})+\mathrm{A}\times(\nabla\times\mathrm{v})$$

$$= (v \cdot \nabla)A + v \times B \tag{3.88}$$

$$\nabla(v \cdot E) = (v \cdot \nabla)E + (E \cdot \nabla)v + v \times (\nabla \times E) + E \times (\nabla \times v)$$
$$= (v \cdot \nabla)E + v \times (\nabla \times E). \tag{3.89}$$

$$\nabla(v \cdot B) = (v \cdot \nabla)B + (B \cdot \nabla)v + v \times (\nabla \times B) + B \times (\nabla \times v)$$
$$= (v \cdot \nabla)B + v \times (\nabla \times B). \tag{3.90}$$

***Cross Product:* $(\boldsymbol{v} \times \boldsymbol{\nabla}) \cdot \boldsymbol{A}$*, * $(\boldsymbol{v} \times \boldsymbol{\nabla}) \cdot \boldsymbol{E}$*, and* $(\boldsymbol{v} \times \boldsymbol{\nabla}) \cdot \boldsymbol{B}$**

$$(Y \times \nabla) \cdot Z = Y \cdot (\nabla \times Z). \tag{10}$$

Substituting $(Y = v,\ Z = A)$, or $(Y = v, Z = E)$, or $(Y = v, Z = B)$, respectively, Equation (10) gives new field equations,

$$(v \times \nabla) \cdot A = v \cdot (\nabla \times A) = v \cdot B. \tag{3.91}$$

$$(v \times \nabla) \cdot E = v \cdot (\nabla \times E) = -v \cdot \partial B/\partial t. \tag{3.92}$$

$$(v \times \nabla) \cdot B = v \cdot (\nabla \times B) = v \cdot j + v \cdot \partial E/\partial t. \tag{3.93}$$

***Cross Product:* $(\boldsymbol{v} \times \boldsymbol{\nabla}) \times \boldsymbol{A}$*, * $(\boldsymbol{v} \times \boldsymbol{\nabla}) \times \boldsymbol{E}$*, and* $(\boldsymbol{v} \times \boldsymbol{\nabla}) \times \boldsymbol{B}$**

$$(Y \times \nabla) \times Z = Y \times (\nabla \times Z) + (Y \cdot \nabla)Z - Y(\nabla \cdot Z) \tag{11}$$

Substituting $(Y = v,\ Z = A)$, or $(Y = v, Z = E)$, or $(Y = v, Z = B)$, respectively, Equation (11) gives new field equations:

$$(v \times \nabla) \times A = v \times (\nabla \times A) + (v \cdot \nabla)A - v(\nabla \cdot A) = v \times B + \partial A/\partial t - v(\nabla \cdot A). \tag{3.94}$$

$$(v \times \nabla) \times E = v \times (\nabla \times E) + (v \cdot \nabla)E - v(\nabla \cdot E) = v \times (\nabla \times E) + \partial E/\partial t - \rho v. \tag{3.95}$$

$$(v \times \nabla) \times B = v \times (\nabla \times B) + (v \cdot \nabla)B - v(\nabla \cdot B) = v \times (\nabla \times B) + \partial B/\partial t. \tag{3.96}$$

***Divergence of Curl of Vector Field:* $\boldsymbol{\nabla} \times (\boldsymbol{\varphi Y})$**

$$\nabla \cdot \big(\nabla \times (\varphi Y)\big) = \nabla \cdot [\varphi(\nabla \times Y) + (\nabla\varphi) \times Y] = 0 \tag{12}$$

Substituting $(Y = v)$, Equation (12) gives new field equations:

$$\nabla \cdot \big(\nabla \times (\varphi v)\big) = \nabla \cdot [(\nabla\varphi) \times v] = 0 \tag{3.97}$$

Curl of Curl of Vector Field: $(\varphi \boldsymbol{Y})$

$$\nabla \times \big(\nabla \times (\varphi Y)\big) = \nabla[\varphi(\nabla \cdot Y) + Y \cdot (\nabla\varphi)] - \nabla^2(\varphi Y) \tag{13}$$

Substituting $(Y = v)$, Equation (13) gives new field equations:

$$\nabla \times \big(\nabla \times (\varphi v)\big) = \nabla[v \cdot (\nabla\varphi)] - \nabla^2(\varphi v) \tag{3.98}$$

***Divergence of Cross Product of Two Vector Fields:* $(\boldsymbol{\varphi Y}) \times \boldsymbol{Z}$**

$$\nabla \cdot \big((\varphi Y) \times Z\big) = \varphi(\nabla \times Y) \cdot Z - \varphi(\nabla \times Z) \cdot Y + [(\nabla\varphi) \times Y] \cdot Z \tag{14}$$

Substituting $(Y = v,\ Z = A)$, or $(Y = v, Z = E)$, or $(Y = v, Z = B)$, respectively, Equation (14) gives new field equations:

$$\nabla \cdot \big((\varphi v) \times A\big) = -\varphi(\nabla \times A) \cdot v + [(\nabla\varphi) \times v] \cdot A \tag{3.99}$$

$$\nabla \cdot \big((\varphi v) \times E\big) = -\varphi(\nabla \times E) \cdot v + [(\nabla\varphi) \times v] \cdot E \tag{3.100}$$

$$\nabla \cdot \big((\varphi Y) \times B\big) = -\varphi(\nabla \times B) \cdot v + [(\nabla\varphi) \times v] \cdot B \tag{3.101}$$

Curl of Cross Product of Two Vector Fields: $(\boldsymbol{\varphi Y}) \times \boldsymbol{Z}$

$$\nabla \times \big((\varphi Y) \times Z\big) = \varphi Y(\nabla \cdot Z) - \varphi Z(\nabla \cdot Y) - Z[Y \cdot (\nabla\varphi)] + (Z \cdot \nabla)(\varphi Y) - \varphi(Y \cdot \nabla)Z \tag{15}$$

Substituting $(Y = v,\ Z = A)$, or $(Y = v, Z = E)$, or $(Y = v, Z = B)$, respectively, Equation (15) gives new field equations:

$$\nabla \times \big((\varphi v) \times A\big) = \varphi v(\nabla \cdot A) - A[v \cdot (\nabla\varphi)] + (A \cdot \nabla)(\varphi v) - \varphi(v \cdot \nabla)A \tag{3.102}$$

$$\nabla \times \big((\varphi v) \times E\big) = \varphi v(\nabla \cdot E) - E[v \cdot (\nabla\varphi)] + (E \cdot \nabla)(\varphi v) - \varphi(v \cdot \nabla)E \tag{3.103}$$

$$\nabla \times \big((\varphi v) \times B\big) = -B[v \cdot (\nabla\varphi)] + (B \cdot \nabla)(\varphi v) - \varphi(v \cdot \nabla)B \tag{3.104}$$

Gradient of Dot Product of Two Vector Fields: $(\boldsymbol{\varphi Y} \cdot \boldsymbol{Z})$.

$$\nabla\big((\varphi Y) \cdot Z\big) = \big((\varphi Y) \cdot \nabla\big)Z + (Z \cdot \nabla)(\varphi Y) + (\varphi Y) \times (\nabla \times Z) + Z \times \big(\nabla \times (\varphi Y)\big) \tag{16}$$

Substituting $(Y = v,\ Z = A)$, or $(Y = v, Z = E)$, or $(Y = v, Z = B)$, respectively, Equation (16) gives new field equations:

$$\nabla\big((\varphi v) \cdot A\big) = \big((\varphi v) \cdot \nabla\big)A + (A \cdot \nabla)(\varphi v) + (\varphi v) \times (\nabla \times A) + A \times \big(\nabla \times (\varphi v)\big) \tag{3.105}$$

$$\nabla\big((\varphi v) \cdot E\big) = \big((\varphi v) \cdot \nabla\big)E + (E \cdot \nabla)(\varphi v) + (\varphi v) \times (\nabla \times E) + E \times \big(\nabla \times (\varphi v)\big) \tag{3.106}$$

$$\nabla\big((\varphi v) \cdot B\big) = \big((\varphi v) \cdot \nabla\big)B + (B \cdot \nabla)(\varphi v) + (\varphi v) \times (\nabla \times B) + B \times \big(\nabla \times (\varphi v)\big) \tag{3.107}$$

Cross Product: $((\boldsymbol{\varphi Y}) \times \boldsymbol{\nabla}) \cdot \boldsymbol{Z}$

$$\big((\varphi Y) \times \nabla\big) \cdot Z = \varphi Y \cdot (\nabla \times Z). \tag{17}$$

Substituting $(Y = v,\ Z = A)$, or $(Y = v, Z = E)$, or $(Y = v, Z = B)$, respectively, Equation (17) gives new field equations:

$$\big((\varphi v) \times \nabla\big) \cdot A = \varphi v \cdot (\nabla \times A). \tag{3.108}$$

$$\big((\varphi v) \times \nabla\big) \cdot E = \varphi v \cdot (\nabla \times E). \tag{3.109}$$

$$\big((\varphi v) \times \nabla\big) \cdot B = \varphi v \cdot (\nabla \times B). \tag{3.110}$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \cdot (\boldsymbol{\varphi Z})$

$$(Y \times \nabla) \cdot (\varphi Z) = Y \cdot (\nabla \times (\varphi Z)) \tag{18}$$

Substituting $(Y = v,\ Z = A)$, or $(Y = v, Z = E)$, or $(Y = v, Z = B)$, respectively, Equation (18) gives new field equations:

$$(v \times \nabla) \cdot (\varphi A) = v \cdot (\nabla \times (\varphi A)) \tag{3.111}$$

$$(v \times \nabla) \cdot (\varphi E) = v \cdot (\nabla \times (\varphi E)) \tag{3.112}$$

$$(v \times \nabla) \cdot (\varphi B) = v \cdot (\nabla \times (\varphi B)) \tag{3.113}$$

Cross Product: $((\boldsymbol{\varphi Y}) \times \boldsymbol{\nabla}) \times \boldsymbol{Z}$

$$((\varphi Y) \times \nabla) \times Z = \varphi Y \times (\nabla \times Z) + \varphi(Y \cdot \nabla)Z - \varphi(\nabla \cdot Z)Y. \tag{19}$$

Substituting $(Y = v,\ Z = A)$, or $(Y = v, Z = E)$, or $(Y = v, Z = B)$, respectively, Equation (19) gives new field equations:

$$((\varphi v) \times \nabla) \times A = \varphi v \times (\nabla \times A) + \varphi(v \cdot \nabla)A - \varphi(\nabla \cdot A)v. \tag{3.114}$$

$$((\varphi v) \times \nabla) \times E = \varphi v \times (\nabla \times E) + \varphi(v \cdot \nabla)E - \varphi(\nabla \cdot E)v. \tag{3.115}$$

$$((\varphi v) \times \nabla) \times B = \varphi v \times (\nabla \times B) + \varphi(v \cdot \nabla)B. \tag{3.116}$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \times (\boldsymbol{\varphi Z})$

$$(Y \times \nabla) \times (\varphi Z) = Y \times \left[\left(\nabla \times (\varphi Z)\right)\right] + Y \times [(\nabla\varphi) \times Z] + (Y \cdot \nabla)(\varphi Z) - Y[\varphi(\nabla \cdot Z) + Z \cdot (\nabla\varphi)] \tag{20}$$

