

Comparison of Non-Linear Height-Diameter Models of *Tectona grandis* L.f. in a Rainforest Site in Southern Nigeria

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ABSTRACT

This study was carried out to select the best model for predicting height for effective management of *Tectona grandis* stand in the study area. Five 3-parameters well-established non-linear models were used to compare their suitability in predicting height, evaluate and select the best H-D model based on test statistics criteria such as R^2 , RMSE, AIC, and BIC. Models with the lowest RMSE, AIC and BIC and highest R^2 values are judged the best. One hundred (100) trees were enumerated and were subjected for analysis in the study. A plot size of 0.04ha (20m x 20m) with an area of 0.64ha was chosen for this study and 10 sample plots were purposively selected from the study area. The result indicated that schnute model had the highest R^2 value of 99.9% but it also had the highest RMSE, AIC and BIC values, hence the worst candidate in predicting height in the study area. Weibull model although did not have the largest R^2 of 97.2% when compared to schnute, had the smallest RMSE, AIC and BIC (1.37596E-14, -832.6836 and -823.9054 respectively) and hence is the best candidate model. The Chapman-Richards, Logistic and Korf models also performed well based on the evaluation criterion. Weibull function with 3-parameters has been confirmed to provide a secure estimate of total tree height for *Tectona grandis* in the rainforest site of the study area of Southern Nigeria.

Keywords: Comparison, fit statistics, Weibull model, parameter-estimates.

INTRODUCTION

The relationship between tree height and diameter at breast height (DBH) is an important element of forest structure that has long been used to characterize important aspects of tree growth, stand development and that helps define relationships among individuals. Height diameter models are used to calculate measurable quantities needed to characterize forest growth and yield [1] especially volume, weight and biomass. For inventory purposes, it is common practice to measure DBH on all trees in a plot while limiting the measurement of height to a subsample of trees because measurements of height can be more difficult and time consuming [2]. The relationship between tree DBH and total height is a structural characteristic of a tree that describes key elements of stem form, and thus the volume of the harvestable stem. [3]

The height-diameter relationship describes the correlation between height and diameter of the tree in a stand on a given site and this can be represented by a linear or non-linear mathematical model. Developing forest inventory estimates often involves predicting tree volumes from only diameter at breast height (dbh.) and/or merchantable height. [3]. Tree height and diameter at breast height (DBH; that is, diameter outside of the bark of tree taken at 1.3m above ground level) are fundamental tree characteristics used in forest measurements in order to come up with estimates for timber volume, site index and other important variables related to forest growth and yield, succession and carbon budget models [4]

DBH can be determined easily and accurately at little cost and time using such instruments as: diameter tapes, callipers, Biltmore stick and bark gauge. "Information on tree heights is essential in forest inventories for computing tree volumes. However, compared to measuring tree diameter, field measurement of tree height is rather tedious. This is why many forest inventories save time and effort by predicting tree heights using height-diameter (H-D) models instead of direct measurements. To improve prediction of the local H-D curve, height measurements from a subsample of trees on each sample plot or sample plot cluster may be utilized. Predicting total tree-height based on observed diameter at breast height outside bark is routinely in practical management and silvicultural research work" [5]

The *Tectona grandis* L.f. is a fast-growing tropical hardwood tree species belonging to the family of Verbenaceae. Teak is one of the most widely cultivated exotic species in Nigeria because of its good anatomical and physical properties [6]. It is also multipurpose tree species and as such, there is continuous demand for its products [6]. The teak plantation in the study area is being managed for research purpose. There are dearth of information on non-linear H-D models for teak plantation in Nigeria which is important for the management of the plantation. The main aim of the study therefore was to compare five non-linear height-diameter models for height prediction and prescribing silvicultural option for the effective management of the teak plantation stand.

