



# Assessment of RCS-specific SNR and Loglikelihood Function in Detecting Low-observable Targets and Drones Illuminated by a Low Probability of Intercept Radar Operating in Littoral Regions

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## ABSTRACT

The objective of this study is to deduce signal-to-noise ratio (SNR) based loglikelihood function involved in detecting low-observable targets (LoTs) including drones illuminated by a low probability of intercept (LPI) radar operating in littoral regions. Detecting obscure targets and drones and tracking them in near-shore ambient require ascertaining signal-related track-scores determined as a function of radar cross section (RCS) of the target. The stochastic aspects of the RCS depend on non-kinetic features of radar echoes due to target-specific (geometry and material) characteristics; as well as, the associated radar signals signify randomly-implied, kinetic signatures inasmuch as, the spatial aspects of the targets could fluctuate significantly as a result of random aspect-angle variations caused by self-maneuvering and/or by remote manipulations (as in drones). Hence, the resulting mean RCS value would decide the SNR and loglikelihood ratio (LR) of radar signals gathered from the echoes and relevant track-scores decide the performance capabilities of the radar. A specific study proposed here thereof refers to developing computationally-tractable algorithm(s) towards detecting and tracking by an LPI radar, the hostile LoTs and/or drones flying at low altitudes over the sea (at a given range, R) in littoral regions. Estimation of relevant detection-theoretic parameters and deciding track-scores in terms of maximum likelihood (ML) estimates are presented and discussed.

**Key Words:** Low-observable/Stealth Target, Radar Cross-section (RCS), Low Probability of Interception (LPI) Radar, Ultrawideband (UWB) Radar, Loglikelihood Function, Maximum Likelihood (ML) Estimation

## INTRODUCTION

The study proposed here refers to modeling the performance details of radars capable of detecting low-observable targets (LoTs) and drones. Concomitantly the test radar is presumed to possess the adjunct efficacy of low-probability of interception (LPI) by hostile reconnaissance systems. In all, relevant modeling being addressed refers to an LPI radar system commissioned to detect effectively, the low-observable targets (LoTs) as well as, they are

designed to counter serious threats from hostile electronic attacks with the ability “to see and not to be seen” [1].

In addition, also considered in the proposed modeling is that the operational ambient of test LPI radars refers to near-shore regions posing significant sea-clutter background; and, the associated target echoes thereof are presumably submerged in the “diffused” scattering of radar-sourced electromagnetic (EM) waves from the sea/land surface posing a unique class of clutter prevailing in near-shore/littoral regions.

Typically, the LPI radars being considered do not easily lend themselves to be detected by hostile electronic reconnaissance systems [2]. Such “quiet radars” can, however be designed to possess effective performance capabilities characteristics in tracking LoTs with a high probability of detection ( $P_d$ ) even under adverse clutter environment.

This paper is organized as follows: Described in the following section is a typical operational ambient of LPI radars in near-shore regions. In the subsequent section, the LPI radar related signal characteristics and associated radar cross section (RCS) features of fluctuating LoTs being tracked are described. Relevant statistical aspects of underlying detection-theoretics are identified and described subsequently along with pertinent analytical details on signal-to-noise ratio (SNR) and loglikelihood function involved; and, related analytical considerations on maximum likelihood (ML) estimations are furnished along with computational assessment of track-score details.

### NEAR-SHORE DEPLOYMENT OF LPI RADAR

Illustrated in Figure 1 is an LPI radar that views a low-altitude flying LoT (or stealth target) at low-grazing angles in a littoral, near sea-shore region. The LoT being tracked poses fluctuating RCS due to its kinematic states of varying aspect angles. Further, its non-kinematic details of RCS include stealth attributes due to geometry, as well as, low observance deliberations rendered *via* EM energy absorption by target-surface and/or by active (deceptive) on-board jamming. In modern contexts, drones (unmanned aerial vehicles or UAVs) and high-altitude, long-endurance (HALE) stealth drone are also deployed in warfronts causing perceivable threats [3] [4]. Such drones denote targets of small size and low RCS. Further, in the design of the passive or active systems dedicated for drone detection, the RCS value of such obscure potential targets is not known *a priori*. [5]-[7]

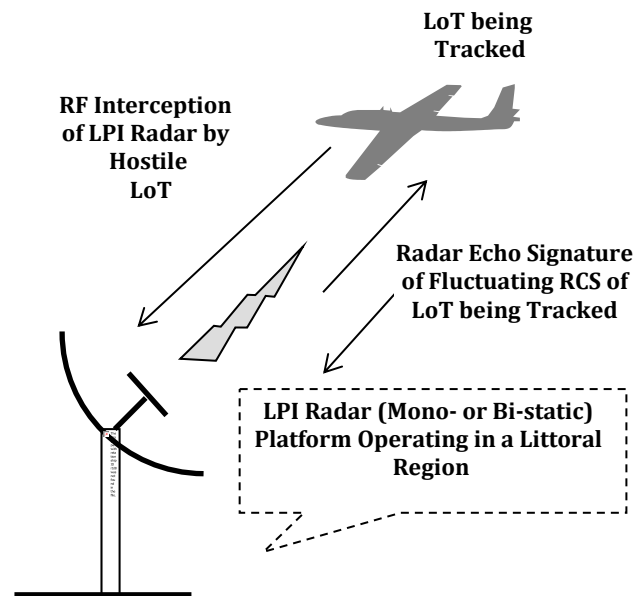


**Figure 1. Conceived operational ambient of an LPI radar viewing a low-altitude flying LoT (or stealth target) at low-grazing angles in the vicinity of littoral, near sea-shore regions. The LoT being tracked poses fluctuating RCS due to the kinetics of its varying aspect angles; further, the shape/geometry of the hostile target under surveillance could possess low observance features improvised *via* radar absorbing materials (RAMs) on the target-surface and/or by deploying active (deceptive) on-board jamming.**

Illumination of the LoT being tracked results in echo signatures at the LPI radar (mono- or bi-static version) constituted by specularly-reflected fluctuating echoes as well as, signals due to diffused scattering of EM waves from statistically-rough sea surface. In summary, the echo signature being received and processed at the LPI radar system depicts a random signal constituted by: (i) fluctuating target scattering details plus an interference pattern due to multipath echo-reflections modulated by stochastic attributes of the sea-scatter (from the clutter-cells).

Further, the system considerations of LPI radar deployed towards tracking LoTs can be briefly stated as follows: The LPI radars are intended to survive countermeasures envisaged by the hostile systems to detect the presence of radars (especially in battlefield ambient) *via* electronic reconnaissance techniques designed to intercept EM/RF emissions from the radars. That is, the LPI radars are enabled to mask their presence so as not to be seen but, at the same time possessing high performance capabilities “to see” the target by detecting and ranging it even under low observable target obscurity improvised.

In general, LPI radars are rendered to escape detection by hostile interception by using antennas that have a transmit radiation pattern with sidelobes of ultra-low levels of  $-45$  dB [8]; and, the radiated energy of the LPI radar is spread over a wide spectrum of frequencies so that the intercepting hostile receiver is compelled to search a large bandwidth to find/locate the LPI radar. Further, the LPI radar buries itself in the environmental noise by exploiting time-bandwidth product to reduce its peak transmitted power. As such, with a mismatch in waveforms for which the intercepting hostile system is tuned, the LPI radar effectively masks itself and becomes invisible [9] [10].

The adjunct mission of LPI radars is to detect even low-observable targets (or LoTs), which return echoes that are characterized by low signal-power levels; as well as, such target echoes could fluctuate randomly due to the kinetics of continuously changing aspects angle of the LoT. Hence, in relevant modeling exercises, the associated details of the signal-power level are specified in terms of stochastic attributes of fluctuating RCS representing the target. The contributing statistics of RCS depends on the dynamic state (or time-dependent random variations in the aspect angle) of the maneuvering LoT viewed by the LPI radar. In addition, LoTs represent a special category of targets treated for low-observance and mostly rendered stealthy by means of their RCS control *via* shaping and linear-sizing (relative to wavelength of operation) plus appropriate RAMs selected for surface treatment. In addition, the LoT echo signatures can be contaminated with RF jamming emissions stemming from on-board electronics. In general, the LoTs can be of physically small types (like small aircrafts, helicopters or even drones).

Another crucial consideration of performance of LPI radar refers to an ambient of operation as in Figure 1, as regard to its location in a near-coastal, littoral region; and, the LoT involved may represent a low-altitude flying target at low-grazing angles with its back- and/or forward-scattered echoes illuminating the sea surface and getting scattered yielding clutter signatures.

