ISSN: 2634-9221
European Journal of Applied Sciences; Vol. 14 No. 03 (2026) (313-317)
https://doi.org/10.14738/aivp.1403.10805

The Yearly Nutation: The Euler's Mechanical Moment Related to the Earth's Centre

Monica Zoe Ciobanu

1. Retired Astronomer

Abstract: The paper attempts, only approximately, to present when and where a wobble may appear and for how long it may persist along the Earth's rotation axis, with external forces assumed to be the Moon's tangential forces on its real orbit and the Sun's on its apparent orbit. The paper also attempts to determine whether there are moments when no wobbles exist, and when the Earth's rotation axis and the Earth's kinetic moment are both aligned with the Oz terrestrial axis, pointing together to the real terrestrial north pole. The period investigated is the great nutation period of 18,6 years. An approximate calculation may suggest that this nutation with a periodicity of 18,6 years is due to wobbles caused by the Moon and the Sun.

Keywords: nutation, wobbles, Chandler's periods.

INTRODUCTION

It is well known that during the 18th century, James Bradley (1693-1762) observed an 18.6year variation in the positions of the stars, similar to the retrograde motion of the lunar nodal point. Assuming the stars represent an inertial system of reference, it is obvious that the Earth's rotation axis varied in position over the 18,6 years period.

Leonard Euler (1707–1783), supposing that the connection between the two periods was caused by some forces of the Moon, namely the tangential component of the Moon's force acting on its orbit, formulated the formula for the moment of inertia. In fact, in his theory, the Earth is no longer a simple point with mass, as in Newtonian mechanics, but a solid-rigid body with three principal axes of inertia moment, chosen as the Earth's reference axes "Oxyz". In 1803, his important work "The theory of a solid-rigid body with a fixed point" was published.

In this case, the fundamental equation of classical mechanics is:

$$DK/Dt = Mfex$$

Where K is the kinetic momentum of the solid-rigid body with a fixed point, DK/Dt is the derivative of K with respect to a fixed frame of reference; "Mfex" is Euler's mechanical momentum, related to the fixed point, of the resultant of those external forces which do not pass through the fixed point.

To understand how Euler's mechanical moment acts on Earth's rotation axis (ERA), it must be known as its components in the equatorial coordinate system "Oxyz" (chosen to be situated along the principal axes of Earth's moment of inertia, with the Oz axis directed along the ERA). For beginning, supposing also the existence of an ecliptic system of axes "Oxyz" having the origin also in "O" in Earth's centre of inertia.

THE COMPONENTS OF THE EULER'S MOMENT AROUND THE EARTH'S CENTRE OF INERTIA, "MFEX", IN THE EQUATORIAL PLANE

Suppose a fictitious celestial body of mass "M" moving under the Earth's attraction in a circular orbit, situated in the ecliptic plane. In this case, the Eulerian mechanical moment "Mfex" will be centred on the Earth's centre of inertia, perpendicular on the ecliptic plane, constant in magnitude, and directed towards the northern part of the celestial sphere.

The component of "Mfex" in the celestial equatorial plane is very important; it depends on the orbit's inclination relative to the terrestrial equatorial plane, and it is straightforward to calculate to demonstrate the presence of a wobble.

Indeed, given the celestial equatorial coordinates, alpha and delta, of the celestial body as it moves along its orbit, and the orbit's inclination (obl) relative to the celestial equatorial plane, it is straightforward to determine the components of the Euler mechanical moment, "Mfex", on the terrestrial equatorial plane as "Mfex sin(delta)". In this case, during an orbital period, the extreme values of "Mfex sin(delta)" are "+Mfex sin(obl)" and "-Mfex sin(obl)", but it is null when the celestial body crosses the equinoctial line.

Supposing the obliquity is constant, for instance at +23.5 °, then the equatorial component of Euler's moment "Mfex" lies between: [-0,3827 "Mfex", +0.3827 "Mfex"] and is null when the celestial body crosses the equinoctial line.

