



Planck-Scale GR Quantum Models of the Universe: Vacuum as a Coalescence of Micro-Universes; Particles as Micro-Photon Spheres Calculation of: Alpha; Lambda

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Abstract: This article is a new completed version of EJAS 11804 (Vol. 10, N°1, Feb 25, 2022). It presents in a synthetic way three articles published in JHEPGC ([1], [2]), EJAS [12] and develops some aspects. Two hypotheses are studied. In the first one, the vacuum is endowed with a quantum structure in which the vacuum particles are Friedmann-Planck micro-universes. For this, the article introduces a quantization of a closed Friedmann universe, then a quantization of the photon spheres filling this universe. This approach gives a numerical value consistent with cosmological measurements for the current dark energy density and the cosmological constant of our Universe. Next, the second hypothesis takes the content of a model published in Physics Essays in 2013 [3], assuming that elementary particles are Schwarzschild photon spheres; these could be derived from the Friedmann photon spheres composing the vacuum particles. It is further recalled that the model presents a unified structure of elementary particles and allows us to calculate the value of the elementary electric charge and the fine structure constant. The masses of some elementary particles are calculated in a complementary model. Finally, this article summarizes a model of closed cyclic universe described in reference [2]. This universe begins as one alone Friedmann-Planck micro-universe, then multiplying to constitute our Universe. Further a Big-Rip suddenly transforms it into a Friedmann-Planck macro-universe on a much larger scale. This one is the beginning of a new Big-Bang with the same evolution as ours. This process can be assumed to explain the existence of the initial FP micro-universe: it would be the result of a Big-Rip at the end of the evolution of a much smaller scale universe.

Keywords: Friedmann Universe, Planck, Schwarzschild photon spheres, Cosmology, Elementary particle masses, Dark energy, PACS: 04.60.-m.

INTRODUCTION

The contraction of a homogeneous and isotropic universe such as our own, governed by a Friedmann evolution equation, implies that it passes through a radiative phase dominated by radiation. If this universe is closed, its contraction will bring it to the size of Planck, the ultimate phase before a possible singularity.

This observation is at the origin of the hypothesis developed in this article: that the constituent elements of vacuum, or vacuum particles, of the current Universe, could be quantum Friedmann micro-universes.

Our hypothesis is developed by writing a Schrödinger equation of the Friedmann universe starting from the equation of evolution in general relativity. It results in a quantization whose fundamental state is characterized by a radius, an energy, and a density of energy close to the corresponding Planck units.

The article then proceeds to a quantization of the photons of the Friedmann-Planck micro-universe studied previously, which shows that the photons are structured in

concentric spheres, later called Fp-spheres. The number of these spheres is calculated by comparing their cumulative total energy and the energy of the Friedmann-Planck micro-universe.

The result of this calculation leads to the observation that the dark energy density of our Universe is equal to the energy density induced by the lightest photon sphere of each micro-universe.

The following section of this article uses the reasoning developed in an article in Physics Essays 2013 [4], illustrating it with the results obtained on the structure of the vacuum. That article developed the hypothesis that the elementary particles consist of spheres of self-gravitating photons in a Schwarzschild field (later called Sp-spheres). This approach makes it both possible to calculate the value of the elementary electric charge and to propose a representation of all the elementary particles.

It is here that the coherence between the model describing the vacuum and the one describing the elementary particles appears: the elementary particles, or Sp-spheres of our Universe, appear as excited states of the Fp-spheres contained in the FP micro-universes.

Two papers published in Physics Essays [4] and in Nova Science Publisher [5] propose a derived model for calculating the masses of the charged leptons for the first and those of all the elementary particles for the second; see a summary presentation of this model in the appendix.

QUANTIZATION OF A CLOSED RADIATIVE FRIEDMANN UNIVERSE

A Friedmann universe is characterized, in the case where it is closed, by a dynamic equation expressed here in the following form [4]:

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{8\pi G}{3c^4} a^2 d \quad (1)$$

where a is the radius of the universe, a function of time and d the energy density. In its radiative phase, the following relation characterizes the universe:

$$a^4 d = C^{te} \quad (2)$$

We can write it thus:

$$\frac{8\pi G a^4 d}{3c^4} = C^t = a_0^2 \quad (3)$$

Equation (1) becomes:

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{a_0^2}{a^2} \quad (4)$$

The total energy of the universe is defined by: $\mathcal{E} = 2\pi^2 a^3 d$ (5)

By putting $\Gamma = \mathcal{E}a$, we obtain from (3) the following relations:

$$\frac{4G\mathcal{E}a}{3\pi c^4} = \frac{4G\Gamma}{3\pi c^4} = a_0^2 \quad (6)$$

The quantization proposed consists of writing a Schrödinger equation of the Universe from equation (4), sometimes called the Wheeler- DeWitt equation. For this, it is necessary

to define a quantity homogeneous to a momentum in order to introduce a quantum operator. We will assume that this quantity can be written:

$$p = \mathcal{E} \frac{\dot{a}}{c^2} \quad (7)$$

We can thus write equation (4) in the following form:

$$p^2 c^2 + \mathcal{E}^2 \left(1 - \frac{a_0^2}{a^2}\right) = 0 \quad (8)$$

By introducing the constant quantity Γ defined above, (8) becomes:

$$p^2 c^2 + \frac{\Gamma^2}{a^2} \left(1 - \frac{a_0^2}{a^2}\right) = 0 \quad (9)$$

The Schrödinger equation is obtained in the usual way, using the quantum operator $p \rightarrow -i\hbar d/da$ acting on the wave function ψ :

$$-\hbar^2 c^2 a^2 \psi'' + \Gamma^2 \left(1 - \frac{a_0^2}{a^2}\right) \psi = 0 \quad (10)$$

By using the variable $x = a/a_0$ and the constant $\gamma = \Gamma/\hbar c$, (10) becomes:

$$-x^2 \psi'' + \gamma^2 \left(1 - \frac{1}{x^2}\right) \psi = 0 \quad (11)$$

The asymptotic solution of this differential equation when $x \rightarrow \infty$ is:

$$\psi \propto x^k; \gamma^2 = k(k-1) \quad (12)$$

If we look for solutions in the form of an entire series in x , it appears that $k = -n$ with n integer; moreover, $n \geq 2$ is the condition for the average value of x to have a finite value. Therefore:

$$\gamma^2 = n(n+1); n = 2 \Rightarrow \gamma^2 = 6 \quad (13)$$

In this way we see that a quantum Friedmann universe is characterized by a quantization of the quantity that remains constant during the evolution of the universe: the product of its total energy by its radius. The fundamental level corresponds to $n = 2$.