Substituting $(Y = v,\ Z = A)$, or $(Y = v, Z = E)$, or $(Y = v, Z = B)$, respectively, Equation (20) gives new field equations:

$$(v \times \nabla) \times (\varphi A) = v \times \left[\left(\nabla \times (\varphi A)\right)\right] + v \times [(\nabla\varphi) \times A] + (v \cdot \nabla)(\varphi A) - v[\varphi(\nabla \cdot A) + A \cdot (\nabla\varphi)] \tag{3.117}$$

$$(v \times \nabla) \times (\varphi E) = v \times \left[\left(\nabla \times (\varphi E)\right)\right] + v \times [(\nabla\varphi) \times E] + (v \cdot \nabla)(\varphi E) - v[\varphi(\nabla \cdot E) + E \cdot (\nabla\varphi)] \tag{3.118}$$

$$(v \times \nabla) \times (\varphi B) = v \times \left[\left(\nabla \times (\varphi B)\right)\right] + v \times [(\nabla\varphi) \times B] + (v \cdot \nabla)(\varphi B) + B \cdot (\nabla\varphi)] \tag{3.119}$$

Electric Field and Magnetic Field Created by Electric Charge Moving with Acceleration a

Divergence of Cross Product of Acceleration a and Vector Fields: $\boldsymbol{\nabla} \cdot (\boldsymbol{a} \times \boldsymbol{A}), \boldsymbol{\nabla} \cdot (\boldsymbol{a} \times \boldsymbol{E})$, ***and*** $\boldsymbol{\nabla} \cdot (\boldsymbol{a} \times \boldsymbol{B})$

$$\nabla \cdot (Y \times Z) = (\nabla \times Y) \cdot Z - Y \cdot (\nabla \times Z) \tag{1}$$

Substituting (Y = a and Z = A), (Y = a and Z = E), and (Y = a and Z = B), respectively, Equation (1) gives New field equations:

$$\nabla \cdot (a \times A) = (\nabla \times a) \cdot A - a \cdot (\nabla \times A) = -a \cdot B. \tag{3.120}$$

$$\nabla \cdot (a \times E) = (\nabla \times a) \cdot E - a \cdot (\nabla \times E) = a \cdot (\partial B/\,\partial t). \tag{3.121}$$

$$\nabla \cdot (\mathrm{a} \times \mathrm{B}) = (\nabla \times \mathrm{a}) \cdot \mathrm{B} - \mathrm{a} \cdot (\nabla \times \mathrm{B}) = -\mathrm{a} \cdot \mathrm{j} - \mathrm{a} \cdot (\partial \mathrm{E}/\partial \mathrm{t}). \tag{3.122}$$

Curl of Cross Product of Acceleration a and Vector Fields: $\boldsymbol{\nabla} \times (\boldsymbol{a} \times \boldsymbol{A})$***,*** $\boldsymbol{\nabla} \times (\boldsymbol{a} \times \boldsymbol{E})$***, and*** $\boldsymbol{\nabla} \times (\boldsymbol{a} \times \boldsymbol{B})$

$$\nabla \times (\mathrm{Y} \times \mathrm{Z}) = \mathrm{Y}(\nabla \cdot \mathrm{Z}) - \mathrm{Z}(\nabla \cdot \mathrm{Y}) + (\mathrm{Z} \cdot \nabla)\mathrm{Y} - (\mathrm{Y} \cdot \nabla)\mathrm{Z} \tag{2}$$

Substituting (Y = a and Z = A), (Y = a and Z = E), and (Y = a and Z = B), respectively, Equation (2) gives NEW field equations:

$$\nabla \times (\mathrm{a} \times \mathrm{A}) = \mathrm{a}(\nabla \cdot \mathrm{A}) - \mathrm{A}(\nabla \cdot \mathrm{a}) + (\mathrm{A} \cdot \nabla)\mathrm{a} - (\mathrm{a} \cdot \nabla)\mathrm{A} = \mathrm{a}(\nabla \cdot \mathrm{A}) - (\mathrm{a} \cdot \nabla)\mathrm{A}. \tag{3.123}$$

$$\nabla \times (\mathrm{a} \times \mathrm{E}) = \mathrm{a}(\nabla \cdot \mathrm{E}) - \mathrm{E}(\nabla \cdot \mathrm{a}) + (\mathrm{E} \cdot \nabla)\mathrm{a} - (\mathrm{a} \cdot \nabla)\mathrm{E} = \mathrm{qa} - (\mathrm{a} \cdot \nabla)\mathrm{E}. \tag{3.124}$$

$$\nabla \times (\mathrm{a} \times \mathrm{B}) = \mathrm{a}(\nabla \cdot \mathrm{B}) - \mathrm{B}(\nabla \cdot \mathrm{a}) + (\mathrm{B} \cdot \nabla)\mathrm{a} - (\mathrm{a} \cdot \nabla)\mathrm{B} = -(\mathrm{a} \cdot \nabla)\mathrm{B}. \tag{3.125}$$

Divergence of Product of Acceleration a and Scalar Potential $\boldsymbol{\varphi}$***:*** $\boldsymbol{\nabla} \cdot (\boldsymbol{\varphi a})$

$$\nabla \cdot (\varphi \mathrm{Y}) = \varphi(\nabla \cdot \mathrm{Y}) + \mathrm{Y} \cdot (\nabla \varphi) \tag{7}$$

Substituting Y = a into Equation (7), we obtain NEW field equation:

$$\nabla \cdot (\varphi \mathrm{a}) = \varphi(\nabla \cdot \mathrm{a}) + \mathrm{a} \cdot (\nabla \varphi) = \mathrm{a} \cdot (\nabla \varphi) \tag{3.126}$$

Curl of Product of Acceleration a and Scalar Field $\boldsymbol{\varphi}$***:*** $\boldsymbol{\nabla} \times (\boldsymbol{\varphi a})$

$$\nabla \times (\varphi \mathrm{Y}) = \varphi(\nabla \times \mathrm{Y}) + (\nabla \varphi) \times \mathrm{Y}. \tag{8}$$

Substituting Y = a into Equation (8), we obtain NEW field equation:

$$\nabla \times (\varphi \mathrm{a}) = \varphi(\nabla \times \mathrm{a}) + (\nabla \varphi) \times \mathrm{a} = (\nabla \varphi) \times \mathrm{a} \tag{3.127}$$

Gradient of Scalar Product of Acceleration a and Vector Fields: $\boldsymbol{\nabla}(\boldsymbol{a} \cdot \boldsymbol{A})$***,*** $\boldsymbol{\nabla}(\boldsymbol{a} \cdot \boldsymbol{E})$***, and*** $\boldsymbol{\nabla}(\boldsymbol{a} \cdot \boldsymbol{B})$

$$\nabla(\mathrm{Y} \cdot \mathrm{Z}) = (\mathrm{Y} \cdot \nabla)\mathrm{Z} + (\mathrm{Z} \cdot \nabla)\mathrm{Y} + \mathrm{Y} \times (\nabla \times \mathrm{Z}) + \mathrm{Z} \times (\nabla \times \mathrm{Y}). \tag{9}$$

Substituting (Y = a, Z = A), (Y = a, Z = E), and (Y = a and Z = B), into Equation (9), respectively, we obtain NEW field equations:

$$\begin{aligned} \nabla(\mathrm{a} \cdot \mathrm{A}) &= (\mathrm{a} \cdot \nabla)\mathrm{A} + (\mathrm{A} \cdot \nabla)\mathrm{a} + \mathrm{a} \times (\nabla \times \mathrm{A}) + \mathrm{A} \times (\nabla \times \mathrm{a}) \\ &= (\mathrm{a} \cdot \nabla)\mathrm{A} + \mathrm{a} \times \mathrm{B} \end{aligned} \tag{3.128}$$

$$\begin{aligned} \nabla(\mathrm{a} \cdot \mathrm{E}) &= (\mathrm{a} \cdot \nabla)\mathrm{E} + (\mathrm{E} \cdot \nabla)\mathrm{a} + \mathrm{a} \times (\nabla \times \mathrm{E}) + \mathrm{E} \times (\nabla \times \mathrm{a}) \\ &= (\mathrm{a} \cdot \nabla)\mathrm{E} + \mathrm{a} \times (\nabla \times \mathrm{E}). \end{aligned} \tag{3.129}$$

$$\begin{aligned} \nabla(\mathrm{a} \cdot \mathrm{B}) &= (\mathrm{a} \cdot \nabla)\mathrm{B} + (\mathrm{B} \cdot \nabla)\mathrm{a} + \mathrm{a} \times (\nabla \times \mathrm{B}) + \mathrm{B} \times (\nabla \times \mathrm{a}) \\ &= (\mathrm{a} \cdot \nabla)\mathrm{B} + \mathrm{a} \times (\nabla \times \mathrm{B}). \end{aligned} \tag{3.130}$$

Cross Product: $(\boldsymbol{a} \times \boldsymbol{\nabla}) \cdot \boldsymbol{A}$***,*** $(\boldsymbol{a} \times \boldsymbol{\nabla}) \cdot \boldsymbol{E}$***, and*** $(\boldsymbol{a} \times \boldsymbol{\nabla}) \cdot \boldsymbol{B}$

$$(\mathrm{Y}\times\nabla)\cdot\mathrm{Z}=\mathrm{Y}\cdot(\nabla\times\mathrm{Z}). \tag{10}$$

Substituting $(\mathrm{Y}=\mathrm{a}$ and $\mathrm{Z}=\mathrm{A})$, $(\mathrm{Y}=\mathrm{a}\ \text{and}\ \mathrm{Z}=\mathrm{E})$, and $(\mathrm{Y}=\mathrm{a}\ \text{and}\ \mathrm{Z}=\mathrm{B})$, respectively, Equation (10) gives new field equations,

$$(\mathrm{a}\times\nabla)\cdot\mathrm{A}=\mathrm{a}\cdot(\nabla\times\mathrm{A})=\mathrm{a}\cdot\mathrm{B}. \tag{3.131}$$

$$(\mathrm{a}\times\nabla)\cdot\mathrm{E}=\mathrm{a}\cdot(\nabla\times\mathrm{E})=-\mathrm{a}\cdot\partial(\mathrm{B})/\partial\mathrm{t}. \tag{3.132}$$

$$(\mathrm{a}\times\nabla)\cdot\mathrm{B}=\mathrm{a}\cdot(\nabla\times\mathrm{B})=\mathrm{a}\cdot\mathrm{j}+\mathrm{a}\cdot\partial(\mathrm{E})/\partial\mathrm{t}. \tag{3.133}$$

Cross Product: $(\boldsymbol{a}\times\boldsymbol{\nabla})\times\boldsymbol{A}$***,*** $(\boldsymbol{a}\times\boldsymbol{\nabla})\times\boldsymbol{E}$***, and*** $(\boldsymbol{a}\times\boldsymbol{\nabla})\times\boldsymbol{B}$

$$(\mathrm{Y}\times\nabla)\times\mathrm{Z}=\mathrm{Y}\times(\nabla\times\mathrm{Z})+(\mathrm{Y}\cdot\nabla)\mathrm{Z}-\mathrm{Y}(\nabla\cdot\mathrm{Z}) \tag{11}$$

