

Deblurring Acoustic Signals for Optimum Perception

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ABSTRACT

This work presents a preliminary study on the effect that a blurring mechanism has on the quality of acoustic signals. The mechanism is mathematically modeled by means of a matrix that distorts the clear (blur and noise-free) digitized signal. Then, a combined deblurring-denoising scheme, adopted from image processing, is introduced and is shown to be very efficient for the simultaneous deblurring and denoising of the acoustic signal in cases where the blurring mechanism expressed through the corresponding matrix is known. Approximate matrices close to the actual one may be proven adequate for specific applications but it is important that the type of the matrix is approximated as much as possible. Comparison of raw and clear signals is made using a statistical characterization scheme, which is also appropriate for cases where the signal features have to be exploited for applications involving machine-learning. The deblurring of acoustic signals improve their quality and enhance their perceptibility, in favor of their exploitation for a wide range of applications, including human communication.

Keywords: Signal processing, Blurring of acoustic signals, Machine learning

INTRODUCTION

During the COVID-19 pandemic video-calls and video-conferencing were the main if not the only way to safely socialize, go to school, and work remotely. Even though advancements in technology have made such transmission of video and audio very accessible, it became apparent that the quality of the received video and audio was far from optimal. Blur and noise altered both types of signals. Having distorted image and sound signals was a big obstacle in the social life of people all over the world and it is important that we find a way to bring the signals to their original form, which is the form that we perceive during in-person communication.

Of course the issue of “cleaning” or “denoising” acoustic signals or images is not new. Many research studies are devoted to this objective as the use of clear signals has been a very important prerequisite for several applications that affect our social life. Yet, the recent pandemic revealed in a more emphatic way that when modern technology is used to bring

people together, the tools of communication through electronic means should be such that the flavor and full appreciation of discussions among people should be kept to a highest level of quality and if possible the sound (or image) reaches the human sensors in a way that resemble the in-person interaction.

The purpose of this paper is not to review methods that have been developed so far for treating the problem of signal improvement but concentrate on a specific case, that is of acoustic signal blurring and suggest a technique for signal cleaning. It should be noted that this case is discussed here as an academic example rather than a comprehensive treatment of the problem of acoustic signal blurring and de-blurring, which is an ambitious goal.

Blurring is a mechanism affecting both images and sound signals. The source of blurring is mainly the sensors of the signal capturing or signal reproduction instrumentation, which are never perfect but other mechanisms may also be responsible for the distortion of the signals that lose clarity. Although blurring of images is more extensively studied, blurring of acoustic signals is less widely studied.

In particular, blur as a mechanism affecting acoustic signals, has been studied, at least to the authors' knowledge, mainly for signals recorded in the air. Blur is recognized as one of the reasons that cause degradation of the signals and affects the amplitude and frequency patterns induced by selective frequency filtering, reverberation, and atmospheric turbulence [1,2]. A typical example of an application that is affected by the amount of blur in the acoustic signal is the recognition of birds species by means of their songs. In this respect, Dabelsteen et al. [3] studied the effects of communication sounds and bird location on the blurring of the birds' song recordings at a specific environment. No specific studies on the processing of the blurred signals to obtain a clear signal are reported, and this was the main motivation for performing this study.

An additional motivation came through our involvement in acoustical oceanography applications. To the knowledge of the authors at least, there are no references related to the analytic study of blurring mechanisms of underwater acoustic signals. Apparently, blurring as a deterministic signal degradation mechanism in underwater acoustics has not been considered as important as it is for acoustic signals in the air. On the other hand, underwater noise has been extensively studied (see for instance [4]) and has been considered as one of the major components of the sonar equation that controls reception or proper use of the underwater acoustic signals [5]. However, preliminary studies performed by the authors [6] showed that blurring of underwater acoustic signals might be a serious problem when these signals are exploited for a large class of problems related to underwater acoustics, including inverse problems, communication problems and problems of marine bioacoustics (study of the behavior of marine mammals). There is no need to present further details on this subject here, as they fall beyond the scope of this article.

This paper presents an overview of the work performed by the authors to model the blurring mechanism applied to an acoustic signal and the effectiveness of a suggested deblurring scheme for its cleaning. The study presented here is of theoretical nature, due to the lack of real data, available to the authors, associated with signals affected by blur with an underlying analysis of the blurring mechanism. The case of a simulated underwater signal has been used here only for illustration purposes and in association with a signal characterization scheme [7] which has been applied for ocean acoustic

tomography or geoacoustic inversions. As already stated, this is an area of possible application of the acoustic signals measured underwater [8,9]. It should be emphasized though, that the method presented is by no means restricted to underwater acoustic signals and the signal studies could be any signal of finite duration recorded in the air, which is contaminated by noise and blur.

To model the blurring of acoustic signals, an approach applied in images is considered and was adapted to acoustic signals. In image processing, a blurring mechanism is mathematically described by means of a simple model involving a blurring matrix which is applied to the clear image. The suggested deblurring procedure is based on the knowledge of the blurring matrix, which is a rather restrictive requirement and involves a simultaneous denoising action. Deblurring with unknown blurring matrix is called blind deblurring and requires the knowledge of different features of the image or signal which are out of the scope of this study. An effective and efficient deblurring-denoising scheme based on statistical optimal filtering using the Singular Value Decomposition and Statistical Analysis of the noise has been presented by Taroudaki and O' Leary [10,11] and consists the background for application of a similar scheme for acoustic signals.

Notice that since acoustic blurring mechanisms in the air or in the water are not extensively studied and the blurring matrices cannot be determined with confidence, it was desirable to study how effective and efficient the deblurring method is, when there is a necessity to approximate blurring matrices based on some qualitative information on the blurring mechanism. For the comparison of the various signals, the signal characterization method presented in [7] has been used. The acoustic signal is characterized using the parameters describing the statistics of the multi-level 1-D wavelet decomposition coefficients which follow symmetric α -stable distributions [12]. The comparison is based on the Kullback Leibler Divergence (KLD) [13], which is considered an optimal choice for the comparison of signals characterized by statistical or probabilistic features. For the readers interested in the aforementioned approach of signal characterization, a brief introduction is included here (4th Section). A detailed presentation of the statistical signal characterization of acoustic signals can be found in [7].

The results presented here, provide a preliminary indication of the importance of the blurring mechanisms on the quality of the acoustic signals. An additional conclusion from our study is that it is very important that the blurring mechanism is appropriately modeled. We hope that this work will motivate further studies of this effect in acoustic signals of general type, including of course audio signals used for human communication.