When x is close to zero, the real asymptotic solution is:

$$\psi \propto \cos\left(\frac{\gamma}{x} + \theta\right) \quad (14)$$

θ is an arbitrary constant; the relation (6) allows us to write additionally:

$$\frac{4G\hbar\gamma}{3\pi c^3} = a_0^2 \quad (15)$$

Showing the Planck length unit: $a_p = \sqrt{\frac{G\hbar}{c^3}}$, we obtain:

$$a_0^2(n) = \frac{4\gamma a_p^2}{3\pi} = \frac{4\sqrt{n(n+1)} a_p^2}{3\pi} \quad (16)$$

At the fundamental level ($n = 2$):

$$a_0^2 = \frac{4\sqrt{6}}{3\pi} a_p^2 \sim 1,04 a_p^2 \quad (17)$$

Thus, the characteristic dimension of a Friedmann closed quantum universe in its ground state is close to the Planck length unit. Its energy and energy density are given by relationships:

$$\mathcal{E}_F = \frac{\sqrt{6}\hbar c}{a}; d_F = \frac{\sqrt{6}\hbar c}{2\pi^2 a^4} \quad (18)$$

The "radius" of this micro-universe is a quantum variable whose average value is close to that of the Planck radius. We will refer to it as the "Friedmann-Planck micro-universe (FPmu)". Its density, given by formula (18), is also of the order of magnitude of Planck's density. If the micro-universes are the vacuum particles and are contiguous (granular space), the energy density of the macro-universe (ours) would also be on the order of magnitude of the Planck density. Other theoretical approaches lead to the same result on vacuum energy, that is to say an energy density close to the Planck density, in flagrant contradiction with the experimental data, which give a very low value for vacuum energy (see below).

Here, we suppose that in order to resolve this contradiction the energy of each of these Friedmann-Planck micro-universes constituting our macro-universe is confined within it and is not an observable of our Universe. The relations (18) correspond to virtual values of energy and energy density of the vacuum particles. Only a tiny part of this energy -- dark energy -- appears on the outside in our macro-universe. An alternative explanation will be considered below.

QUANTIZATION OF THE PHOTONS OF A RADIATIVE CLOSED FRIEDMANN UNIVERSE

Quantization of the Photon Trajectories

The aim of this section is to show, in a quantum Friedmann-Planck micro-universe, the equivalent of the "spherical shells" of self-gravitating photons (see below).

We will use the following expression of a Friedmann universe metric:

$$ds^2 = \left(1 - \frac{r^2}{a^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r^2}{a^2}} - r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \quad (19)$$

r , θ and φ denote the spherical coordinates with respect to an arbitrary point.

This expression differs from the expression usually used in cosmology for temporal component of the metric tensor. In effect, assuming that the micro-universe at this stage consists only of photons, we cannot use coordinates related to matter at each point. Here, integration of the metric was carried out in the same way as for that of Schwarzschild, but considering a non-zero constant energy-momentum tensor.

We are interested in the circular trajectories of photons, assuming that they allow a complete quantum description. The following relation expresses the relation between the energy and the angular momentum of a light ray (see [6]):

$$\mathcal{E} = \frac{Mc}{r} \sqrt{1 - \frac{r^2}{a^2}} \quad (20)$$

Let ρ be the distance to the center of the trajectory of the light ray, defined according to the metric by the differential element:

$$d\rho = \frac{dr}{\sqrt{1 - \frac{r^2}{a^2}}} \quad (21)$$

We first consider the trajectories in the planes $\varphi = Ct$.

M can be written $M = p_\theta \rho, p_\theta$ designating the momentum tangent to the trajectory in one of these planes. The relation (21) is then written:

$$\mathcal{E} = \frac{p_\theta \rho}{r} \sqrt{1 - \frac{r^2}{a^2}} \quad (22)$$

The relation:

$$\mathcal{E} = p_\theta c \quad (23)$$

is an invariant for a photon and implies following (24):

$$\frac{\rho}{r} \sqrt{1 - \frac{r^2}{a^2}} = \pm 1 \text{ or } \frac{\rho}{r} = \pm \frac{dr}{a^2} \quad (24)$$

To solve this equation, we proceed to the variable change and the following conversions:

$$r = a \cos u; r = 0; \rho = 0 \pmod{\pi} \Rightarrow \rho_n = \int \frac{dr}{\sqrt{1 - \frac{r^2}{a^2}}} = \left(-u + \frac{\pi}{2} + n\pi\right) a \quad (25)$$

$$\rho_{-n} = -\left(u + \frac{\pi}{2} + n\pi\right) a$$

Equation (24) gives two equations:

$$\left(-u + \frac{\pi}{2} + n\pi\right) \tan u = 1; \left(u + \frac{\pi}{2} + n\pi\right) \tan u = 1$$

For $n \geq 1$ it can thus be seen that solution u is a weak angle, hence:

$$\begin{aligned} \tan u_n &\cong u_n \cong \frac{1}{\left(n + \frac{1}{2}\right)\pi} \\ \rho_n &= \left[\left(n + \frac{1}{2}\right)\pi - u_n\right] a; -\rho_{-n} = \left[\left(n + \frac{1}{2}\right)\pi + u_n\right] a \\ r_n &\cong a \left[1 - \frac{1}{2\left(n + \frac{1}{2}\right)^2 \pi^2}\right] \end{aligned} \quad (26)$$

For $n=0$, we have also two equations;

$$\left(-u + \frac{\pi}{2}\right) \tan u = 1; \left(u + \frac{\pi}{2}\right) \tan u = 1; \quad (27)$$

The solution of the first equation is: $u = \frac{\pi}{2}$; $\rho = 0$. The solution of the second is:

$$\begin{aligned} u_0 &= b_0 \pi = 0.458; b_0 = 0.146 \\ -\rho_{-0} &= \left(b_0 + \frac{1}{2}\right) \pi a; \end{aligned} \quad (28)$$

These relations therefore define a GR quantization of the trajectories of the photons.