Substituting $(\mathrm{Y}=\mathrm{a}$ and $\mathrm{Z}=\mathrm{A})$, $(\mathrm{Y}=\mathrm{a}\ \text{and}\ \mathrm{Z}=\mathrm{E})$, and $(\mathrm{Y}=\mathrm{a}\ \text{and}\ \mathrm{Z}=\mathrm{B})$, respectively, Equation (11) gives new field equations:

$$\begin{aligned}(\mathrm{a}\times\nabla)\times\mathrm{A}&=\mathrm{a}\times(\nabla\times\mathrm{A})+(\mathrm{a}\cdot\nabla)\mathrm{A}-\mathrm{a}(\nabla\cdot\mathrm{A})\\&=\mathrm{a}\times\mathrm{B}+(\mathrm{a}\cdot\nabla)\mathrm{A}-\mathrm{a}(\nabla\cdot\mathrm{A}).\end{aligned} \tag{3.134}$$

$$\begin{aligned}(\mathrm{a}\times\nabla)\times\mathrm{E}&=\mathrm{a}\times(\nabla\times\mathrm{E})+(\mathrm{a}\cdot\nabla)\mathrm{E}-\mathrm{a}(\nabla\cdot\mathrm{E})\\&=\mathrm{a}\times(\nabla\times\mathrm{E})+(\mathrm{a}\cdot\nabla)\mathrm{E}-\rho\mathrm{a}.\end{aligned} \tag{3.135}$$

$$\begin{aligned}(\mathrm{a}\times\nabla)\times\mathrm{B}&=\mathrm{a}\times(\nabla\times\mathrm{B})+(\mathrm{a}\cdot\nabla)\mathrm{B}-\mathrm{a}(\nabla\cdot\mathrm{B})\\&=\mathrm{a}\times(\nabla\times\mathrm{B})+(\mathrm{a}\cdot\nabla)\mathrm{B}.\end{aligned} \tag{3.136}$$

Divergence of Curl of Vector Field: $\boldsymbol{\nabla}\times(\boldsymbol{\varphi Y})$

$$\nabla\cdot\big(\nabla\times(\varphi\mathrm{Y})\big)=\ \nabla\cdot[\varphi(\nabla\times\mathrm{Y})+(\nabla\varphi)\times\mathrm{Y}]=0 \tag{12}$$

Substituting $(\mathrm{Y}=\mathrm{a})$, Equation (12) gives new field equations:

$$\nabla\cdot\big(\nabla\times(\varphi\mathrm{a})\big)=\ \nabla\cdot[(\nabla\varphi)\times\mathrm{a}]=0 \tag{3.137}$$

Curl of Curl of Vector Field: $(\boldsymbol{\varphi Y})$

$$\nabla\times\big(\nabla\times(\varphi\mathrm{Y})\big)=\nabla[\varphi(\nabla\cdot\mathrm{Y})+\mathrm{Y}\cdot(\nabla\varphi)]-\nabla^2(\varphi\mathrm{Y}) \tag{13}$$

Substituting $(\mathrm{Y}=\mathrm{a})$, Equation (13) gives new field equations:

$$\nabla\times\big(\nabla\times(\varphi\mathrm{a})\big)=\nabla[\mathrm{a}\cdot(\nabla\varphi)]-\nabla^2(\varphi\mathrm{a}) \tag{3.138}$$

Divergence of Cross Product of Two Vector Fields: $(\boldsymbol{\varphi Y})\times\boldsymbol{Z}$

$$\nabla\cdot\big((\varphi\mathrm{Y})\times\mathrm{Z}\big)=\varphi(\nabla\times\mathrm{Y})\cdot\mathrm{Z}-\varphi(\nabla\times\mathrm{Z})\cdot\mathrm{Y}+[(\nabla\varphi)\times\mathrm{Y}]\cdot\mathrm{Z} \tag{14}$$

Substituting $(\mathrm{Y}=\mathrm{a}$ and $\mathrm{Z}=\mathrm{A})$, $(\mathrm{Y}=\mathrm{a}\ \text{and}\ \mathrm{Z}=\mathrm{E})$, and $(\mathrm{Y}=\mathrm{a}\ \text{and}\ \mathrm{Z}=\mathrm{B})$, respectively, Equation (14) gives new field equations:

$$\nabla\cdot\big((\varphi\mathrm{a})\times\mathrm{A}\big)=-\varphi\mathrm{B}\cdot\mathrm{a}+[(\nabla\varphi)\times\mathrm{a}]\cdot\mathrm{A} \tag{3.139}$$

$$\nabla\cdot\big((\varphi\mathrm{a})\times\mathrm{E}\big)=-\varphi(\nabla\times\mathrm{E})\cdot\mathrm{a}+[(\nabla\varphi)\times\mathrm{a}]\cdot\mathrm{E} \tag{3.140}$$

$$\nabla \cdot \big((\varphi a) \times B\big) = -\varphi(\nabla \times B) \cdot a + [(\nabla\varphi) \times a] \cdot B \tag{3.141}$$

Curl of Cross Product of Two Vector Fields: $(\boldsymbol{\varphi Y}) \times \boldsymbol{Z}$

$$\nabla \times \big((\varphi Y) \times Z\big) = \varphi Y(\nabla \cdot Z) - \varphi Z(\nabla \cdot Y) - Z[Y \cdot (\nabla\varphi)] + (Z \cdot \nabla)(\varphi Y) - \varphi(Y \cdot \nabla)Z \tag{15}$$

Substituting ($Y = a$ and $Z = A$), ($Y = a \text{ and } Z = E$), and ($Y = a \text{ and } Z = B$), respectively, Equation (15) gives new field equations:

$$\nabla \times \big((\varphi a) \times A\big) = \varphi a(\nabla \cdot A) - A[a \cdot (\nabla\varphi)] + (A \cdot \nabla)(\varphi a) - \varphi(a \cdot \nabla)A \tag{3.142}$$

$$\nabla \times \big((\varphi a) \times E\big) = \varphi a(\nabla \cdot E) - E[a \cdot (\nabla\varphi)] + (E \cdot \nabla)(\varphi a) - \varphi(a \cdot \nabla)E \tag{3.143}$$

$$\nabla \times \big((\varphi a) \times B\big) = -B[a \cdot (\nabla\varphi)] + (B \cdot \nabla)(\varphi a) - \varphi(a \cdot \nabla)B \tag{3.144}$$

Gradient of Dot Product of Two Vector Fields: $(\boldsymbol{\varphi Y} \cdot \boldsymbol{Z})$.

$$\nabla\big((\varphi Y) \cdot Z\big) = \big((\varphi Y) \cdot \nabla\big)Z + (Z \cdot \nabla)(\varphi Y) + (\varphi Y) \times (\nabla \times Z) + Z \times \big(\nabla \times (\varphi Y)\big) \tag{16}$$

Substituting ($Y = a$ and $Z = A$), ($Y = a \text{ and } Z = E$), and ($Y = a \text{ and } Z = B$), respectively, Equation (16) gives new field equations:

$$\nabla\big((\varphi a) \cdot A\big) = \big((\varphi a) \cdot \nabla\big)A + (A \cdot \nabla)(\varphi a) + (\varphi a) \times B \tag{3.145}$$

$$\nabla\big((\varphi a) \cdot E\big) = \big((\varphi a) \cdot \nabla\big)E + (E \cdot \nabla)(\varphi a) + (\varphi a) \times (\nabla \times E) \tag{3.146}$$

$$\nabla\big((\varphi a) \cdot B\big) = \big((\varphi a) \cdot \nabla\big)B + (B \cdot \nabla)(\varphi a) + (\varphi a) \times (\nabla \times B) \tag{3.147}$$

Cross Product: $((\boldsymbol{\varphi Y}) \times \boldsymbol{\nabla}) \cdot \boldsymbol{Z}$

$$\big((\varphi Y) \times \nabla\big) \cdot Z = \varphi Y \cdot (\nabla \times Z). \tag{17}$$

Substituting ($Y = a$ and $Z = A$), ($Y = a \text{ and } Z = E$), and ($Y = a \text{ and } Z = B$), respectively, Equation (17) gives new field equations:

$$\big((\varphi a) \times \nabla\big) \cdot A = \varphi a \cdot (\nabla \times A). \tag{3.148}$$

$$\big((\varphi a) \times \nabla\big) \cdot E = \varphi a \cdot (\nabla \times E). \tag{3.149}$$

$$\big((\varphi a) \times \nabla\big) \cdot B = \varphi a \cdot (\nabla \times B). \tag{3.150}$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \cdot (\boldsymbol{\varphi Z})$

$$(Y \times \nabla) \cdot (\varphi Z) = Y \cdot (\nabla \times (\varphi Z)) \tag{18}$$

Substituting ($Y = a$ and $Z = A$), ($Y = a \text{ and } Z = E$), and ($Y = a \text{ and } Z = B$), respectively, Equation (18) gives new field equations:

$$(a \times \nabla) \cdot (\varphi A) = a \cdot (\nabla \times (\varphi A)) \tag{3.151}$$

$$(a \times \nabla) \cdot (\varphi E) = a \cdot (\nabla \times (\varphi E)) \tag{3.152}$$

$$(a \times \nabla) \cdot (\varphi B) = a \cdot (\nabla \times (\varphi B)) \tag{3.153}$$

Cross Product: $((\boldsymbol{\varphi Y}) \times \boldsymbol{\nabla}) \times \boldsymbol{Z}$

$$((\varphi Y) \times \nabla) \times Z = \varphi Y \times (\nabla \times Z) + \varphi(Y \cdot \nabla)Z - \varphi(\nabla \cdot Z)Y. \tag{19}$$

Substituting ($Y = a$ and $Z = A$), ($Y = a$ and $Z = E$), and ($Y = a$ and $Z = B$), respectively, Equation (19) gives new field equations:

$$((\varphi a) \times \nabla) \times A = \varphi a \times (\nabla \times A) + \varphi(a \cdot \nabla)A - \varphi(\nabla \cdot A)a. \tag{3.154}$$

$$((\varphi a) \times \nabla) \times E = \varphi a \times (\nabla \times E) + \varphi(a \cdot \nabla)E - \varphi(\nabla \cdot E)a. \tag{3.155}$$

$$((\varphi a) \times \nabla) \times B = \varphi a \times (\nabla \times B) + \varphi(a \cdot \nabla)B. \tag{3.156}$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \times (\boldsymbol{\varphi Z})$

$$(Y \times \nabla) \times (\varphi Z) = Y \times \left[\left(\nabla \times (\varphi Z)\right)\right] + Y \times [(\nabla\varphi) \times Z] + (Y \cdot \nabla)(\varphi Z) - Y[\varphi(\nabla \cdot Z) + Z \cdot (\nabla\varphi)] \tag{20}$$

Substituting ($Y = a$ and $Z = A$), ($Y = a$ and $Z = E$), and ($Y = a$ and $Z = B$), respectively, Equation (20) gives new field equations:

$$(a \times \nabla) \times (\varphi A) = a \times \left[\left(\nabla \times (\varphi A)\right)\right] + a \times [(\nabla\varphi) \times A] + (a \cdot \nabla)(\varphi A) - a[\varphi(\nabla \cdot A) + A \cdot (\nabla\varphi)] \tag{3.157}$$

$$(a \times \nabla) \times (\varphi E) = a \times \left[\left(\nabla \times (\varphi E)\right)\right] + a \times [(\nabla\varphi) \times E] + (a \cdot \nabla)(\varphi E) - a[\varphi(\nabla \cdot E) + E \cdot (\nabla\varphi)] \tag{3.158}$$

$$(a \times \nabla) \times (\varphi B) = a \times \left[\left(\nabla \times (\varphi B)\right)\right] + a \times [(\nabla\varphi) \times B] + (a \cdot \nabla)(\varphi B) - a[B \cdot (\nabla\varphi)] \tag{3.159}$$