Note that, blurring of acoustic signals or images as it is described here, concerns a distortion of a set of discrete data (the elements of the digitized signal or image). If we consider that big data analysis is a major challenge of Machine learning which has started to be employed in Social and Political Sciences [14,15] we can assume that the use of clear data is expected to improve predictions of future states of a specific situation, based on some model and the suitable interpretation of existing data. Although "blurring" mechanism, as it is described here seems to be applied to images or acoustic signals only, there is a high probability that data cleaning methods based on some a-priori knowledge of some deterministic or systematic data distorting mechanisms, expressed mathematically similarly to blurring as it is presented here, may improve predictions of future states, for a large class of problems for which machine learning seems to be an efficient treatment tool.

The paper is structured as follows: The next section introduces the mathematical treatment of the blurring mechanism. The subsequent session presents the theory of acoustical deblurring, followed by a section that presents the quantification of the difference between raw (blurred) and clear (deblurred) signals and introduces the statistical characterization scheme and the Kulback Leibler Divergence which are the means for signal comparison. The section after presents an example of signal deblurring-denoising, illustrating how efficient the method is with a known blurring matrix, while next section is devoted to a short study on the use of approximate blurring matrices for signal deblurring, showing the sensitivity of the deblurring mechanism on the blurring matrix. The last section presents some comments on the study presented here.

BLURRING OF ACOUSTIC SIGNALS

We will consider an acoustic signal measured in the time domain. Blur is mathematically modeled as convolution of a real signal with the mechanism distorting it [16], which is expressed through an appropriate function when the mechanism is known. Blurring is important in image processing. According to Nagy and O'Leary [17], the natural mathematical model of the recording operation is an integral equation of the first kind:

$$\int_{\Omega} a(s,t) x^*(t) dt + e(s) = b(s) , \quad (1)$$

where the spatial coordinates are $s \in \sim^2, t \in \sim^2$ and Ω is a closed region containing the domain of the image, $a: \sim^2 \rightarrow \sim^2$ is the Point Spread Function (PSF) (kernel of the blur) that convolves the original image x^* , and e is the additive noise. Since the measured image is only captured at a discrete number of pixels, convolution can be used to obtain a discrete model for the blurring of an image in the form of a matrix equation. The discrete model is then $\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{e}$ where $\mathbf{A}$ is the blurring matrix produced by the PSF.

In our analysis we will consider acoustic signals measured in the time domain and digitized, which allows us to avoid working in the continuous domain and base our work in the matrix equation model as above.

We will therefore consider an acoustic signal $\mathbf{b}$ given in digital format as an array with n elements. The elements of the array correspond to the amplitude of the signal in subsequent time steps. Following the convolution model for images as above, the blurring model for the acoustic signal is expressed through the equation:

$$\mathbf{b} = \mathbf{A}\mathbf{x}_{true} + \mathbf{e} \quad (2)$$

where : $\mathbf{x}_{true}$ is a vector of n elements representing the signal not affected by noise or blur that will be denoted in the remaining of this paper as "clear signal", $\mathbf{e}$ is a vector of size $n \times 1$ that represents the noise, and $\mathbf{A}$ is a real $n \times n$ matrix modeling the blurring mechanism.

To illustrate the effect of the blurring mechanism as expressed by equation 2 we consider a simple case of an underwater acoustic signal that has been "recorded" in a shallow water range-independent environment. Although the details of the source and the environment have no influence on the study of the blurring or deblurring mechanisms, we present here the details of the signal simulation for the sake of completeness.

The source excitation function is assumed to be Gaussian which is a model widely adopted for simulating actual acoustic sources of general type used in applications of acoustical oceanography.

The signal is due to a point source of central frequency 100 Hz and bandwidth of 40 Hz. The environmental and operational parameters and a sketch of the environment are presented in Appendix. To produce the simulated clear signal a normal-mode acoustic propagation model, implemented through the simulated clear signal is obtained using the code "MODE1" [18] which provides the system transfer function (acoustic pressure at each frequency) within the frequency bandwidth followed by inverse Fourier transform to obtain the signal $p(t)$ in the time domain (see Appendix I). The signals shown in the figures of this article correspond to the digitized version of the continuous signal and are therefore given in vector format.

The simulated clear signal $\mathbf{x}_{true}$ is presented in Fig. 1. The horizontal axis represents time steps and does not indicate actual time values. The vertical axis presents the amplitude of the clear signal.

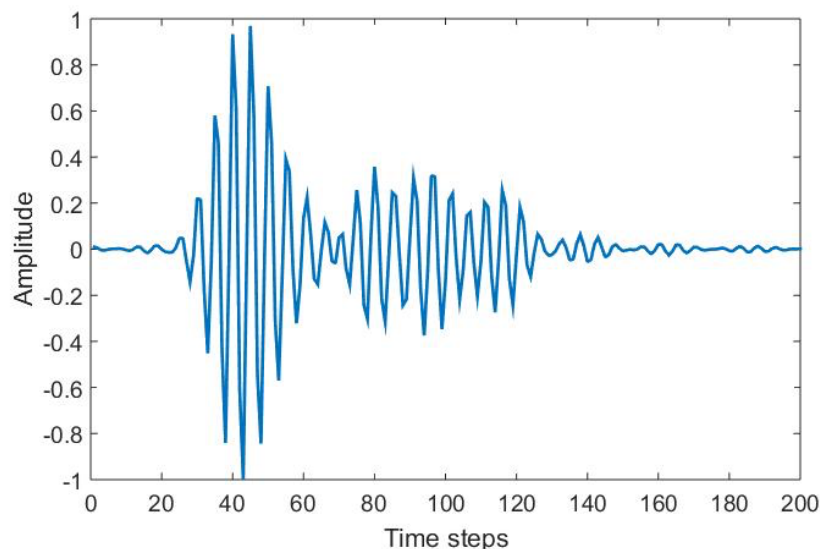


Figure 1 The original (clear) signal.

Then, blur and noise were added with blur modeled through the five-diagonal matrix $\mathbf{A}_1$:

$$\mathbf{A}_1 = \begin{bmatrix} 0.5 & 0.35 & 0.15 & 0 & \dots & 0 \\ 0.175 & 0.5 & 0.175 & 0.15 & \ddots & \\ 0.075 & 0.175 & 0.5 & 0.175 & 0.075 & \ddots \\ & \ddots & \ddots & \ddots & & \\ \ddots & 0.075 & 0.175 & 0.5 & 0.175 & 0.075 \\ & & 0.15 & 0.175 & 0.5 & 0.175 \\ 0 & \dots & 0 & 0.15 & 0.35 & 0.5 \end{bmatrix}$$

and Gaussian white noise of standard deviation $s = 0.02$. This particular type of blurring matrix indicates a fair amount of blurring. The blurred-noisy signal is presented in Fig. 2.