Quantization of the Photon Spheres

To quantify photon spheres where the trajectory is characterized by n , we return to equation (23), considering it as a quantum equation between the operators associated with the energy and momentum of the photon under consideration.

$$\hat{\mathcal{E}} = c\hat{p}_\theta; \quad (29)$$

By explaining the equation with the wave function ψ , we obtain:

$$\mathcal{E}\psi = -\frac{\hbar c}{\rho} \frac{d\psi}{d\theta} \quad (30)$$

$$\psi \propto \exp\left(\frac{i\mathcal{E}\rho\theta}{\hbar c}\right) \quad (31)$$

The wave function must be invariant with respect to a rotation of π , taking into account the spherical symmetry, which leads to the following quantification relation (k integer):

$$\mathcal{E}_k = \frac{2k\hbar c}{\rho} \quad (32)$$

It should be taken into account, however, that any point on the surface of the sphere is defined by two angular parameters. We must therefore also write a quantum equation similar to (27) with the angle φ as variable; we will obtain a quantization of the same type for each energy level k of the photon and then write (l integer):

$$\mathcal{E}_{kl} = \frac{4kl\hbar c}{\rho} \quad (33)$$

This relation defines all energy levels of a photon whose trajectory is contained in the surface of the sphere of radius ρ (n) relative to the arbitrary "center" of the universe. This distance is itself quantized as defined above, which makes it possible to write:

$$\mathcal{E}_{kl}(\pm n) = \frac{4kl\hbar c}{\left((n+\frac{1}{2})\pi \pm u_n\right)a} \quad (34)$$

In this way we obtain all the energy levels of the photons whose trajectory is quantized by n. Now we can give the expression of the total energy of a set of photons n by summing all the energy levels k and l by means of the infinite sum (as for the calculation of the Casimir effect, see appendix):

$$\sum_1^\infty k = -\frac{1}{12}; \quad \sum_1^\infty kl = \sum_1^\infty k \sum_1^\infty l = \left(-\frac{1}{12}\right)^2 = \frac{1}{144} \quad (35)$$

The double summation of k and l leads to the expression:

$$\mathcal{E}_S(\pm n) = \frac{\hbar c}{36\left[\left(n+\frac{1}{2}\right)\pm u_n\right]a} \quad (36)$$

We assume that the energy of the photon sphere for quantum number n is:

$$\mathcal{E}_S^*(n) = \frac{1}{2}(\mathcal{E}_S(n) + \mathcal{E}_S(-n)) \cong \frac{\hbar c}{36\pi\left(n+\frac{1}{2}\right)a}; \quad (37)$$

This expression, therefore, gives the energy of the photon sphere for quantum number n.

For $n = 0$, we have the value calculated above for the trajectory of the photon, but we have also another value:

$$\text{For } u = \pm \frac{\pi}{2}; \quad r_0 = a \cos u = 0; \quad \rho_0 = \pm \frac{\pi}{2}a; \quad (38)$$

As above, the mean energy of the 0 photon sphere is:

$$\mathcal{E}_s^*(0) = \frac{1}{2}(\mathcal{E}_s(0) + \mathcal{E}_s(-0)) = \frac{\hbar c}{2 \times 36a} \left(\frac{2}{\pi} + \frac{1}{(b_0 + \frac{1}{2})\pi} \right) = \frac{\hbar c}{36\pi a} \left(1 + \frac{1}{1+2b_0} \right); \quad (39)$$

DENSITY OF THE DARK ENERGY OF OUR UNIVERSE

We now calculate the total energy of all spheres of n-index photons of the quantum Friedmann-Planck universe, up to the maximum value N:

$$\mathcal{E}_T = \frac{\hbar c}{36\pi a} \left(1 + \frac{1}{1+2b_0} + \sum_1^N \frac{1}{n+\frac{1}{2}} \right) = \frac{\hbar c}{36\pi a} \left(1 + \frac{1}{1+2b_0} + \ln N - C_E \right) \quad (40)$$

$$C_E = \lim(N \rightarrow \infty); \ln N - \sum_1^N \frac{1}{n+\frac{1}{2}} = 0.054 \quad (41)$$

Considering that the sum of the energies of all the photon spheres contained in the Friedmann-Planck quantum universe is equal to the energy of this universe (eq.18), we obtain the following equality, determining N, which is the number of photon spheres contained in each Friedmann-Planck micro-universe:

$$\ln N = 36\pi\sqrt{6} - \left(1 + \frac{1}{1+2b_0} \right) + C_E = 36\pi\sqrt{6} - 1.72; \quad (42)$$

We calculate now with eq.(37) the energy of the N photon sphere and the corresponding energy density in our Universe, taking into account the volume of the FPMu for the value of the average radius given in eq.(17):

$$\mathcal{E}_s^*(N) = \frac{\hbar c}{36\pi N a_0}; \quad d_s(N) = \frac{1}{2\pi^2} \cdot \frac{\hbar c}{36\pi N a_0^4}; \quad a_0^4 = \frac{96}{9\pi^2} a_p^4; \quad (43)$$

We find:

$$d_s(N) = \frac{d_p}{8\pi} \cdot \frac{\exp(-36\pi\sqrt{6}+1.72)}{96}; \quad (44)$$

Numerically:

$$d_s(N) = 1.125 \times 10^{-123} d_p = 0.574 \times 10^{-29} g/cm^3$$

This value for density corresponds precisely to that of the experimental measure of the dark energy of our universe. This means that in the context of our model, indicated above, only appears at the macroscopic level the energy density corresponding to the photon sphere of the maximum quantum number N, the lightest of each FPMu (vacuum particle), that is to say the most "superficial". The energy of all other photon spheres is masked and does not appear at the macroscopic level of our universe. The energy of the N sphere may be considered as the binding energy between the vacuum particles, which results in a repulsive force between them.

