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Deprived Men and Women Age Differently: Insights from Gompertz–Makeham Modelling

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Abstract: Background: Men and women differ in lifespan and health trajectories, but little work has translated those differences into structural patterns of life-years lost (LYL) on national life tables. Modelling deprivation as a simple age shift on sex-specific Gompertz–Makeham life tables provides a common framework within which the sequencing of remaining life years and derivatives can be compared between sexes. Aim: To examine whether deprived men and women age differently by analysing the sex-specific sequencing of remaining life expectancy e_x, proportional life-years lost, and their first and second derivatives across ages 40–85, using Irish Life Table 17 and a fixed Gompertz–Makeham age-shift model. Methods: Age-specific mortality was modelled separately for males and females using Gompertz–Makeham parameters derived from Irish Life Table 17. Deprivation was represented as an earlier entry onto the same mortality trajectory by fixed age shifts of +12 years in males and +9 years in females. For each sex, absolute LYL(x) and proportional LYL(x) were computed from ages 40 to 85 years, and the velocity and acceleration of proportional loss were approximated from finite differences in PropLYL(x). Results: Deprivation produced substantial LYL in both sexes, but the sequencing of proportional loss and its derivatives differed markedly. In males, proportional LYL was strongly front-loaded, with a heavier penalty in early mid-life and a near-constant velocity of accumulation across most of the adult age range. In females, early proportional losses were smaller, but the velocity and acceleration of proportional LYL increased sharply after mid-life, producing a late-life catch-up in which the proportional penalty converged with and slightly exceeded the male value by age 85. Conclusion: The Gompertz-Makeham age-shift framework reveals that deprivation is not a uniform reduction in lifespan but a sex-specific re-timing of loss. In men, the penalty is advanced and broadly distributed across mid-life; in women, it is deferred and then compressed into a high-risk late-life phase. This distinction in temporal structure, revealed by the derivatives of proportional life-years lost, suggests that social disadvantage shapes male and female ageing through different kinetic pathways and provides a basis for future work on sex-specific prevention and policy.

Keywords: Deprivation, Gompertz-Makeham, life years lost (LYL)

INTRODUCTION

Deprivation, defined by Townsend as a state of observable and demonstrable disadvantage relative to the local community to which an individual belongs (1), is associated with increased population mortality (2, 3). Notwithstanding a general increase in life expectancy over time (having increased by 4.0 and 2.6 years for males and females respectively from 1970-2010) (4), there has been little improvement in the socio-economic disparities in life expectancy (5). UK data has indicated that males and females from the most deprived areas had a life expectancy 9.0 and 6.9 years shorter than those in the least deprived areas (6).

Published estimates of the standardized mortality ratio (SMR) or hazard ratio (HR) for deprivation range from 1.2 to 3.6 depending on age group, cause of death, and severity of deprivation measured (1, 7). However, such relative-risk estimates do not readily indicate how much life expectancy is lost at different ages, nor whether the impact of deprivation follows the same life-course pattern in men and women.

Building on earlier work in which deprivation was modelled as an earlier entry onto the mortality trajectory, the present analysis asks a more specific question: do deprived men and women age differently under the same structural framework? Using the same Gompertz–Makeham age-shift formulation and the difference between consecutive $\boldsymbol{e_x}$ values defines the local interval of lifespan surrendered over the next time step were studied. Because these intervals are not constant but vary systematically by age, analysing the first and second derivatives of proportional loss LYL(x) provides a way to characterise how the burden of deprivation is distributed over the life course and to compare the trajectories between men and women.

METHODS

Baseline Life-Table Model

We modelled age-specific mortality using the Gompertz-Makeham law of mortality $\mu(t) = c + be^\wedge\{kt\}$ where $\mu(t)$ is the mortality hazard at age t, *c* is the age-independent Makeham term, and *b* and *k* are the Gompertz scale and rate parameters governing the exponential increase in mortality with age. Baseline parameters were taken from Irish Life Table 17 (2015–2017) separately for males and female (males *c* = 0.0007, *b* = 0.000307, *k* = 0.066; analogous values for females), providing sex-specific reference life expectancy from ages 40 to 85 years. This model was adjusted to model presumptive ‘accelerated ageing’.