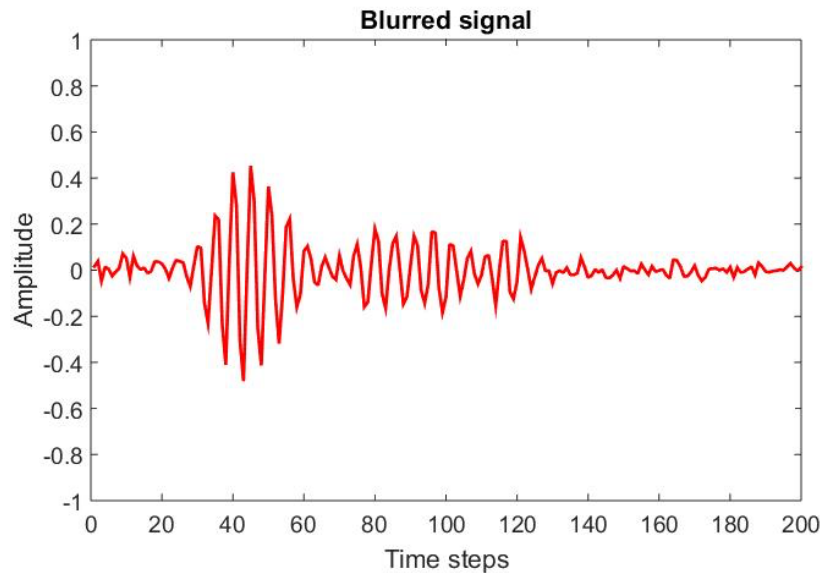


Figure 2. Blurred signal using $\mathbf{A}_1$ and $s=0.02$.

We note that the main character of the signal is retained; however the presence of noise and blur distorts the energy distribution throughout the signal duration, which is a serious problem when the signal has to be used for communication or for exploitation of its statistical features. In case of an audio signal in the air it is obvious that blurring seriously affects the clearness and the perceptibility of the signal. This is why denoising combined with deblurring is considered a necessary pre-processing action in relevant applications.

If instead of using matrix $\mathbf{A}_1$ we consider matrix $\mathbf{A}_2$:

$$\mathbf{A}_2 = \begin{bmatrix} 0.7 & 0.3 & 0 & \dots & 0 \\ 0.15 & 0.7 & 0.15 & \ddots & \\ 0 & \ddots & \ddots & \ddots & 0 \\ & \ddots & 0.15 & 0.7 & 0.15 \\ 0 & \dots & 0 & 0.3 & 0.7 \end{bmatrix},$$

with added noise of $s=0.04$, the blurred-noisy signal is presented in Fig. 3. Since matrix $\mathbf{A}_2$ indicates a lighter blurring mechanism, the blurred signal in this case seems to be closer to the clear one despite the higher level of noise. Later in this paper we will introduce a means to quantify the difference between signals, based on their statistical characterization.

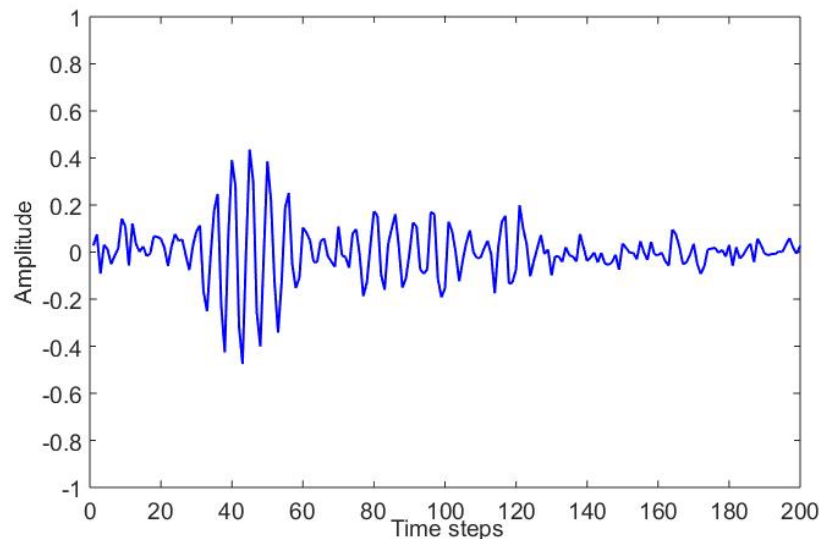


Figure 3. Blurred signal using A_2 and $s=0.04$.

SIMULTANEOUS DEBLURRING AND DENOISING OF ACOUSTIC SIGNALS

In order to restore a measured signal in a form that contains no artifacts, which in our case are due to noise and blur, a deconvolution procedure should be applied. We will apply this procedure in the time domain. Note that, at least to our knowledge, simultaneous deblurring and denoising of acoustic signals has not been presented so far at least under the concept adopted in this article that consider blur and noise as two distinct signal deterioration factors.

Using the matrix equation model to describe blurring, it is reasonable to adopt a statistical near-optimal filtering method presented for images by Taroudaki and O'Leary [10,11], modified for acoustic signals, which is using the same model to describe blurring. Adaptation of this method to acoustic signals is straightforward. The basic difference between these two approaches is the dimension of the processed signals, which in the case of images is two, while in acoustic signals is one. Apart from that, the deblurring-denoising procedure is similar. Note that in the absence of noise, which, of course, is a non realistic case, knowing the blurring matrix and in the case that the blurring matrix is invertible, the clear signal could be obtained directly by solving the linear system

$$\mathbf{b} = \mathbf{A}\mathbf{x}_{true} , \quad (3)$$

as

$$\mathbf{x}_{true} = \mathbf{A}^{-1}\mathbf{b} . \quad (4)$$

The presence of noise does not allow such a simple treatment, so a more elaborate scheme should be applied. The Statistical Near-Optimal Filtering (SOF) method briefly presented below, is a method that simultaneously removes blur and noise from a signal. The signal resulted as output from the SOF method is denoted as deblurred signal.

The Statistical Near-Optimal Filtering (SOF) method

We present here the basics of the SOF method that can be applied to signals contaminated by noise and blur as described by equation 2. The basic equations presented below are taken from a previous work by Taroudaki and O'Leary related to images [10,11], to which

we refer for additional information and detailed analysis. Also, preliminary results of our study have been presented in a conference [19].

Following the study on blurring of images, the smaller singular values of the blurring matrix $\mathbf{A}$, which is a square real matrix of dimension $n \times n$ correspond to blurring and noise effects, while the larger singular values correspond to the significant detailed part of the image. We assume that the same is true for acoustic signals. It is therefore reasonable to apply the Singular Value Decomposition (SVD) of the blurring matrix $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$, to determine the significant parts of the signal and the ill-conditioning of the blurring matrix. The diagonal elements of the matrix $\mathbf{\Sigma}$ are the singular values of the matrix $\mathbf{\Sigma}$ in decreasing order ($\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n > 0$).

