



The Motion and Deposition of Small Particles in The Human Lung Acinus, Part A: Micron-size, Infinitely Long Airway

Yakir Vanunu

Israel Institute of Technology

Shimon Haber

Israel Institute of Technology

ABSTRACT

A simple geometrical model of the human lung acinus is investigated. The goal is to get a preliminary insight of the deposition of various size particles. The model comprises of micron-size spherical particles suspended in air, moving inside micron-size, infinitely long pipes. The particle motion is affected by gravity, fluid convection and Brownian motion. Simulations were performed for a 360 mm diameter pipe, a typical size of airways in the human lung acinus. Applying a MATLAB program, it is found that the motion of *small* particles, (10^{-9} , 10^{-8} m) in diameter, is mainly controlled by diffusion, *medium* size particles (10^{-7} , 10^{-6} m) are mainly affected by fluid convection and *large* particles ($> 2.5 \times 10^{-6}$ m) are dominated by gravity. Ninety percent of *small* particles deposit within 1 mm of the pipe's length and during 0.1 seconds, *medium* size particles require 115 mm and 17 seconds and *large* particles need 8 mm and 1.5 seconds.

INTRODUCTION

This article investigates the motion and deposition of submicron and micron-size rigid spherical particles in an infinitely-long pipe. The particles are convected by air flowing in a pipe and are affected by gravity and Brownian forces. It is a preliminary study in which we investigate the fate of small particles in the human lung acinus. The human lung model according to Weibel (1963) includes a binary system of 23 generations of airway pipes of various sizes, where each pipe bifurcates into two pipes to form the next generation. The pulmonary acinus is the deepest region in the lung which includes pulmonary alveoli (generations 15-23) [1]. When a particle cloud moves inside a tiny pipe, some of the particles reach the pipe walls and deposit there, similar to the deposition of inhaled particles on the airways and alveolar walls. Particle motion and deposition on the pipe walls depend on the particle physical properties (diameter, density, diffusivity, electric charge) and on the dynamics of the surrounding fluid flow [2]. Small particles generally follow the flow streamlines, so deposition on the pipe walls requires their deviation from the streamlines. Such a deviation can occur due to different mechanisms: inertial impaction (due to particle momentum), gravitational settling (due to particle mass) and Brownian motion (due to the stochastic forces exerted by the fluid molecules). During the motion of the particles in the pipe, mass transfer between the particles and the fluid may occur (such as nucleation, condensation, evaporation

and hygroscopy) and inter-particulate interactions may exist (such as coagulation and hydrodynamic coupling) [4]. In the absence of hygroscopic growth and electrostatic forces, particle motion is governed by three transfer mechanisms: convection (due to the motion of the surrounding fluid), diffusion (due to Brownian forces) and gravitation [5]. The relative importance of these mechanisms is determined by the following dimensionless numbers: The convective Péclet number, which defines the ratio between the fluid characteristic convective transfer rate and the diffusive transfer rate,

$$Pe_c = LV / D_p \quad [1a]$$

where L is the flow characteristic length, V is the characteristic fluid velocity and D_p is the molecular diffusivity of the mass carried in the fluid [1].

The gravitational Péclet number, which defines the ratio between the gravitational settling velocity of the particle and its diffusion,

$$Pe_g = LV_{TS} / D_p \quad [1b]$$

where, at low Reynolds numbers, the particle's terminal settling velocity, V_{TS} , is the steady velocity that a particle reaches under gravity while submerged in a quiescent fluid. This velocity is achieved when the drag force and the gravitational force, exerted on the particle, are in opposite directions and equal.

$$V_{TS} = (\rho_p - \rho) d_p^2 g C_c / 18\mu \quad [2a]$$

where ρ_p , d_p denotes particle density and diameter, respectively, ρ , μ denote fluid density and dynamic viscosity, respectively, g is the gravitational acceleration and C_c is the Cunningham correction factor (Davies 1945),

$$C_c = 1 + 2\lambda / d_p (1.257 + 0.4 \exp(-0.55 d_p / \lambda)) \quad [2b]$$

Here λ is the mean free path of the fluid molecules ($\lambda = 69 \text{ nm}$ in air at 310K).

Equation [2a] tacitly assumes that the drag force exerted by the fluid is calculated for a particle moving at low Reynolds number, $Re < 1$, where,

$$Re = \rho d_p V_{TS} / \mu \quad [2c]$$

In the following tables 1 and 2 we use various pipe diameters, as the characteristic length L in [1a] and [1b], common to airway dimensions which exist in the human lung. V in [1a] is assumed to equal the mean air velocity in the human lung acinar pipes while breathing normally (Weibel, 1963). We further assume that the fluid surrounding the particles possess human body temperature and that the particle density is 1000 kg/m^3 . Various particle diameters, that may be inhaled and reach the acinus of the human lung, are used to calculate the particle diffusivity

D_p the terminal settling velocity, V^{TS} and eventually, the convective and gravitational Péclet numbers, Pe_c , Pe_g

Table 1 – Convective Péclet number for various pipe diameters and particle sizes Pe_c

Pe_c $d_p = 2.5$ μm	Pe_c $d_p = 2$ μm	Pe_c $d_p = 1$ μm	Pe_c $d_p = 0.5$ μm	Pe_c $d_p = 0.4$ μm	Pe_c $d_p = 0.3$ μm	Pe_c $d_p = 0.2$ μm	Pe_c $d_p = 0.1$ μm	Pe_c $d_p = 0.075$ μm	Pe_c $d_p = 0.05$ μm	Pipe diameter , mm
3478968	2738749	1268141	551580	413558	279892	154743	49881	30022	14280	0.5
1729739	1361703	630518	274245	205621	139162	76938	24801	14927	7100	0.5
835634	657837	304602	132487	99335	67229	37169	11981	7211	3430	0.4
510638	401990	186136	80960	60702	41082	22713	7321	4407	2096	0.4
248108	195318	90439	39337	29494	19961	11036	3557	2141	1018	0.3
126295	99423	46037	20024	15013	10161	5618	1811	1090	518	0.3
56326	44342	20532	8930	6696	4532	2505	808	486	231	0.3
21147	16647	7708	3353	2514	1701	941	303	182	87	0.3
11304	8899	4121	1792	1344	909	503	162	98	46	0.2

Table 2 – Gravitational Péclet number Pe_g for various pipe diameters and particle sizes