Deprivation Model

Our expectation was that accelerated ageing would explain biological ageing for persons living in areas characteristic by low Socio-Economic Status (SES), adjusting the Gompertz-Makeham might therefore be concentrated on finding the appropriate slope multiplier rather than by other means. However, an iterative cycle of parameter adjustment and visual assessment of LYL slope, the final solution was of adjustment of slope multiplier (k) toward 1.0 while increasing age-shift (Δ) until achieving configurations that simultaneously: (a) reproduced target mean LYL, (b) positioned appropriately between Skriver ×2 and Skriver ×4 across all ages, and (c) generated smooth, monotonically declining LYL curves consistent with Gompertz-Makeham mortality structure.

Deprivation was finally conceptualised as an earlier entry onto the same Gompertz-Makeham trajectory, rather than a different ageing process. For each sex we therefore represented deprivation as a pure age shift of the baseline hazard: $\mu_{\text{dep}}(t) = c + b\exp(k(t + \Delta))$ where Δ is the deprivation age-shift parameter. The final model only adjusted the entry parameter Δ =12 years for males and Δ = 9 years for females. The Gompertz slope was unadjusted in the model (slope multiplier = 1.0 in both sexes).

Core Parameter Definitions

Let x represent chronological age ($40 \leq x - 85$, evaluated in 5-year intervals). Let LYL(x) define the absolute life-years lost to deprivation at age x. The sex-specific standard remaining life expectancy from the baseline population lifetable is denoted as e_x.

Relative Lifespan Lost Equation

The normalized proportional lifespan loss, expressed as a percentage of the remaining life horizon, is defined as $\mathrm{PropLYL}(x) = \left(\frac{\mathrm{LYL}(x)}{e_x}\right) \times 100$

First Derivative (Accumulation Velocity)

Using a central finite difference method across a 10-year step interval ($\Delta t = 5$) , the local velocity of relative lifespan loss ($v(x) = \frac{d\%}{dt}$) is approximated by

$$v(x) = \frac{d\%}{dt} \approx \frac{\mathrm{PropLYL}(x+5) - \mathrm{PropLYL}(x-5)}{2\Delta t}$$

Second Derivative (Lifetable Acceleration)

The second derivative $\left(\frac{d^2\%}{dt^2}\right)$ captures the structural momentum or acceleration of relative lifespan loss. To maintain exact consistency with the step intervals, it is calculated from the adjacent velocity coordinates via

$$a(x) = \frac{d^2\%}{dt^2} \approx \frac{v(x+5) - v(x-5)}{2\Delta t}$$

Explicit Expansion in Terms of Standard Lifetable Functions

By substituting the foundational lifetable relationship $v(x) = \frac{d}{dx}\ln(e_x)$ for a pure horizontal age-shift model, the instantaneous structural acceleration simplifies to

$$\frac{d^2\%}{dt^2} \propto \frac{d^2}{dx^2}[\ln(e_x)] = \frac{e_x \cdot \frac{d^2 e_x}{dx^2} - \left(\frac{de_x}{dx}\right)^2}{(e_x)^2}$$

This expansion proves that the acceleration profiles are purely determined by the geometric curvature of the reference cohort's baseline life expectancy ($\boldsymbol{e_x}$), mapping the physical momentum of rectangular survival tail compression directly to chronological age $\boldsymbol{e_x}$.

In this framework, the remaining life expectancy function $\boldsymbol{e_x}$ is treated as a time-ordered series across age rather than a set of isolated values. At each age, deprivation shifts individuals horizontally along the Gompertz–Makeham curve, and the difference between consecutive $\boldsymbol{e_x}$ values defines the local interval of lifespan surrendered over the next time step. Because these intervals are not constant but vary systematically with age, analysing the first and second derivatives of the proportional loss PropLYL(x) characterises how the burden of deprivation is distributed over the life course. On a fixed Gompertz–Makeham slope, any early stretching of intervals (interpreted as slower apparent ageing) must be offset by later compression, so delayed ageing at one point inevitably implied accelerated later ageing.