As the noise vector $\mathbf{e}$ is unknown, it was shown in [6] that a solution to the problem of determining the clear signal from the measured (raw) one $\mathbf{b}$ is given by means of a spectral filter $\varphi_i(\lambda)$ the singular values σ_i , and the orthogonal square matrices $\mathbf{U}$ and $\mathbf{V}$, using the formula:

$$\mathbf{x}_{filt} = \sum_{i=1}^n \varphi_i(\lambda) \frac{\mathbf{u}_i^T \mathbf{b}}{\sigma_i} \mathbf{v}_i \quad (5)$$

This solution is called “filtered solution” and depends on the choice of the filter with parameter λ .

Motivated by the aforementioned work for image deblurring, we are using the so called Truncated SVD filter (TSVD filter) [20] which is expressed through the simple formula:

$$\varphi_i(\lambda) = \begin{cases} 1 & \text{if } 1 \leq i \leq \lambda \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

The definition of the parameter λ is based on the minimization of the norm of the error

$$\|\mathbf{x}_{true} - \mathbf{x}_{filt}\|^2 = g(\lambda), \quad (7)$$

where $\mathbf{x}_{true}$ is the true solution of the problem derived from equation 4.

$$\mathbf{x}_{true} = \sum_{i=1}^n \frac{\mathbf{u}_i^T (\mathbf{b} - \mathbf{e})}{\sigma_i} \mathbf{v}_i. \quad (8)$$

Substituting expressions for the $\mathbf{x}_{true}$ and the $\mathbf{x}_{filt}$, an analytic expression for the function $g(\lambda)$ can be derived.

Now, a new parameter is introduced called “Picard Parameter”. It is defined for blurred signals satisfying the discrete Picard condition [21] which dictates that the blurred signal coefficients decay, on average, faster than the singular values of the blurring matrix $\mathbf{A}$. It is shown that in these cases there exists a parameter k , such that $\mathbf{u}_i^T \mathbf{b} \approx \mathbf{u}_i^T \mathbf{e}$ for $i \geq k$. We will assume that this condition is satisfied for the acoustic signals considered in our study.

Following the introduction of the Picard Parameter, the function to be minimized, after some simple algebra and neglecting terms of $g(\lambda)$ which are not dependent on λ , becomes:

$$g'(\lambda) = \sum_{i=\lambda+1}^n \left(\frac{\beta_i}{\sigma_i} \right)^2 - 2s^2 \sum_{i=\lambda+1}^{k-1} \frac{1}{\sigma_i^2}, \quad (9)$$

where $\beta = U^T \mathbf{b}$ and the filtered solution is :

$$\mathbf{x}_{flt} = \mathbf{x}_{TSVD} = \sum_{i=1}^{\lambda^*} \frac{\beta_i}{\sigma_i} \mathbf{v}_i, \quad (10)$$

where λ^* is the minimizer of $g'(\lambda)$ with the condition that it is less than or equal to the Picard Parameter.

The estimation of the Picard Parameter for the signals studied was based on an automatic method described in [10] and an appropriate optimization algorithm has been applied for the determination of the parameter λ^* .

Note that the use of the Truncated SVD with Picard Parameter (TSVDk) is one of several alternative approaches (filters) that could be applied to estimate a solution to the signal deblurring-denoising problem. More information on this issue can be found in related literature [11]. The use of this particular filter for the deblurring of acoustic signals was adopted for demonstration reasons only, observing that basic assumptions are commonly used for studies on signals.

It is important to also note, that by exploiting the fact that the signal after the Picard parameter index consists practically of noise only, the statistical parameters of noise, namely the mean and the standard deviation, can be estimated using elements of the signal with index larger than that of the Picard parameter.

Extra layer of denoising

Note that following this approach, there is the option to neglect any extra denoising, or apply an extra step of denoising either before or after the method described above and leading to the filtered solution.

To do so, we estimate the mean and the standard deviation of the noise using the Picard Parameter and Picard condition as in [10,11,22] from the blurred and noisy (recorded) signal.

Then, if at a specific time step, the absolute value of the difference of the frequency of the blurred signal (in the denoised-deblurred case) or the output of the SOF method (in the deblurred-denoised case) and the estimated mean of the noise is less than three times the standard deviation of the noise, the signal is approximated to be silent there, i.e., the frequency is 0 as the recorded blurred signal or the deblurred signal after the SOF is approximated to be just noise.

The efficiency of these different approaches will be illustrated in the last section of the paper just before the presentation of the conclusions.

COMPARISON BETWEEN BLURRED AND DEBLURRED SIGNALS

A significant difference between applications related to deblurring images and those of deblurring acoustic signals, is related to the fact that in the case of images a comparison of deblurred and clear signals could be merely vision based, as the use of images is related to the human perception through the vision, although methods for quantifying these differences have also been proposed [10,11] to provide more universal results that do not depend on the human eye. In the case of acoustic signals, the necessity for their post processing, dictates the use of methods ideally employing the features of the signal which are to be possibly used for the subsequent analysis.

In real applications of course, we don't know the actual, "clear", signal. However, by means of simulations we can test the suggested deblurring procedure on the basis of its capability to reproduce the clear signal when we have full control of all type of signals (actual, blurred, deblurred).

Having in mind applications for which statistical features of the signal are exploitable; the comparison of the signals will be based on their statistical characterization and the use of the Kullback-Leibler Divergence (KLD) as the similarity measure. To this end, both the clear and the deblurred-denoised signals are statistically characterized as described in the next session to get their features that will be eventually exploited and compared through the KLD to reveal their similarity.

The statistical signal characterization scheme

The sound sources used in experiments conducted in the marine environment as it the case of the simulated sound source considered in the current article, and intended for the estimation of environmental parameters are of different types with respect to their bandwidth and excitation function. Taroudakis et al. [12] showed that signals emitted by a typical class of these sources and recorded in the marine environment can be statistically characterized using a multi-step process.

At a first step we apply a 1-D Discrete Wavelet Transform (DWT) to the discrete signal $\mathbf{x}$ using Daubechies' (db4) wavelet. Then the signal is passed through a High-Pass and a Low-Pass filter to get two sets of coefficients called detailed $\mathbf{d}_i[n; \mathbf{x}]$ and $\mathbf{a}_i[n; \mathbf{x}]$ approximate, correspondingly, with n indicating the size of the signal vector and index 1, indicating the order of the processing level.