In effect, dark energy is repulsive and provokes an acceleration of the expansion of the Universe. In our model, the result obtained for dark energy density shows that only the external photon sphere of quantum number N is the cause. It can be explained by considering that each photon sphere, considered as a thin membrane, produces a double radiative pressure, one centripetal and the other centrifugal. If we suppose - the spheres being concentric - that the centripetal radiative pressure of the n sphere balances the centrifugal pressure of the n-1 sphere, only the radiative pressure of the N sphere constituting the envelope (of the FPMu) appears in our macro-universe.

COSMOLOGICAL CONSTANT

When the energy density d is constant, we can write the Friedmann equation (1) by introducing the cosmological constant corresponding to dark energy:

$$3\left(\frac{\dot{a}^2}{c^2} + 1\right) = \Lambda a^2; \quad \Lambda = \frac{8\pi d}{a_p} a_p^{-2}; \quad (45)$$

With the value for d of eq. (44) we find for Λ :

$$\Lambda(model) = \frac{\exp(-36\pi\sqrt{6}+1.72)}{96} a_p^{-2} = 2.828 \times 10^{-122} a_p^{-2}; \quad (46)$$

The experimental value is:

$$\Lambda(exp.) = 2.846 \pm 0.076 \times 10^{-122} a_p^{-2};$$

Eq.(46) gives a very good value for the cosmological constant of our Universe

ELEMENTARY PARTICLES: SELF-GRAVITATING PHOTON SPHERES IN A SCHWARZSCHILD FIELD

We presented in Physics Essays [4] a model assuming that elementary particles are self-gravitating photon spheres in a Schwarzschild field. Here we resume the reasoning in synthetic and more rigorous form to show its consistency with the above presentation.

Calculation of the Electric Charge Associated with a Polarized Photon Sphere

Wheeler and others have studied spheres of self-gravitating photons (sometimes called photon shells) in the context of general relativity without considering the quantum aspects. We were not aware of this work when writing the Physics Essays article. We cite as reference a recent article on the subject that evokes the many others that preceded it [7].

To calculate the electric charge generated by a sphere, it is assumed that polarization of the electric vector of the electromagnetic waves constitutive of the sphere is radial and the classical properties of the field are used.

The starting point is the expression of the Schwarzschild metric:

$$ds^2 = \left(1 - \frac{r_0}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_0}{r}} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (47)$$

We use the distance in the center beginning with its differential element:

$$d\rho = \frac{dr}{\sqrt{1 - \frac{r_0}{r}}} \Rightarrow \rho = r \sqrt{1 - \frac{r_0}{r}} + r_0 \ln \left(\sqrt{\frac{r}{r_0}} + \sqrt{\frac{r}{r_0} - 1} \right) + C; \quad C = r_0 \quad (48)$$

From the equation of the light ray motion in the field we are studying, we draw the following relation between its energy W and its angular momentum M , for constant r (see [6]):

$$\frac{W}{c} = \frac{M}{r} \sqrt{1 - \frac{r_0}{r}} \quad (49)$$

On the other hand, the energy density of the electromagnetic field as a function of the vectors $\mathbf{E}$ and $\mathbf{H}$ is:

$$\frac{dW}{dV} = \frac{E^2 + H^2}{8\pi}; \quad (51)$$

We continue to the derivative with the following elements:

$$dV = \frac{4\pi r^2 dr}{\sqrt{1 - \frac{r_0}{r}}}; \quad E^2 = H^2 \quad (52)$$

In this way we obtain:

$$q^2(r) = E^2 r^4 = Mc \left(1 - \frac{3r_0}{2r}\right) \quad (53)$$

Eq.(53) thus giving the expression of the electric charge associated with a photon sphere (whose electric vector is radially polarized) of radius r , whose angular momentum is M .

Calculation of the Elementary Electric Charge

To calculate the elementary electric charge based on the relationship (53) for a sphere of photons effectively, we refer to the double quantization of photons as we considered it above in the Friedmann micro-universe; this makes it possible to obtain the quantum expression for the angular momentum of a photon sphere, using infinite sums of integers as for the calculation of the Casimir effect:

$$M = (\sum_1^\infty k \sum_1^\infty l) \hbar = \left(-\frac{1}{12}\right)^2 \hbar = \frac{1}{144} \hbar \quad (54)$$

We can also express M conventionally as the product of the momentum p (tangent to the sphere) by the distance to the center: $M=p\rho$; equation (40) then becomes:

$$\frac{W}{c} = \frac{p\rho}{r} \sqrt{1 - \frac{r_0}{r}} \quad (55)$$

For a single photon or a set of photons, the relation $W/c = p$ is an invariant, which implies (44):

$$\frac{\rho}{r} \sqrt{1 - \frac{r_0}{r}} = 1; \text{ or } \frac{\rho}{r} = \frac{d\rho}{dr} \quad (56)$$

We have the same relation, applied here to the Schwarzschild metric, as for the trajectories of the photons in the FP micro-universe.