Electric Field E and Magnetic Field B Created by Spin S of Electric Charge

Divergence of Cross Product of Spin and Vector Field: $\boldsymbol{\nabla} \cdot (\boldsymbol{S} \times \boldsymbol{A})$***,*** $\boldsymbol{\nabla} \cdot (\boldsymbol{S} \times \boldsymbol{E})$***, and*** $\boldsymbol{\nabla} \cdot (\boldsymbol{S} \times \boldsymbol{B})$

$$\nabla \cdot (Y \times Z) = (\nabla \times Y) \cdot Z - Y \cdot (\nabla \times Z) \tag{1}$$

Substituting (Y = S (Spin), Z = A), (Y = S, Z = E), (Y = S, Z = B), respectively, Equation (1) gives New field equation

$$\nabla \cdot (S \times A) = (\nabla \times S) \cdot A - S \cdot (\nabla \times A) = -S \cdot B. \tag{3.160}$$

$$\nabla \cdot (S \times E) = (\nabla \times S) \cdot E - S \cdot (\nabla \times E) = S \cdot \partial(B)/\,\partial t. \tag{3.161}$$

$$\nabla \cdot (S \times B) = (\nabla \times S) \cdot B - S \cdot (\nabla \times B) = -S \cdot j - S \cdot \partial(E)/\,\partial t \tag{3.162}$$

Curl of Cross Product of Spin and Vector Fields: $\boldsymbol{\nabla} \times (\boldsymbol{S} \times \boldsymbol{A})$***,*** $\boldsymbol{\nabla} \times (\boldsymbol{S} \times \boldsymbol{E})$***, and*** $\boldsymbol{\nabla} \times (\boldsymbol{S} \times \boldsymbol{B})$

$$\nabla \times (Y \times Z) = Y(\nabla \cdot Z) - Z(\nabla \cdot Y) + (Z \cdot \nabla)Y - (Y \cdot \nabla)Z \tag{2}$$

Substituting (Y = S and Z = A), (Y = S and Z = E), and (Y = S and Z = B), respectively, Equation (2) gives NEW field equations:

$$\nabla \times (S \times A) = S(\nabla \cdot A) - A(\nabla \cdot S) + (A \cdot \nabla)S - (S \cdot \nabla)A = S(\nabla \cdot A) - (S \cdot \nabla)A. \quad (3.163)$$

$$\nabla \times (S \times E) = S(\nabla \cdot E) - E(\nabla \cdot S) + (E \cdot \nabla)S - (S \cdot \nabla)E = \rho S - (S \cdot \nabla)E. \quad (3.164)$$

$$\nabla \times (S \times B) = S(\nabla \cdot B) - B(\nabla \cdot S) + (B \cdot \nabla)S - (S \cdot \nabla)B = -(S \cdot \nabla)B. \quad (3.165)$$

Divergence of Product of Scalar Potential φ and Spin S: $\nabla \cdot (\varphi S)$

$$\nabla \cdot (\varphi Y) = \varphi(\nabla \cdot Y) + Y \cdot (\nabla\varphi) \quad (7)$$

Substituting Y = S (Spin) into Equation (7), we obtain NEW field equations:

$$\nabla \cdot (\varphi S) = \varphi(\nabla \cdot S) + S \cdot (\nabla\varphi) = S \cdot (\nabla\varphi) \quad (3.166)$$

Curl of Product of Scalar Field φ and Spin S: $\nabla \times (\varphi S)$

$$\nabla \times (\varphi Y) = \varphi(\nabla \times Y) + (\nabla\varphi) \times Y. \quad (8)$$

Substituting Y = S into Equation (8), we obtain NEW field equations:

$$\nabla \times (\varphi S) = \varphi(\nabla \times S) + (\nabla\varphi) \times S = (\nabla\varphi) \times S \quad (3.167)$$

Gradient of Scalar Product of Spin and Vector Fields: $\nabla(S \cdot A)$, $\nabla(S \cdot E)$, and $\nabla(S \cdot B)$

$$\nabla(Y \cdot Z) = (Y \cdot \nabla)Z + (Z \cdot \nabla)Y + Y \times (\nabla \times Z) + Z \times (\nabla \times Y). \quad (9)$$

Substituting ($Y = S$ and $Z = A$), ($Y = S$ and $Z = E$), and ($Y = S$ and $Z = B$), into Equation (9), respectively, we obtain NEW field equation:

$$\nabla(S \cdot A) = (S \cdot \nabla)A + (A \cdot \nabla)S + S \times (\nabla \times A) + A \times (\nabla \times S)$$
$$= (S \cdot \nabla)A + S \times B \quad (3.168)$$

$$\nabla(S \cdot E) = (S \cdot \nabla)E + (E \cdot \nabla)S + S \times (\nabla \times E) + E \times (\nabla \times S)$$
$$= (S \cdot \nabla)E + S \times (\nabla \times E). \quad (3.169)$$

$$\nabla(S \cdot B) = (S \cdot \nabla)B + (B \cdot \nabla)S + S \times (\nabla \times B) + B \times (\nabla \times S)$$
$$= (S \cdot \nabla)B + S \times (\nabla \times B). \quad (3.170)$$

Cross Product: $(S \times \nabla) \cdot A$, $(S \times \nabla) \cdot E$, and $(S \times \nabla) \cdot B$

$$(Y \times \nabla) \cdot Z = Y \cdot (\nabla \times Z). \quad (10)$$

Substituting ($Y = S$ and $Z = A$), ($Y = S$ and $Z = E$), and ($Y = S$ and $Z = B$), respectively, Equation (10) gives new field equation,

$$(S \times \nabla) \cdot A = S \cdot (\nabla \times A) = S \cdot B. \quad (3.171)$$

$$(S \times \nabla) \cdot E = S \cdot (\nabla \times E) = -S \cdot \partial(B)/\partial t. \quad (3.172)$$

$$(S \times \nabla) \cdot B = S \cdot (\nabla \times B) = S \cdot j + S \cdot \partial(E)/\partial t. \quad (3.173)$$

Cross Product: $(S \times \nabla) \times A$, $(S \times \nabla) \times E$, and $(S \times \nabla) \times B$

$$(\mathrm{Y} \times \nabla) \times \mathrm{Z} = \mathrm{Y} \times (\nabla \times \mathrm{Z}) + (\mathrm{Y} \cdot \nabla)\mathrm{Z} - \mathrm{Y}(\nabla \cdot \mathrm{Z}) \tag{11}$$

Substituting $(\mathrm{Y} = \mathrm{S}$ and $\mathrm{Z} = \mathrm{A})$, $(\mathrm{Y} = \mathrm{S} \text{ and } \mathrm{Z} = \mathrm{E})$, and $(\mathrm{Y} = \mathrm{S} \text{ and } \mathrm{Z} = \mathrm{B})$, respectively, Equation (11) gives new field equations:

$$(\mathrm{S} \times \nabla) \times \mathrm{A} = \mathrm{S} \times (\nabla \times \mathrm{A}) + (\mathrm{S} \cdot \nabla)\mathrm{A} - \mathrm{S}(\nabla \cdot \mathrm{A})$$
$$= \mathrm{S} \times \mathrm{B} + (\mathrm{S} \cdot \nabla)\mathrm{A} - \mathrm{S}(\nabla \cdot \mathrm{A}). \tag{3.174}$$

$$(\mathrm{S} \times \nabla) \times \mathrm{E} = \mathrm{S} \times (\nabla \times \mathrm{E}) + (\mathrm{S} \cdot \nabla)\mathrm{E} - \mathrm{S}(\nabla \cdot \mathrm{E})$$
$$= \mathrm{S} \times (\nabla \times \mathrm{E}) + (\mathrm{S} \cdot \nabla)\mathrm{E} - \rho\mathrm{S}. \tag{3.175}$$

$$(\mathrm{S} \times \nabla) \times \mathrm{B} = \mathrm{S} \times (\nabla \times \mathrm{B}) + (\mathrm{S} \cdot \nabla)\mathrm{B} - \mathrm{S}(\nabla \cdot \mathrm{B})$$
$$= \mathrm{S} \times (\nabla \times \mathrm{B}) + (\mathrm{S} \cdot \nabla)\mathrm{B}. \tag{3.176}$$

Divergence of Curl of Vector Field: $\boldsymbol{\nabla} \times (\boldsymbol{\varphi Y})$

$$\nabla \cdot \big(\nabla \times (\varphi \mathrm{Y})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{Y}) + (\nabla\varphi) \times \mathrm{Y}] = 0 \tag{12}$$

Substituting $(\mathrm{Y} = \mathrm{S})$, $(\mathrm{Y} = \mathrm{A})$, and $(\mathrm{Y} = \mathrm{E}) \text{ and } \mathrm{Y} = \mathrm{B})$, respectively, Equation (12) gives new field equations:

$$\nabla \cdot \big(\nabla \times (\varphi \mathrm{S})\big) = \nabla \cdot [(\nabla\varphi) \times \mathrm{S}] = 0 \tag{3.177}$$

$$\nabla \cdot \big(\nabla \times (\varphi \mathrm{A})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{A}) + (\nabla\varphi) \times \mathrm{A}] = 0 \tag{3.178}$$

$$\nabla \cdot \big(\nabla \times (\varphi \mathrm{E})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{E}) + (\nabla\varphi) \times \mathrm{E}] = 0 \tag{3.179}$$

$$\nabla \cdot \big(\nabla \times (\varphi \mathrm{B})\big) = \nabla \cdot [\varphi(\nabla \times \mathrm{B}) + (\nabla\varphi) \times \mathrm{B}] = 0 \tag{3.180}$$

Curl of Curl of Vector Field: $(\boldsymbol{\varphi Y})$

$$\nabla \times \big(\nabla \times (\varphi \mathrm{Y})\big) = \nabla[\varphi(\nabla \cdot \mathrm{Y}) + \mathrm{Y} \cdot (\nabla\varphi)] - \nabla^2(\varphi \mathrm{Y}) \tag{13}$$

Substituting $(\mathrm{Y} = \mathrm{S})$, $(\mathrm{Y} = \mathrm{A})$, $(\mathrm{Y} = \mathrm{E})$, and $(\mathrm{Y} = \mathrm{B})$, respectively, Equation (13) gives new field equations:

$$\nabla \times \big(\nabla \times (\varphi \mathrm{S})\big) = \nabla[\mathrm{S} \cdot (\nabla\varphi)] - \nabla^2(\varphi \mathrm{S}) \tag{3.181}$$

$$\nabla \times \big(\nabla \times (\varphi \mathrm{A})\big) = \nabla[\varphi(\nabla \cdot \mathrm{A}) + \mathrm{A} \cdot (\nabla\varphi)] - \nabla^2(\varphi \mathrm{A}) \tag{3.182}$$

$$\nabla \times \big(\nabla \times (\varphi \mathrm{E})\big) = \nabla[\varphi(\nabla \cdot \mathrm{E}) + \mathrm{E} \cdot (\nabla\varphi)] - \nabla^2(\varphi \mathrm{E}) \tag{3.183}$$

$$\nabla \times \big(\nabla \times (\varphi \mathrm{B})\big) = \nabla[\mathrm{B} \cdot (\nabla\varphi)] - \nabla^2(\varphi \mathrm{B}) \tag{3.184}$$

Divergence of Cross Product: $(\boldsymbol{\varphi S}) \times \boldsymbol{Z}$, $(\boldsymbol{\varphi Y}) \times \boldsymbol{S}$

$$\nabla \cdot \big((\varphi \mathrm{Y}) \times \mathrm{Z}\big) = \varphi(\nabla \times \mathrm{Y}) \cdot \mathrm{Z} - \varphi(\nabla \times \mathrm{Z}) \cdot \mathrm{Y} + [(\nabla\varphi) \times \mathrm{Y}] \cdot \mathrm{Z} \tag{14}$$