If this procedure is continued and we convolve the new vectors of coefficients with the High-Pass and Low-Pass filters, we eventually come-up with K vectors representing the transformed version of the signal. These vectors $\mathbf{d}_i[n; \mathbf{x}]$ and $\mathbf{a}_i[n; \mathbf{x}]$, $i=1,..K$ are not appropriate for signal characterization, so further analysis is required. Using techniques usually employed in Machine Learning, we are using statistical analysis to derive specific features of these vectors that are able to uniquely describe the signal. It was shown [8] that for the acoustic signals considered in applications of acoustical oceanography such as the signals considered here, the elements of the vectors representing the transformed version of the signal obey a Symmetric α -Stable distribution (SaS) described by its characteristic function:

$$\Phi(t) = \exp\left(i\delta t - \gamma^\alpha |t|^\alpha\right), \quad (11)$$

where $0 < a \leq 2$, $-\infty < \delta < +\infty$, γ are the characteristic parameters of the distribution and t is the value of the sub-band wavelet coefficient. Note that t here, does not represent time. For a detailed description of the characteristic parameters, the interested reader is referred to the work by Taroudakis et al [8].

For practical applications of statistical signal characterization using the procedure described above, there is no need to keep all the vectors resulted from the signal decomposition. Thus, in the applications we have studied so far, we keep the detailed coefficients of all levels, plus the approximate coefficients of the final level. In addition, by carefully examining the curves representing the characteristic function of the SaS distribution for the acoustic signals we can assign the value 0 to the local parameter δ for all the acoustic signals considered in our analysis. This observation, in connection with the use of the parameters describing the approximate coefficients of the final level only, reduces significantly the number of features describing an acoustic signal, through its statistical characterization based on the SaS distributions.

Thus, the signal $\mathbf{x}$ is eventually associated with a single feature vector $\mathbf{d}$ of dimension $2K+2$ as follows:

$$\mathbf{x} \leftrightarrow \mathbf{d} = [a_1, \gamma_1, \alpha_2, \gamma_2, \dots, \alpha_{K+1}, \gamma_{K+1}]^T, \quad (12)$$

where K is the total number of levels used in the multilayered analysis. Note that parameters with index $K+1$ correspond to the statistical parameters of the approximate coefficients of the final level, while all the others correspond to the detailed coefficients of each one of the K levels.

Although this specific characterization scheme has been used mainly with underwater acoustic signals, it was shown that it has the potential to be applied for the characterization of other types of signals, including seismic ones [23] and therefore can be viewed as a method to be applied with other types of signals as well.

Using the Kullback-Leibler Divergence

Using the above mentioned characterization scheme, an acoustic signal is associated with a characteristic vector $\mathbf{d}$, the elements of which represent its features. Similarity of the signals should be measured using an appropriate norm (or pseudonorm) comparing their characteristic vectors. As the vectors correspond to parameters of a probability distribution an appropriate measure of their similarity is the Kullback-Leibler divergence [13]. In the case of the symmetric alpha stable distributions and in the case of the sub-band analysis presented above, the fitness function is the sum of the KLD distances of the selected sub-band coefficients between the actual and replica signals and takes the analytic form:

$$D_s(\mathbf{x}_1, \mathbf{x}_2) = \sum_{k=0}^L \left\{ \ln \left(\frac{c_{2,k}}{c_{1,k}} \right) - \frac{1}{a_{1,k}} + \left(\frac{\gamma_{2,k}}{\gamma_{1,k}} \right) \frac{\alpha_{2,k} \Gamma \left(\frac{\alpha_{2,k} + 1}{\alpha_{1,k}} \right)}{\Gamma \left(\frac{1}{\alpha_{1,k}} \right)} \right\}, \quad (13)$$

where, $\Gamma(x)$ is the Gamma function and

$$c_{i,k} = \frac{2\Gamma\left(\frac{1}{\alpha_{i,k}}\right)}{a_{i,k}\gamma_{i,k}}, i = 1, 2 \quad k = 0, \dots, L. \quad (14)$$

$\mathbf{x}_1$ and $\mathbf{x}_2$ denote the two signals to be compared. In the remaining text, we will refer to this function simply as KLD. When the distributions describing the statistics of the wavelet sub-band coefficients are identical, the KLD, (D_s), takes its minimum value 0.

Blurred and deblurred signals will be compared in this work using the KLD.

DEBLURRING USING THE EXACT BLURRING MATRIX

This is an optimal case. While in image processing the blurring mechanisms related to the recording devices have been studied and can be estimated as a blurring matrix after experimenting, in applications of acoustics this is not the case, especially due to the fact that blurring mechanisms have not been studied a lot and can be in addition to the recording devices be due to the environmental parameters and conditions. However, for simulation purposes, and in order to test the deblurring-denoising scheme, it is desirable to consider cases that the blurring matrix is known. Since the present study is based on simulated signals only, studying the ideal case is possible. Next section will be devoted to deblurring using approximate blurring matrices.

We will consider the signal presented in Fig. 2 as the measured signal $\mathbf{b}$ which is contaminated by noise of standard deviation $s=0.02$ and blur represented by matrix $\mathbf{A}_1$ and the signal presented in Fig. 1 as the clear signal $\mathbf{x}_{true}$.

Applying the deblurring-denoising method already presented here, with blurring matrix $\mathbf{A}_1$, the optimum filtered solution $\mathbf{x}_{filt}$ is presented in Fig. 4 (blue curve) along with the clear signal (red curve). It is obvious that the optimum filtered solution is practically identical to the original (clear) one.

In order to quantify the difference between them, the two signals are characterized using the signal characterization method presented earlier in this article and are compared using the KLD. The results are presented in Table I. We also present in same table the KLD between the clear and the blurred noisy signal. It is obvious that the deblurred-denoised signal is very close to the actual (true) one as the KLD between them is small, much smaller than that between the blurred-noisy signal and the clear one.

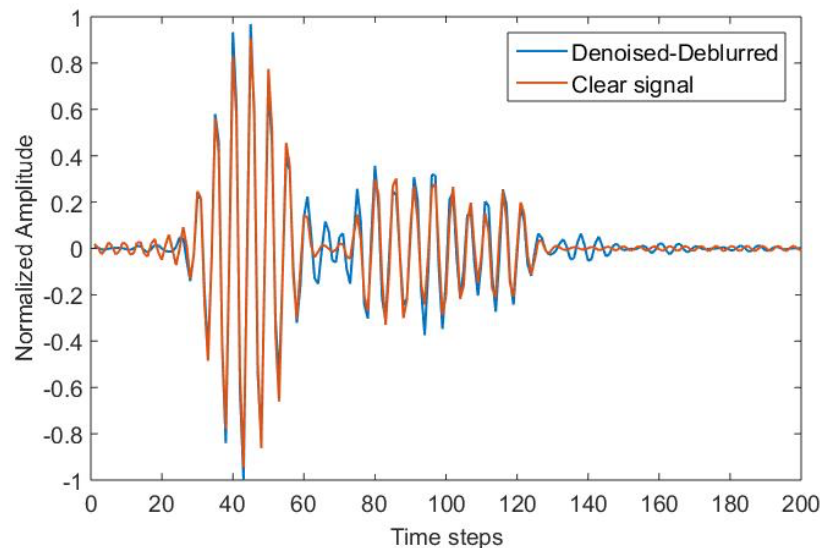


Figure 4. Deblurred-Denoised signal using blurring matrix A_1 in comparison with the clear signal

Table I. KLD values between the clear signal, the blurred-noisy and the deblurred-denoised one. Blurring matrix A_1

$\mathbf{x}_{true}$ VS	$\mathbf{b}$	$\mathbf{x}_{filt}$
D_s	311.31994	0.37151

DEBLURRING USING APPROXIMATE BLURRING MATRICES

When the blurring matrix cannot be modeled or determined with confidence but the blurring mechanism is more or less known, approximate matrices might help to get a deblurred signal which is closer to the actual one, under the condition that the blurring matrix is of a form describing, to a great extend, the blurring mechanism when noise added in the signal is very low. We demonstrate the validity of our statements with an illustrative experiment.