### **RADAR SIGNAL PROCESSING AT LPI RADAR**

With reference to the details as above, the echo signature being received and processed by the LPI radar, would refer to a random signal decided by the statistical aspects of: (i) EM scattering from the kinetics of fluctuating aspects of the LoT being tracked; (ii) stealthy and/or low-observance RCS details associated with the LoT; (iii) interference pattern from multipath echo-reflections/diffused scattering randomly modulated by stochastic attributes of the sea clutter-cells; and, possible jamming emissions from on-board electronics in the LoT. Pertinent statistical aspects of signal details as above are described in the following subsections for subsequent considerations in relevant analyses later.

#### **Kinetics of fluctuating LoT aspects and associated statistics of EM scattering**

The LoT being tracked can be equivalently modeled by a surface-area that represents the extent of (effective) EM energy scattered (by the target) and received as an echo signal by the viewing (LPI) radar. If the transmitting and receiving (viewing) radars are at the same locale, the received EM energy is contributed by EM backscattering (at the target); and, the corresponding echo corresponds to monostatic, back-scattering cross-section of the target; alternatively, if the transmitting radar, (which illuminates the target) and the receiving radar are separated and placed at different locales, the received EM energy is caused by forward-scattering at the target; hence, the relevant echo depicts the signature of bistatic, forward-scattering cross-section of the target [11][12]. The equivalent surface-area mentioned above (considered *in lieu* of the target) represents the extent of (effective) EM energy scattered by the target and received as an echo by the viewing radar and well-known as the radar cross-section (RCS, in  $\text{m}^2$ ) of the target.

The RCS of a target denotes a random variable (RV) and its statistical aspects are specified by appropriate models of relevant probability distributions [11] [13]. The RCS (of both monostatic and bistatic systems) as stated before is dependent essentially on the EM scattering characteristics of the target as decided by shape, size, surface impedance of the target as well

as by the kinetics of randomly changing aspect (or spatial attitude) of the target as seen by the viewing radar. Also, frequency and polarization characteristics of the EM energy would influence RCS values.

Thus, considering the temporal variation of the RCS ( $\sigma$ ), it can be modeled as a RV depicting a function of time,  $\sigma(t)$ ; and traditionally, the stochastic attributes of  $\sigma(t)$  are decided by: (i) Pulse-to-pulse fluctuations due to target(s) changing their aspects rapidly; and, (ii) scan-to-scan fluctuations resulting from the slowly-varying changes (per scan) in target aspects. In either case, the RCS fluctuations represent random phenomena with statistical attributes specified *via* appropriate probability density functions (pdfs).

Considering conventional radars that track non-stealthy targets, the associated stochastic aspects of RCS fluctuations are described by the so-called Swerling-Marcum models [14] - [19]. In contrast, the present study is concerned with LPI radars tracking LoTs exhibiting fluctuating RCS with stealthy attributes. Further, as will be detailed later, the test LoTs being tracked by the LPI are illuminated by ultra-wideband (UWB), impulse EM energy; and, corresponding statistics of RCS is deduced in this study. Further, pertinent LPI radar operation is tailored to study the specific ambient of littoral near-shore regions and low-altitude flying LoTs, possibly housed with on-board reconnaissance electronics designed to intercept the LPI radars in the vicinity. Presented in this study is an outline on relevant considerations and hence, models are proposed. Prior to such descriptions, it is necessary to review first as described in the following section, the salient aspects of conventional Swerling-Marcum models [14-18] indicated above.

### **RCS fluctuation models of non-stealthy targets illuminated by a conventional radar**

In general, radar targets like aircrafts, ships etc. depict complicated combinations of surfaces having irregular shapes, sizes and material characteristics; as such, unique representation of their RCS values cannot be, *per se* specified deterministically. Further, due to random changes in the aspect of the target due to inevitable maneuvering, only a statistical and mostly nondeterministic description can be presented to describe the echo fluctuations and model the associated RCS ( $\sigma$ ) involved. That is,  $\sigma$  denotes a RV with an appropriate probability density function (pdf); and, the underlying heuristics of RCS modeling (specified *via* pdf) with reference to conventional radar systems and targets are described below.

The problem of scattering of a plane EM wave from an arbitrary surface has been given considerable attention in the past [20] due to its significance for applications in radar techniques of detection [19]. When an object is illuminated by an EM wave, a portion of its energy is absorbed and the remainder is scattered in many different directions. The portion of the reradiated energy scattered towards the direction of the radar is of chief interest in radars. As mentioned earlier, an equivalent area of cross-section thereof refers to the RCS (in units of  $m^2$ ); and, it is normally defined to describe the scattered power intercepted by the radar. Stating again, RCS is the area intercepting that amount of power, which when scattered equally in all directions, produces an echo at the radar equal to that from the target. Extensive studies have been made in the past to analyze and determine RCS values of simple and complex targets both theoretically and experimentally [13]-[19].

The RCS value is uniquely defined with reference to backscattered energy in monostatic radars and in terms of forward-scattered energy in the case of bistatic systems. Relevant monostatic radar range equation as follows:  $R_{\max} = \{P_t \times G \times A_e \times \sigma / [(4\pi)^2 \times S_{\min}]\}^{1/4}$  where,  $R_{\max}$  = Maximum monostatic radar range;  $P_t$  = Peak transmitted power watts);  $G$  = Gain of the trans-receive antenna.;  $A_e$  = Effective aperture of receiving antenna ( $m^2$ );  $S_{\min}$  = Minimum detectable signal (watts) and  $\sigma$  = RCS ( $m^2$ ) of the target. Similarly, considering relevant radar range equation of bistatic radar systems, the RCS can be specified in terms of forward-scattering cross-section parameter of the target.

For maneuvering targets with complex shape, size and material properties, EM scattering returns random echoes and as such, only a statistical description for the associated RCS is feasible; relevantly, the statistical aspects of the RCS are specified in terms of a pdf function depicting the probability of finding the random value of RCS between,  $\sigma$  and  $(\sigma + d\sigma)$ . Further, the RCS seen fluctuating with changing spatial coordinate of the target aspect with respect to time ( $t$ ), can be specified as a time-dependent random function,  $\sigma(t)$  and corresponding pdf can be modeled as outlined below:

Swerling-Marcum [14] [19] heuristics proposed thereof implies four different fluctuating RCS models in terms of pdf functions describing the scattering behavior of complex targets. In two of the four cases, it is assumed that fluctuations are completely correlated during a particular scan but, completely uncorrelated from scan-to-scan. In the other two cases, the fluctuations are assumed to be more rapid and uncorrelated from pulse-to-pulse. Briefly, relevant cases as above are as follows:

Case 1: Echo pulses received from a target on any single scan are of constant amplitude (throughout the entire scan period) but, are independent from scan-to-scan. Echo fluctuation of this type is known as scan-to-scan fluctuation. Pertinent pdf of the RCS is given by the power form of Rayleigh density function, namely,  $p(\sigma) = (1/\sigma_{av}) \times \exp(-\sigma/\sigma_{av})$ ,  $\sigma \geq 0$  where  $\sigma_{av}$  denotes the average cross-section over all target fluctuations.

Case 2: The probability density function,  $p(\sigma)$  here is same as above (as in Case 1), but the fluctuations are presumed to be more rapid and are taken to be independent from pulse-to-pulse in a given scan.

Case 3: In this case, the fluctuations are assumed to be independent from scan-to-scan and relevant pdf,  $p(\sigma)$  is given by:  $p(\sigma) = [4 \times \sigma / (\sigma_{av})^2] \times \exp(-2\sigma/\sigma_{av})$ ,  $\sigma \geq 0$ .

Case 4: The probability density function,  $p(\sigma)$  is same as above (as in Case 3) with the fluctuations assumed to be independent from pulse-to-pulse in the scan duration.

Swerling viewed that Cases (3 and 4) are relevant to fluctuation statistics where the target echo is caused by the complex target consisting of a pair of two-component scatterers, namely, a distinct, “dominant scatterer” being present along with a cluster of “small scatterers”; and, relevant to such a two-component complex target model, the echoes that cause a predetector signal-to-receiver noise power ratio (SNR) at the receiver is specified as  $X$ ; further, it is presumed that each of the two-component entities (dominant- and small-scatterers) independently return a random echo variable, so that the resulting value of  $X$  can be defined in terms of a sum of a pair of independent gaussian variables. That is,  $X = \sum_i (U_i)^2$ , ( $i = 1, 2, \dots, k$ ),

where,  $U_i$  represents the field or echo parameter returned from an independent fluctuating  $i^{\text{th}}$  scatterer; and, the mean-square value of  $U_i$  namely,  $\langle U_i^2 \rangle$  is equal to  $\psi_0$  proportional to the RCS and the average value of  $U_i$  is equal to zero. Relevantly, the pdf of  $X$  can also be specified by a general family of Chi-squared pdf given by,  $p(X) = \{X^{k-1} / [(2\psi_0)^k \times \Gamma(k)]\} \times \exp[-X / (2 \times \psi_0)]$ ,  $X \geq 0$  and  $\Gamma(\cdot)$  is the gamma function. An alternate form of this  $p(X)$  is given by:  $p(X) = \{k / [X_{av} \times (k-1)!]\} \times \{(k \times X / X_{av})^{k-1}\} \times \{\exp[-k \times X / X_{av}], X \geq 0$  where,  $X_{av}$  denotes the mean value of  $X$ .