Substituting $(\mathrm{Y} = \mathrm{S}$ and $\mathrm{Z} = \mathrm{A})$, $(\mathrm{Y} = \mathrm{S} \text{ and } \mathrm{Z} = \mathrm{E})$, and $(\mathrm{Y} = \mathrm{S} \text{ and } \mathrm{Z} = \mathrm{B})$, respectively, Equation (14) gives new field equations:

$$\nabla \cdot \big((\varphi \mathrm{S}) \times \mathrm{A}\big) = -\varphi(\nabla \times \mathrm{A}) \cdot \mathrm{S} + [(\nabla\varphi) \times \mathrm{S}] \cdot \mathrm{A} \tag{3.185}$$

$$\nabla \cdot \big((\varphi \mathrm{S}) \times \mathrm{E}\big) = -\varphi(\nabla \times \mathrm{E}) \cdot \mathrm{S} + [(\nabla\varphi) \times \mathrm{S}] \cdot \mathrm{E} \tag{3.186}$$

$$\nabla \cdot \big((\varphi S) \times B\big) = -\varphi(\nabla \times B) \cdot S + [(\nabla\varphi) \times S] \cdot B \tag{3.187}$$

Curl of Cross Product of Two Vector Fields: $(\boldsymbol{\varphi S}) \times \boldsymbol{Z}, (\boldsymbol{\varphi Y}) \times \boldsymbol{S}$

$$\nabla \times \big((\varphi Y) \times Z\big) = \varphi Y(\nabla \cdot Z) - \varphi Z(\nabla \cdot Y) - Z[Y \cdot (\nabla\varphi)] + (Z \cdot \nabla)(\varphi Y) - \varphi(Y \cdot \nabla)Z \tag{15}$$

Substituting $(Y = A,\ Z = S)$, $(Y = E, Z = S)$, and $(Y = B, Z = S)$, respectively, Equation (15) gives new field equations:

$$\nabla \times \big((\varphi A) \times S\big) = -\varphi S(\nabla \cdot A) - S[A \cdot (\nabla\varphi)] + (S \cdot \nabla)(\varphi A) \tag{3.188}$$

$$\nabla \times \big((\varphi E) \times S\big) = -\varphi S(\nabla \cdot E) - S[E \cdot (\nabla\varphi)] + (S \cdot \nabla)(\varphi E) \tag{3.189}$$

$$\nabla \times \big((\varphi B) \times S\big) = -S[B \cdot (\nabla\varphi)] + (S \cdot \nabla)(\varphi B) \tag{3.190}$$

Substituting $(Y = S$ and $Z = A)$, $(Y = S \text{ and } Z = E)$, and $(Y = S \text{ and } Z = B)$, respectively, Equation (15) gives new field equations:

$$\nabla \times \big((\varphi S) \times A\big) = \varphi S(\nabla \cdot A) - A[S \cdot (\nabla\varphi)] + (A \cdot \nabla)(\varphi S) - \varphi(S \cdot \nabla)A \tag{3.191}$$

$$\nabla \times \big((\varphi S) \times E\big) = \varphi S(\nabla \cdot E) - E[S \cdot (\nabla\varphi)] + (E \cdot \nabla)(\varphi S) - \varphi(S \cdot \nabla)E \tag{3.192}$$

$$\nabla \times \big((\varphi S) \times B\big) = -B[S \cdot (\nabla\varphi)] + (B \cdot \nabla)(\varphi S) - \varphi(S \cdot \nabla)B \tag{3.193}$$

Gradient of Dot Product of Two Vector Fields: $(\boldsymbol{\varphi Y} \cdot \boldsymbol{Z})$.

$$\nabla\big((\varphi Y) \cdot Z\big) = \big((\varphi Y) \cdot \nabla\big)Z + (Z \cdot \nabla)(\varphi Y) + (\varphi Y) \times (\nabla \times Z) + Z \times \big(\nabla \times (\varphi Y)\big) \tag{16}$$

Substituting $(Y = S$ and $Z = A)$, $(Y = S \text{ and } Z = E)$, and $(Y = S \text{ and } Z = B)$, respectively, Equation (16) gives new field equations:

$$\nabla\big((\varphi S) \cdot A\big) = \big((\varphi S) \cdot \nabla\big)A + (A \cdot \nabla)(\varphi S) + (\varphi S) \times (\nabla \times A) + A \times \big(\nabla \times (\varphi S)\big) \tag{3.194}$$

$$\nabla\big((\varphi S) \cdot E\big) = \big((\varphi S) \cdot \nabla\big)E + (E \cdot \nabla)(\varphi S) + (\varphi S) \times (\nabla \times E) + E \times \big(\nabla \times (\varphi S)\big) \tag{3.195}$$

$$\nabla\big((\varphi S) \cdot B\big) = \big((\varphi S) \cdot \nabla\big)B + (B \cdot \nabla)(\varphi S) + (\varphi S) \times (\nabla \times B) + B \times \big(\nabla \times (\varphi S)\big) \tag{3.196}$$

Dot Product: $((\boldsymbol{\varphi Y}) \times \boldsymbol{\nabla}) \cdot \boldsymbol{Z}$

$$\big((\varphi Y) \times \nabla\big) \cdot Z = \varphi Y \cdot (\nabla \times Z). \tag{17}$$

Substituting $(Y = S$ and $Z = A)$, $(Y = S \text{ and } Z = E)$, and $(Y = S \text{ and } Z = B)$, respectively, Equation (17) gives new field equations:

$$\big((\varphi S) \times \nabla\big) \cdot A = \varphi S \cdot B. \tag{3.197}$$

$$\big((\varphi S) \times \nabla\big) \cdot E = \varphi S \cdot (\nabla \times E). \tag{3.198}$$

$$\big((\varphi S) \times \nabla\big) \cdot B = \varphi S \cdot (\nabla \times B). \tag{3.199}$$

Dot Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \cdot (\boldsymbol{\varphi Z})$

$$(Y \times \nabla) \cdot (\varphi Z) = Y \cdot (\nabla \times (\varphi Z)) \tag{18}$$

Substituting $(Y = S \text{ and } Z = A)$, $(Y = S \text{ and } Z = E)$, and $(Y = S \text{ and } Z = B)$, respectively, Equation (18) gives new field equations:

$$(S \times \nabla) \cdot (\varphi A) = S \cdot (\nabla \times (\varphi A)) \tag{3.200}$$

$$(S \times \nabla) \cdot (\varphi E) = S \cdot (\nabla \times (\varphi E)) \tag{3.201}$$

$$(S \times \nabla) \cdot (\varphi B) = S \cdot (\nabla \times (\varphi B)) \tag{3.202}$$

Cross Product: $((\boldsymbol{\varphi Y}) \times \boldsymbol{\nabla}) \times \boldsymbol{Z}$

$$((\varphi Y) \times \nabla) \times Z = \varphi Y \times (\nabla \times Z) + \varphi(Y \cdot \nabla)Z - \varphi(\nabla \cdot Z)Y. \tag{19}$$

Substituting $(Y = S \text{ and } Z = A)$, $(Y = S \text{ and } Z = E)$, and $(Y = S \text{ and } Z = B)$, respectively, Equation (19) gives new field equations:

$$((\varphi S) \times \nabla) \times A = \varphi S \times B + \varphi(S \cdot \nabla)A - \varphi(\nabla \cdot A)S. \tag{3.203}$$

$$((\varphi S) \times \nabla) \times E = \varphi S \times (\nabla \times E) + \varphi(S \cdot \nabla)E - \varphi(\nabla \cdot E)S. \tag{3.204}$$

$$((\varphi S) \times \nabla) \times B = \varphi S \times (\nabla \times B) + \varphi(S \cdot \nabla)B. \tag{3.205}$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \times (\boldsymbol{\varphi Z})$

$$(Y \times \nabla) \times (\varphi Z) = Y \times \left[\left(\nabla \times (\varphi Z)\right)\right] + Y \times [(\nabla\varphi) \times Z] + (Y \cdot \nabla)(\varphi Z) - Y[\varphi(\nabla \cdot Z) + Z \cdot (\nabla\varphi)] \tag{20}$$

Substituting $(Y = S \text{ and } Z = A)$, $(Y = S \text{ and } Z = E)$, and $(Y = S \text{ and } Z = B)$, respectively, Equation (20) gives new field equations:

$$(S \times \nabla) \times (\varphi A) = S \times \left[\left(\nabla \times (\varphi A)\right)\right] + S \times [(\nabla\varphi) \times A] + (S \cdot \nabla)(\varphi A) - S[\varphi(\nabla \cdot A) + A \cdot (\nabla\varphi)] \tag{3.206}$$

$$(S \times \nabla) \times (\varphi E) = S \times \left[\left(\nabla \times (\varphi E)\right)\right] + S \times [(\nabla\varphi) \times E] + (S \cdot \nabla)(\varphi E) - S[\varphi(\nabla \cdot E) + E \cdot (\nabla\varphi)] \tag{3.207}$$

$$(S \times \nabla) \times (\varphi B) = S \times \left[\left(\nabla \times (\varphi B)\right)\right] + S \times [(\nabla\varphi) \times B] + (S \cdot \nabla)(\varphi B) - S[B \cdot (\nabla\varphi)] \tag{3.208}$$

Summery

In Section 3, we show that the vector calculus identities are powerful tool in re-deriving existing physical laws, and in discovering novel physical laws. We rederived mathematically the following existing physical laws by the vector calculus identities:

1. Poynting's Theorem;
2. Gauss's Law for Magnetism;
3. Charge density **C**ontinuity equation;
4. Wave equations of potentials φ and **A**, electric field **E**, magnetic field **B**;

5. Faraday's Law of Induction;
6. Time changes of energy density of the magnetic field;
7. Time changes of energy density of the electric field;
8. Energy density of the magnetic field.

Also, in Section 3, applying the vector calculus identities, we further *mathematically* derived/proposed many new field equations of both electric fields and magnetic fields with sources moving at velocity v, acceleration a, and spin S, respectively. The new field equations need to be tested experimentally.

CLASSICAL GRAVITY TO GRAVITOELECTROMAGNETICS

Newton Gravitational Field g to Gravitoelectromagnetic Field g and B_g

Newton Gravitational field, g, is created by static gravitational charge density ρ_g (mass density), as shown by Equation (4.1a):

$$\nabla \cdot g = -\rho_g. \tag{4.1a}$$

$$\nabla \cdot E = \rho_e. \tag{4.1b}$$

Equation (4.1b) is the Coulomb law of the electric field, which has the exactly same form as that of Equation (4.1a). This similarity between the two suggests us to study the effects of a moving gravitational charge density $\rho_g v$.