METHODOLOGY

Study Area

The study area is located in the University of Benin which was formerly called the institute of technology. It was founded on Saturday 23rd of November 1970. It occupies an area of about 13000 hectares. The study area is located between latitudes 6°24'08 N and longitudes 5°37'38 E. It is under the Ovia North East Local Government Area, Edo State, Nigeria. The climate condition around the area varies all year round. In the wet season, it is warm, oppressive and overcast and the dry season is hot and muggy. Over the course of the year, the temperature typically varies from 67°F to 88°F and is rarely below 60°F or above 91°F. It has a tropical climate. During most months of the year, there is significant rainfall. The average annual temperature is 25.7°C. About 2679mm/ 105.5 inch of precipitation falls annually.

Data Collection

A total of 30 sample plots of 0.04ha (20m x 20m) with an area of 0.64ha were chosen for this study. Ten sample plots were purposively selected from the total sample plots. Within each plot, the following measurements were taken; diameter at breast height (dbh), diameter at top (dt),

midpoint diameter (dm), total height (tht), merchantable height (mht) were taken with spiegel relaskop. Observed volume by Newton and basal area were computed.

Data Cleaning

Data cleaning is the process of detecting and correcting inaccurate records from a data set. It involves fixing or removing outliers, incorrectly formatted, duplicate or incomplete data within a dataset. The duplicated data were cleaned and incorrect computations were adjusted. After inputting the data into the system, each datum was crosschecked to ensure that there no wrong data was inputted and also to ensure that there were no duplicate data.

The Applied Nonlinear Models

From past published studies on H-D modeling (Table 1), five (5) sets of 3- parameters nonlinear models were chosen for this study. In other to avoid problems with back transformation bias, only models that expressed tree height without transformations were used. The models were parameterized so that parameter b_0 defines the scale of the H-D models, which is the major dimension of variability among sample plots. To generate parameter estimates, the datasets were handled with R-script using package 'nlme' of R-environment [7].

Table 1: The five non-linear 3-parameters height-diameter models chosen are;

Model Form	Reference Eq.
Chapman Richards $HT = 1.3 + b_0(1 - e^{-b_1.DBH})^{b_2}$	Chapman 1961, Richards 1959 [1]
Weibull $HT = 1.3 + b_0(1 - e^{-b_1.DBH^{b_2}})$	Yang <i>et al.</i> 1978 [2]
Schnute; $HT = \left[1.3^{b_1} + (b_2^{b_1} - 1.3^{b_1}) \frac{1 - e^{-b_0(DBH - DBH_0)}}{1 - e^{-b_0(DBH_2 - DBH_0)}} \right]^{\frac{1}{b_1}}$	Schnute 1981 [3]
Logistic $HT = 1.3 + \frac{b_0}{(1 + b_1^{-1}.DBH^{-b_2})}$	Hung <i>et al.</i> , 1992 [4]
Korf/Lundqvist; $HT = 1.3 + b_0.e^{(-b_1.DBH^{-b_2})}$	Stage 1963, Zeide 1989 [5]

b_0, b_1, b_2 = model parameters to be estimated, e = base of the natural logarithm, HT = total tree height (m); DBH = diameter at breast height (cm)

Evaluation Statistics

The models used in this were adjudged base on four test statistics; Root Mean Square Error (RMSE), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The best model was adjudged on the criterion that the smaller the values of the evaluation statistics, the better the fitting model. The model evaluation criterion is indicated below:

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} \quad [6]$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n}} \quad [7]$$