If the mass of the celestial body is null or negligible, this means Mfex = null, no notable wobble can occur, and then the vector ERA and the Earth's kinetic moment (the vector K) retain the same direction, being both situated on the support of axis Oz, pointing towards the terrestrial north pole. Consequently, neither free nutation nor free wobble can exist.

Nevertheless, if the Euler moment around the Earth's centre is large, the Earth's kinetic moment "K" will not be aligned with the Oz axis; it will point towards what is called the instantaneous north pole, and then, the Earth, together with its ERA, will be submitted to a wobble around the new position of the Earth's kinetic moment. If this wobble becomes a periodic nutation, it will cause a periodic variation in the terrestrial latitude.

REMARKS ON THE DECLINATIONS OF A CELESTIAL BODY MOVING UNDER THE EARTH'S ATTRACTION IN THE ECLIPTIC PLANE, ASSUMED TO HAVE A YEARLY ORBITAL PERIOD AND A CIRCULAR ORBIT

Supposing the fictive celestial body crosses the equinoctial line in an instant, its equatorial coordinates, alpha and delta, are null and no wobble exists; but next, as it climbs towards the turning point, its ascension and declination begin to rise, and the component of Euler's moment, "Mfex", on the equatorial plane also rises.

Naturally, before reaching the turning point, the gradient of the declination decreases, becoming null in the same turning point; then, before and after the turning point, the declination remains approximately the same over a time interval. For instance, in this case, with a yearly orbital period and an obliquity of 23.5 °, the declination oscillates near 20 days around 23 °. Likewise, the component of Euler's moment, "Mfex" sin(delta), acting during those 20 days, will oscillate around its greatest value, "Mfex" sin23°. Being interested in the nutation phenomena, it may be concluded that, around the turning point, in this case, the wobbles act on ERA not only instantaneously but also over about 20 days, keeping the same great value for about 20 days.

Therefore, the Earth's kinetic moment will point around 20 days, not like the ERA towards the terrestrial north pole, but towards some instantaneous north poles, and a nutation phenomenon may appear. Regarding the terrestrial latitude variation, it results that only after crossing the equatorial line does a wobble appear and grow until the celestial body arrives ten days before the turning point; then, the wobbles, having now the great value "Mfex" sin(23º) during also the next 10 days they will maintain the same great value during the next 10 days, and during that period the new latitude will keep the same value due to the nutation phenomenon.

After the celestial body began to descend from the turning point, the wobbles diminished. When the celestial body crossed the equinoctial line again, the ERA and the K points together again to the only one same terrestrial north pole. In fact, this suggests, according to Euler's theory, that the Earth, supposed to be a rigid solid body, undergoes a yearly nutation.

In 1892, Chandler [1] announced the first detection of a variation in terrestrial latitude with a yearly period.

SUPPOSING A FICTITIOUS CELESTIAL BODY WITH MASS "M" RUNNING UNDER THE EARTH'S ATTRACTION IN A CIRCULAR ORBIT, BUT HAVING ONLY FOUR WEEKS ORBITAL PERIOD AND THE OBLIQUITY OF THE ECLIPTIC PLANE BEING SLIGHTLY VARIABLE

Also, in this fictitious case, when the celestial body crosses the celestial equatorial plane, its declination is null. However, after approaching its turning point, it will retain the same declination for some time before beginning to descend. Obviously, the wobbles grow with the celestial body's declination, becoming more pronounced around its turning point (declination between 18.5° and 28.5 °).

Nevertheless, in contrast with the yearly period (when the stationary period of declination around the turning point may last nearly 20 days), in this case (the celestial body having only a fourweek orbital period), the wobbles act powerfully on ERA only for two to three days near the turning point.

Let us admit that when the fictitious celestial body has a yearly period, the radius of its orbit is equal to one astronomical unit; in that case, it may be accepted that when the Sun crosses the equinoctial line, no wobble exists.

Supposing now that the radius of the other celestial body's orbit is about 380,000 km, it is interesting to determine where and when the Moon crosses the celestial equatorial plane. If the Moon crosses it along the equinoctial line together with the Sun, at that moment the vectors K and ERA are both aligned with the Oz axis, and no wobble acts on ERA.