With reference to the signal considered in the 2nd section, we study the case where the simulated signal is blurred by means of the matrix A_1 . We use different levels of noise to study how noise affects the procedure. We consider Gaussian noise with standard deviation $s = 0.005, 0.02, 0.04$ added to produce 4 different simulated “measured” signals $\mathbf{b}_i, i = 1, 2, 3, 4$ including a noise-free signal. For convenience, we refer to the case of noise-free signals as $s = 0$.

Then the deblurring-denoising procedure was applied assuming that the blurring matrices were A_1 and A_2 and the following additional blurring matrix A_3 .

$$A_3 = \begin{bmatrix} 0.9 & 0.1 & 0 & 0 & 0 \\ 0.05 & 0.9 & 0.05 & \ddots & \\ 0 & \ddots & \ddots & \ddots & 0 \\ & \ddots & 0.05 & 0.9 & 0.05 \\ 0 & & 0 & 0.1 & 0.9 \end{bmatrix}$$

Matrix $\mathbf{A}_3$ is close to identity indicating little or practically no blur in the signal.

We used the three matrices along with all combinations of added noise and order of performing deblurring-denoising to assess the performance of the scheme.

We observed significant differences in the performance of the deblurring-denoising algorithm when matrices different than the matrix causing the blur were used for the deblurring procedure and for different noise levels. Figures 5-8 illustrate this observation. Fig. 5 presents original, blurred and deblurred acoustic signals when blurring is due to matrix $\mathbf{A}_1$, while deblurring is performed by means of matrix $\mathbf{A}_3$ with added noise of $s=0.04$. The horizontal axis corresponds to time steps. It is obvious that since the matrix used in the deblurring procedure assumed little blur, the “clear” signal obtained is not much different than the noisy-blurred signal. The deblurring procedure is somehow improved when matrix $\mathbf{A}_2$ is used as the deblurring matrix as it can be seen in Fig. 6. However, this improvement is not considered significant. Of course, the best performance is observed when the blurring matrix is the same that is used for the deblurring as it can be seen by inspection of Fig. 7. It is also expected that the less noisy the signal is, the better deblurred-denoised results are obtained as seen in Fig. 8, for which added noise of $s=0.02$ has been considered. This case corresponds to the one presented in the previous section.

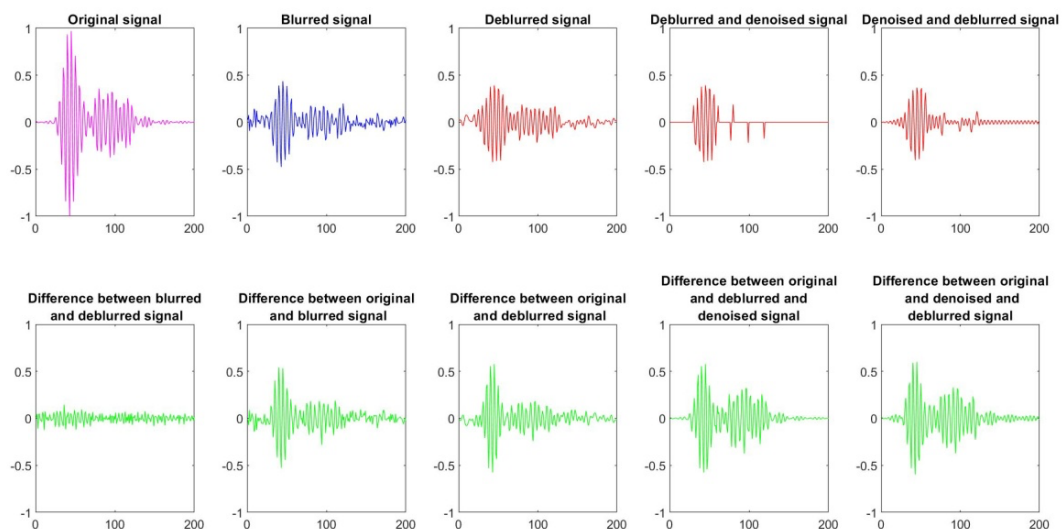


Figure 5. Original blurred and deblurred signals with blurring matrix $\mathbf{A}_1$ and deblurring matrix $\mathbf{A}_3$ with noise of $s=0.04$.

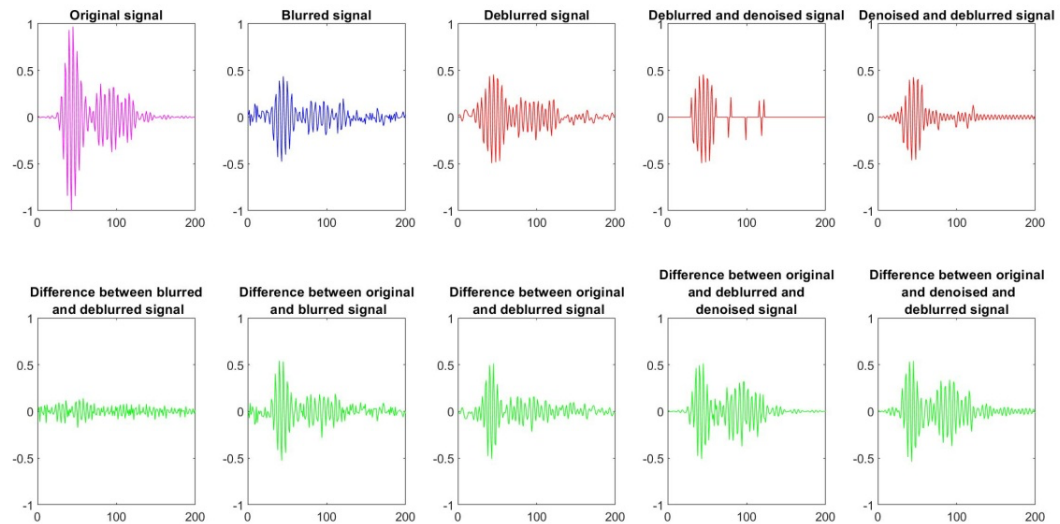


Figure 6. Original, blurred and deblurred signals with blurring matrix A_1 and deblurring matrix A_2 with noise of $s=0.04$.

