

Discovery Teaching About Calculating Side Lengths of Right Triangles Using Pythagoras Theorem With the Help of Geogebra Software

Ngoc-Giang Nguyen

Faculty of Economic Mathematics

Banking University Ho Chi Minh City, Ho Chi Minh City, Vietnam

<https://orcid.org/0000-0001-7560-7972>

Thuy Hong-Anh Nguyen

Math Team, Dong Nai Pedagogical Practice High School Bien Hoa City

Dong Nai province, Vietnam

ABSTRACT

Discovery teaching is an active teaching method that encourages learners to create their own knowledge. Students who participate in the discovery learning process will remember longer than the conventional teaching method. This teaching method is very important in the process of teaching innovation in Vietnam when changing from a teaching method that focuses on knowledge transmission to one that focuses on developing learners' capacity. Our study proposes an exploratory teaching method for calculating the side lengths of right triangles using the Pythagoras theorem with the help of GeoGebra software. By the method of empirical investigation, we surveyed 51 students in Bien Hoa city and used descriptive statistics with the help of SPSS software, the research has shown that the discovery teaching of calculating side lengths of right triangles using Pythagoras theorem with the help of GeoGebra software is needed. However, many teachers still do not fully understand the discovery teaching method. The research results also show that there are three groups of factors affecting the way of discovery teaching. First, the effort of teachers in teaching discovery is huge. Second, the time spent on discovery teaching is relatively large. Third, the current teaching environment has not yet met the requirements of discovery teaching. The findings from the research results are important for proposing a way of discovery teaching about calculating the side lengths of right triangles using Pythagoras theorem with the help of GeoGebra software.

Keywords: Discovery teaching, calculating the side lengths of a right triangle, Pythagoras theorem, process, GeoGebra software.

INTRODUCTION

Learning only really makes sense, and learners only remember longer when learners participate in the construction of knowledge themselves. Learners are more active and dynamic in teamwork (Morgan K Williams, 2017). Teaching must encourage learners to discover knowledge for themselves. Learning is an active process in which learners construct new ideas or concepts on the basis of existing knowledge (Jerome S Bruner, 1960). Instead of just listening

to lectures, learners have the opportunity to apply different skills in activities and master their learning. Learning is not a rigid process that cannot be changed, learners assimilate new knowledge into their own knowledge base (Tracy Bicknell-Holmes Hoffman, 2000). Discovery teaching focuses on active learning activities, creating opportunities for learners to explore and solve problems on their own. The focus of learning is on learning how to analyze and synthesize information to understand what's going on, rather than just coming up with exact answers from rote memorization. It pushes students to a deeper level of understanding (Joyce Castronova, 2002).

In Vietnam, there are also many studies on discovery teaching. To use this teaching method, it is necessary to first build lessons designed with questions, so that when answering or trying to solve component questions, students gradually see how to solve the initial problem. Finding the answer goes from easy to difficult, from things that are obvious, conspicuous, and explicit to uncovering hidden rules and concepts (Tran Ba Hoanh, 2004). A discovery activity is a thinking process including observation, analysis, evaluation, hypothesis and inference in order to discover and acquire knowledge of the subject. It is usually done through a system of activities, which will gradually appear a path to knowledge when students answer and do (Le Vo Binh, 2007). This teaching method is a process in which, under the role of the teacher's orientation, learners take the initiative in their own learning, form questions posed in thinking, and expand research and search. From there, new insights and knowledge are built. This knowledge helps learners answer questions, find different methods of solving problems, prove a theorem or point of view (Pho Đức Hoa and Ngo Duy Son 2008). The teaching method encourages students to ask questions and find answers on their own, or to draw principles from examples or practical experiences. In addition, discovery teaching can be defined as a learning situation in which the main content to be learned is not introduced first but must be discovered by the students themselves, making the students an active participant in the learning process (Nguyen Phu Loc, 2010).