Substituting $(Y = v,\ Z = g)$ into Equation 2 of Vector Calculus Identity,

$$\nabla \times (Y \times Z) = Y(\nabla \cdot Z) - Z(\nabla \cdot Y) + (Z \cdot \nabla)Y - (Y \cdot \nabla)Z \tag{2}$$

we obtain

$$\nabla \times (v \times g) = -\rho_g v - \partial g/\,\partial t. \tag{4.2}$$

Comparing Equation (4.2) with Ampere-Maxwell law (Equation (4.3),

$$\nabla \times B = \rho_e v + \partial E/\,\partial t, \tag{4.3}$$

we propose that Equation (4.2) shows that the mass current $\rho_g v$ creates the gravitomagnetic field B_g, where the gravitomagnetic field, B_g, is defined as,

$$B_g \equiv -v \times g. \tag{4.4}$$

Equation (4.2) becomes

$$\nabla \times B_g = J_g + \partial g/\,\partial t. \tag{4.5}$$

$$J_g = \rho_g v.$$

Following the electromagnetics, let define the gravitomagnetic field B_g in term of the gravitational vector potential A_g, and define the gravitational field g in terms of the gravitational vector potential A_g and the gravitational scalar potential φ_g,

$$B_g \equiv \nabla \times A_g, \tag{4.6}$$

$$g \equiv -\nabla\varphi_g - \partial A_g/\partial t. \tag{4.7}$$

Equation (4.6) and Equation (4.7) give,

$$\nabla \cdot \mathrm{B_g} = 0, \tag{4.8}$$

$$\nabla \times \mathrm{g} = -\partial \mathrm{B_g}/\partial \mathrm{t} \tag{4.9}$$

Equation (4.1a), Equation (4.5), Equation (4.8) and Equation (4.9) are Maxwell-type equations for gravity field.

Note: by introducing the definition of the g field, Equation (4.7), Equation (4.1a) leads to the wave Equation (4.1c) of the scaler gravitation potential φ_g,

$$\nabla \cdot \mathrm{g} = -\nabla^2 \varphi_g + \frac{\partial^2 \varphi_g}{\partial t^2} = -\rho_g. \tag{4.1c}$$

Next, we study the gravitoelectric field g and gravitomagnetic field $\mathrm{B_g}$ further in the same way of studying the electric field E and magnetic field B in Section 3.

Divergence of Cross Product of Vector Fields: $\nabla \cdot (g \times B_g)$, $\nabla \cdot (A_g \times B_g)$, $\nabla \cdot (A_g \times g)$

$$\nabla \cdot (\mathrm{Y} \times \mathrm{Z}) = (\nabla \times \mathrm{Y}) \cdot \mathrm{Z} - \mathrm{Y} \cdot (\nabla \times \mathrm{Z}) \tag{1}$$

Poynting-type Gravitational Theorem:

Substituting (Y = g, Z = $\mathrm{B_g}$), Equation (1) gives Poynting-type Gravitational Theorem:

$$\nabla \cdot (\mathrm{g} \times \mathrm{B_g}) = -\mathrm{g} \cdot \mathrm{j_g} - \frac{1}{2}\partial(\mathrm{g}^2 + \mathrm{B_g}^2)/\partial t \tag{4.10}$$

Where $\nabla \times \mathrm{B_g} = \mathrm{j_g} + (\partial \mathrm{g}/\partial t)$ and $\mathrm{j_g} = \rho_g \mathrm{v}$.

New Field Equation:

Substituting (Y = $\mathrm{A_g}$, Z = $\mathrm{B_g}$) and (Y = $\mathrm{A_g}$, Z = g), respectively, Equation (1) gives NEW field equation:

$$\nabla \cdot (\mathrm{A_g} \times \mathrm{B_g}) = \mathrm{B_g^2} - \mathrm{A_g} \cdot \mathrm{j_g} - \mathrm{A_g} \cdot (\partial \mathrm{g}/\partial t). \tag{4.11}$$

$$\nabla \cdot (\mathrm{A_g} \times \mathrm{g}) = \mathrm{g} \cdot \mathrm{B_g} + \mathrm{A_g} \cdot (\partial \mathrm{B_g}/\partial t).. \tag{4.12}$$

Equation (4.11) represent the energy of magnetic field. In electromagnetics, the term $\mathrm{E} \cdot \mathrm{B}$ is the Lorentz invariant. Similarly, the term, "$\mathrm{g} \cdot \mathrm{B_g}$", of Equation (4.12) is Lorentz scalar invariant.

Curl of Cross Product of Two Vector Fields: $\nabla \times (g \times B_g)$, $\nabla \times (A_g \times B_g)$, and $\nabla \times (A_g \times g)$

$$\nabla \times (\mathrm{Y} \times \mathrm{Z}) = \mathrm{Y}(\nabla \cdot \mathrm{Z}) - \mathrm{Z}(\nabla \cdot \mathrm{Y}) + (\mathrm{Z} \cdot \nabla)\mathrm{Y} - (\mathrm{Y} \cdot \nabla)\mathrm{Z} \tag{2}$$

Substituting (Y = g, Z = $\mathrm{B_g}$), (Y = $\mathrm{A_g}$, Z = $\mathrm{B_g}$), and (Y = $\mathrm{A_g}$, Z = g), respectively, Equation (2) gives NEW field equations:

$$\nabla \times (\mathrm{g} \times \mathrm{B_g}) = \rho_g \mathrm{B_g} + (\mathrm{B_g} \cdot \nabla)\mathrm{g} - (\mathrm{g} \cdot \nabla)\mathrm{B_g} \tag{4.13}$$

$$\nabla \times (\mathrm{A_g} \times \mathrm{B_g}) = -\mathrm{B_g}(\nabla \cdot \mathrm{A_g}) + (\mathrm{B_g} \cdot \nabla)\mathrm{A_g} - (\mathrm{A_g} \cdot \nabla)\mathrm{B_g} \tag{4.14}$$

$$\nabla \times (\mathrm{A_g} \times \mathrm{g}) = -\rho_g \mathrm{A_g} - \mathrm{g}(\nabla \cdot \mathrm{A_g}) + (\mathrm{g} \cdot \nabla)\mathrm{A_g} - (\mathrm{A_g} \cdot \nabla)\mathrm{g}. \tag{4.15}$$

Divergence of Curl of Vector Field: $\nabla \times Y$

$$\nabla \cdot (\nabla \times \mathrm{Y}) = 0 \tag{3}$$

Gauss-type Law for Gravitomagnetics (1):

Substituting Y = $\mathrm{A_g}$, Equation (3) gives Gauss-type Law for gravitomagnetics:

$$\nabla \cdot (\nabla \times \mathrm{A_g}) = \nabla \cdot \mathrm{B_g} = 0 \tag{4.16}$$

Equation (4.16) mathematically indicates no gravitomagnetic monopoles.

Gauss-type Law for Gravitomagnetics (2):

Substituting Y = g, Equation (3) gives:

$$\nabla \cdot (\nabla \times \mathrm{g}) = 0 \tag{4.17}$$

Substituting Equation (4.9) into Equation (4.17), we obtain:

$$\nabla \cdot (\nabla \times \mathrm{g}) = \nabla \cdot (-\partial \mathrm{B_g}/\partial t) = -\frac{\partial}{\partial t}(\nabla \cdot \mathrm{B_g}) = 0. \tag{4.18}$$

Equation (4.18) has two solutions:

1. The gravitomagnetic field $\mathrm{B_g}$ is a constant field:

$$\partial \mathrm{B_g}/\partial t = 0. \tag{4.19}$$

2. Gauss-type Law for gravitomagnetics

$$\nabla \cdot \mathrm{B_g(t)} = 0. \tag{4.20}$$

Gravity-Charge Density Continuity Equation:

Substituting Y = $\mathrm{B_g}$, Equation (3) gives:

$$\nabla \cdot (\nabla \times \mathrm{B_g}) = 0 \tag{4.21}$$

Substituting Equation (4.5) into Equation (4.21), we obtain the Gravity-charge Density Continuity Equation,

$$\nabla \cdot (\nabla \times \mathrm{B_g}) = \nabla \cdot \mathrm{J_g} - \frac{\partial}{\partial t}\rho_g = 0 \tag{4.22}$$

Curl of Curl of Vector Field: Wave Equation

$$\nabla \times (\nabla \times \mathrm{Y}) = \nabla(\nabla \cdot \mathrm{Y}) - \nabla^2 \mathrm{Y} \tag{4}$$

Wave Equation of Gravity Vector Potential A_g:

Substituting Y = $\mathrm{A_g}$, Equation (4) gives:

$$\nabla \times (\nabla \times \mathrm{A_g}) = \mathrm{j_g} + \frac{\partial}{\partial t}\mathrm{g} \tag{4.23a}$$

and

$$\nabla(\nabla \cdot \mathrm{A_g}) - \nabla^2 \mathrm{A_g} = \frac{\partial}{\partial t}\mathrm{g} + \frac{\partial^2}{\partial t^2}\mathrm{A_g} - \nabla^2 \mathrm{A_g} \tag{4.23b}$$

Combining Equation (4.23a) and Equation (4.23b), we obtain wave equation of vector potential $\mathrm{A_g}$:

$$\frac{\partial^2}{\partial t^2}\mathrm{A_g} - \nabla^2 \mathrm{A_g} = \mathrm{j_g} \tag{4.23c}$$

Wave Equation of Gravitoelectric Field g:

Substituting Y = g, Equation (4) gives

$$\nabla \times (\nabla \times \mathrm{g}) = -\frac{\partial}{\partial t}\mathrm{j_g} - \frac{\partial^2}{\partial t^2}\mathrm{g} \tag{4.24a}$$

and

$$\nabla(\nabla \cdot \mathrm{g}) - \nabla^2 \mathrm{g} = \nabla \rho_\mathrm{g} - \nabla^2 \mathrm{g} \tag{4.24b}$$

Combining Equation (4.24a) and Equation (4.24b), we obtain wave equation of gravitoelectric field g,

$$\frac{\partial^2}{\partial t^2}\mathrm{g} - \nabla^2 \mathrm{g} = -\frac{\partial}{\partial t}\mathrm{j_g} - \nabla \rho_\mathrm{g} \tag{4.24c}$$

Wave Equation of Gravitomagnetic Field B_g:

Substituting Y = B, Equation (4) gives

$$\nabla \times (\nabla \times \mathrm{B_g}) = \nabla \times \mathrm{j_g} - \frac{\partial^2}{\partial t^2}\mathrm{B_g} \tag{4.25a}$$

And

$$\nabla(\nabla \cdot \mathrm{B_g}) - \nabla^2 \mathrm{B_g} = -\nabla^2 \mathrm{B_g} \tag{4.25b}$$

Combining Equation (4.25a) and Equation (4.25b), we obtain wave equation of $\mathrm{B_g}$,

$$\frac{\partial^2}{\partial t^2}\mathrm{B_g} - \nabla^2 \mathrm{B_g} = \nabla \times \mathrm{j_g} \tag{4.25c}$$

Divergence of Gradient of Scalar Field: Wave Equation of Gravitation Scalar Potential φ_g

$$\nabla \cdot (\nabla \varphi) = \nabla^2 \varphi \tag{5}$$

Substituting the definition of gravitoelectric field, $\mathbf{g} = -\nabla \varphi_\mathrm{g} - \frac{\partial}{\partial t}\mathbf{A}_\mathrm{g}$ into Equation (5). We obtain wave equation of Gravitation potential φ_g:

$$\nabla \cdot (\nabla \varphi_g) = \rho_g + \frac{\partial^2}{\partial t^2}\varphi_g = \nabla^2 \varphi_g \tag{4.26a}$$

i.e.,

$$\frac{\partial^2}{\partial t^2}\varphi_g - \nabla^2 \varphi_g = -\rho_g. \tag{4.26b}$$

Curl of Gradient of Scalar Field: Faraday-type Gravitation Law of Induction

$$\nabla \times (\nabla \varphi) = 0 \tag{6}$$

Substituting the definition of the gravitoelectric field ,$g \equiv -\nabla \varphi_g - \partial A_g/\partial t$, into Equation (6).