For high astrometric precision, it is interesting to know whether there exists a moment when the Sun and the Moon cross the equinoctial line together, and no wobble occurs.

Indeed, it is well known that, approximately, using only mean values, the ecliptic longitude between two lunar nodes that intersect the equinoctial line diminishes by 18.35 °; but the Sun's ecliptic longitude during a tropical year (equinox to equinox) increases by 360 °. If at $T=t_o$ the Moon and the Sun crossed the equinoctial line together, they will cross it together again after 18,35 tropical years.

If at T=to the Moon and the Sun cross the equinoctial line together, in the past they also crossed together 18,35 x 360 at T=6705 days ago (Indeed 18,35 o x 365,242 =6705). That means around 18,6 tropical years ago.

This approximate value of 18,6 tropical years suggests that the great nutation period of 18,6 years is caused by the wobbles created by the tangential forces of the Sun on its apparent orbit and of the Moon on its real orbit.

REMARKS ON EULER'S MECHANICAL MOMENT OF THE EXTERNAL FORCES AROUND THE EARTH'S INERTIA CENTRE

It is well known that the mechanical moment around a point is defined as the vector product of the external force vector and the position vector relative to the fixed point, with the resultant vector perpendicular to both.

The value of the mechanical moment about a point depends first on the mass of the celestial body moving along its orbit and on its distance from the fixed point. Therefore, the Sun's large mass and distance cause the most significant variation in the direction of the ERA, which in turn leads to the first discovered periodic variation of the terrestrial latitude, namely the yearly period (Chandler, 1892); the most important moment occurs during the solstices, when the ERA undergoes the strongest wobble over 20 days.

The Moon's mass and its distance from the Earth are smaller; the wobbles caused by its tangential forces are negligible compared with those created by the Sun's tangential forces, but there is an exception. In contrast with the small eccentricity of the Sun's apparent orbit (0.0167), the Moon's orbital eccentricity (0.0549) is very large, and its orbit is far from circular. (The Earth's Moon distance varies between 360000km at perigee and 400000km at apogee.)

Indeed, the Moon's orbit, which has a large eccentricity, means that there are moments when the Moon, being in one of its extreme positions, is also in syzygy with the Sun and the Earth; at those moments, the wobbles due to the Moon act together with the yearly wobbles, and the resultant wobble may be greater and detectable.

Indeed, the synodic month (new Moon to new Moon) is 29,531 days, and the anomalistic month (perigee to perigee) is 27,555 days, resulting in an interval of 813,7 days between the Moon being at perigee and being in syzygy.

Assuming the Moon's apogee occurs midway through this interval, the interval between the Moon's two extreme positions in a syzygy is 407 days (13,5 months). At those moments, the two wobbles (from the Moon and the Sun) act together, producing a greater, detectable resultant wobble that may explain Chandler's period of around 14 months. Naturally, due to the great perihelion advance of 40° and the retrogradation of the lunar nodal point, the value of the 14-month period is not always the same. However, it is certainly close to that value.

In a previous paper (2), the author argued that only at the Moon's perigee can a special wobble occur.

Regarding the Moon's wobbles as a problem of three bodies in celestial mechanics, each Moon's wobble is unique. However, acting together with a Sun's wobble at syzygy moments, when the Moon is at apogee, it may also cause a 14-month period, similar to that at perigee.

CONCLUSION

To be concise, in the syzygy moments, when the Moon is near its extreme position (perigee or apogee), the resultant wobble, created by the Sun and the Moon acting together, may cause a 14month variation in terrestrial latitude, discovered by Chandler. The most values of the instantaneous north pole position are under 0,2''

The need to know the position of the ERA with great accuracy led to the establishment of several international services beginning a century ago: the Bureau International de l'Heure in 1912, the International Polar Motion Service in 1987, and the International Earth Rotation and Reference Systems Service in 2003.

BIBLIOGRAPHY

1. Chandler, S. On the variation of latitude, VII, Astron, J. 12, 1892,97j
2. Ciobanu, M.Z. Remarques about Euler's theory regarding a body with a fixed point SCIREA Journal of Physics Vol 10, Issue 6, December 2025