It can be noted that, with  $k = 1$ ,  $p(X) = (1/X_{av}) \times \exp(-X/X_{av})$ ,  $X \geq 0$ ; and, with  $k = 2$ ,  $p(X) = [4 \times X / (X_{av})^2] \times \exp(-2X/X_{av})$ ,  $X \geq 0$  denoting respectively Cases 1 and 3 of Swerling-Marcum models indicated above. Here, the function,  $p(X) = [4 \times X / (X_{av})^2] \times \exp(-2X/X_{av})$ ,  $X \geq 0$  depicts the power-form of Rayleigh pdf. And, corresponding Swerling's heuristics denote the statistical aspects of a two-component model of a complex target made of a cloud of independent and identical small-scatterers randomly phased plus a coexisting dominant reflector, all yielding a total mean (echo) power,  $2\psi_0$ .

Swerling offered the above relation on Rayleigh pdf, namely,  $p(X) = [4 \times X / (X_{av})^2] \times \exp(-2X/X_{av})$ ,  $X \geq 0$  on RCS fluctuation of a complex target (comprised of two-components namely, a cloud of small-scatterers plus a dominant scatterer) to depict a conventional complex target such as, a maneuvering aircraft. However, the underlying consideration does not explicitly account for the distinguishability of relative sizes of "dominant reflector" *versus* "small-scatterer cloud" (assumed to depict the complex structure); and, the aforesaid pdf (as proposed by Swerling) was applied indiscriminately towards RCS studies on complex targets (such as an aircraft) with its frequent citations in literature for years and even in standard text books [11].

It can be noted that the said pdf function of Swerling's two-component of the RCS statistics ignores the actual ratio of physical sizes between the associated two components involved and it had been mis-applied in practical studies until Scholefield [21] indicated more realistic and physically-meaningful models that represent the following descriptions of complex targets with appropriate modifications on the associated pdf functions. The proposal by Scholefield has been tested experimentally as in [13] and further expanded in the studies of [22] [23].

Relevant considerations, in essence account for the relative extents of dominant *versus* small-scatterers in the complex structure of a target; as such, the following distinct classes are specified: (i) A cloud of independent scatterers all having approximately equal echoing area and are of stationary (non-moving) types (for example, ground-based targets). (ii) A dominant scatterer together with a set of small reflectors all being stationary (non-moving) (for example, a mountainous terrain with clusters of trees, etc.). (iii) Number of independent, randomly-phased, small fluctuating (moving) targets with equal echoing area (for example, chaff or precipitation). (iv) A dominant fluctuating target together with a cloud of small, fluctuating and randomly-phased scatterers (for example, a propeller driven aircraft, helicopter); and, (v) small set of reflectors remaining stationary (non-moving) with respect to an associated dominant scatterer (for example, jet aircraft, missile, rocket, etc.)

Swerling's two-component model, in general refers to resolving radar signal into four orthogonal components arising from the two-component constituents of the target yielding the

fluctuating echoes; however, with reference to a target consisting of a randomly-phased dominant scatterer with a cluster background randomly-phased small (and almost identical type of) scatterers, the pre-detector power  $X$  received is decided by peak-power amplitude,  $d$  contributed by the dominant scatterer. Relevant power form of the so-called Rice probability distribution function is given by:  $p(X) = [1/2\psi_0] \times [\exp\{-(X + d^2)/(2\psi_0)\}] \times I_0[d \times X^{1/2}/\psi_0]$ ,  $X \geq 0$  and  $d \geq 0$ . Here  $I_0$  denotes the modified Bessel function. And, the mean value of  $X$  is:  $X_{av} = (2\psi_0 + d^2)$ . A simple empirical way of matching the correct Rician pdf as above with an appropriate member of Chi-squared distribution is to equate their mean values. Hence, it follows that:  $\kappa = [1 + d^2/2\psi_0] \equiv (1 + \alpha)$ , where  $\alpha$  represents the dominant-to-background power ratio. Correspondingly, the Rician pdf can be rewritten as:  $p(X) = \{X^\alpha/[2\psi_0]^{1+\alpha} \times \Gamma(1 + \alpha)\} \times \{\exp(-X/2\psi_0)\}$ ,  $X \geq 0$ . When  $\alpha$  tends to zero, this pdf function reduces to,  $p(X) = (1/X_{av}) \times \exp(-X/X_{av})$ ,  $X \geq 0$ ; and, when  $\alpha$  tends to unity,  $X_{av} = 4\psi_0$  leading to,  $p(X) = [4 \times X/(X_{av})^2] \times \exp(-2 \times X/X_{av})$ ,  $X \geq 0$ ; This implies that Swerling's pdf of RCS of two-components (of one dominant plus a set of small scatterers) model of a complex target applies approximately only to a particular case, whenever the dominant scatterer power is approximately equal to that from the background or the associated small scatterers. Relevant considerations proposed by Scholfield in [21] was experimentally verified as indicated in [13]. Though Swerling-Marcum model seems to have been applied and empirically fitted to some experimental data of RCS fluctuation statistics of a complex target (such as an aircraft), it has become more logical to consider Scholfield's model being appropriate in such cases.

### RCS fluctuation models of low-observable or stealth targets

As will be detailed in the next section, detection methods of low-observable/stealth targets are facilitated by deploying ultrawideband (UWB) radars, which illuminate the hostile targets with impulse EM energy. Under such circumstances, the underlying RCS details of interest are characterized by the following: (i) Feeble EM scattering (signatures) by the target present low-observable echoes available for detection at the tracking radar. (ii) Relevant echoes are also uniquely dependent on UWB frequency spectrum of impulse illumination of the targets. (iii) RCS of the target thereof, is a characteristic function of frequency over the UWB spectrum. (iv) RCS features are dependent on monostatic or bistatic arrangements of the radar detection system. (v) Presence of any active emissions facilitated as an on-board jamming electronic counter-measure (ECM) can significantly alter the RCS statistics. (vi) The cohort presence of dominant and/or small-scatterers as constituent entities of the stealth-target/LoT decide the RCS pdf. (vii) Impulse-to-impulse fluctuations, if any exist, can also decide the effective RCS pdf; and, (viii) statistics of the underlying RCS could be viewed differently in time- or in frequency-domain.

The major consideration in detecting low-observable/stealth targets exposed to UWB illumination (*via* impulse radar deployment towards tracking) is that the target irradiation sweeps through the UWB spectral constituents of the impulse; and hence, the corresponding RCS posed by the target spans the Rayleigh, Mie (resonance) and geometrical optics regimes of scattering signatures; and, such signatures are decided by the corresponding scattering features of the target at each (spot) frequency (spectral) component. At the higher end of the spectrum, the cross-section presented by the target in question could be reduced considerably due to intense EM absorption by possible RAM treatment of the target surface. In the middle range of the frequency spectrum (corresponding to Mie/resonance region), the target-shaping

implemented deliberately to offer stealthy and low-observability, could cause destructive interferences, leading to suppressed RCS details. At low frequencies, the Rayleigh scattering in general, would inherently provide low observability, that is, a low RCS.