Numerically, this condition is fulfilled for a value of r such that:

$$\frac{r_1}{r_0} = 1.544\,994\,44 \quad (57)$$

We obtain with (53), (54), and (57):

$$q^2(r_1) = \frac{\hbar c}{144} \cdot \left(1 - \frac{3r_0}{2r_1}\right) = \frac{\hbar c}{6^2} \cdot \frac{1}{137.3498} \quad (58)$$

Fine Structure Constant

We see that the second factor in Eq.(58) is close to the **fine structure constant** $\alpha = 1 / 137.036$ proportional to the square of the elementary electric charge e . This value is further

approximated by introducing the radiative corrections to Coulomb's law (see appendix) which give the following relation (59) where the FPMu average radius appears:

$$\alpha^{-1} = 4 \left(1 - \frac{3r_0}{2r_1}\right)^{-1} \left[1 + \frac{2\alpha}{3\pi} \left(\ln \frac{2a_p}{a_0} - 0.577 - \frac{5}{6}\right)\right]^2 = 137.036\,658\,7$$

The last experimental value (2020) for $1/\alpha$ is:

$$\alpha^{-1}(exp.) = 137.035\,999\,206 \quad (81)$$

The accuracy of the value given by the model is then $<1/100\,000$. The difference can be explained by the fact that the radiative corrections used here are only the first term of the perturbation theory.

Minimal Elementary Electric Charge

Eq. (58) shows that the model gives for the elementary charge $1/6$ of the electron charge:

$$q^2(r_1) \cong \left(\frac{e}{6}\right)^2 \quad (60)$$

Structure and Representation of Leptons

According to this model, the most basic electric charge is $|e|/6$, while the smallest fractional charge found in quarks is $|e|/3$.

This observation leads us to suppose that the fundamental element constituting the leptons is an electrically charged photon sphere at: $\pm e/6$. These elements must be grouped in pairs. One pair may therefore have the charge: $+e/3$, $-e/3$ or 0 , so a lepton is composed of three pairs of photon spheres in this model, i.e., six fundamental spheres all having the same charge and the same center of gravity.

We will represent the fundamental sphere of charge $-e/6$ by the symbol $\ominus$ and that of charge $+e/6$ by $\oplus$.

They may be represented as follows:

Charged leptons (electron, muon, tau): electric charge: $-e$: $\ominus \ominus \mid \ominus \ominus \mid \ominus \ominus$

Charged anti-leptons: electric charge: $+e$: $\oplus \oplus \mid \oplus \oplus \mid \oplus \oplus$

Neutrinos:

Case 1: three zero pairs: $\ominus \oplus \mid \ominus \oplus \mid \ominus \oplus$

Case 2: one null pair and two pairs of opposite charges: $\oplus \oplus \mid \ominus \oplus \mid \ominus \ominus$

Color Charge

We assume that the color charge can be considered as a magnetic dipole created by a radial magnetic vector of the photon spheres. We then consider for the color charge a relationship analogous to the relationship (53) written for the electric charge:

$$C^2(r) = H^2 r^4 = M_c \left(1 - \frac{3r_0}{2r}\right) \quad (53b)$$

The difference between the two calculations lies in the consideration of the moment M . We assume here that only the negative values of the angular momentum ($-n\hbar$) of the photon spheres must be taken into account and only for an infinite sum of integers (because the axial symmetry of a magnetic field) counted two times (because we consider a dipole).

$$M = 2 \sum_{1}^{\infty} -n\hbar = \frac{\hbar}{6}$$

Then we have:

$$C^2(r_1) = \frac{\hbar c}{6} \left(1 - \frac{3r_0}{2r_1}\right) = \frac{6\hbar c}{3^2} \cdot \frac{1}{137.3498} \quad (58b)$$

With the same reasoning as for the electric charge, it appears that we obtain an elementary color charge equal to $C/3$ with:

$$\alpha_C = 6\alpha$$

This relationship applies only in the zone of asymptotic freedom, close to the particle. We know that the color force intensity increases from the distance to the particle.

We found this relationship empirically in other works (EJAS 16092)

Structure and Representation of Quarks

We shall use the symbol Δ , ($\bar{\Delta}$ for the anti-color) to represent a fundamental color sphere. Taking into account the above result, a color part of a quark corresponds to only a fundamental sphere (against one pair for an electric part). The spheres all have the same center of gravity. Then we represent the quarks as below.

- Quarks: Up, Charm, Top; electric charge: $-e/3$: $\Theta \Theta | \Delta | \Delta$
- Quarks: Down, Strange, Bottom; electric charge: $+2e/3$ $O O | O O | \Delta$

Application of the Model: Calculation of Elementary Particle Masses

It should be noted that the masses of particles in this model are not related to the "Schwarzschild radius" of photon spheres constituting the elementary particles. A derived model was performed to calculate the masses of charged leptons in an article in Physics Essays [4] in 2014. More recently, an application to the calculation of the masses of all elementary particles (and those of hypothetical sterile neutrinos) has been included in a collection published by Nova Science Publishers [5]. An overview of the model for calculating particle masses is given in the appendix.

INTERPRETATION OF THE QUANTUM THEORY RESULTING FROM THE MODEL

Appearance and Disappearance of Elementary Particles from Vacuum Particles (FPmu)

Let us first consider the pairs of virtual particles appearing and disappearing in permanence in a vacuum. In the framework of the model, they may be considered spontaneous excitations of quantified photon spheres in the Friedmann metric (Fp-spheres) structure belonging to the Friedmann-Planck micro-universes. These excited states would thus

correspond to a transformation of Friedmann's photon spheres into Schwarzschild photonspheres (Sp-spheres), allowing them to exist in our macro-universe.

Viewed from a more formal perspective, the integration of the Friedmann and Schwarzschild metrics from the Einstein field equations may be carried out following the same mathematical process. The sole difference is that the energy-momentum tensor considered is zero in the second case, whereas it is constant non-zero in the first. In other words, the fact that the energy-momentum tensor becomes zero following a quantum fluctuation in FPmu implies that Fp-spheres are transformed into Sp-Spheres, that is to say, particles of matter, real or virtual.

For real particles to appear, an additional condition is necessary. It requires a supply of energy by free photons so that the excited states become stable.