$$AIC = n \ln \left(\frac{RSS}{n} \right) + 2p \quad [8]$$

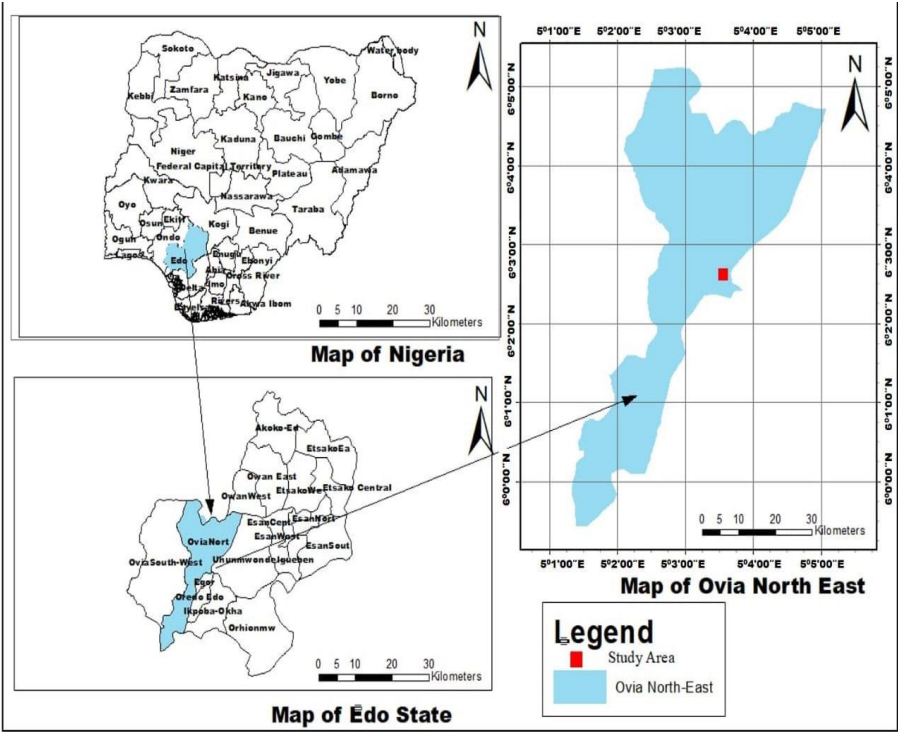
$$BIC = n \ln\left(\frac{RSS}{n}\right) + p \ln n$$

[9]

Map of the Study Area



Plate 1: Satellite image of the study area.



RESULTS

The estimated parameters of the five fitted H-D models and their corresponding evaluation test statistics criterion are summarized in table 2 below. Models with the lowest RMSE, AIC and BIC and highest R² values are judged the best. Schnute model had the highest R² value of 99.9% but it also had the highest RMSE, AIC and BIC values (see table 2), hence performed poorly in predicting height in the study area. Weibull model although did not have the largest R² of 97.2% when compared to schnute, it had the smallest RMSE, AIC and BIC (1. 37596E-14, -832.6836

and -823.9054 respectively) and hence is the best fit model. The Chapman Richards, Logistic and Korf models also performed well based on the evaluation criterion.

Table 2: Parameters and fit indices of the height prediction models for the teak data

H-D Models	Parameters			Fit statistics			
	b_0	b_1	b_2	R^2	RMSE	AIC	BIC
Chapman-Richards	27.846	0.636416	18311.12	0.911	3.149	35.894	29.894
Korf	30.4928	0.063406	9.066469	0.9219	5.3368	49.6369	45.4151
Logistic	0.0000004	0.075	0.635506	0.9953	16.2201	78.6031	74.3813
Weibull	27.846	0.0000003	0.612322	0.9719	1.37596E-14	-832.6836	-823.9054
Schnute	27.846	0	1.0635	0.999	2.23593E+15	15.3494	922.7465

b_0 , b_1 , b_2 – parameter estimates, R^2 – coefficient of determination, $RMSE$ – root mean square error, AIC – Akaike information criteria, BIC – Bayesian information criterion.

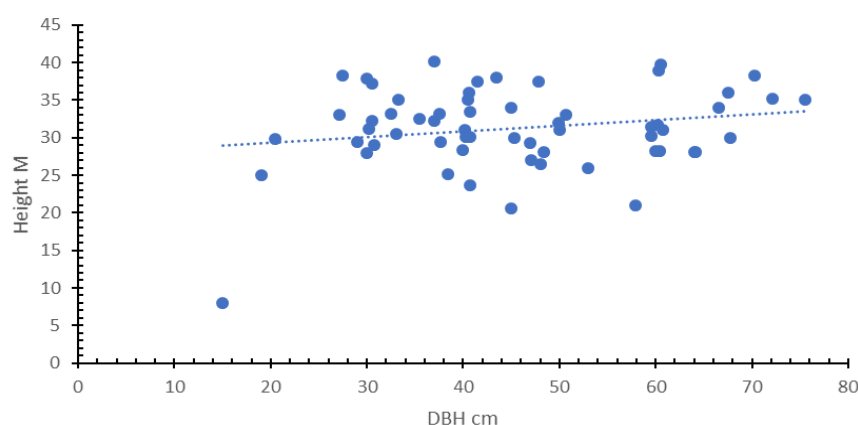


Fig.1: Scatter plot of total height (HT) against diameter at breast height (DBH) for *Tectona grandis*