In order to specify and/or characterize the pdf details of stealth/low-observable target fluctuation statistics and compare them *vis-à-vis* conventional (Swierling-Marcum and Scholefield) models mentioned earlier, it is necessary to consider the following questions:

*What are the corresponding features of low-observable/stealthy targets (under UWB illumination) equitable to pulse-to-pulse fluctuation statistics of conventional targets tracked by a single frequency radar?*

*What is the equitable description of low-observable/stealthy targets (under UWB excitation) statistics vis-à-vis scan-to-scan fluctuation randomness observed in the case of conventional targets tracked by a single-frequency radar?*

The first question corresponds to radar detection situation where the target changes its aspect rapidly (in the time-frame of about 1ms, which corresponds to a pulse repetition frequency (prf) of about 1 kHz). The second question refers to radar detection option in which the received pulses during a single-scan are almost identical; however, a partial correlation in the set of pulses may exist corresponding to the previous and/or next scan. These two, distinct versions of time-dependency of radar echoes posing fluctuations in the received signal were described earlier with reference to Swierling-Marcum and/or Rician distributions in terms of the statistics of RCS *versus* time; and, relevant considerations are now revised in the present study to model the LoTs being tracked by a UWB -LPI radar. That is, the underlying modeling currently refers to the question of specifying the corresponding EM-scattering cross-sections that represent the stochastic aspects of fluctuating LoTs (and/or stealth-targets) as viewed by a monostatic impulse (UWB) radar representing the LPI shown in Figure 1. The unique aspects of such UWB radars are: (a) Transmission and reception of impulse (carrier-free) EM energy and (b) inter-impulse-interval is presumed to be significantly large. Relevantly, the significantly narrow, impulse illumination of the test target leads to echo signatures (depicting stochastically-defined attributes of fluctuations associated RCS,  $\sigma$ ) having a power spectral density (PSD) characteristics of the spectral components,  $\omega = 2\pi \times$  (frequency,  $f$ ) in the ultrawideband range involved; and, a discretized-width  $\Delta\omega$  of this spectrum pertinent to  $\sigma(\omega)$  is considered in modeling the test targets so as to characterize the associated stochastic aspects as follows: The RCS,  $\sigma$  is a RV that admits a pdf,  $p(\sigma)$  with  $F[\sigma(j\omega)]$  denoting the Fourier transform (FT) of  $(\sigma)$ . The associated characteristic function (CF) concurrently depicts the FT of the pdf; (that is, when the RV that admits a density function, then the characteristic function is its dual, in the sense that each of them is a FT of the other). As such, The CF can be considered alternatively as an implicit parameter of the statistical details pertinent to RCS fluctuations viewed in  $j\omega$ -plane. Hence, the Swierling-Marcum and Rician models of RCS mentioned earlier (in the case of traditional radars tracking non-stealthy (conventional) targets), can now be remodeled equivalently to represent RCS fluctuations of  $\sigma$  in  $j\omega$ -plane as follows:

Rayleigh distribution:

$$\begin{aligned} p_{Ra}(\sigma) &= 0 & -\infty < \sigma < 0 \\ &= \left[ \frac{\sigma}{s^2} \right] \times \exp\left(-\frac{\sigma^2}{2s^2}\right) & 0 < \sigma < +\infty \end{aligned}$$

where  $s^2$  is the variance of  $s$ ; and, the corresponding pdf of the power-form of  $\sigma$  is given by:

$$g_{\text{Ra}}\left(\frac{\sigma}{\sigma_{\text{av}}}\right) = 0 \quad -\infty < \left(\frac{\sigma}{\sigma_{\text{av}}}\right) < 0$$

$$= \left(\frac{\sigma}{\sigma_{\text{av}}}\right) \times \exp\left(\frac{\sigma}{\sigma_{\text{av}}}\right) \quad 0 < \left(\frac{\sigma}{\sigma_{\text{av}}}\right) < +\infty$$

where  $\sigma_{\text{av}}$  is the mean value of  $\sigma$ . Hence, the characteristic function of Rayleigh distribution statistics (of pdf in power form) is given by:

$$\Phi_{\text{Ri}, \frac{\sigma}{\sigma_{\text{av}}}}(j\omega) = \left(\frac{1}{\sigma}\right) \int_0^\infty \exp\left(\frac{\sigma}{\sigma_{\text{av}}}\right) \exp\left(j\omega \frac{\sigma}{\sigma_{\text{av}}}\right) d\left(\frac{\sigma}{\sigma_{\text{av}}}\right) \quad 0 < \left(\frac{\sigma}{\sigma_{\text{av}}}\right) < +\infty$$

Rician distribution

$$p_{\text{Ri}}(\sigma) = 0 \quad -\infty < \sigma < 0$$

$$= \left[\frac{\sigma}{s^2}\right] \times \exp\left(-\frac{(\sigma^2 + A^2)}{2s^2}\right) \times I_0\left(\frac{A\sigma}{s^2}\right) \quad 0 < \sigma < +\infty$$

where,  $s^2$  denotes the variance of  $\sigma$  and  $A^2$  refers to the power contributed in the echo by the dominant scatterer; and, the corresponding power-form of  $\sigma$  is given by:

$$g_{\text{Ri}}\left(\frac{\sigma}{\sigma_{\text{av}}}\right) = 0 \quad -\infty < \left(\frac{\sigma}{\sigma_{\text{av}}}\right) < 0$$

$$= \left(\frac{1+S}{\sigma_{\text{av}}}\right) \times \exp\left[\left(-S - \left(\frac{\sigma}{\sigma_{\text{av}}}\right) \times (1+S)\right) \times I_0\left(2 \times \left[\left(\frac{\sigma}{\sigma_{\text{av}}}\right) \times S \times (S+1)\right]^{1/2}\right)\right]$$

$$0 < \left(\frac{\sigma}{\sigma_{\text{av}}}\right) < +\infty$$

where,  $S = (A^2/s^2)$ , which refers to the ratio of: (Dominant scatterer echo power) - to - (variance of the echo power from small-scatterers). Corresponding characteristic function of Rician distribution (in power form) is given by:

$$\phi_{\text{Ri}, \sigma/\sigma_{\text{av}}}(j\omega) = [(S+1)/(S+1-j\omega)] \times \exp\left[\frac{j\omega S}{S+1-j\omega}\right] \quad S > 0$$

Chi-squared distribution

The pdf (in power form) of  $\chi^2$ -distribution of RCS is given by:

$$g_{\chi^2}(\sigma/\sigma_{\text{av}}) = \left[\frac{m}{\sigma_{\text{av}}} \times \frac{1}{(m-1)!}\right] \times \left[\frac{m \times \sigma}{\sigma_{\text{av}}}\right]^{(m-1)} \times \exp\left[-\frac{m \times \sigma}{\sigma_{\text{av}}}\right] \quad (\sigma/\sigma_{\text{av}}) > 0$$

$$= 0 \quad \text{Otherwise}$$

where, the parameter,  $(2m)$  depicts the degrees-of-freedom of  $\chi^2$ -distribution; and, the corresponding characteristic function,  $\phi_{\chi^2}$  of Chi-squared distribution is given by:

$$\phi_{\chi^2, \frac{\sigma}{\sigma_{\text{av}}}}(j\omega) = \left[\frac{m^m}{\sigma_{\text{av}}}\right] \times [m - j\omega]^{(-m)} \quad (m > 0)$$

If Cases (1 and 3) of Swerling-Marcum models (pertinent to scan-to-scan fluctuations of non-stealthy targets) are specified to stealthy/low-observable targets, it would then correspond to impulse-to-impulse fluctuations specified in time-domain. Likewise, considering Cases (2 and

4) of Swerling-Marcum model of non-stealthy targets (pertinent to pulse-to-pulse fluctuations), the equitable model of stealthy/low-observable targets would refer to the statistics of echo fluctuations viewed in discrete spectral "slices/windows" or "widths" across the frequency-domain; and, the underlying statistical features can be implicitly specified *via* characteristics functions of the pdf of the RCS indicated above.