Pe_g $d_p = 2.5$ μm	Pe_g $d_p = 2$ μm	Pe_g $d_p = 1$ μm	Pe_g $d_p = 0.5$ μm	Pe_g $d_p = 0.4$ μm	Pe_g $d_p = 0.3$ μm	Pe_g $d_p = 0.2$ μm	Pe_g $d_p = 0.1$ μm	Pe_g $d_p = 0.075$ μm	Pe_g $d_p = 0.05$ μm	Pipe diameter , mm
9365	4795	599	74.922	38.360	16.183	4.795	0.599	0.253	0.075	0.5
9365	4795	599	74.922	38.360	16.183	4.795	0.599	0.253	0.075	0.5
9178	4699	587	73.423	37.593	15.859	4.699	0.587	0.248	0.073	0.4
7492	3836	479	59.937	30.688	12.946	3.836	0.479	0.202	0.060	0.4
7118	3644	456	56.941	29.154	12.299	3.644	0.456	0.192	0.057	0.3
6743	3452	432	53.944	27.619	11.652	3.452	0.432	0.182	0.054	0.3
6368	3261	408	50.947	26.085	11.005	3.261	0.408	0.172	0.051	0.3
5806	2973	372	46.451	23.783	10.034	2.973	0.372	0.157	0.046	0.3
5432	2781	348	43.455	22.249	9.386	2.781	0.348	0.147	0.043	0.2

Table 1 demonstrates that big convective Péclet numbers are obtained in all the acinar generations, which means that convection is much more dominant than diffusion, for all the particle sizes tested. As the generation number increases, the convection becomes weaker and the Péclet number becomes smaller. As the particle diameter increases, the diffusion becomes weaker and the Péclet number becomes bigger.

Table 2 demonstrates that a wide range of the gravitational Péclet numbers is obtained, that are strongly dependent on particle sizes. Since this Péclet number, reflects the ratio between gravitation and radial diffusion in the pipes, we can conclude that for particles smaller than $0.1 \mu m$, $Pe_g \ll 1$, hence, diffusion is the dominant sedimentation mechanism. For particles larger than, $0.4 \mu m$, $Pe_g \gg 1$, gravity is dominant. For medium size particles, $0.1 \mu m < d_p < 0.4 \mu m$, $Pe \sim O(1)$ thus, both gravity and diffusion have the same order effect on particle sedimentation.

Obviously, we missed two very important factors. The foregoing analysis does not account for pipe's length and breathing time. We assume that pipe's length and duration of inhalation are sufficiently long for sedimentation to take place. The goal of this paper is to provide estimates for inhalation duration and pipe length required for significant sedimentation to occur.

Model

Governing Equations: An infinitely long circular pipe of micron-size diameter D contains fluid of dynamic viscosity μ and density ρ . The fluid moves at constant velocity, $\mathbf{u}$, and possesses Poiseuille distribution. Small spherical particles of diameter d_p are embedded inside the fluid and are subjected to random Brownian forces, gravity, and to drag force that the fluid imposes on the particles. The particles deposit on the pipe's walls when its center distance from the wall becomes smaller than $d_p/2$. Particle trajectories are determined by the following differential equation [5],

$$\begin{aligned} d\mathbf{r}_p/dt &= \mathbf{u}_p \\ m_p d\mathbf{u}_p/dt &= \mathbf{F}_D + \mathbf{F}_G + \mathbf{F}_B \end{aligned} \quad [3]$$

where m_p is the particle mass, $\mathbf{r}_p$, $\mathbf{u}_p$ denote particle location and velocity, respectively, t is time, and $\mathbf{F}_D$, $\mathbf{F}_G$ and $\mathbf{F}_B$ stand for the drag, gravity and Brownian forces, respectively. The drag force can be calculated according to Schiller and Naumann (1935),

$$\mathbf{F}_D = C_D \frac{1}{2} \rho |\mathbf{u} - \mathbf{u}_p| (\mathbf{u} - \mathbf{u}_p) \pi d_p^2 / 4 \quad [4a]$$

Where ρ is fluid density and C_D is the drag coefficient,

$$C_D = \begin{cases} \frac{24}{\text{Re}_p} (1 + 0.15 \text{Re}_p^{0.687}) & \text{Re}_p \leq 1000 \\ 0.44 & \text{Re}_p > 1000 \end{cases} \quad [4b]$$

and Re_p is the Reynolds number based on the relative convective velocity and the kinematic viscosity of the fluid, ν ,

$$\text{Re}_p = d_p |\mathbf{u} - \mathbf{u}_p| / \nu \quad [4c]$$

The gravity force is simply,

$$\mathbf{F}_g = m_p \mathbf{g} \quad [5]$$

where $\mathbf{g}$ is the gravity acceleration constant.

The Brownian force as suggested by Li and Ahmadi (1992) is,

$$\mathbf{F}_B = \zeta \sqrt{\frac{1}{D_p} \frac{2k_B^2 T_f^2}{\Delta t}} \quad [6a]$$

where ζ is a random vector, equally probable in every direction, its length is governed by a Gaussian distribution of zero mean and unit standard deviation, k_B is the Boltzmann constant, T_f is the fluid temperature and the particle diffusivity D_p is [7],

$$D_p = k_B T_f C_c / 3\pi\mu d_p \quad [6b]$$

The time increment, Δt , is a small fraction of the minimum of the particle convection characteristic time, τ_{conv} , the diffusion characteristic time, τ_{diff} , and gravity characteristic time, τ_g ,

$$\Delta t = 0.0025 \cdot \min \{ \tau_{conv}, \tau_{diff}, \tau_g \} \quad [7a]$$

where,

$$\tau_{conv} = D / u_{conv} \quad [7b]$$

$$\tau_{diff} = R^2 / 2D_p \quad [7c]$$

$$\tau_g = 18\mu R / (\rho_p - \rho) d^2 p g C_c \quad [7d]$$

For instance, Koullapis *et al* (2020) suggested $\Delta t = 10^{-5}$ sec that proved well for flow inside lung airways [6].

A Cartesian coordinate system is assumed, where the x-coordinate is along the pipe axis, the z-coordinate is along the negative gravity direction, and the y coordinate is perpendicular to both of them and forms a right hand system.

To simplify the solution procedure, we assume that at any time t the particle initially assumes the fluid velocity sans gravity and Brownian forces. The effect of the latter is superimposed and result in particle location at a time $t + \Delta t$. Equation [1] is further simplified neglecting particle inertia. Thus, particle displacement during time Δt can be calculated accounting for particle convection, diffusion and gravity as follows,

$$\begin{aligned} \Delta x_p &= u_{conv} \Delta t + \Delta_{diff} \\ \Delta y_p &= \Delta_{diff} \\ \Delta z_p &= u_g \Delta t + \Delta_{diff} \end{aligned} \quad [8a]$$

where the particle convective velocity, u_{conv} is assumed to equal the flow velocity along the x direction. Since the Reynolds number of the flow is much less than 2000, a Poiseuille flow distribution can be assumed,

$$u_{conv} = 2u (1 - r^2 / R^2) \quad [8b]$$

where $R = D/2$.

Theoretically, the RMS (Root Mean Square) value for the displacement of a cloud of spherical particles, with diffusivity D_p , in a three-dimensional infinite stationary

medium, under Brownian motion, is $l_{RMS} = \sqrt{6D_p t}$ [7]. This value is a superposition of the three **one-dimensional** values: The RMS value for particle displacement under onedimensional Brownian motion is $\sqrt{2D_p t}$ [3].