Quantum and Classical Particle Systems

The quantum wave-to-particle duality leads us to consider that the excited states of the FPmu producing the elementary particles (Fp-photon spheres) propagate in the FPmu substrate in the form of waves, which can be assimilated to the de Broglie-Schrödinger wave functions. In other words, the excited states move from one FPmu to another during the movement of the quantum system whose wave packet may concern one or more particles (entangled states).

In the case of conventional systems comprising many particles, the current explanation is that their wave functions collapse due to the complexity of these systems (decoherence). In the context of this model, it is necessary to consider that the elementary particle associated with a FPmu fixes it to the classical system, which drives it in its movement (defined in the framework of special relativity).

COSMOLOGICAL DEVELOPMENTS

In this approach, the expansion of the Universe implies the appearance of new FPmus if they remain contiguous, because their dimension is constant.

For a finite (closed) universe, the Big Bang might start from a single FPmu that evokes G. Lemaitre's "primeval atom". It would then multiply according to a mechanism reminiscent of cell multiplication in biology. The exact nature of this mechanism remains to be studied. We can only take note of the following relation, resulting from equation (13):

$$\gamma^2(3) = 12 = 2\gamma^2(2)$$

This relationship suggests that an FPmu corresponding to an excited state $n = 3$ could result in its decay into two FPmu in the ground state $n = 2$.

A Closed Universe Model

In the article ref. [2] we present a closed Universe model beginning by only one Friedmann-Planck micro-universe which then multiplies identically. We show that this growth takes the form of a quadratic period linked to a quantum approach to the problem.

For that, we assume that each vacuum particle (FPmu) has a Compton wavelength equal to the maximum extension of the Universe at any time. We have so the below relationship between the energy of each vacuum particle (in our Universe) and the maximum extension of the Universe at any time.

$$\pi a = \frac{2\pi\hbar c}{\mathcal{E}(a)} \text{ or } \mathcal{E}(a) = \frac{2\hbar c}{a}; a : \text{Universe radius of curvature}; \quad (61)$$

We introduce the corresponding energy density in the differential equation of Friedmann:

$$1 + \frac{\dot{a}^2}{c^2} = \frac{8\pi G}{3c^4} \rho a^2; \rho(a) = \frac{2\hbar c}{2\pi^2 a_0^3 a}; \quad (62)$$

We obtain the Friedmann equation in this case, and its solution (p index the Planck units):

$$1 + \frac{\dot{a}^2}{c^2} \cong \frac{\pi a}{4a_p}; a \cong \frac{\pi}{16} a_p \left(\frac{t}{t_p}\right)^2 \quad (63)$$

We see that the first period of expansion of the Universe is a quadratic function of time.

Then we assume a new hypothesis: the elementary particles appear when their Compton wavelength equals the Hubble radius of the Universe at this instant (noted t_1)

$$l_m = \frac{2\pi\hbar}{mc}; t_1 = \frac{l_m}{c}; \quad (64)$$

So the elementary particles (ordinary or dark matter particles) with the highest mass appear first. This implies the end of the quadratic period and the beginning of the radiative period, followed by the matter period, of the Universe expansion. Numerically, we find a very fast initial expansion (case where the highest mass is that of the d-quark):

$$t_1 = 5.4 \times 10^{-22} \text{ s}; a_1 = 314000 \text{ km}$$

Model Results

We then use the classical solutions of the Friedmann equation for these periods to calculate the current radius of the Universe, which is a function of the mass of heaviest elementary particles appeared at t_1 (ordinary or dark matter). We have the below relationship:

$$\log \frac{a_3}{r_H} = 4.83 - \frac{3}{2} \log \frac{m_s}{m_e} \quad (65)$$

$$\frac{a_3}{r_H} : \frac{\text{current radius}}{\text{Hubble radius}}; m_s : \text{heaviest mass}; m_e : \text{electron mass}$$

- If the heaviest elementary particle is the d-quark (4.8 MeV), the radius of curvature of the Universe is 2344 Hubble radius, or 32350 billion light-years.
- If the heaviest elementary particle is a dark-quark with a mass of 8.16 MeV (see appendix) we obtain for the radius of the current Universe 1060 Hubble radius or 14620 billion light-years.
- The radius of curvature of the current Universe equals the Hubble radius if the heaviest particle is a darkmatter elementary particle with a mass of 848 MeV

In the two first cases the Universe appears flat on our scale, in accordance with the observations; but this is not true in the third case. The reasoning only concerns the elementary particles (ordinary or dark matter); composite particles made up of elementary particles after they were created do not intervene in the calculation of the Universe radius. We can assume a composite dark matter particle (a “darkon” composed of 3 dark-quarks, see appendix) with a mass of 2566 MeV. If the Universe contains two “darkons” for one proton, we have the numerical relationship:

$$\frac{2m(\text{darkon})}{m(\text{proton})} = 5.47 = \frac{26.8 (\% \text{dark matter})}{4.9 (\% \text{ordinary matter})}; \quad (66)$$

Evolution of the Universe in the Future

In the future, the density of dark energy becomes dominant and determines the expansion of the Universe in the form of a “soft” inflation, identical to that generated by a cosmological constant. In our model, the density of dark energy is constant because it is a characteristic of the vacuum particles (see above). The Friedmann equation is now:

$$1 + \frac{\dot{a}^2}{c^2} = \frac{a^2}{Ra_p^2}; \quad R = 1.06 \times 10^{122}; \quad (67)$$