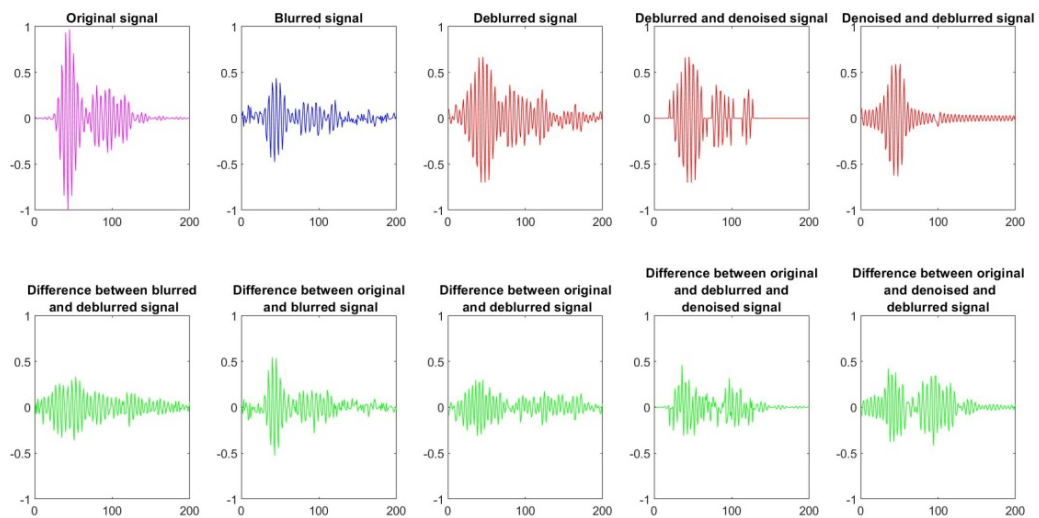


Figure 7. Original blurred and deblurred signals with blurring matrix A_1 and deblurring matrix A_1 with noise of $s=0.04$.

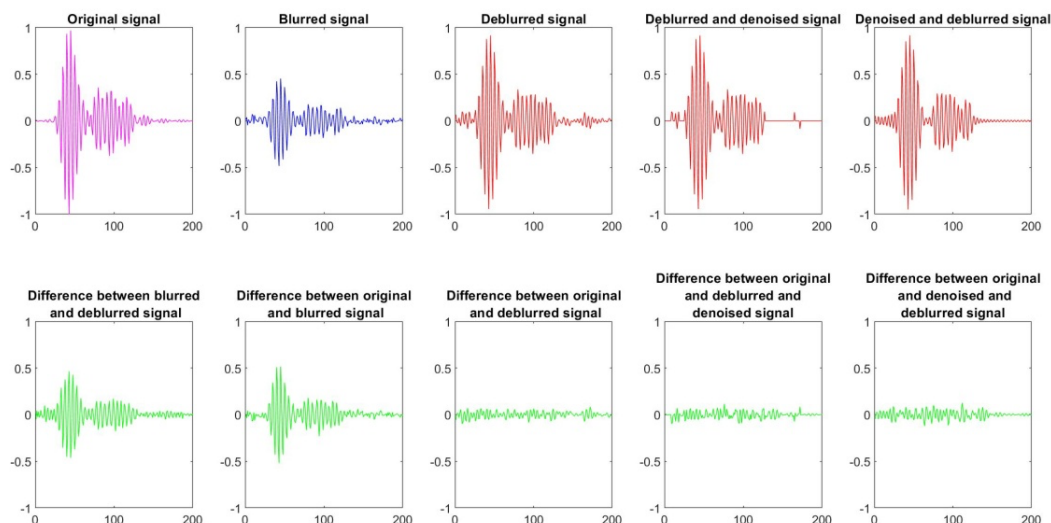


Figure 8. Original blurred and deblurred signals with blurring matrix A_1 and deblurring matrix A_1 with noise of $s=0.02$.

Table II shows the KLD values when comparing the original signal with the signals after deblurring using matrices A_1, A_2, A_3 for all noise scenarios. Note that **Deb** is referred to the standard deblurring-denoising. **Deb & Den** is referred to the **extra** denoising after the standard deblurring-denoising scheme has been applied, while **Den & Deb** is referred to the preliminary denoising **before** the standard deblurring-denoising scheme has been applied. As it was expected, the general trend is that the closer our assumed blurring matrix to the actual one is, the smaller KLD is obtained, and the further our blurring matrix to the original one is, the larger KLD is calculated. This conclusion was consistent with all the different combinations studied. Note that, when there is no noise in the measurements, the deblurring algorithm is able to reproduce the clear signal almost perfectly (last row in the table) when the exact matrix is assumed, but deblurring with the other matrices is also satisfactory. It is also evident that when denoising is performed prior to deblurring, the recovered signal is much closer to the original one in practically all cases of noise. An additional observation is that noise is an important factor, as when it is ignored or not removed carefully, deblurring only is not capable of retrieving the clear signal with good accuracy. This can be easily observed by studying the first column of the results shown in table II presenting the KLD difference between the clear and filtered signal when performing deblurring and simultaneous denoising only without any additional noise removal.

A final note is that if we concentrate our attention to the last column, showing very low KLD values in all cases, we can conclude that the recovery of the clear signal is in general satisfactory when denoising is performed prior to standard deblurring-denoising.

Table II. KLD values for the test case when the signal has original blur defined by matrix A_1 .

Noise	Matrix	Deb	Deb & Den	Den & Deb
s=0.04	A_3	279.5528	9.3391	4.4647
	A_2	214.0648	10.9428	4.4714
	A_1	42.1638	2.2314	4.3099
s=0.02	A_3	28.7002	7.3798	0.97381
	A_2	31.2693	5.3745	0.49438
	A_1	13.171	4.8222	0.58638
s=0.005	A_3	3.2791	1.0307	0.48744
	A_2	2.1998	0.56849	0.39936
	A_1	0.56102	0.31506	0.42317
s=0.00	A_3	0.55639	0.56295	0.55703
	A_2	0.3753	0.37522	0.37586
	A_1	2.214×10^{-5}	2.254×10^{-4}	6.7846×10^{-5}

DISCUSSION

Motivated by the necessity for obtaining audio files and images which are clear from any distorted artifacts, which is necessary for the improvement of human communication using modern audio and image technology, we concentrated our work on the specific case of acoustic signal blurring. We presented a preliminary study of the blurring as a degradation mechanism for acoustic signals based on a simple scenario for the modeling of the blurring mechanism through a matrix that distorts the clear (blur-free) digitized signal. As a test case we have used a simulated underwater acoustic signal, mainly due to the fact that we have full control of its production, while possible blurring of this type of signals has not been studied. A combined deblurred-denoised scheme adopted from image processing has been introduced, which has been shown to be very effective for the deblurring of the acoustic signal in cases that the blurring mechanism expressed through the corresponding matrix is known. The use of approximate matrices in the deblurring procedure, which are close to the “actual” blurring matrix, has been proven to be less effective for deblurring purposes, as expected.