The random displacement due to diffusion, Δ_{diff} is assumed equal in all directions, and its value is determined by a Gaussian distribution of zero mean and standard deviation of $\sqrt{2Dp\Delta t}$

The particle velocity due to gravity, u_g , is assumed parallel to the gravity vector (opposite the z coordinate direction and perpendicular to the pipe axis),

$$u_g = -(\rho_p - \rho) d_p^2 g C_c / 18\mu \quad [8c]$$

Initial Conditions: At $t=0$ all particles possess the local flow velocity, are spread evenly at the cross-section $x=0$ (See Figure 1), and no interactions exist between adjacent particles.

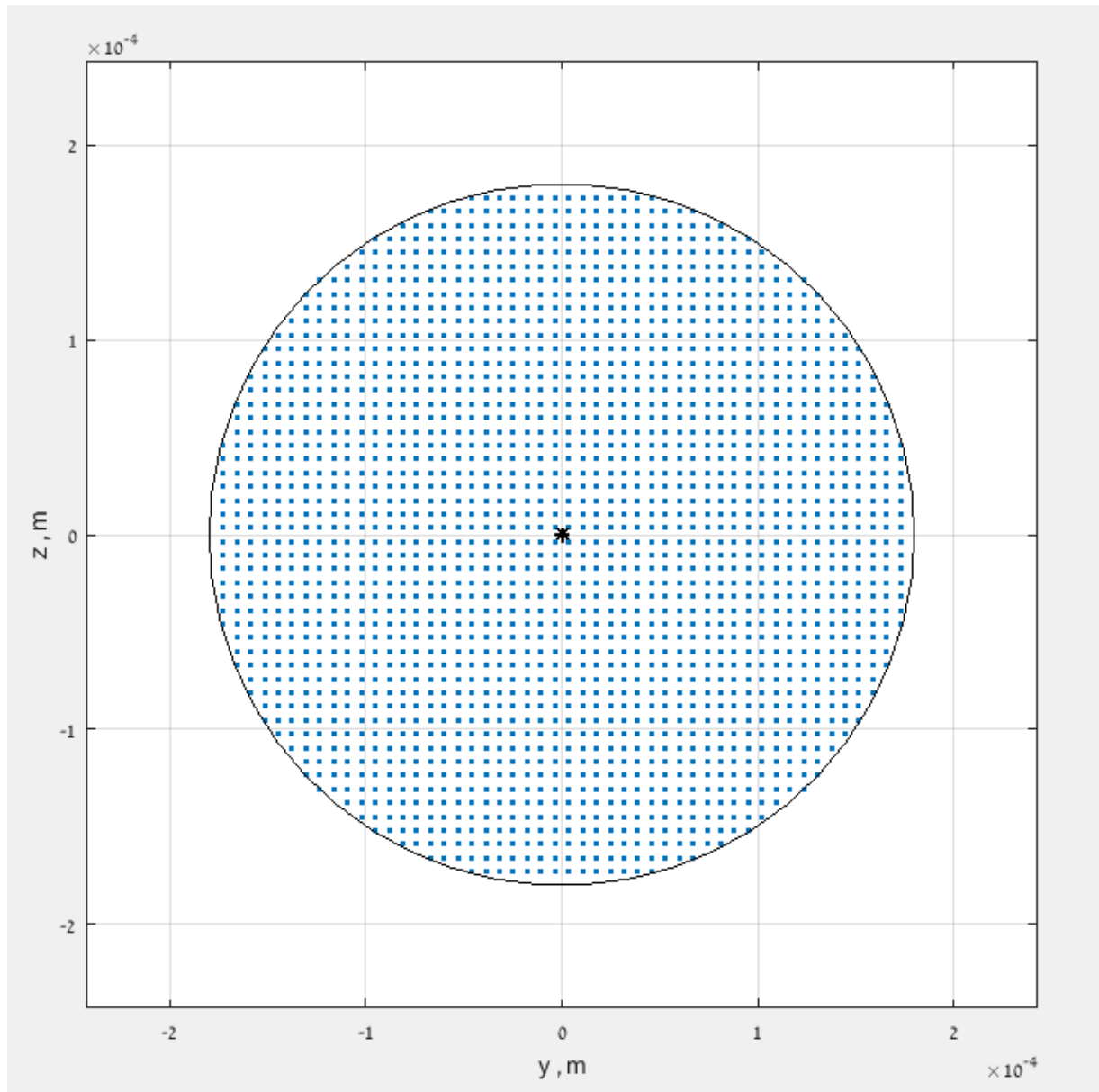


Figure 1 – Initial uniform distribution of particles inside the pipe's cross-section $x=0$

The fluid flow inside the pipe does not change with time and is not affected by the suspended particles.

Boundary Conditions: The laminar fluid is assumed to stick to the pipe walls. The particles are allowed to move within the fluid, but if their distance from the walls becomes less than $dp/2$, they deposit and no longer can move within the fluid domain.

SOLUTION AND RESULTS

The purpose of the simulation is to examine the progress distances of the particle cloud and the deposition times, for particles of different sizes, and thus to draw some conclusions about the general behavior of motion and deposition of particles in a micron-size pipe.

A numerical simulation based upon the MATLAB code was written and is provided in Appendix A. The diameter of the pipe $D=360\mu m$ and the mean velocity of airflow at 310K, $u=0.36\text{ cm/s}$ were assumed. The Reynolds number of the flow (0.0777) is much less than 2000.

The foregoing geometrical and flow values were picked since they pertain to generation 20 of the human lung acinus while breathing normally (Weibel, 1963).

The gravity force along the z coordinate was assumed perpendicular to the pipe axis to maximize its importance in particle deposition.

Under the foregoing conditions the fate of spherical particles of different diameters,

$d_p = \{10^{-3}, 10^{-2}, 0.1, 0.5, 1, 2.5\} \mu m$ was examined. Particles larger than $2.5\mu m$ were not chosen since they normally don't reach the acinus zone of the lung and pose no harm to human health [4]. Particle density of 1000 kg/m^3 was assumed.

Deposition Fraction (DF) is a number, normalized between 0 and 1, which is defined as the ratio between the number of particles deposited on the walls of the system, and the number of particles that entered the system [4].

In the following figures we show particle position in 3 different times: the time required (t_{obs}) for a Deposition Fraction (DF) of about 10%, 50% and 90%.

Deposited particles are colored orange while freely moving particles are shown as blue points.