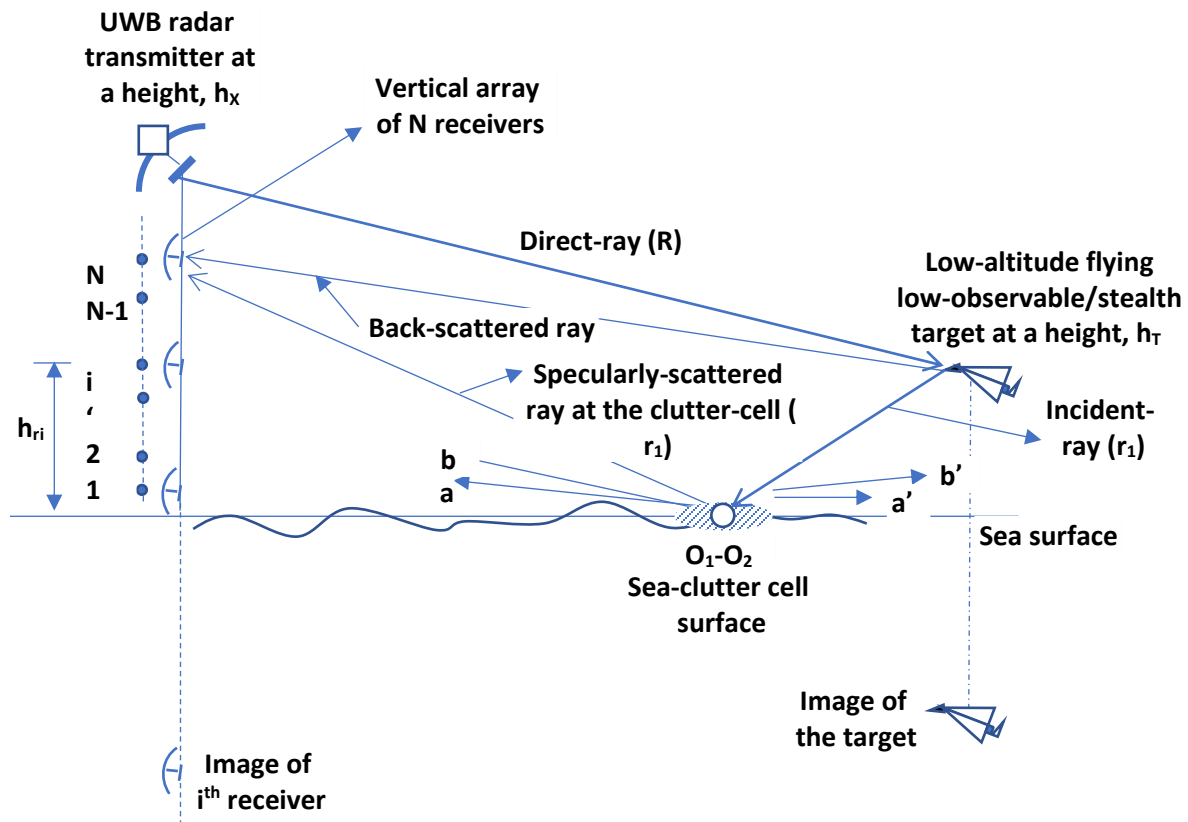


**Figure 2: UWB LPI radar tracking a stealth/low-observable target flying at low-altitude with a sea-clutter background. Multipath rays of EM waves: (a & a'): Sea-swell based specular reflections; and, (b & b'): Scattered Norton components of EM waves at a clutter-cell**

The objective of the proposed effort refers to developing computationally-tractable algorithm(s) to detect stealth/low-observable target in terms of the aforesaid considerations and estimate the associated parameters specified *via* SNR and likelihood ratio LR functions using a set of data gathered at  $N$  receiving units placed as shown in Figure 2. Hence, the feasibility aspects of detecting and tracking, a stealth/low-observable target flying at different heights over the sea (at a given range,  $R$ ) by an LPI radar are elucidated. Hence, considering a vertical set of antennas disposed beneath the UWB radar, the detection efforts to be envisaged refer to constructing a loglikelihood function and maximize it with reference to sensing the target echo plus clutter returns as sensed by the antenna array elements. Further, the stochastic attributes of scattering of EM energy in the sea-shore ambient (Figure 2) are included in the conceived model appropriately.

In all the scope of the proposed work can be summarized as follows: Conventional detection methods of targets by a radar cannot be *per se* applied to track low-observable, stealth targets due to feeble/subtle observability considerations. Therefore, a method to detect such LoTs is proposed here to deploy an ultra-wideband (UWB) radar system, which illuminates the targets with a base-band impulse; and, from the corresponding broad spectral characteristics of echo-returns, a system of antenna-array is suggested to detect and track of such (low-altitude flying) targets even under clutter-infested littoral ambient, as well as, when the (hostile) target supports on-board jamming emissions. The underlying detection-theoretics are detailed below.

### DETECTION THEORETICS OF RCS-BASED TARGET TRACKING: SNR AND ML ESTIMATION

Complexity of detecting low-flying, stealthy/low-observable targets (or even non-stealthy/conventional targets) becomes immensely augmented in the presence of any coexisting clutter such as, unspecified scattered EM energy from the sea-surface [24]; hence, certain neoteric techniques are required in the underlying detection strategies as proposed here. Relevant method refers to using a set of antenna array-processors constituting a UWB radar receiving system (Figure 2). The set of antennas of the arrays is located at different heights above the sea where, "blind-spots" due to possible destructive interference of the signal are absent.

Hence, an SNR-based log-likelihood detection is considered in processing the radar signals received at the array sensors and pertinent parameters are estimated. The proposed strategy aims at detecting multiple EM fields due to multipath, baseband (impulse) response, which depicts the echo signature(s) from a stealthy/low-observable target flying at low altitudes over a sea/ocean surface as illustrated in Figure 2. The detection technique relies on using radar signal features and estimate SNR details gathered from RCS measurements acquired at the UWB LPI radar). The tracking method envisaged yields track-scores based on two measures namely, (i) SNR and (ii) likelihood ratio (LR). The likelihood ratios involved *vis-à-vis* specific RCS fluctuation models can be derived and RCS tracking score plus the multi-model RCS values can be decided as in [25]- [27]. Relevantly, the following notations are used in the analyses pursued.

$LR^{ij}(k)$ : Loglikelihood ratio (LR) function associated with measurement- $i$  and track- $j$  at time  $t_k$

$L^{ij}_K(k)$ : Kinematic details of loglikelihood function

$L^{ij}_S(k)$ : Signal portion of the loglikelihood function

$SNR_{jk}$ : Observed SNR of the  $j^{th}$  measurement (in linear units) at time  $t_k$

$X_{jk}$ : RCS of  $j^{th}$  measurement (in  $m^2$ ) at time  $t_k$

$\sigma_{jk}$ : RCS of  $j^{th}$  measurement (in dBsm) at time  $t_k$

$\sigma^{*i|k-1}$ : Estimate of target RCS associated with  $i^{th}$  track (in dBsm) at time  $t_k$  using data through time  $t_{k-1}$ .

The track-score (in terms of LR) for the  $i^{th}$  track updated with the  $j^{th}$  measurement at time  $t_k$  is given by [25]:  $L^{ij}(k) = L^{ij}(k-1) + \Delta L^{ij}(k)$  where  $\Delta L^{ij}(k)$  is equal to  $[\ln(1 - P_d)]$  when there is no up data on track-score; and,  $\Delta L^{ij}(k) = \Delta L^{ij}_K(k) + \Delta L^{ij}_S(k)$  with track-score updated. Explicitly the term,  $\Delta L^{ij}_K(k) = [\ln(1 - P_d)]$  refers to kinematic details of the score-updates while,  $\Delta L^{ij}_S(k)$  is the incremental signal of the score-update while tracking. It is given by:

$$\Delta L^{ij}_S(k) = \ln(P_d/P_{fa}) + \ln \left[ \frac{p(SNR_{jk}^j | H_1 \quad SNR_{jk}^j > SNR_{th})}{p(SNR_{jk}^j | H_0 \quad SNR_{jk}^j > SNR_{th})} \right] \text{ where, } SNR_{jk} \text{ as listed above refers to observed}$$

SNR of the  $j^{\text{th}}$  measurement (in linear units) at time  $t_k$ ; and  $P_d$  and  $P_{fa}$  denote respectively, the detection and false-alarm probabilities. Further,  $p(\text{SNR}_k^j | H_1; \text{SNR}_k^j > \text{SNR}_{th})$  is the pdf for the SNR given the hypothesis,  $H_1$  that a signal (due to received echo) is present and the observed SNR exceeds the SNR detection threshold,  $\text{SNR}_{th}$ . Likewise,  $p(\text{SNR}_k^j | H_0; \text{SNR}_k^j > \text{SNR}_{th})$  depicts the pdf for the SNR conditioned on the alternate hypothesis,  $H_0$  that noise (or, clutter) alone is present and the observed SNR exceeds the SNR detection threshold,  $\text{SNR}_{th}$ . And, in terms of the values of  $P_d$  and  $P_{fa}$  assigned to the hypotheses  $H_1$  and  $H_0$  respectively,  $\Delta L_{js}^i(k) = \ln \left[ \frac{p(\text{SNR}_k^j | H_1)}{p(\text{SNR}_k^j | H_0)} \right] + [\ln(P_d/P_{fa})]$ , inasmuch as,  $p(\text{SNR}_k^j | H_1) = p(\text{SNR}_k^j | H_1/P_d)$  and

$$p(\text{SNR}_k^j | H_0) = p(\text{SNR}_k^j | H_0/P_{fa}).$$

Further, deduced in [25] are explicit values of  $\Delta L_{js}^i(k)$  depicting incremental updates on signal-score based on classical Swerling-Marcum models of fluctuating, conventional targets being tracked by traditional radar systems. The underlying heuristics are considered here to elucidate the detection algorithm expressed in terms of maximum likelihood (ML) estimate of the unknown receiver parameter vector,  $\mathbf{r}$  that maximizes the conditional probability density  $p(\text{RCS}|\mathbf{r})$  associated with the observations. Pertinent details are outlined in the following sections.

## DETECTION OF A LOW-ALTITUDE FLYING STEALTH/LOW-OBSERVABLE TARGET WITH A SEA-CLUTTER BACKGROUND

As stated earlier, the proposed study refers to detecting a stealth/low-observable target flying at low-altitudes over a sea-clutter surface illuminated by an ultra-wideband (UWB) LPI radar. In the relevant detection ambient, the radar signal received corresponds to "stealthy" echoes due to target RCS submerged in the "diffused" EM-scattering from the sea surface as illustrated in Figure 2. The track-score involved refers to the assessment of loglikelihood function deduced from a vertical set of antenna array (Figure 2) placed beneath a UWB LPI radar; and the detection effort involves estimating the loglikelihood function and maximizing it with reference to sensing the target RCS-based echo plus sea-clutter returns sensed by the antenna array elements.