Following the analysis presented here, there are several issues that should be considered and studied for a more comprehensive assessment of the importance of the blurring mechanism in acoustic signals and the development of optimal methods for acoustic signal deblurring. In particular, since the mathematical description of the blurring mechanism, which is a prerequisite for the application of a deblurring algorithm, is of primary importance, it is desirable that a systematic study of the blur mechanisms for typical applications employing acoustic signals is performed. This study is expected to provide reliable blurring matrices to be employed for the deblurring procedure. Although this is definitely an ambitious goal, we believe that it will add knowledge about the actual mechanisms that degrade the quality of the acoustic signals due to blur and will trigger further studies of de-blurring methods that will eventually enhance the quality of acoustic signals used for communication purposes.

As an acoustic signal can be considered as the means to get input data to some inverse problem of acoustics, for instance sound source identification and/or localization, the

necessity for exploiting “clear” signals is of profound importance. This gives additional directions to the research associated with blurring and deblurring of acoustic signals. For instance, it is possible to consider the blurring matrix as an additional unknown of the inverse problem under consideration. Modern machine learning techniques might be the appropriate vehicle to treat such a problem which is really challenging as these techniques seem to work for the identification of the underlying signal degradation model as well.

The definition of confidence intervals has been proven to be very useful when assessing the quality of image deblurring when limited information on the blurring mechanism exists. Confidence intervals are essential when the clear signal is to be used for inverse problems of acoustics and it is the purpose of the authors to concentrate their future studies on this objective.

Finally, it should be noted that the idea of acoustic signal deblurring may trigger research work on the improvement of the quality of data to be used for studies related to totally different applications. Machine learning is a research area continuously providing new tools for handling big data related to problems of practically every science including of course Social sciences. Uncertainties in the quality of data to be exploited and analyzed may be proven a serious handicap in the production of reliable predictions. Studying data corruption by means of some deterministic mechanism, such as the blurring mechanism considered in this work, may motivate data quality improvement by recantation of the degradation cause using of course techniques specifically tuned to the problem under consideration. This would be definitely a big improvement in data handling.

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APPENDIX

The Simulated Underwater Acoustic Signal

The signal used in this article has been calculated using Normal-Model theory of acoustic propagation underwater [24]. A broadband source has been considered emitting a signal which is described by a Gaussian excitation function

$$S(\omega) = \exp\left(-\frac{1}{2}\left(\frac{\omega - \omega_0}{\Delta\omega}\right)^2\right), \quad [A1]$$

where, ω is the circular frequency ($\omega = 2\pi f$), f is the frequency in Hz, ω_0 is the central frequency and $\Delta\omega$ is the source bandwidth.

The environment under consideration is a shallow range-independent water waveguide with homogeneous fluid sea-bed and pressure release surface. The source is located at depth z_0 in the water column and a receiver is considered at depth z and range r also in the water column.

Following standard theory of underwater acoustic propagation, the acoustic pressure $H(\omega)$ calculated at a single frequency ω is given as a sum of normal modes that characterize acoustic propagation in this medium, using formula

$$H(\omega) = B \sum_{n=1}^N u_n(z_0) u_n(z) H_0^{(1)}(\kappa_n r), \quad [A2]$$

where, u_n are the eigenfunctions and κ_n are the eigenvalues of an appropriate “depth” problem, defined for the environment under consideration, $H_0^{(1)}(x)$ is the Hankel function of first type and zero order and B is a constant depending on the geometry of the problem. Further analysis is beyond the scope of this article and the interested reader can find the details in [24].

For a broadband source the acoustic pressure is $P(\omega) = H(\omega)S(\omega)$.

To obtain the signal in the time domain, an inverse Fourier transform is applied.

$$p(t) = \mathfrak{F}^{-1}(P(\omega)). \quad [A3]$$

The signal considered in this work has been calculated considering a point source of central frequency 100 Hz and bandwidth of 40 Hz. The environmental parameters are presented in Table A.I and the environment is shown in figure Fig. A.1. We assumed source depth $z_0 = 100$ m, receiver depth $z = 100$ m, and a range $r = 5000$ m.

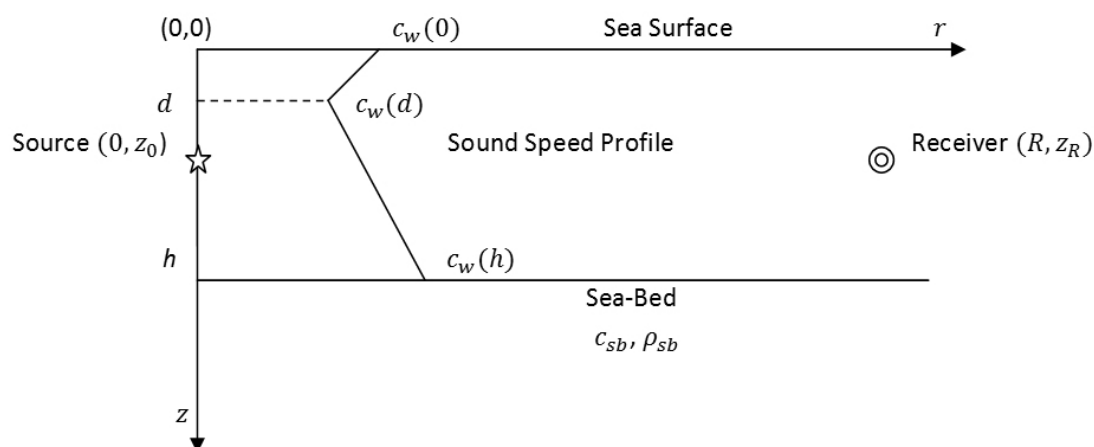


Figure A.1 The model environment

Table A.I : The environmental parameters considered for calculating the test signal.

Parameter	Symbol	Value
Water depth	h	200 m
Sound speed at depth 0	$c_w(0)$	1500 m/sec
Minimum sound speed	$c_w(d)$	1490 m/sec
Sound speed at depth h	$c_w(h)$	1515 m/sec
Depth of minimum sound speed	d	50 m
Bottom density	ρ_{sb}	1200 kg/m ³
Sound speed in the bottom	c_{sb}	1600 m/sec

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