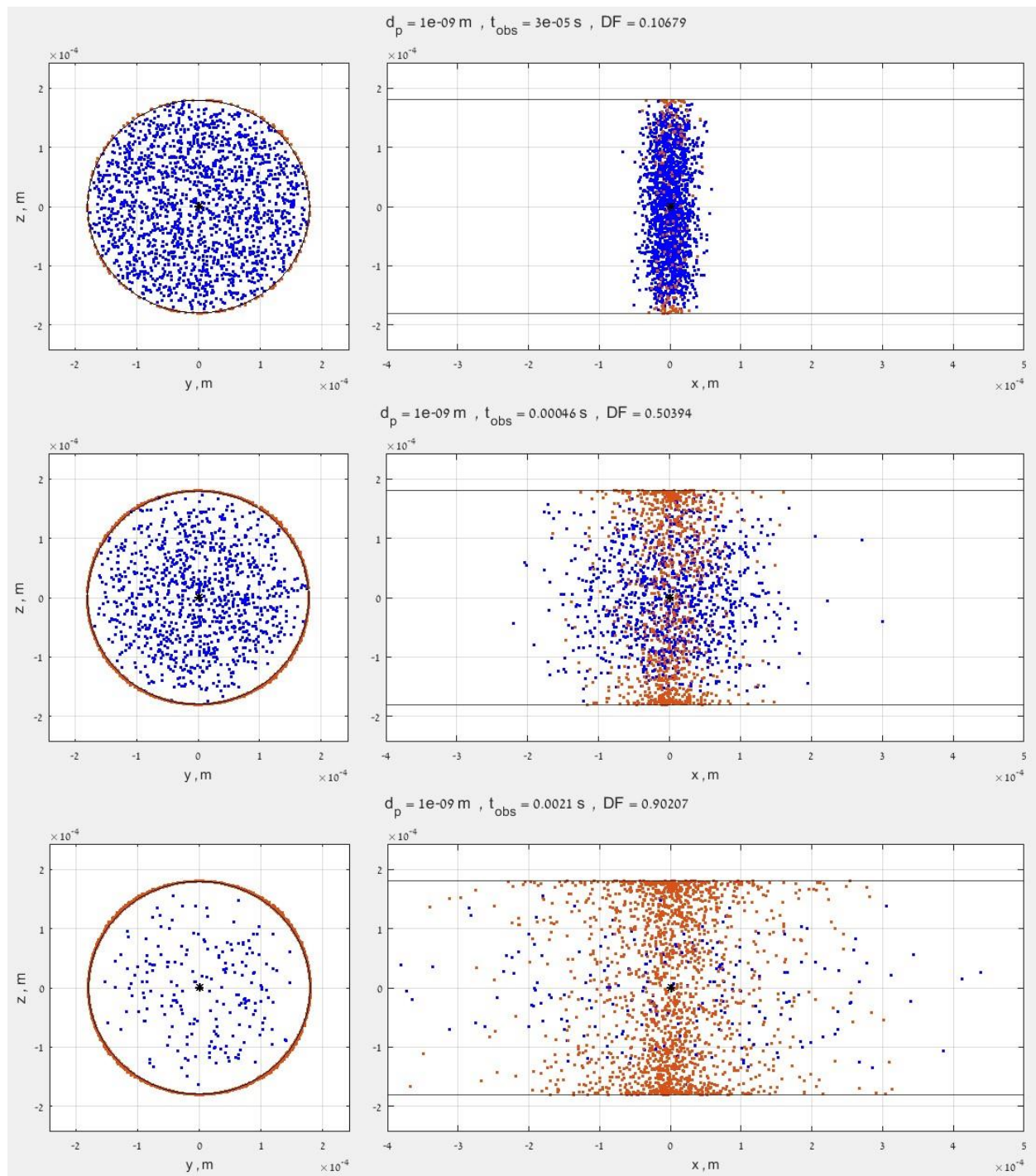


Figure 2 – Snapshots of $0.001 \mu\text{m}$ particles progressing in a micron-size pipe

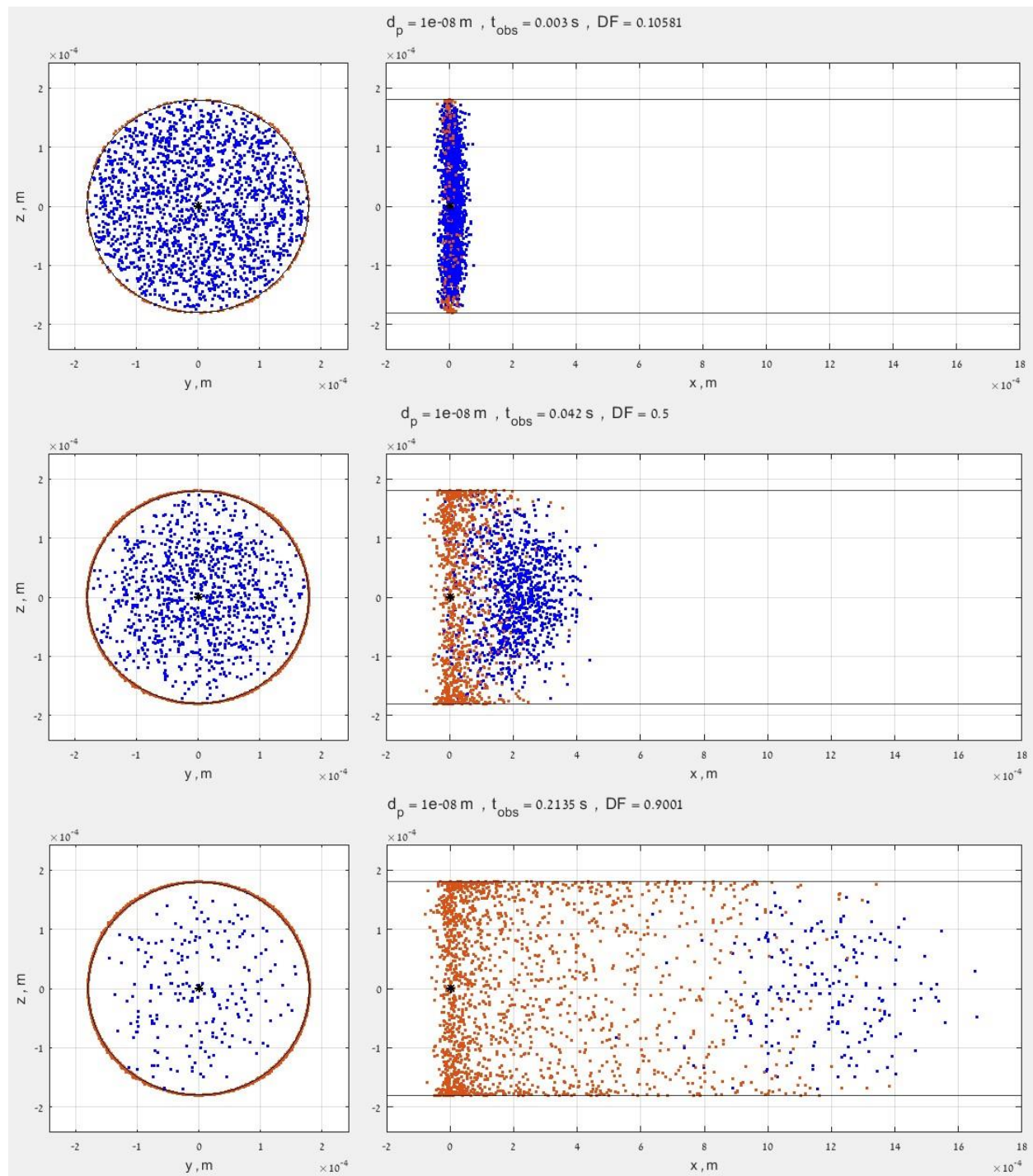


Figure 3 – Snapshots of $0.01 \mu\text{m}$ particles progressing in a micron-size pipe