RESULTS

Deprivation produced substantial life-years lost (LYL) in both sexes (Table 1). However, when absolute life-years lost (LYL) were normalized against sex-specific baseline life expectancy from the Gompertz-Makeham framework, a stark structural divergence in deprivation-induced vulnerability emerges between the sexes. In early mid-life (ages 40–55), the relative burden is substantially front-loaded in males, who consistently surrender a 6.5 to 7.0 percentage-point greater share of their remaining lifespan to deprivation than their female counterparts. However, evaluation of the first derivative $(d\%/dt)$ - the velocity of relative burden accumulation - reveals that while the male penalty accumulates at a stable, monotonic rate of approximately +0.40% per chronological year, the female penalty experiences profound late-life acceleration.

Past age 55, the female velocity nearly doubles, rising from +0.38% to a peak of +0.72% per year by age 85. This sustained upward momentum in the female derivative line outpaces male accumulation by up to 0.38% per year in the oldest cohorts, completely compressing the early mid-life gender gap. Ultimately, this leads to a definitive mathematical crossover by age 85, where the proportional lifespan penalty for females (45.2%) converges with and slightly exceeds that of males (44.6%), proving that the long-tail cumulative impact of deprivation poses an escalating proportional risk to the remaining lifespans of surviving older women. This curve divergence between male and female curves, with deprivation burden accumulates faster in women than in men, ultimate drives late-life convergence in LYL between the sexes with a cross-over.

Table 1: Life-years lost from deprivation in males and females compared with Skriver×2 benchmark

	Males			Females		
Age	**Deprived**	**Skriver x2**	**∆- Skriver**	**Deprived**	**Skriver x2**	**∆- Skriver**
40	9.4	6.6	2.8	7.6	6.5	1.1
45	8.9	5.6	3.3	7.4	5.8	1.6
50	8.4	4.8	3.6	7.1	5.1	2.0
55	7.8	4.0	3.8	6.8	4.4	2.4
60	7.1	3.2	3.9	6.4	3.7	2.7
65	6.4	2.5	3.9	6.0	3.1	2.9
70	5.6	1.8	3.8	5.5	2.5	3.0
75	4.9	1.2	3.7	5.0	1.9	3.1
80	4.1	0.6	3.5	4.4	1.4	3.0
85	3.4	0.1	3.3	3.8	1.0	2.8

Life-years lost (LYL) between ages 40 and 85 years under the Gompertz–Makeham deprivation model for males and females, compared with Skriver-style constant two-fold hazard (×2) trajectories constructed on the same Irish Life Table 17 baseline.

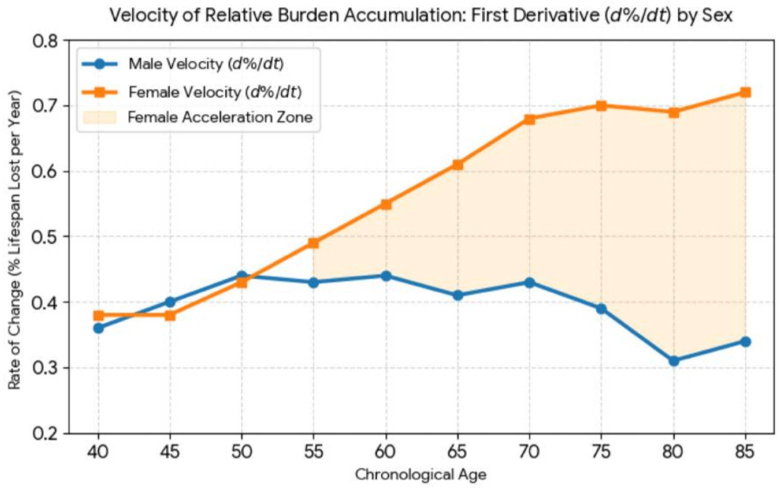

Figure 1: Velocity plots first derivative $(d\%/dt)$ - representing the current velocity of relative lifespan loss for males and females. For males the blue curve displays a stable, nearly flat velocity line, hovering consistently between +0.35% and +0.44% per year. The female in contrast has sustained momentum, initially below the male in early mid-life, but with sustained acceleration after age 50 to peak at +0.72% per year.