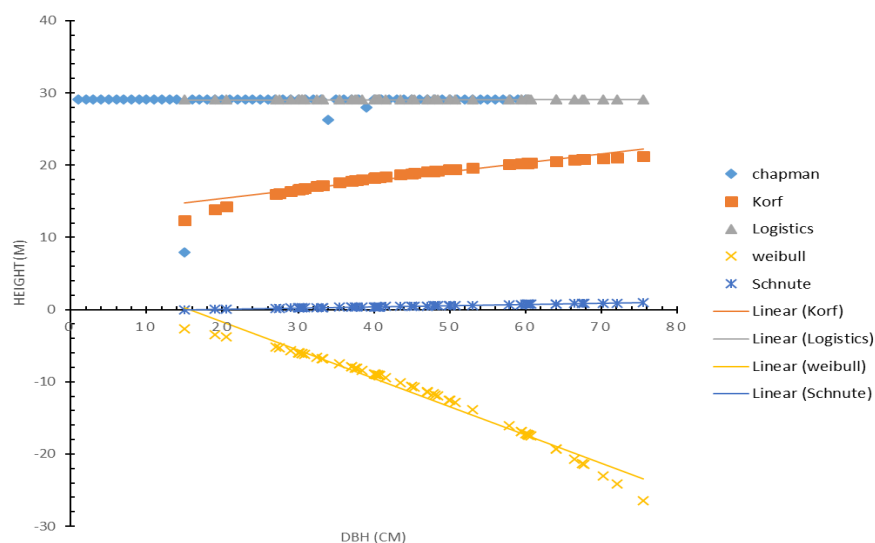


Figure 2: Scatter plot and fitted height trajectories of the five H-D models.

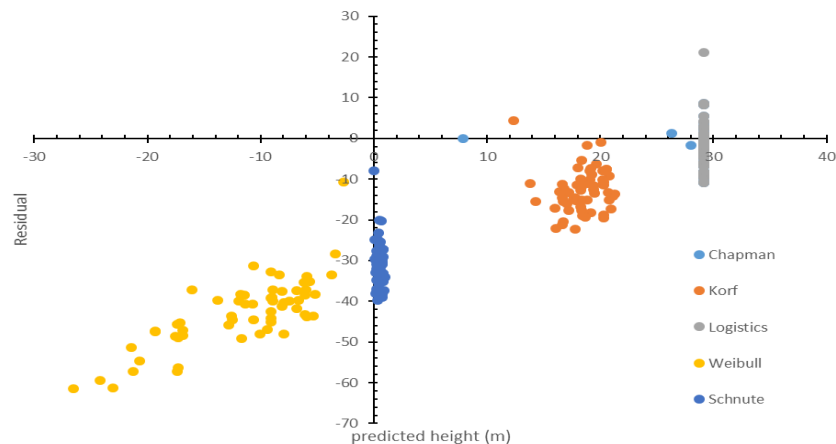


Figure 3: Residual plot of the five H-D models.

Table 3: Summary statistics of the data

STATISTICS	DBH (cm)	THT(m)	BA(m ²)	VOL.(m ³)
Mean	44.17	33.10	0.17	4.34
Standard Error	1.38	0.52	0.01	0.26
Standard Deviation	13.78	5.20	0.10	2.60
Kurtosis	-0.57	4.32	0.02	0.61
Skewness	0.38	-1.36	0.91	0.83
Minimum	15	8	0.0177	0.136
Maximum	75.5	40.5	0.4478	13.609

DISCUSSION

Summary of the growth variables are listed in table 3. The minimum and maximum diameter at breast height was 15 cm and 75 cm respectively. The minimum height was 8m while the maximum height was 40.5m. The standard error for the various parameters ranged from 0.01 to 1.69. The result revealed a strong relationship between the growth parameters for the pooled data.