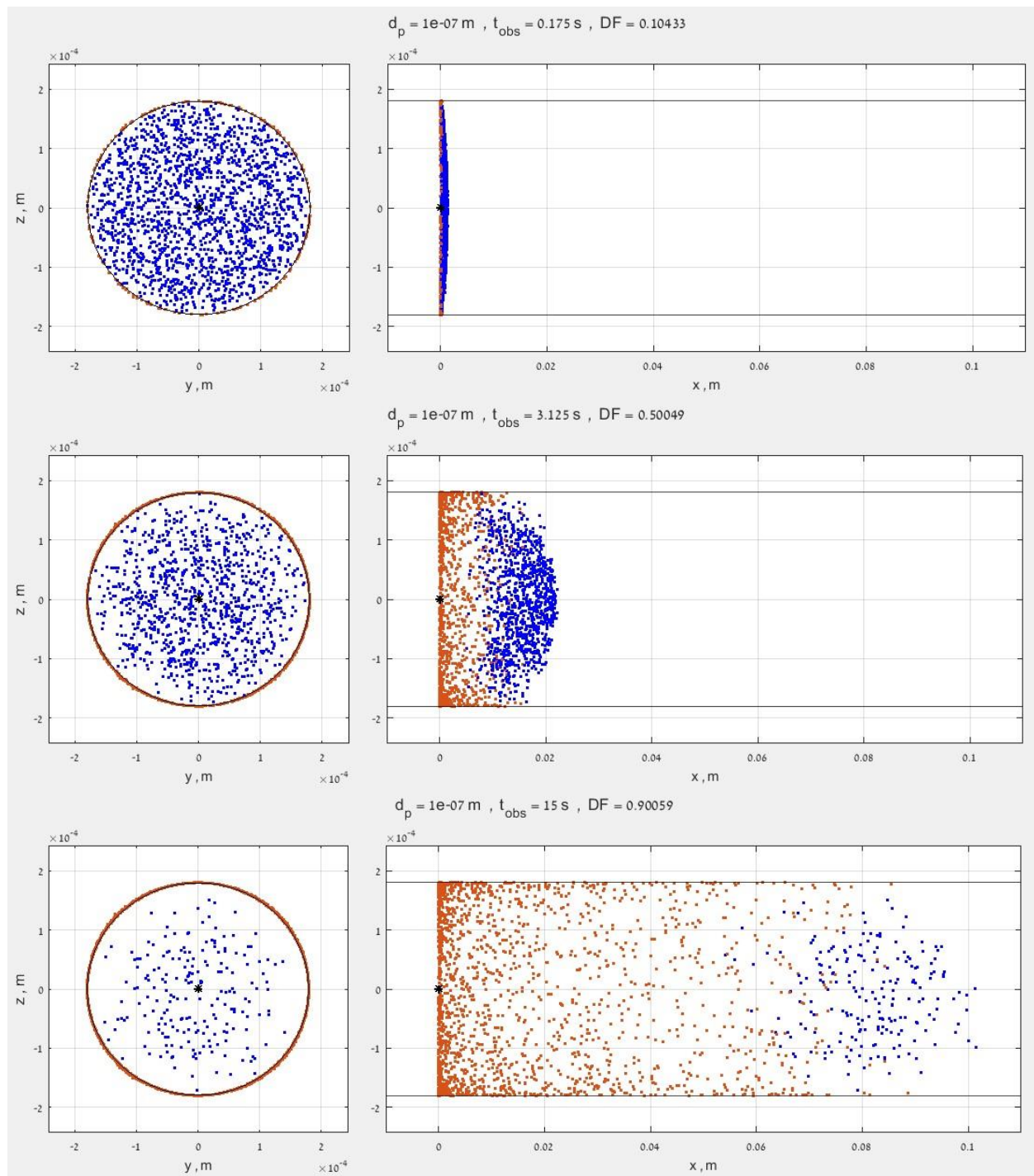


Figure 4 – Snapshots of $0.1 \mu\text{m}$ particles progressing in a micron-size pipe