This value of R is experimental, but we calculated it above (eq. 46):

$$R = \frac{3}{\Lambda a_p^2}$$

The solution is exponential (index 0 for the present time):

$$a(t) = a_o \exp\left(\frac{t-t_o}{\sqrt{R}t_p}\right); \quad (68)$$

We formulate now a speculative hypothesis: when the acceleration of the Universe reaches the Planck acceleration, the structure of the vacuum is destroyed as well as the internal structure of each vacuum particle, made up of photons spheres, because the inertial force of these elements exceeds the Planck force. This destruction might be immediately followed by a restructuring of the Universe in the form of a Friedmann-Planck super-universe, whose dimension would be that of our Universe at the time of this event. We could call this event “Big Rip” (different than Caldwell). These considerations can be translated as follows:

$$\ddot{a}(t_R) = \frac{a_o}{Rt_p^2} \exp\left(\frac{t_R-t_o}{\sqrt{R}t_p}\right) = \frac{c}{t_p}; \quad (69)$$

We deduct from this equation the instant of the “Big Rip” and the radius of the Universe at this instant:

$$a_R = Ra_p; \quad t_R = \frac{\sqrt{R}(\ln R - C)t_p}{2}; \quad C = 11.8; \quad (70)$$

This event would take place after 135 times the current age of the Universe, or 1863 billion years. We can imagine that the “Big Rip” becomes a “Big Bang” starting from this Friedmann-Planck super-universe (FPsu), then has an evolution identical as our Universe starting from one FPMu. We describe in ref.2 the reasons that allow this hypothesis. We can calculate the new constants of this FPsu:

$$c_R = \frac{2\sqrt{R} c}{\ln R - C}; \quad \hbar_R = \frac{\sqrt{6}}{2} R^{\frac{3}{2}} (\ln R - C) \hbar; \quad G_R = \frac{4\sqrt{6} G}{3(\ln R - C)}; \quad (71)$$

A Cyclic Universe of Successive Big Bangs-Big Rips of Increasing Scale?

The process thus described is obviously iterative; when the super universe reaches an acceleration equal to the value “super-Planck”, it metamorphoses during a new Big-Rip into the super-super-universe of Friedmann-Planck, which immediately restarts a new Big Bang. We can speak of a cyclic universe, but with a coefficient of expansion R always the same at each cycle.

If the ascending cyclical process described above is true, then it must also apply to the past of our Universe, that is, to the initial Big Bang. We can therefore imagine that the first Friedmann-Planck micro-universe at the origin of our Universe results from the Big Rip of a much smaller universe having undergone its “Planck infra-acceleration”. The characteristics and the fundamental constants of this infra-universe are easy to determine: it suffices to reverse the above relations to pass from our Universe to that which preceded it ($R \Rightarrow 1/R$).

Of course, for the past as for the future, we can imagine that this Planck infra-universe on the scale $1/R$ results from a Planck infra-universe on the scale $1/R^2$, according to an iterative process extended to infinity; this amounts to saying that the Universe might have been born out of nothingness exclusively following the laws of Physics.

Let us follow our reasoning to the end. During the first Planck time of our Universe, an undetermined number of infra-universes could hatch, develop and metamorphose onto infra-universes on a higher scale R before arriving at our Universe. Our length and time scales would not in this case be arbitrary initial conditions; they would be the results of a cyclical process of increasing scale whose beginning is unknown, which can only be attributed to a chance of quantum nature.

CONCLUSION

The complementary models presented in this article seem to give new insights concerning a potential reconciliation of general relativity and quantum theory, which are often considered incompatible.

In fact, Schwarzschild photon spheres (self-gravitating photons) generate the elementary particles and their fundamental properties of interaction. This model allows the calculation of the fine structure constant with an accuracy of 0.5 for 100 000

$$\alpha^{-1} = 4 \left(1 - \frac{3r_0}{2r_1} \right)^{-1} \left[1 + \frac{2\alpha}{3\pi} \left(\ln \frac{2a_p}{a_0} - 0.577 - \frac{5}{6} \right) \right]^2$$

$$\frac{r_1}{r_0} = 1.544\,994\,44 \Leftrightarrow \int_{r_0}^{r_1} \frac{dr}{\sqrt{1 - \frac{r_0}{r_1}}} + r_0 = \frac{r_1}{\sqrt{1 - \frac{r_0}{r_1}}}; \quad \frac{a_p}{a_0} = 0.980\,772$$

$$\alpha^{-1}(\text{model}) = 137.036\,658\,7$$

$$\alpha^{-1}(\text{exp.}) = 137.035\,999\,206$$

Conversely, these interactions are at the origin of elementary particle mass: the masses are all defined by the exponential of a linear function of α , constant of fine structure. In summary, in this model, gravity is at the origin of the charges (and their interactions), while the charges are at the origin of the masses.

The space structure of Friedmann-Planck's micro-universes is also a consequence of both general relativity and quantum theory. The probing numerical result is the value of the cosmological constant corresponding to dark energy:

$$\Lambda = \frac{\exp(-36\pi\sqrt{6} + 1.72)}{96} a_p^{-2} = 2.828 \times 10^{-122} a_p^{-2}$$

$$\Lambda(exp.) = 2.846 \times 10^{-122} a_p^{-2}$$

The micro-universes contain Friedmann photon spheres; by turning into Schwarzschild photon spheres, they appear as elementary particles in our Universe. The theoretical justification for this approach is that the integration of Einstein's equations for a central symmetric field gives either a Friedmann metric or a Schwarzschild metric according to the value of the two integration constants (0 or $\neq 0$). Quantum fluctuations can therefore produce these transformations.

The validity of the models presented here is based thus on their coherence and on the results obtained: the calculation of the cosmological constant, the calculation of the fine structure constant, as well as calculation of the masses of some elementary particles and of hypothetical dark matter particles.

APPENDICES

A1- Model Providing the Masses of Elementary Particles

The basic principle of this model, developed initially for charged leptons, is to first quantize their electrostatic field starting from the expression of its energy, defined as the difference between the electrostatic energy of the lepton in the field it generates and its mass energy:

$$E_f(r) = \frac{\alpha \hbar c}{2r} - mc^2$$

and then to define a momentum quantum operator by the relation:

$$\hat{A}c = \pm 4\pi\theta \hat{E}_f(r)$$

The parameter θ characterizes each elementary particle.