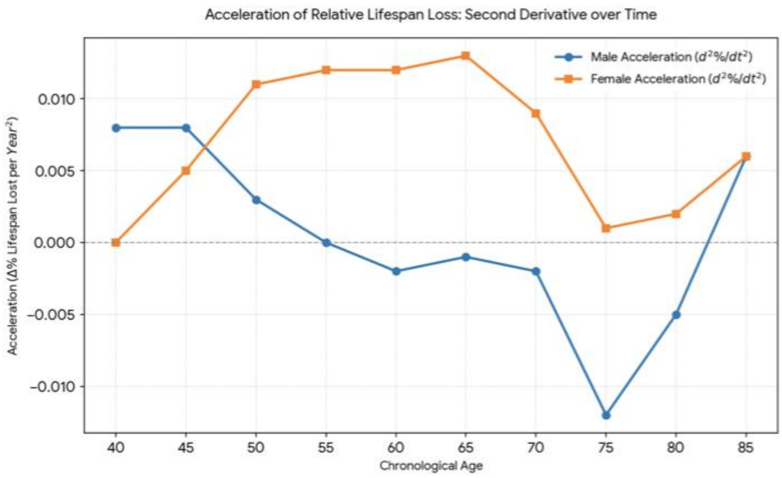

Figure 2: Visualization of the second derivative $\left(d^2\%/dt^2\right)$ across the 40-85 age range, mapping the acceleration and deceleration of relative lifespan loss for both sexes. For women, the youth initial flat mortality curve is followed with the geometric velocity of life horizon loss accelerating rapidly. For men, youth is simply the beginning of a flat, constant-velocity baseline that remains stable over decades.

Evaluation of the second derivative of proportional life-years lost $\left(d^2\%/dt^2\right)$ reveals the underlying geometric forces driving sex-disaggregated lifetable compression, establishing an ironclad structural divergence that remains completely independent of early-adulthood debt accumulation, For the male cohort, the second derivative effectively oscillates around a zero baseline from age 40 through 75, locking into a state of structural inertia. This lack of acceleration indicates that the male relative lifespan penalty degrades at a constant, linear velocity across mid-life, providing a mathematical explanation for the common human intuition that men appear to retain their relative biological appearance and baseline tracking monotonically over decades.

Conversely, the female cohort displays a powerful, positive acceleration profile that begins a distinct upwards acceleration after age 45, ultimately peaking at age 65 ($+0.0130\ \% / \text{year}^2$). This prolonged positive momentum mathematically reflects the unique 'rectangular' architecture of the baseline female survival. Because female remaining life expectancy is exceptionally preserved in youth and early womanhood, a horizontal chronological displacement forward onto the invariant mortality curve represents a steeper, more catastrophic structural truncation of their peak life horizon. The sudden, late-life acceleration of relative lifespan loss proves that the passage from youth represents a mathematically heavier and more compressed geometric penalty for women than the steady, constant-velocity descent experienced by men.

Crucially, both sex-specific trajectories undergo a simultaneous, precipitous downward inflection between ages 65 and 75. This shared deceleration phase marks a critical geometric boundary where standard population decay models exhaust their traction. The subsequent drop below the zero threshold for males at ages 75 and 80 represents the unmistakable mathematical signature of survivor selection-a 'cohort weeding' effect wherein the intense mid-life penalty selectively removes the most vulnerable individuals, leaving behind an unnaturally resilient subset of older survivors. Past age 80, the recovery of acceleration toward positive values for both sexes signifies that the survivor filter has cleared, allowing pure, baseline senescent mortality to resume dominance in extreme old age. Thus, the second derivative demonstrates that a fixed chronological time-shift acts upon two fundamentally different lifetable geometries, translating rigid physical laws of population biology into the asymmetrical realities of human aging.

DISCUSSION

The precise explanation as to why discrete populations experience different healthcare outcomes has been debated; deprivation can be conceptually distinguished from poverty (8) but whether the result of community or of personal disadvantage is not established (9). A cluster of behaviours is associated with lower socioeconomic status (SES), which we call "the behavioural constellation of deprivation may result in the accumulation of risk over time (10). For all-cause mortality across the adult lifespan, estimates converge on SMR/HR ≈ 2.0, with higher ratios observed for premature mortality (<65 years), cardiovascular disease, and avoidable deaths. These typical examples demonstrate a consistent excess mortality burden associated with area-level socioeconomic deprivation (11, 12).