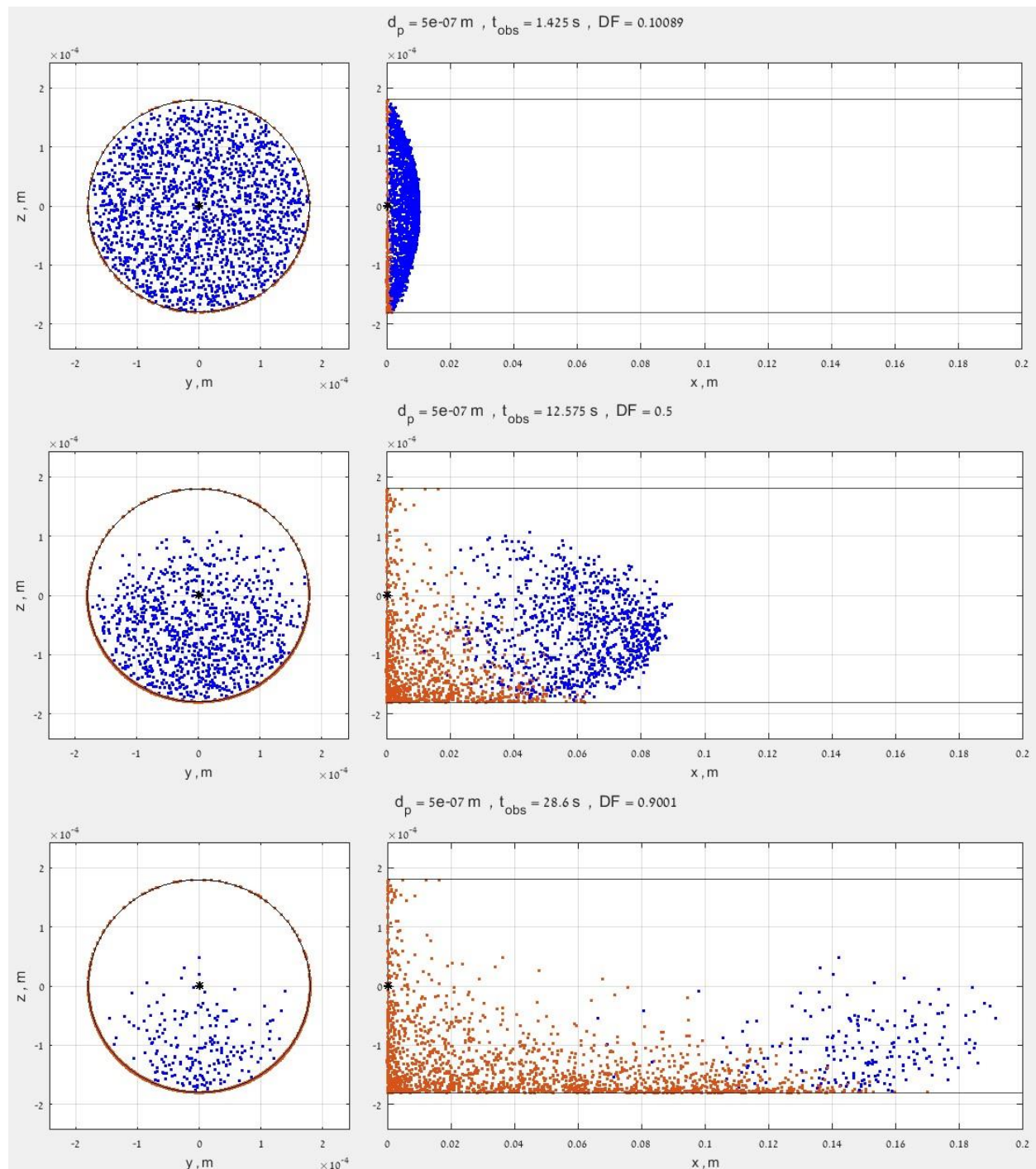


Figure 5 – Snapshots of $0.5 \mu\text{m}$ particles progressing in a micron-size pipe

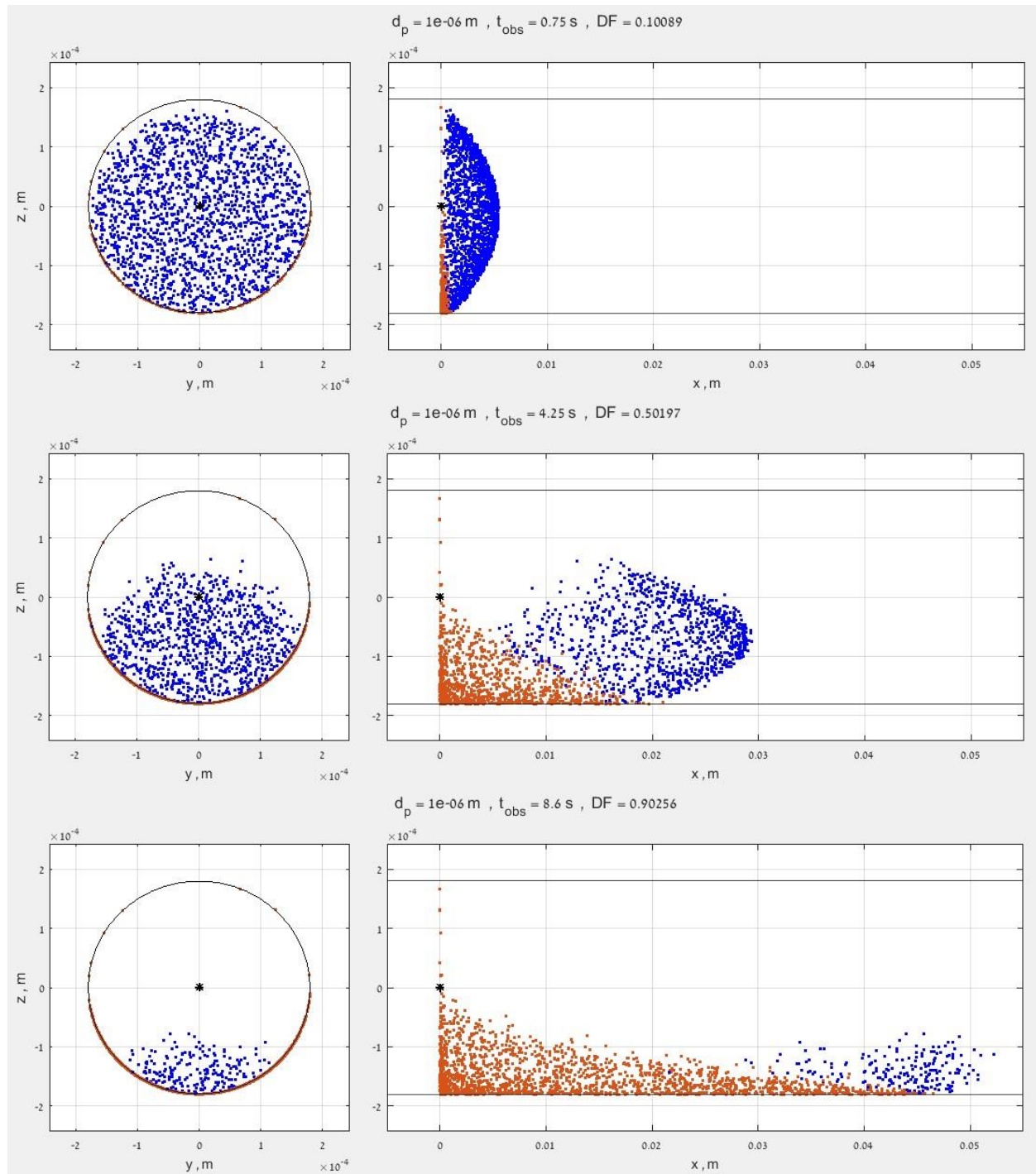


Figure 6 – Snapshots of $1\mu\text{m}$ particles progressing in a micron-size pipe