The results of goodness for predicted height are shown in table 2 along with the performance criteria (R^2 , RMSE, AIC and BIC) for the five models. Models with the lowest RMSE, BIC and AIC values closest to unity are known to perform best (Ahmadi *et al.* 2013). The R^2 values as indicated in table 2 showed that Logistic, Weibull and Schnute models had the highest R^2 values of 99.5%, 97.2% and 99.9% respectively, hence they were selected as the best models for the H-D prediction which also agrees with the findings of [8]. This implies that 99.5%, 97.2% and 99.9% variation in height were explained by the diameter. Also, the curve was best captured by the Chapman-Richards, Logistics and Schnute fit (figure 3). The predicted value follows the same nonlinear pattern and fall close to the line of best fit without much outliers.

The Weibull model however, did not have the largest R^2 but had the smallest RMSE, AIC and BIC (1.37596E-14, -832.6836 and -823.9054 respectively) and hence was chosen as the best fit model. Although Weibull model did not produce the highest R^2 value, it was still chosen as the best model because according to [9], R^2 is a poor tool for model selection as it almost always

favors the most complex models. Also [10] stated that R^2 cannot be used to select the best model because it increases as predictors are added i.e. overfitting is guaranteed. Hence, R^2 cannot be used as a sole indicator for determining the best model. The Chapman Richards and the Korf models were also well fitted to the tree H-D data.

The Logistics model yielded the maximum RMSE which implies that the estimator is not as efficient compared to other growth curves used for the study, this is in sharp contrast in the work by [8], who compared ten nonlinear H-D models and Logistic model out-performed all the other models used in the study with the lowest RMSE. According to [11] determination of fixed and random effects parameters in a model is a flexible decision subject to debate. While [12] were of the opinion that all parameters in a model should first be considered mixed if convergence is possible. However, [13] suggest those parameters with high variability and less overlap in confidence intervals obtained by fitting at each plot should be considered mixed if the convergence is not achieved when considering all the parameters. Hence the results of this study suggest that the ability of DBH in determining height is strong enough as observed among Chapman-Richards, Logistics and Schnute model's ability for predictive purposes.

Weibull was the most suitable model for the teak plantation in this study, agreeing with the work of [14]. The fitness of Weibull model in predicting tree height diameter distribution is well documented. For example, [4] found the Weibull model to be adequate in predicting tree height diameter data of nine major tree species in Ontario's boreal forest. However, some researchers such as [15] and [16] in their study had Weibull model as the least fit due to its inconsistency in describing the tree height diameter relationship of trees. The analysis of the plot of residual and predicted values demonstrated that there was little or no systematic bias towards over-or underestimation of the tree total height (Fig. 3). Plotting the residuals also showed that the models were randomly distributed and had heterogeneous residuals. The five H-D models produced tree height trajectories that are consistent with biological realism: monotonic increment, inflection point and asymptote (Fig. 2). A number of researchers found that adding stand variables to the height-diameter equations and using the generalized height-diameter models increase the precision [17]; [18]. Although Weibull model out-performed the others H-D models in this study, Chapman-Richards, Schnute and Logistics models explained over 90% of the total variance in their corresponding response variables for the data sets. Hence, five models used in this study have shown to be very flexible and have been used extensively in growth and yield studies for describing height-age, diameter-age, and volume-age relationships" [19]

CONCLUSIONS

From this study, we found out that the Weibull model was best suitable for the teak plantation in the study site. When compared to previous studies, this H-D model was found to be more accurate and applicable for predicting height-diameter relationship. However, height-diameter relationship varies within a region, depending on local environmental conditions [20], [21]. The models used in this study do not take into account the effects of climatic and ecological factors on height-diameter relationships within different ecological sites. Hence further development of ecoregion-based individual tree height-diameter models is critical for accurate models on which to base forest management decisions [20], [21].

Competing Interest

The authors have declared that no competing interests exist.

Financial Disclosure

The authors declared that this study has received no financial support.

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