By showing the wave function, the equation above becomes:

$$i\hbar c \frac{d(r\psi)}{rdr} = \pm 4\pi\theta E_f(r)\psi$$

The ensemble of these solutions is expressed as:

$$r\psi(r) \propto \exp\left(\pm \frac{4\pi i\theta}{\hbar c} \int_{r_0}^r E_f(r) dr\right)$$

The value of r , which cancels the energy of the electrostatic field, thus cancels the derivative of the real wave function; it therefore corresponds to an optimum of the real

wave function, which is a cosine function. This reasoning leads to the following quantized expression (n integer) of the masses of the particles into which Planck's length and mass units were introduced:

$$\ln \frac{m_p}{m} = \frac{na}{2\theta} + 1 + \ln(2a) - \ln \frac{a_p}{r_0}; \quad a = \frac{1}{\alpha} = 137.036$$

$$\frac{m}{m_p} = \exp\left(-\frac{na}{2\theta} + K\right); \quad K = -1 - \ln 2a + \ln \frac{a_p}{r_0}$$

The lower bound of integration is supposed to correspond to the Schwarzschild radius of basic photonspheres, whose energy is equal to 1/6 of the total energy of the six photon spheres composing each particle. It is calculated from relation (40):

$$r_0 = \frac{2GW}{6c^4} = \frac{G\hbar}{3 \times 144 c^3 r_1} \sqrt{1 - \frac{r_0}{r_1}}; \quad \rightarrow r_0 \cong \frac{a_p}{33.5}$$

For the electron, we are empirically led to retain the following values for n and θ :

$$\ln \frac{m_p}{m_e} = 51.53 \rightarrow \theta = 3\sqrt{2}; \quad n = 3$$

The relationship between the mass of the particle and α , initially established for the charged leptons in [2], is applicable to all elementary particles [3]. The parameters θ were determined empirically for the first family of elementary particles and calculated according to the composition rules of the pairs of photon spheres for the other two families. Thus, it was possible to calculate the masses of all the other elementary particles (and also those of the hypothetical sterile neutrinos). In practice these calculations have been carried out simply by referring for the mass of the particle x to the mass of the electron according to the formula:

$$\ln \frac{m_x}{m_e} = \frac{a}{2} \left(\frac{n_e}{\theta_e} - \frac{n_x}{\theta_x} \right)$$

All the results concerning the masses of the three families of particles (this number being justified by the model) are presented in the referenced article [3]. Only results relating to the first family of particles, the muon and the tau (with radiative corrections) are shown here.

Particle	Parameter θ	Quantum number	θ/n	Model mass	Experimental mass
Neutrino e	3	3	1	0.983 meV	< 60 meV
Electron	$3\sqrt{2}$	3	1.414	Reference	0.511 MeV
Up quark	$3 + 2\sqrt{2}$	4	1.457	2.13 MeV	2.01 MeV
Down quark	$6 + \sqrt{2}$	5	1.483	4.81 MeV	4.79 MeV
Muon	$4 - \sqrt{6}/3$	2	1.592	105.9 MeV	105.7 MeV
Tau	$2 + \sqrt{2}$	2	1.707	1781 MeV	1777 MeV

Considering these data, we assume in the article ref. [11] a dark quark (with only a colored charge and then without electric charge) with the below characteristics:

$$\theta = 9; \quad n = 6; \quad m_{dq} = 8.163 \text{ MeV}$$

And a neutral dark baryon composed of 3 dark quarks:

$$m_{db} = 3 \left(\frac{9\alpha}{12} + 2 \right) m_{dq} = 2566 \text{ GeV}$$

The experimental values for Up and Down quarks are those provided by C. Davies following a QCD calculation [10] on experimental masses of hadrons. The empirical values of θ found for the first family are simple and consistent with each other. They made it possible to define the values of the pairs and quadruplets of photon spheres whose linear combinations constitute the parameters of the other particles.

A2- Radiative Corrections to Coulomb's Law

The radiative corrections to Coulomb's law, due to the polarization of the vacuum essentially by the virtual pairs e^+ , e^- were used in the two articles of Physics Essays cited to precisely calculate the value of the elementary electric charge. We refer here only to the relation defining the value of the electrical charge at a distance r with respect to the charge measured at infinity as a function of the mass of the electron [8].

$$q(r) = q_\infty \left[1 + \frac{2\alpha}{3\pi} \left(\ln \frac{\hbar}{m_e c r} - C - \frac{5}{6} \right) \right]; C = 0.577.. (\text{Euler constant})$$

This relation shows that the electric charge increases when r tends towards 0. But when r is close to the Planck radius, the virtual mass of e^+ , e^- increases and we assume that, when r tends toward the average radius of the FPmu calculated above:

$$\lim(m_e c r) = \frac{\hbar a_0}{2a_p}; r \rightarrow a_0; a_0 = \left[\frac{4\sqrt{6}}{3\pi} \right]^{\frac{1}{2}} a_p = 1.0196 a_p$$

We obtain for α :

$$\alpha^{-1} = \alpha^{-1}(r_1) \left[1 + \frac{2\alpha}{3\pi} \left(\ln \frac{2a_p}{a_0} - 0.577 - \frac{5}{6} \right) \right]^2$$

$\alpha^{-1}(r_1)$: value of α for r_1 calculated above

A3- Infinite Sum

This article uses the infinite sum:

$$\sum_1^\infty n = -\frac{1}{12} \quad n: \text{integer}$$

Euler formulated this relationship for the first time; Ramanujan also cited it. It is obviously questionable mathematically. From this point of view, the most rigorous expression that can be given to it and to consider it as the analytic extension of the Riemann function:

$$\zeta(s) = \sum_1^\infty n^{-s}; s = -1$$

The relation has been used several times in physics (see ref. [9]) as:

- Justification for the number of dimensions (26) of the space of non-supersymmetrical strings
- Calculation of the attractive force between two plates in the vacuum due to the Casimir effect
- Calculation of the elementary electric charge in article [1].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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