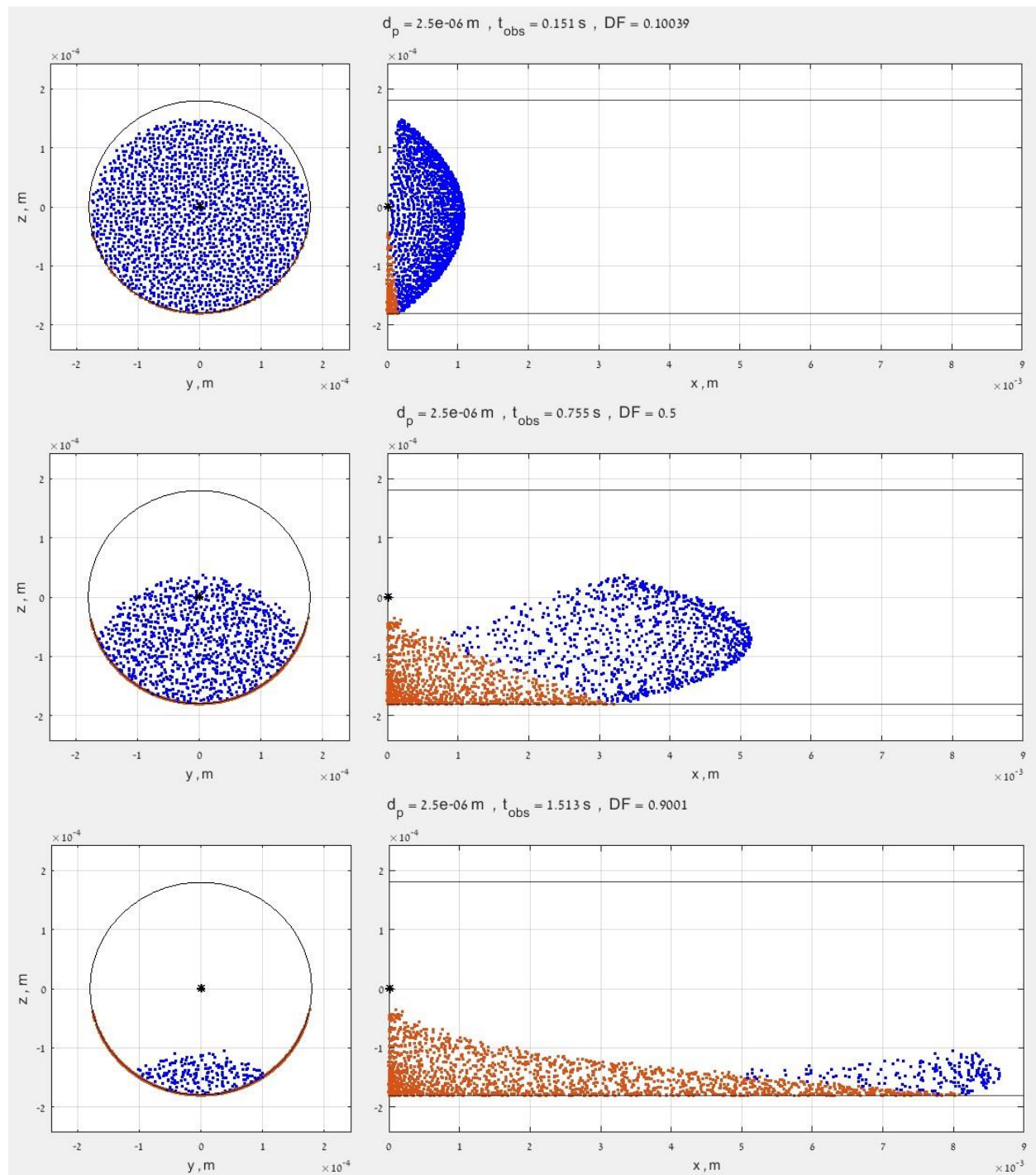


Figure 7 – Snapshots of $2.5 \mu\text{m}$ particles progressing in a micron-size pipe

In summary, the above results are shown in the following table,

Table 3 – Particle deposition in a micron-size pipe, results of numerical simulation

90% deposition		50% deposition		10% deposition		Particle Diameter [m]
Time [s]	Axial distance [m]	Time[s]	Axial distance [m]	Time[s]	Axial Distance[m]	
0.00210	0.00044	0.00046	0.00030	0.00003	0.00006	$1 \cdot 10^{-9}$
0.21350	0.00166	0.04200	0.00046	0.00300	0.00008	$1 \cdot 10^{-8}$
15.00000	0.10154	3.12500	0.02216	0.17500	0.00129	$1 \cdot 10^{-7}$
28.60000	0.19139	12.57500	0.08947	1.42500	0.01027	$5 \cdot 10^{-7}$
8.60000	0.05219	4.25000	0.02935	0.75000	0.00539	$1 \cdot 10^{-6}$
1.51300	0.00866	0.75500	0.00514	0.15100	0.00109	$2.5 \cdot 10^{-6}$

CONCLUSIONS

Several conclusions can be obtained from this model of the motion of small rigid particles in micron-size pipes: Very small particles (10^{-9} - 10^{-8} m) are mainly affected by diffusion.

Thereby, a particle cloud reaches the pipe's wall within a very short time. Consequently, the distance it travels downstream is very limited. It is fully deposited close to its initial position despite being carried by the flow and the walls are almost evenly covered despite the effect of the gravitational force.

Particles of medium size (10^{-7} - 10^{-6} m) travel the longest distance downstream and are affected mainly by flow convection and gravity. The latter causes them to deposit mainly at the lower side of the pipe's walls despite the effect of diffusion.

Large particles ($2.5 \cdot 10^{-6}$ m) travel a short distance before being deposited. They are mainly affected by gravity and consequently cover mainly the lower side of the pipe hardly affected by either diffusion or convection.

Finally, particles of diameter $5 \cdot 10^{-7}$ m (the size of cigarette smoke) seem to travel the longest distance downstream. Consequently, they are more inclined to reach the acinus of the human lung during inhalation time (about 2s).

References

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Appendix A – MATLAB Code for Simulation of Motion and Deposition of Small Particles in a Micron-size Pipe

Note: Particle diameter, d_p , and observation time t_{obs} , were manually changed in each simulation.

```

clc; clear; close all;
rng(2021);

% Data for Generations #15-#23:

% Pipes' radius , m
R = [0.000250 0.000250 0.000245 0.000200 0.000190 0.000180 0.000170 0.000155 0.000145];

% Pipes' length , m
L = [0.00140 0.00133 0.00112 0.00093 0.00083 0.00070 0.00070 0.00070 0.00070];

% Mean axial air velocity in pipes , m/s
v_m = [0.0714 0.0355 0.0175 0.0131 0.0067 0.0036 0.0017 0.0007 0.0004];

alpha_g = pi/2; % Gravity angle of the pipe , rad

% Constants:
g = 9.81; % Earth standard gravity , m/s^2
mfp = 6.9e-8; % Mean free path of air molecules at sea level, m
k_B = 1.3806488e-23; % Boltzmann constant , J/K
T_body = 310; % Body temperature (36.85C), K
Mu = 1.893e-5; % Air dynamic viscosity at 310 K and atmospheric pressure , Pa*s
rho = 1.139; % Air density at 310 K and atmospheric pressure, kg/m^3

% Pipe Generation: (select Generation nubmer: from 15 to 23)
k = 20;

% Particle Properties:
d_p = 1e-9; % Particle diameter , m
rho_p = 1000; % Particle density , kg/m^3
N_p = 2000; % Number of particles

% Initial state:

delta = 2*R(k-14) / (round(sqrt(4*N_p/pi) + 1)); % constant gap for initial uniform distribution on YZ plane

disp('delta =');
disp(delta);

N_p_actual = 0; % Actual number of particles

for y = -R(k-14):delta:R(k-14)
    for z = -R(k-14):delta:R(k-14)
        if (y^2 + z^2) < R(k-14)^2 % if the point is inside the circle (r<R)
            N_p_actual = N_p_actual + 1;
        end
    end
end

disp('N_p_actual =');
disp(N_p_actual);

X = zeros(N_p_actual,1); % Particles' X-coordinate vector
Y = zeros(N_p_actual,1); % Particles' Y-coordinate vector
Z = zeros(N_p_actual,1); % Particles' Z-coordinate vector
vlv = 0; % vector-location variable

for y = -R(k-14):delta:R(k-14)
    for z = -R(k-14):delta:R(k-14)
        if (y^2 + z^2) < R(k-14)^2 % if the point is inside the circle (r<R)
            vlv = vlv+1;
            Y(vlv) = y;
            Z(vlv) = z;
        end
    end
end

deposited_particles = 0; % Counter for the number of particles deposited on walls