Conventional radar detection methods cannot *per se* be applied to track the stealth/low-observable targets due to low-observability considerations [28]-[31] arising from highly frequency-dependent RCS of such targets. Therefore, a method to detect such stealth targets is to deploy an UWB LPI radar system, which illuminates the (low observable) targets with a base-band impulse; and, from the corresponding broad spectral characteristics of the echo-returns, an antenna-array based detection and tracking of such targets is done.

### Sea-clutter environment and multipath considerations of radar-specific EM waves

Detecting low-altitude flying stealth/low-observable targets (or even the conventional targets) normally would pose detection problems in the presence of intense sea-clutter [24]. Hence, proposed here is to use a set of array processors constituting a UWB LPI radar receiving system. [32] [33]. Such arrays can be located at different heights above the sea where "blind-spots" due

to destructive interference are absent; and, loglikelihood detection by these array sensors is proposed currently towards target detection and track-score estimation purposes.

The underlying methodology involves detecting multiple EM fields resulting from multipath, baseband (impulse) EM echo signature(s) due to the test target presumed to be flying at low-altitudes over a sea/ocean surface. The multipath returns are typically composed of specular reflection(s) at the ocean surface and scattered EM waves stemming from statistically rough sea-surface. Illustrated in Figure 2 is the scenario under consideration where an UWB LPI radar views a (low-altitude flying) LoT at low-grazing angles. The UWB radar illuminates the test target with baseband impulses of nanosecond width ( $\tau$ ). Pertinent to such nanosecond regimes of the EM impulse, the sea-clutter would present a high range- resolution (typically of the order of range less than about ten meters); and, the scattering of EM energy from the stretch of long, wind-driven gravity waves in a scattering patch (clutter-cell) would correspond to a Bragg scattering profile, which increases linearly with wind-speed for a given angle of incidence.

Referring to Figure 2, the target is illuminated by a direct-ray from the transmitter as well as, by a specularly-reflected component (a-a') and a set of scattered Norton components of EM waves component (b-b') at the clutter-cell on the sea-surface. Similarly, the back-scattered echo returned by the target has a direct-ray component reaching the  $i^{\text{th}}$ -receiver; further, relevant specularly-reflected components plus scattered Norton components from the clutter-cell are also concurrently intercepted by the  $i^{\text{th}}$  receiver.

Under low-altitude flying conditions, the scattered components (b-b') indicated above would mostly correspond to contributions of EM energy scattered by Norton's surface waves. Further, considering the relative heights, ( $h_{ri}/h_T$ ) (where,  $h_{ri}$  denotes the height of the  $i^{\text{th}}$  receiver and  $h_T$  depicts the height of the target, both heights referred with respect to mean sea-level,) the specular components may cancel against with their respective direct-ray components at the receiver and at the locales of the moving target (respectively). In other words, depending on the relative height, ( $h_{ri}/h_T$ ), the target illumination leads to an interference pattern along the vertical direction; and, the received signals as a function of height would constitute an interference profile exhibiting maxima and minima. That is, a given unit in the collinear set of vertically disposed receivers intercepts a complex signal decided by the interference pattern indicated above.

In all, referring to the target-tracking scheme of Figure 2, the associated sea-clutter is viewed as a clutter-patch illuminated by the EM energy under "swell" sea conditions. That is, the water waves are assumed to continue on after the wind stops or they change their direction constituting a swell. It is a common phenomenology seen across ocean surfaces. In essence, a swell is constituted normally by a mixture of longitudinal and transverse wave motion and its phase velocity is controlled by earth's gravity. Both impulse excitation of the target by the radar and echo returns from the target are specularly reflected at the sea-clutter cell; as well as, they are scattered by statistically-implied rough sea surface in a diffused manner. The net echo signature is thus constituted by an interference pattern due to multipath echo-reflections modulated by stochastic attributes of the sea-scatter (clutter-cell); and, relevant echo signature is viewed in the vertical plane by an array of  $N$  collinear receivers (located at heights  $h_1, h_2, \dots, h_{ri}, \dots, h_{rN} = h_{\text{XTR}}$ ) beneath the UWB LPI radar transmitter (Figure 2).

## Detection of LoTs and track-scoring algorithm

The set of receiving array processors constituting a UWB LPI system are placed at different heights above the sea-surface where possible "blind-spots" due to destructive interference are absent. Hence target detection considerations and LoT track-score estimation details can be considered as outlined below. Considering Swerling-Marcum heuristics of traditional (non-stealthy) targets flying at low-altitudes over the sea-surface, Bosse et al. have proposed in [34], a target observation vector ( $\mathbf{S}$ ) to represent the time-domain "snap-shot" of observed epoch. It is given by,  $\mathbf{S} = [\mathbf{B} \times \mathbf{F}(h) + \boldsymbol{\eta}]$  at the outputs of  $N$  receiving units. Further,  $\mathbf{B}$  and  $\mathbf{F}$  denote propagation functions due to direct- and specularly- reflected rays from the sea-surface; and, the coefficient,  $\mathbf{B}$  specifies both deterministic (nonfluctuating case) or random (fluctuating case). The entity  $\boldsymbol{\eta}$  depicts a noise parameter assumed to be stationary, additive, spatially white and independent of target signals,  $(\mathbf{S}, \mathbf{F}, \boldsymbol{\eta}) \in \mathcal{R}^{N-1}$  and  $\mathbf{B} \in \mathcal{R}^{1 \times 1}$ . Using far-field approximation of underlying EM waves,  $\mathbf{B}_i$  and  $\mathbf{F}_i$  can be derived as functions for the observation at  $i^{\text{th}}$ -receiver at the height,  $h_{ri}$ . Hence, viewing several snap-shots (in time-domain) of the observation model, the  $\mu^{\text{th}}$  instant snap-shot at the  $i^{\text{th}}$ -receiver can be specified as:  $\mathbf{S}_{\mu i} = \mathbf{B}_{\mu i} \mathbf{F}_{\mu i}(h_{ri}) + \boldsymbol{\eta}_{\mu i}$ ,  $\mu = 1, 2, \dots, (K = \tau/\Delta\tau)$ , where  $\tau$  is the total time of observation sliced into  $(K \times \Delta\tau)$  instants. The ML estimate of the unknown parameter vector  $\boldsymbol{\theta}$  that maximizes the conditional density  $p(\mathbf{S}|\boldsymbol{\theta})$  of the observations; and, relevant loglikelihood function  $L(\boldsymbol{\theta})$  given by:  $L(\boldsymbol{\theta}) = -\sum_{\mu=1}^K (\|\mathbf{S}_{\mu} - \mathbf{B}_{\mu} \mathbf{F}_{\mu}\|^2 / \rho_{\mu}^2)$  with  $\boldsymbol{\theta} = [h_{ri}, B_1, B_2, \dots, B_K]$  and  $(\rho_{\mu}^2)$  represents the variance of the snap-shot signals. A necessary condition for  $L(\boldsymbol{\theta})$  to be maximized is that the partial derivatives of  $L(\boldsymbol{\theta})$  with respect to  $\mathbf{B}_{\mu}$  is set equal to zero. This condition yields an estimation of  $\mathbf{b}_{\mu}$  given by:  $\mathbf{B}_{\mu} = \frac{\mathbf{F}_{\mu}^H \mathbf{S}_{\mu}}{\|\mathbf{F}_{\mu}\|^2}$  with  $\mathbf{H}$  denoting the Hermitian; and, taking into consideration of propagation assumptions on  $\mathbf{B}_{\mu}$ , an optimal receiver decides ML estimation on track-scores *vis-à-vis* the statistics of the target fluctuations involved.

## LPI-BASED ML ESTIMATION: LOW-ALTITUDE FLYING LoT WITH A SEA-CLUTTER BACKGROUND

When a stealth/low-observable target is illuminated by a UWB LPI radar, the associated target fluctuations viewed in time-domain can be presumed to be almost nil during the nanosecond regime of impulse EM excitation involved. As such, in the case of nonfluctuating RCS namely, the so-called Marcum-0 model can be invoked and pertinent ML estimation can be carried out as described below.