We obtain Faraday-type gravitation Law of Induction:

$$\nabla \times (\nabla \varphi_g) = -\nabla \times g - \partial B_g/\,\partial t = 0. \tag{4.27a}$$

i.e.,

$$\nabla \times g = -\partial B_g/\,\partial t. \tag{4.27b}$$

Divergence of Product of Scalar Gravitation Potential φ_g with Vector Gravitation Potential A_g, Gravitoelectric Field g, Gravitomagnetic Field B_g, Respectively

$$\nabla \cdot (\varphi Y) = \varphi(\nabla \cdot Y) + Y \cdot (\nabla \varphi) \tag{7}$$

Substituting Y = A_g, Y = g, and Y = B_g into Equation (7), respectively, we obtain NEW field equations:

$$\nabla \cdot (\varphi_g A_g) = \varphi_g(\nabla \cdot A_g) + A_g \cdot (\nabla \varphi_g) \tag{4.28}$$

$$\nabla \cdot (\varphi_g g) = -\rho_g \varphi_g + g \cdot (\nabla \varphi_g) \tag{4.29}$$

$$\nabla \cdot (\varphi_g B_g) = B_g \cdot (\nabla \varphi_g) \tag{4.30}$$

Curl of Product of Scalar Gravity Potential φ_g and Vector Gravity Field: A_g, g, B_g

$$\nabla \times (\varphi Y) = \varphi(\nabla \times Y) + (\nabla \varphi) \times Y. \tag{8}$$

Substituting Y = A_g, Y = g, and Y = B_g into Equation (8), respectively, we obtain NEW field equations:

$$\nabla \times (\varphi_g A_g) = \varphi_g B_g + (\nabla \varphi_g) \times A_g \tag{4.31}$$

$$\nabla \times (\varphi_g g) = -\varphi_g \partial B_g/\,\partial t + (\nabla \varphi_g) \times g \tag{4.32}$$

$$\nabla \times (\varphi_g B_g) = J_g \varphi_g + \varphi_g \partial g/\,\partial t + (\nabla \varphi_g) \times B_g \tag{4.33}$$

Gradient of Scalar Product of Two Vector Fields: $\nabla(g \cdot B_g)$, $\nabla(A_g \cdot B_g)$, and $\nabla(A_g \cdot g)$

$$\nabla(Y \cdot Z) = (Y \cdot \nabla)Z + (Z \cdot \nabla)Y + Y \times (\nabla \times Z) + Z \times (\nabla \times Y). \tag{9}$$

Substituting ($Y = g$, $Z = B_g$), ($Y = A_g$, $Z = B_g$), ($Y = A_g$, $Z = g$), ($Y = Z = B_g$), ($Y = Z = A_g$), ($Y = Z = g$), into Equation (9), respectively, we obtain NEW field equations:

$$\nabla(g \cdot B_g) = (g \cdot \nabla)B_g + (B_g \cdot \nabla)g + g \times J_g \tag{4.34}$$

$$\nabla(A_g \cdot B_g) = (A_g \cdot \nabla)B_g + (B_g \cdot \nabla)A_g + A_g \times J_g + A_g \times \partial g/\partial t \tag{4.35}$$

$$\nabla(A_g \cdot g) = (A_g \cdot \nabla)g + (g \cdot \nabla)A_g - A_g \times \partial B_g/\partial t + g \times B_g \tag{4.36}$$

$$\nabla(A_g \cdot A_g) = 2(A_g \cdot \nabla)A_g + 2A_g \times B_g \tag{4.37}$$

$$\nabla(g \cdot g) = 2(g \cdot \nabla)g - 2g \times (\partial B_g/\partial t) \tag{4.38}$$

$$\nabla(B_g \cdot B_g) = 2(B_g \cdot \nabla)B_g + 2B_g \times (\nabla \times B_g). \tag{4.39}$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \cdot \boldsymbol{Z}$

Equation (10),

$$(Y \times \nabla) \cdot Z = Y \cdot (\nabla \times Z), \tag{10}$$

gives the gravitoelectromagnetics field equations.

Substituting, respectively, ($Y = g$, $Z = B_g$), ($Y = B_g$, $Z = g$), ($Y = A_g$, $Z = B_g$), ($Y = B_g$, $Z = A_g$), ($Y = g$, $Z = A_g$), and ($Y = A_g$, $Z = g$),

$$(g \times \nabla) \cdot B_g = g \cdot J_g + g \cdot \partial g/\partial t \tag{4.40}$$

$$(B_g \times \nabla) \cdot g = -B_g \cdot \partial B_g/\partial t. \tag{4.41}$$

$$(A_g \times \nabla) \cdot B_g = A_g \cdot J_g + A_g \cdot \partial g/\partial t \tag{4.42}$$

$$(B_g \times \nabla) \cdot A_g = B_g^2 \tag{4.43}$$

$$(g \times \nabla) \cdot A_g = g \cdot B_g \tag{4.44}$$

$$(A_g \times \nabla) \cdot g = -\, A_g \cdot \partial B_g/\partial t \tag{4.45}$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \times \boldsymbol{Z}$

$$(Y \times \nabla) \times Z = Y \times (\nabla \times Z) + (Y \cdot \nabla)Z - Y(\nabla \cdot Z) \tag{11}$$

Substituting ($Y = g$, $Z = B_g$), ($Y = B_g$, $Z = g$), ($Y = A_g$, $Z = B_g$), ($Y = B_g$, $Z = A_g$), ($Y = A_g$, $Z = g$), and ($Y = g$, $Z = A_g$), Equation (11) gives new field equations:

$$(g \times \nabla) \times B_g = g \times J_g + (g \cdot \nabla)B_g \tag{4.46}$$

$$(B_g \times \nabla) \times g = -\rho B_g + (B_g \cdot \nabla)g \tag{4.47}$$

$$(A_g \times \nabla) \times B_g = A_g \times J_g + A_g \times \partial g/\partial t + (A_g \cdot \nabla)B_g \tag{4.48}$$

$$(B_g \times \nabla) \times A_g = (B_g \cdot \nabla)A_g + B_g(\partial \varphi_g/\partial t) \tag{4.49}$$

$$(A_g \times \nabla) \times g = -A_g \times \partial B_g/\partial t + (A_g \cdot \nabla)g - \rho A_g \tag{4.50}$$

$$(g \times \nabla) \times A_g = g \times B_g + (B_g \cdot \nabla)A_g + g(\partial \varphi_g/\partial t) \tag{4.51}$$

In Section 4.1, we have mathematically derived physics laws which have the same forms of electromagnetic laws:

(a) Poynting-type gravity Theorem

(b) Gauss-type Law for gravitomagnetism

(c) Gravity-Charge density continuity equation

(d) Faraday-type gravitation Law of Induction

(e) Wave equation of vector gravity potential A_g , gravitoelectric field g and gravitomagnetic field B_g

(f) Energy density of time change of the gravitoelectric field

(g) Energy density of time change of the gravitomagnetic field

(h) Energy density of the gravitomagnetic field

We also derived several new physical laws of gravity.

Gravitoelectric Field and Gravitomagnetic Field Created by Velocity v of Gravitational Mass-charge

We study how Newton gravity field g of rest mass goes to Gravitomagnetic field B_g of moving mass

Divergence of Cross Product of Velocity v and Vector Fields, A_g, g, B_g

$$\nabla \cdot (\mathrm{Y} \times \mathrm{Z}) = (\nabla \times \mathrm{Y}) \cdot \mathrm{Z} - \mathrm{Y} \cdot (\nabla \times \mathrm{Z}) \tag{1}$$

Substituting (Y = v, Z = A_g), (Y = v, Z = g), and (Y = v, Z = B_g), respectively, Equation (1) gives New field equations:

$$\nabla \cdot \left(\mathrm{v} \times \mathrm{A_g}\right) = -\mathrm{v} \cdot \mathrm{B_g}. \tag{4.52}$$

$$\nabla \cdot (\mathrm{v} \times \mathrm{g}) = \mathrm{v} \cdot \partial \mathrm{B_g} / \partial \mathrm{t}. \tag{4.53}$$

$$\nabla \cdot \left(\mathrm{v} \times \mathrm{B_g}\right) = -\mathrm{v} \cdot \mathrm{J_g} - \mathrm{v} \cdot \partial \mathrm{g} / \partial \mathrm{t} \tag{4.54}$$

Curl of Cross Product of Velocity v and Vector Gravitation Fields, A_g, g, B_g

$$\nabla \times (\mathrm{Y} \times \mathrm{Z}) = \mathrm{Y}(\nabla \cdot \mathrm{Z}) - \mathrm{Z}(\nabla \cdot \mathrm{Y}) + (\mathrm{Z} \cdot \nabla)\mathrm{Y} - (\mathrm{Y} \cdot \nabla)\mathrm{Z} \tag{2}$$

Substituting (Y = v, Z = A_g), (Y = v, Z = g), and (Y = v, Z = B_g), respectively, Equation (2) gives Maxwell-type gravitation field equations:

$$\nabla \times \left(\mathrm{v} \times \mathrm{A_g}\right) = \mathrm{v}\left(\nabla \cdot \mathrm{A_g}\right) - \partial \mathrm{A_g} / \partial \mathrm{t}. \tag{4.55}$$

$$\nabla \times (\mathrm{v} \times \mathrm{g}) = \mathrm{v}(\nabla \cdot \mathrm{g}) - \partial \mathrm{g} / \partial \mathrm{t} \tag{4.56}$$

$$\nabla \times \left(\mathrm{v} \times \mathrm{B_g}\right) = \mathrm{v}\left(\nabla \cdot \mathrm{B_g}\right) - \partial \mathrm{B_g} / \partial \mathrm{t}. \tag{4.57}$$

Equation (4.56) and Equation (4.57) are perfect dual to each other.