```

```
% Cunningham correction factor for noncontinuum effects:
Cc = 1 + (2*mfp/d p) * (1.257 + 0.4 * exp(-0.55*d p/mfp));

D_p = (k_B * T_body * Cc) / (3 * pi * Mu * d_p); % Brownian diffusion coefficient of the particle , m^2/s

V_TS = ((rho_p - rho) * d_p^2 * g * Cc) / (18 * Mu); % Terminal Settling Velocity of the particle , m/s

% Times:
t_conv_typ = L(k-14) / v m(k-14); % Typical axial convective time , s

disp('t_conv_typ =');
disp(t_conv_typ);

t_diff_typ = R(k-14)^2 / (2*D_p); % Typical radial diffusive time , s

disp('t_diff_typ =');
disp(t_diff_typ);

% dt=0.05;
dt = 0.0025 * min(t_conv_typ , t_diff_typ); % Time step of the discretization , s

disp('dt =');
disp(dt);

% beta = 0.05;
t_obs = 0.0021; % Observation time , s

% t_obs = beta * max(t_conv_typ , t_diff_typ);

disp('t_obs =');
disp(t_obs);

N_ts = round(t_obs/dt); % Number of time steps

disp('N_ts =');
disp(N_ts);

sigma = sqrt(2*D_p*dt); % Standard deviation of the random walk in each direction

for i = 1:N_ts
    for j = 1:N_p_actual
        if sqrt(Y(j,1)^2 + Z(j,1)^2) < R(k-14) % if the particle hasn't deposited yet (r<R)
            % move X due to convection, diffusion and gravitation
            X(j,1) = X(j,1) + 2*v m(k-14)*(1-((Y(j,1)^2 + Z(j,1)^2)/(R(k-14)^2)))*dt + normrnd(0,sigma) + V_TS * dt
            * cos(alpha g);
            % move Y due to diffusion
            Y(j,1) = Y(j,1) + normrnd(0,sigma);
            % move Z due to diffusion and gravitation
            Z(j,1) = Z(j,1) + normrnd(0,sigma) - V_TS * dt * sin(alpha g);
            if sqrt(Y(j,1)^2 + Z(j,1)^2) > R(k-14) % if the particle got to the wall (r>R)
                Y(j,1) = Y(j,1) * R(k-14) / sqrt(Y(j,1)^2 + Z(j,1)^2); % Normalization to pipe radius, keeping
                the original direction
                Z(j,1) = Z(j,1) * R(k-14) / sqrt(Y(j,1)^2 + Z(j,1)^2); % Normalization to pipe radius, keeping
                the original direction
                deposited particles = deposited particles + 1;
            end
        end
    end
end

DF = deposited particles / N_p_actual; % Deposition fraction

% Plotting the results - Projections on Y-Z and X-Z planes
figure()
tlo = tiledlayout(1,3,'TileSpacing','Compact');

% Projection on Y-Z plane
nexttile
for p=1:N_p_actual
    if sqrt(Y(p,1)^2 + Z(p,1)^2) < R(k-14)
        plot(Y(p,1) , Z(p,1) , 'b. ');
        hold on;
    else
        plot(Y(p,1) , Z(p,1) , '. ', 'color' , [0.8500, 0.3250, 0.0980]);
        hold on;
    end
end
end
```

```

xlim([-1.35*R(k-14) 1.35*R(k-14)]);
ylim([-1.35*R(k-14) 1.35*R(k-14)]);
plot(0,0,'k','LineWidth',1.5);

th = 0:pi/25:2*pi;
xunit = R(k-14) * cos(th);
yunit = R(k-14) * sin(th);
plot(xunit, yunit, 'color', [0 0 0]);

grid on;
xlabel('y , m');
% ylabel('z , m');

% Projection on X-Z plane
nexttile(2,[1 2])
for p=1:N_p_actual
    if sqrt(Y(p,1)^2 + Z(p,1)^2) < R(k-14)
        plot(X(p,1) , Z(p,1) , 'b. ');
        hold on;
    else
        plot(X(p,1) , Z(p,1) , '.' , 'color' , [0.8500, 0.3250, 0.0980]);
        hold on;
    end
end
xlim([-4e-4 5e-4]);
ylim([-1.35*R(k-14) 1.35*R(k-14)]);
plot(0,0,'k','LineWidth',1.5);
yline(-R(k-14) , 'color' , [0 0 0]);
yline(R(k-14) , 'color' , [0 0 0]);
grid on;
xlabel('x , m');
% ylabel('z , m');

tlo.TileSpacing = 'compact';
tlo.Padding = 'compact';

% Display shared title and ylabel
title(tlo,['d_p = ', num2str(d_p), ' m , t_o_b_s = ', num2str(t_obs), ' s , DF = ', num2str(DF)]);
ylabel(tlo,'z , m');

% Plotting the results - Position of Particles in space
figure()
for p=1:N_p_actual
    if sqrt(Y(p,1)^2 + Z(p,1)^2) < R(k-14)
        plot3(X(p,1) , Y(p,1) , Z(p,1) , 'b. ');
        hold on;
    else
        plot3(X(p,1) , Y(p,1) , Z(p,1) , '.' , 'color' , [0.8500, 0.3250, 0.0980]);
        hold on;
    end
end
% xlim([0 5e-3]);
ylim([-1.35*R(k-14) 1.35*R(k-14)]);
zlim([-1.35*R(k-14) 1.35*R(k-14)]);
plot3(0,0,0,'k','LineWidth',1.5);
[Y,Z,X] = cylinder(R(k-14),25);
s = surf(Y,Z,L(k-14)*X,'FaceAlpha',0.5);
s.EdgeColor = 'none';
rotate(s,[0,1,0],90,[0,0,0]);
% xline(0);
% xline(L(k-14));
% yline(-R(k-14));
% yline(R(k-14));
% view(2);
grid on;
title(['d_p = ', num2str(d_p), ' m , t_o_b_s = ', num2str(t_obs), ' s , DF = ', num2str(DF)]);
xlabel('x , m');
ylabel('y , m');
zlabel('z , m');

```