In all, the complex signal vector,  $\mathbf{S}(f)$  at  $i^{\text{th}}$  receiver in the  $N$ -array system (of Figure 2) can be written as:  $\mathbf{S}(f) = \mathbf{B}(f) \mathbf{F}(f, h_{ri}) + \boldsymbol{\eta}(f)$ , where  $\mathbf{S}(f)$  refers to the frequency ( $f$ )-domain observation vector or "snap-shot" of the observed signal at the output of the receiver at any discrete, (Fourier) frequency component of impulse excitation of duration,  $\tau$ . Further,  $\mathbf{B}(f)$  represents a nonfluctuating parameter viewed in the frequency-domain; and, the noise vector  $\boldsymbol{\eta}(f)$  is assumed to be stationary, additive, spatially white and independent of target signals,  $(\mathbf{S}, \mathbf{F}, \boldsymbol{\eta}) \in \mathcal{R}^{N-1}$  and  $\mathbf{B} \in \mathcal{R}^{1 \times 1}$ .

The observation model as above conforms to snapshots, each being statistically-independent from the other and representing a discretized frequency (or spectral width). Relevantly, within the small duration of impulse, a coherency between discretized values of signal in time-domain

is a valid assumption on statistical independency; however, the noise level superimposed on deterministic component of the signal (arising from dominant scatterer) may differ in each frequency “snapshot” inasmuch as, the receiver noise-figure is inevitably frequency dependent. Hence, the complex signal vector at  $i^{\text{th}}$  receiver can be written as:  $\mathbf{S}_{m, \mu, i} = [\mathbf{B}_{m, \mu, i} \mathbf{F}_{m, \mu, i} (f_m) + \eta_m]_{\mu, i}$  where,  $m = (1, 2, \dots, M)$  and it specifies the index for each (discretized) Fourier component of the received signal spectrum;  $\mu = (1, 2, \dots, K)$  indicates the time-domain discretization of the impulse of duration  $\tau$  sec; and,  $i = (1, 2, \dots, N)$  is the reference index of each of  $N$  receivers located at the heights,  $h_{R1}, h_{R2}, \dots, h_{Rii}, \dots, (h_{RN} = h_{XTR})$ . Relevant to the aforesaid discretizations of frequency, time and height frames namely,  $(f_m, \tau_\mu, h_i)$  corresponding discretized entities can be represented as:  $(m \times \Delta f, \mu \times \Delta \tau, i \times \Delta h)$  where,  $\Delta f = (BW/M)$ ,  $\Delta \tau = (\tau/K)$ , and  $\Delta h = (h_{XTR} - h_{RN})/N$  with  $(BW, \tau, h_{RN})$  depicting respectively, the bandwidth, impulse-width and maximum height at which the  $N^{\text{th}}$  receiver is located.

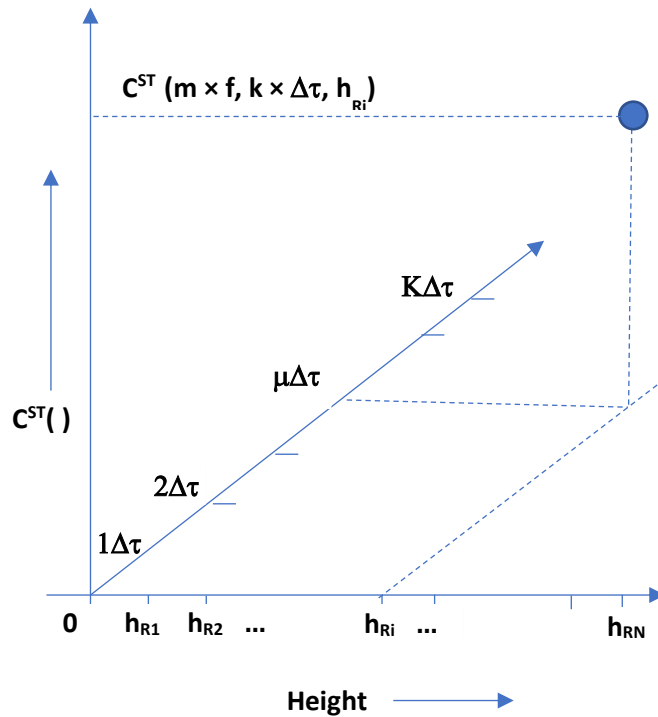


**Figure 3. Estimation of  $C^{\text{ST}}(.)$  at discretized sets of:  $[m \times \Delta$  (frequency,  $f$ ),  $\mu \times \Delta$  (time-duration of the impulse,  $\tau$ ) and  $i \times \Delta$  (receiver heights above the sea-level,  $h$ )]**

In terms of discretizations indicated above, the ML estimator of the signal processed at the height  $(h_{Ri})$  in detecting the stealth/low-observable target and establishing the associated track-score can be written as follows:  $C^{\text{ST}}(f, t, h_R) = \frac{1}{\sum_{m=1, i=1}^{M, N} (\|\mathbf{S}_{k, i}(m \times \Delta f)\|^2 / \eta_{k, i}^2)} \times$

$\sum_{\mu=1}^K \sum_{i=1}^N \sum_{m=1}^M \frac{\|\mathbf{S}_{k, i}^H(m \times \Delta f) \times \mathbf{F}_{\mu, i}(m \times \Delta f)\|^2}{\|\eta_{k, i}^2(m \times \Delta f) \times \mathbf{F}_{\mu, i}(m \times \Delta f)\|^2}$  where, the superscript, ST explicitly denotes the

LoT/stealthy-target; and, the above expression corresponds to the assumption that in the time-frame of the impulse-width, the target is assumed to be nonfluctuating both in its temporal and frequency spectral characteristics. The function  $C^{\text{ST}}(.)$  is a quantitative representation of the track-score on the observed target implicitly depicting receiver signal at the height of interest over the time-frame of the impulse.

## COMPUTED RESULTS AND DISCUSSIONS

Examples of computed details of  $C^{ST}(\cdot)$  are furnished in Figure 3. These results correspond to the illustrated details in Figure 1 as regard to an UWB LPI radar that views a stealth/low-observable target or a drone flying at a low-altitude posing low-grazing angles. That is, the target is illuminated with ultrawideband EM energy, which corresponds to deploying impulse excitation by the test radar. Further, the test radar operation confirms to littoral regions; as such, the echo from the target refers to specularly reflected components as well as, those scattered by statistically-rough sea. Hence, the resulting signal constitutes an interference pattern due to multipath echo-reflections modulated by the stochastic attributes of the sea-scatter (clutter-cell) plus specular direct components; and, the resulting echo signature is viewed in the vertical plane by an array of  $N$  receivers located at the heights  $h_{1r}, h_{2r} \dots, (h_{Nr} = h_{XTR})$  beneath the UWB radar transmitter (Figure 2).

Relevant estimation of  $C^{ST}(\cdot)$  values refers to presentation of their values at discretized set of frequency, time and receiver heights as indicated in Figure 3. Relevantly, this study offers a computationally-tractable algorithms towards detecting the test target and estimating the associated signal details in terms of a track-score expressed *via* ML estimates ( $C^{ST}$ ) gathered at an appropriate receiving unit of Figure 2. Computed results are exemplar details corresponding to test parameters as presented in Figures 3A and 3B. Similar to the computed details of Figures 3A and 3B, a host of results pertinent to assorted sets of values of the height of the UWB LPI radar transmitter ( $h_{XTR}$ ), height of the stealth/low-observable target ( $h_T$ ), radar range ( $R$ ) and bandwidth ( $BW$ ) are considered in [35] and computed results thereof are obtained.

The following is a summary of details on  $C^{ST}$  obtained in the present study and from the computational details due to Mendivil in [35]:

- I. Effect of relative height of the target ( $h_T$ ) with respect to the height of the UWB LPI radar transmitter ( $h_{XTR}$ ) for a given  $BW$  and range on estimated ML values:

For large values of the ratio, ( $h_{XTR}/h_T$ ), there are "blind spots" seen on  $h_{Ri}$  scale wherein,  $C^{ST}$  becomes very low; and, as ( $h_{XTR}/h_T \rightarrow 1$ ), the effect of  $h_{Ri}$  on  $C^{ST}$  is reduced whenever, the range ( $R$ ) is large

- II. Effect of range ( $R$ ) for a given ( $h_{XTR}/h_T$ ) ratio and  $BW$  values on the estimated ML values:

For smaller ranges,  $C^{ST}$  is reduced as the ratio  $h_{XTR}/h_T$  increases from small values. But, when ( $h_{XTR}/h_T \approx 1$ ), the variation of  $C^{ST}$  against  $h_{Ri}$  becomes significant. This may be due to the reason that, reduced ranges could possibly cause stronger interference between various EM-field components that constitute the received signal

- III. Effect of bandwidth ( $BW$ ) for specified values of ( $h_{XTR}/h_T$ ) ratio and range,  $R$  on the estimated ML values:

Reducing receiver bandwidth, (for example, say from 40 GHz to 20 GHz), the aforesaid observations approximately remain invariant except that the "blind-spot" levels of  $C^{ST}$  become prominently more extensive.