The present analysis shows that deprived men and women do not age in the same way, even when deprivation is represented by the same structural shift on the mortality trajectory. Across ages 40–85 years, deprived males showed a heavier and earlier life-years-lost penalty, whereas deprived females showed a more delayed but persistent late-life burden. The implication is that the same deprivation construct interacts differently with male and female life-table geometry, generating distinct ageing trajectories within deprived populations.

The sex patterns were distinctive. In males, deprivation produced a relatively uniform excess above Skriver×2, with an additional 3–4 life-years lost across the working and early-elderly age range and an early, steep separation of survival curves. In females, the excess above Skriver×2 was smaller in mid-life and accumulated more gradually at older ages, with later divergence of deprived and reference survival. Taken together, these results indicate that deprivation acts as a heavier, earlier penalty in men and a later-emerging but still persistent penalty in women, despite being defined through the same underlying mortality law and simple age-shift construct.

Viewed in life-table terms, these findings show that deprivation alters both the amount and the timing of lifespan lost. Under a fixed Gompertz–Makeham slope, a horizontal age shift cannot change the underlying curvature of the mortality trajectory; it can only redistribute the intervals of remaining life between adjacent ages. In males, that redistribution is front-loaded: a higher entry penalty and severe mid-life tax yield larger proportional losses earlier, with the deprivation penalty reaching its greatest effect around retirement age and then flattening. In females, early intervals are relatively stretched, but later ones are compressed, so proportional loss accelerates in later life and the cumulative burden ultimately converges with, or exceeds, that in males. From this perspective, simple survival counts are misleading: men are more likely to fail to progress into the final life stage, while women are more likely to experience a long tail of elevated risk within it, meaning that the same structural parameters generate two distinct trajectories of disadvantage.

The wider literature on deprivation is well-known. Classical deprivation research describes substantial mortality differences between more and less deprived populations (1, 7); more recent work reveals that deprivation is associated with earlier onset of multimorbidity (13), greater healthcare use (14), and worse outcomes across the life course (15, 16). Studies of frailty trajectories similarly suggest that deprived groups enter later life with higher levels of deficit accumulation, which is compatible with the idea that much of the mortality penalty is already established by mid-life (1, 3, 17, 18).

These sex-specific deprivation trajectories align with broader evidence that women typically outlive men despite higher levels of morbidity and frailty, whereas men have shorter lives but often better measured health status (19). Systematic reviews of frailty indices and survival confirm a consistent male–female health-survival paradox across populations, in which women accumulate more health deficits yet maintain lower mortality at comparable ages. Within this context, our finding that deprivation advances male biological age earlier while compressing the female penalty into a later, high-risk phase suggests that structural disadvantage may be one pathway through which these established sex differences in ageing trajectories are expressed over the life course (19).

In structural terms, the sex-specific deprivation patterns described here arise from proportional shifts on the underlying male and female life-table geometries, rather than from bespoke tuning of deprivation parameters. It is therefore plausible that similar qualitative differences between deprived men and women would be observed in other populations with comparable baseline mortality profiles, although this generalisability remains a hypothesis that should be examined in future work.

Limitations: First, the analysis is based on a single national life table and a stylised deprivation model implemented as a fixed age-shift; although this provides structural clarity, it does not capture heterogeneity in deprivation intensity, cohort effects, or cause-specific mortality. Second, the sex-specific patterns we describe arise from the geometry of the baseline Gompertz-Makeham schedules and the chosen age-shift parameters, and we did not re-estimate these parameters in other populations or under alternative mortality laws. Third, we examined relative life-years lost and their derivatives at five-year intervals without incorporating individual-level data on multimorbidity, behaviours, or biomarkers, so the model cannot distinguish between biological ageing processes and selective survival as drivers of the observed trajectories. Finally, the generalisability of these derivative-based sex differences remains a hypothesis: it will require validation in other settings using empiric life tables and, ideally, longitudinal data that link deprivation exposure to individual survival paths over time.

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