Let us introducing the definitions of gravitomagnetic field B_g and gravitoelectric field g:

$$B_g \equiv -v \times g. \tag{4.58}$$

$$g \equiv v \times B_g \tag{4.59}$$

Then Equation (4.56) and Equation (4.57) can be written as:

$$\nabla \times B_g = \rho_g v + \partial g / \partial t \tag{4.60}$$

$$\nabla \times g = -\partial B_g / \partial t. \tag{4.61}$$

Equation (4.60) and Equation (4.61) have the same form as that of Ampere-Maxwell equation and Faraday equation, respectively:

$$\nabla \times B = \rho_e v + \partial E / \partial t.,$$

$$\nabla \times E = -\partial B / \partial t.$$

Gradient of Scalar Product of Velocity v and Vector Fields: $\boldsymbol{\nabla}(\boldsymbol{Y} \cdot \boldsymbol{Z})$

$$\nabla(Y \cdot Z) = (Y \cdot \nabla)Z + (Z \cdot \nabla)Y + Y \times (\nabla \times Z) + Z \times (\nabla \times Y). \tag{9}$$

Substituting (Y = v and $Z = A_g$), or (Y = v and $Z = g$), or (Y = v and $Z = B_g$), into Equation (9), respectively, we obtain NEW field equations:

$$\nabla(v \cdot A_g) = (v \cdot \nabla)A_g + v \times B_g \tag{4.62}$$

$$\nabla(v \cdot g) = (v \cdot \nabla)g + v \times (\nabla \times g). \tag{4.63}$$

$$\nabla(v \cdot B_g) = (v \cdot \nabla)B_g + v \times (\nabla \times B_g). \tag{4.64}$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \cdot \boldsymbol{Z}$

$$(Y \times \nabla) \cdot Z = Y \cdot (\nabla \times Z). \tag{10}$$

Substituting $(Y = v,\ Z = A_g)$, or $(Y = v, Z = g)$, or $(Y = v, Z = B_g)$, respectively, Equation (10) gives new field equations,

$$(v \times \nabla) \cdot A_g = v \cdot B_g. \tag{4.65}$$

$$(v \times \nabla) \cdot g = -v \cdot \partial B_g / \partial t. \tag{4.66}$$

$$(v \times \nabla) \cdot B_g = v \cdot J_g + v \cdot \partial g / \partial t. \tag{4.67}$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \times \boldsymbol{Z}$

$$(Y \times \nabla) \times Z = Y \times (\nabla \times Z) + (Y \cdot \nabla)Z - Y(\nabla \cdot Z) \tag{11}$$

Substituting $(Y = v,\ Z = A_g)$, or $(Y = v, Z = g)$, or $(Y = v, Z = B_g)$, respectively, Equation (11) gives new field equations:

$$(v \times \nabla) \times A_g = v \times B_g + \partial A_g / \partial t - v(\nabla \cdot A_g). \tag{4.68}$$

$$(v \times \nabla) \times g = v \times (\nabla \times g) + \partial g/ \partial t - \rho_g v. \tag{4.69}$$

$$(v \times \nabla) \times B_g = v \times \left(\nabla \times B_g\right) + \partial B_g/ \partial t. \tag{4.70}$$

Gravitoelectric Field and Gravitomagnetic Field Created by Acceleration a of Gravitational-charge

Divergence of Cross Product of Acceleration a and Vector Fields: $\boldsymbol{\nabla} \cdot (\boldsymbol{Y} \times \boldsymbol{Z})$

$$\nabla \cdot (Y \times Z) = (\nabla \times Y) \cdot Z - Y \cdot (\nabla \times Z) \tag{1}$$

Substituting (Y = a, Z = A_g), (Y = a, Z = g), and (Y = a, Z = B_g), respectively, Equation (1) gives New field equations:

$$\nabla \cdot \left(a \times A_g\right) = -a \cdot B_g. \tag{4.71}$$

$$\nabla \cdot (a \times g) = a \cdot (\partial B_g/ \partial t). \tag{4.72}$$

$$\nabla \cdot \left(a \times B_g\right) = -a \cdot J_g - a \cdot (\partial g/ \partial t). \tag{4.73}$$

Curl of Cross Product of Acceleration a and Vector Fields: $\boldsymbol{\nabla} \times (\boldsymbol{Y} \times \boldsymbol{Z})$

$$\nabla \times (Y \times Z) = Y(\nabla \cdot Z) - Z(\nabla \cdot Y) + (Z \cdot \nabla)Y - (Y \cdot \nabla)Z \tag{2}$$

Substituting (Y = a, Z = A_g), (Y = a, Z = g), and (Y = a, Z =B_g), respectively, Equation (2) gives NEW field equations:

$$\nabla \times \left(a \times A_g\right) = a\left(\nabla \cdot A_g\right) - (a \cdot \nabla)A_g. \tag{4.74}$$

$$\nabla \times (a \times g) = \rho_g a - (a \cdot \nabla)g. \tag{4.75}$$

$$\nabla \times \left(a \times B_g\right) = -(a \cdot \nabla)B_g. \tag{4.76}$$

Equation (4.75) shows that an accelerating gravity-charge, ρ_g a, creates a gravitomagnetic-type field, a$\times$ g.

Gradient of Scalar Product of Acceleration a and Vector Fields: $\boldsymbol{\nabla}(\boldsymbol{Y} \cdot \boldsymbol{Z})$

$$\nabla(Y \cdot Z) = (Y \cdot \nabla)Z + (Z \cdot \nabla)Y + Y \times (\nabla \times Z) + Z \times (\nabla \times Y). \tag{9}$$

Substituting (Y = a, Z = A_g), (Y = a, Z = g), and (Y = a, Z = B_g), into Equation (9), respectively, we obtain NEW field equations:

$$\nabla\left(a \cdot A_g\right) = (a \cdot \nabla)A_g + a \times B_g. \tag{4.77}$$

$$\nabla(a \cdot g) = (a \cdot \nabla)g + a \times (\nabla \times g). \tag{4.78}$$

$$\nabla\left(a \cdot B_g\right) = (a \cdot \nabla)B_g + a \times \left(\nabla \times B_{g.}\right). \tag{4.79}$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \cdot \boldsymbol{Z}$

$$(Y \times \nabla) \cdot Z = Y \cdot (\nabla \times Z). \tag{10}$$

Substituting $(Y = a, Z = A_g)$, $(Y = a, Z = g)$, and $(Y = a, Z = B_g)$, respectively, Equation (10) gives new field equations,

$$(a \times \nabla) \cdot A_g = a \cdot B_g. \quad (4.80)$$

$$(a \times \nabla) \cdot g = -a \cdot \partial(B_g)/\partial t. \quad (4.81)$$

$$(a \times \nabla) \cdot B_g = a \cdot J_g + a \cdot \partial(g)/\partial t. \quad (4.82)$$

Cross Product: $(\boldsymbol{Y} \times \boldsymbol{\nabla}) \times \boldsymbol{Z}$

$$(Y \times \nabla) \times Z = Y \times (\nabla \times Z) + (Y \cdot \nabla)Z - Y(\nabla \cdot Z) \quad (11)$$

Substituting $(Y = a, Z = A_g)$, $(Y = a, Z = g)$, and $(Y = a, Z = B_g)$, respectively, Equation (11) gives new field equations:

$$(a \times \nabla) \times A_g = a \times B_g + (a \cdot \nabla)A_g - a(\nabla \cdot A_g). \quad (4.83)$$

$$(a \times \nabla) \times g = a \times (\nabla \times g) + (a \cdot \nabla)g - \rho_g a. \quad (4.84)$$

$$(a \times \nabla) \times B_g = a \times (\nabla \times B_g) + (a \cdot \nabla)B_g. \quad (4.85)$$

Gravitoelectric Field g and Gravitomagnetic Field B_g Created by Spin S of Gravity-charge

Divergence of Cross Product of Spin and Vector Field: $\boldsymbol{\nabla} \cdot (\boldsymbol{S} \times \boldsymbol{A_g})$, $\boldsymbol{\nabla} \cdot (\boldsymbol{S} \times \boldsymbol{g})$, $\boldsymbol{\nabla} \cdot (\boldsymbol{S} \times \boldsymbol{B_g})$

$$\nabla \cdot (Y \times Z) = (\nabla \times Y) \cdot Z - Y \cdot (\nabla \times Z) \quad (1)$$

Substituting (Y = S, Z = A_g), (Y = S, Z = g), and (Y = S, Z = B_g), Equation (1) gives New field equation

$$\nabla \cdot (S \times A_g) = -S \cdot B_g. \quad (4.86)$$

$$\nabla \cdot (S \times g) = S \cdot \partial(B_g)/\partial t. \quad (4.87)$$

$$\nabla \cdot (S \times B_g) = -S \cdot J_g - S \cdot \partial g/\partial t \quad (4.88)$$

Curl of Cross Product of Spin and Vector Fields: $\boldsymbol{\nabla} \times (\boldsymbol{Y} \times \boldsymbol{Z})$

$$\nabla \times (Y \times Z) = Y(\nabla \cdot Z) - Z(\nabla \cdot Y) + (Z \cdot \nabla)Y - (Y \cdot \nabla)Z \quad (2)$$

Substituting (Y = S, Z = A_g), (Y = S, Z = g), and (Y = S, Z = B_g), Equation (2) gives NEW field equation:

$$\nabla \times (S \times A_g) = S(\nabla \cdot A_g) - (S \cdot \nabla)A_g. \quad (4.89)$$

$$\nabla \times (S \times g) = \rho_g S - (S \cdot \nabla)g. \quad (4.90)$$

$$\nabla \times (S \times B_g) = -(S \cdot \nabla)B_g. \quad (4.91)$$

Divergence of Product of Scalar Potential φ and Spin S

$$\nabla \cdot (\varphi Y) = \varphi(\nabla \cdot Y) + Y \cdot (\nabla \varphi) \quad (7)$$

Substituting Y = S into Equation (7), we obtain NEW field equations:

$$\nabla \cdot (\varphi S) = S \cdot (\nabla\varphi) \tag{4.92}$$

Curl of Product of Scalar Field φ and Spin S

$$\nabla \times (\varphi Y) = \varphi(\nabla \times Y) + (\nabla\varphi) \times Y. \tag{8}$$

Substituting Y = S into Equation (8), we obtain NEW field equations:

$$\nabla \times (\varphi S) = (\nabla\varphi) \times S \tag{4.93}$$

Gradient of Scalar Product of Spin and Vector Fields: $\nabla(S \cdot A_g)$, $\nabla(S \cdot g)$, and $\nabla(S \cdot B_g)$

$$\nabla(Y \cdot Z) = (Y \cdot \nabla)Z + (Z \cdot \nabla)Y + Y \times (\nabla \times Z) + Z \times (\nabla \times Y). \tag{9}$$

Substituting ($Y = S$, $Z = A_g$), ($Y = S$, $Z = g$), and ($Y = S$, $Z = B_g$), into Equation (9), respectively, we obtain NEW field equation:

$$\nabla(S \cdot A_g) = (S \cdot \nabla)A_g + S \times B_g \tag{4.94}$$

$$\nabla(S \cdot g) = (S \cdot \nabla)g + S \times (\nabla \times g). \tag{4.95}$$

$$\nabla(S \cdot B_g) = (S \cdot \nabla)B_g + S \times (\nabla \times B_g). \tag{4.96}$$

Cross Product: $(Y \times \nabla) \cdot Z$

$$(Y \times \nabla) \cdot Z = Y \cdot (\nabla \times Z). \tag{10}$$

Substituting ($Y = S$, $Z = A_g$), ($Y = S, Z = g$), and ($Y = S, Z = B_g$), respectively, Equation (10) gives new field equation,

$$(S \times \nabla) \cdot A_g = S \cdot B_g. \tag{4.97}$$

$$(S \times \nabla) \cdot g = -S \cdot \partial(B_g)/\partial t. \tag{4.98}$$

$$(S \times \nabla) \cdot B_g = S \cdot j + S \cdot \partial(g)/\partial t. \tag{4.99}$$

Cross Product: $(Y \times \nabla) \times Z$

$$(Y \times \nabla) \times Z = Y \times (\nabla \times Z) + (Y \cdot \nabla)Z - Y(\nabla \cdot Z) \tag{11}$$

Substituting ($Y = S$, $Z = A_g$), ($Y = S, Z = g$), and ($Y = S, Z = B_g$), respectively, Equation (11) gives new field equations:

$$(S \times \nabla) \times A_g = S \times B_g + (S \cdot \nabla)A_g - S(\nabla \cdot A_g). \tag{4.100}$$

$$(S \times \nabla) \times g = S \times (\nabla \times g) + (S \cdot \nabla)g - \rho_g S. \tag{4.101}$$

$$(S \times \nabla) \times B_g = S \times (\nabla \times B_g) + (S \cdot \nabla)B_g. \tag{4.102}$$

SUMMARY

In this article, we show that the vector calculus identities are powerful tool in both rederiving the existing physical law, such as Maxwell equations, and discovering new physical laws.

The mathematically derived new physical laws need to be tested experimentally.

REFERENCE

[1] Vector calculus identities, Wikipedia.