Further, computations performed on  $C^{ST}$  *versus* the impulse duration ( $\tau$ ) show that the relevant variation remains almost invariant for larger bandwidths (such as, 40 GHz). However, when the bandwidth is reduced (say, to 20 GHz), there is a small variation in  $C^{ST}$  *versus*  $\tau$  at low values of

$h_{Ri}$  (that is, when the receiver units are close to the sea surface). However, as  $h_{Ri}$  is increased,  $C^{ST}$  remains constant as a function of  $\tau$ .

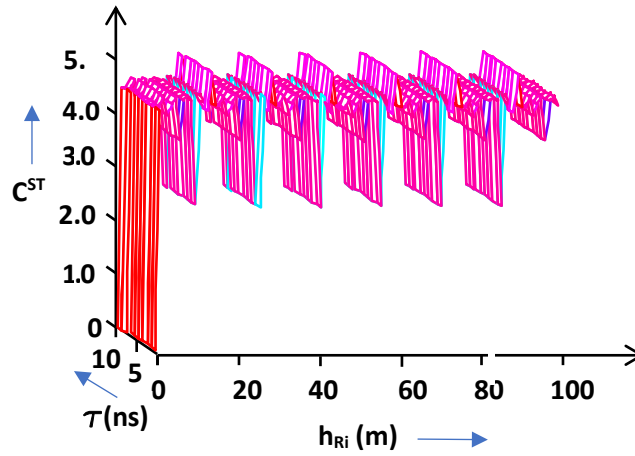


**Figure 3A. Computed ML estimation ( $C^{ST}$ ) over the duration of the transmitted impulse *versus* the height of the receiver. (Data used: Height of the UWB LPI radar transmitter ( $h_{XTR}$ ) = 100 m; height of the stealth/low-observable target = 25 m; radar range ( $R$ ) = 2500 m and bandwidth (BW) = 40 GHz.**

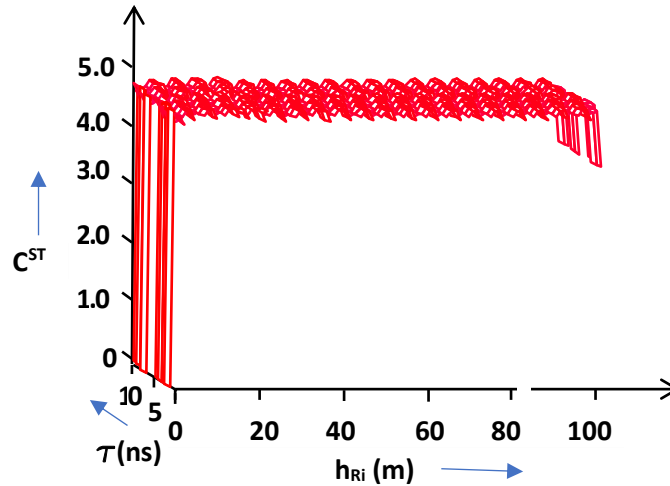


**Figure 3B. Computed ML estimation ( $C^{ST}$ ) over the duration of the transmitted impulse *versus* the height of the receiver. (Data used: Height of the UWB LPI radar transmitter ( $h_{XTR}$ ) = 100 m; height of the stealth/low-observable target ( $h_T$ ) = 75 m; radar range ( $R$ ) = 2500 m and bandwidth (BW) = 40 GHz.**

The validity of the analysis performed in this study can be ascertained by comparing relevant results on loglikelihood calculation(s) performed by Bosse' *et al.* in [34] on conventional target environment with a sea-clutter background; and related, computational details due to Mendivil in [35] specifically indicate the following observations on the variation of  $C^{ST}$  *versus* the impulse duration ( $\tau$ ):

- Observed variations in  $C^{ST}$  with reduced BW: Presence of blind-spots becomes more extensive and there is a small change in  $C^{ST}$  through the duration of the pulse
- For targets flying at lower altitudes,  $C^{ST}$  becomes significantly low
- For targets flying at higher altitudes,  $C^{ST}$  is reduced for large values of target range

- In general, for smaller ranges,  $C^{ST}$  is reduced and it is highly dependent on target height, range and BW
- $C^{ST}$  is essentially controlled by the interference of direct ray, specularly reflected rays(s) and Norton wave components
- Space-diversity reception with multiple receivers as proposed here combats the effects of blind spots.

### CONCLUDING REMARKS

While statistical considerations on fluctuating RCS of conventional targets have been comprehended to a significant extent in the past in terms of Swerling-Marcum heuristics and in terms of Sholefield's extended details, pertinent studies are sparsely addressed in open-literature on stealth/low-observable targets and drones illuminated by UWB radars intended as an LPI platform. Especially, relevant considerations *vis-s-vis* littoral sea-clutter ambient and low-altitude flying conditions are known only to a limited extent in open literature. Therefore, presented here is an effort to address the RCS of low-observable targets exposed to UWB LPI radars; hence, the associated detection-theoretics of the target is indicated in terms of the underlying SNR and ML estimation of the received radar signals. The strategy pursued here in specifying the statistics of the LoT is similar to that of pdf modeling of conventional (non-stealthy) targets in terms of Swerling-Marcum heuristics with the exceptions and modifications as follows: Instead of considering pdf profiles of target fluctuations as in Swerling-Marcum models, the Fourier transform of such pdfs are proposed here instead to specify RCS statistics of stealth/low-observable targets; that is, RCS of low-observable/stealth targets is presumed to exhibit explicitly, characteristic signatures over the spectral band covered by the impulse excitation by the test radar. Relevant adoption of RCS fluctuation statistics thereof specified *via* Fourier transforms of the pdfs corresponds to frequency-domain kinetics of RCS of the test targets. That is, the underlying characteristics of the relevant pdf of the RCS values is indicated as a function of the variable,  $\omega = 2\pi f$  and pertinent ML estimation of signal detection is derived framing the scope of the present study.

Further, unique radar operational ambient existing while tracking the test LoTs flying at low-altitudes on a sea-surface is cohesively inducted into the proposed modeling. Similar ambient corresponds to detecting radar targets in the presence of intense sea-clutter as observed by Neelakanta *et al.* in [24]. The present study however, is an augmented effort on LPI radar based tracking the LoT; further, a set of array processors (located at heights above the sea-level where "blind-spots" are absent) constitutes the UWB LPI radar receiving system being considered. The received radar signals at these receivers/array-sensors are processed to get loglikelihood estimate towards detecting the target and assess the track-score *via* ML estimates.

An exclusive consideration is required to optimize the test radar under consideration to serve as a low-probability interception (LPI) system. As stated before, this is to counter the interception of the test UWB LPI radar by the electronics placed as on- (or off)-board entities in the LPT target being tracked. Towards such LPI radar designs, the EM illumination exercised by the UWB radar (as considered in this study) should have significantly low sidelobe levels [8]-[10].

To extend the scope of the present work, one may consider relevant RCS fluctuation concepts as well as, the associated correlation effects (over the spectral range of the signal handled by the UWB radar) and calculate the false-alarm and detection probability curves in the presence of K-distributed and log-normal versions of sea-clutter. This can be done analogous to the method due to Neelakanta et al. [24] carried out for conventional targets. In essence, the present study offers a directive passage towards exploring neoteric avenues of research concerning stealth/low-observable targets, UWB radar deployment and combating RAM/shape-specifics of low observability *vis-à-vis* targets being tracked. Further, a method of combining the effects of sea-clutter and EM scattering characteristics of low-altitude flying target with a sea-clutter background is a viable futuristic research task.

As mentioned earlier, low-observability or stealth attributes of the hostile target being tracked is usually rendered (as countermeasures on radar detections) *via* appropriate shaping [20] [28]-[31] and/or by coating/making the target with radar absorbing materials (RAMS). Studies on EM energy reflectivity by RAM-coated [36] and/or shaped targets towards RCS evaluations, with UWB illumination of the target using impulses of different shapes are open-questions providing ample scope for future research. [37]-[41].

An inquisitive study that prevails in the state-of-the-art radar technology is to assess comprehensively, RCS details of drone and/or drone-like targets and elucidating detection theoretics of such obscure targets is an open question [4]-[7], [42] [43]. Another explorative effort refers to detecting small targets like drones housed with electromagnetically active-surfaces (EMAS) having embedded false-alarm (FA) generator electronics that could introduce a play-back of optimized FA-signatures into backscattered RF/microwave radiations from hostile sensors. The EMAS can form a conformal sheath on expendable, home-on-jammers (HoJ) [44].

Lastly, without any loss of generality, the present study can be extended (and not just restricted to littoral regions) to any radar-specific ambient of critical locales (such as, a vulnerable air-field) facing hostile drone intrusions and threats [4].

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