GeoGebra software was invented by Austrian educator Markus Hohenwarter, a lecturer at the University of Salzburg in 2001. The software is a combination of Geometry, Algebra, Analysis, and spreadsheet. GeoGebra is free, open-source, and multi-language software (can be used in over 63 languages, including Vietnamese). GeoGebra's interface is friendly and easy to use, with intuitive toolboxes that users can easily manipulate the software (Markus Hohenwarter and Judith Hohenwarter, 2008). GeoGebra has the potential to promote active learning. Teaching with the help of GeoGebra software is a learner-centered teaching method. GeoGebra creates an environment that allows mathematical operations to be performed (Darius Majerek, 2014). GeoGebra is software that can be embedded in an e-learning environment. GeoGebra promotes discovery and self-knowledge (Preiner Judith, 2008). There is a positive relationship between teachers' current perceptions and intention to use GeoGebra in teaching math. In addition, the results confirmed that there was no significant difference between male and female teachers in using GeoGebra in teaching mathematics (Soheila Belgheis and Rosemaliza Kamalludeen, 2018).

In Vietnam, there are also many authors doing research on GeoGebra software. GeoGebra is effective software that supports the proof process, motivating learners to create new knowledge in a sustainable way (Nguyen Danh Nam, 2012). The use of this software will bring many opportunities for students to practice skills and master the content of math knowledge

(Le Viet Minh Triet, 2018). The use of GeoGebra software in teaching mathematics may face a number of difficulties such as limited equipment of some high schools, teachers' low ability to use the software, teachers not being properly guided and trained on the effective exploitation and use of software in teaching mathematics (Nguyen Thi Huong and Le Tuan Anh, 2018). GeoGebra software has many outstanding advantages such as interoperability, allowing the creation of new geometries, problems to be solved instantly, allowing verification of geometry problem results, allowing geometry object movement, and illustrating theorems and properties of animation. Thanks to its outstanding advantages, when combined with discovery learning methods, teaching becomes better and more effective (Nguyen Ngoc Giang, 2016). Discovery teaching using GeoGebra animation software not only helps students acquire knowledge actively, but also creates opportunities for learners to develop self-study, self-discovery, and group cooperation skills, enabling students to assert themselves, thereby improving the quality of teaching and learning in high schools. (Tram Huy Khoi, 2019).

Although there are many studies on discovery teaching, discovery teaching with the support of GeoGebra software, the topic of applying the discovery teaching method to calculate the side lengths of a right triangle by using the Pythagorean theorem with the help of GeoGebra software is still quite new and attractive and needs more research, especially in the current period when the application of information technology in teaching and learning is developing strongly.

LITERATURE REVIEW

Advantages and disadvantages of discovery learning

Advantages

- Students participate actively and voluntarily in the process of learning new knowledge. Exploratory activities stimulate students to ask questions about emerging problems. As a result, students are more attracted and curious, and more eager to learn new knowledge. In performing discovery learning, students also become speakers. They must be able to present their findings on their own and defend them in front of the whole class. By actively seeking information and solving problems, they can accumulate knowledge and create their own capital of experience. (Vu Thi Lan Anh, 2020).
- This method is student-centered and teachers are actively involved in motivating students to voice their ideas. Even the teacher can act as a student or a researcher in the discussion. (Ringgi Candraning Prawerti, 2014).
- This method promotes the development of thinking, skills, and experiences in the process of discovery and learning. (Multi Efrini, 2016).
- Learners learn how to develop their own memory because, in discovery, learners have to mobilize their own knowledge and experience and relate existing knowledge to relationships with the problem to be learned, so they will remember the lesson longer. It is even possible to reconstruct knowledge when relevant information is available. (Novriana Rahma Siagian, 2018).
- Learners learn in interaction, form cooperative relationships, and solve learning tasks together. Students participate in group activities to help solve problems of broader content. Learners will develop social skills and communication skills through these group activities. (Richard E Mayer, 2004).
- Lesson theories are suggested to explore. Students play an important role in learning abstract mathematical concepts, and their will to learn becomes increasingly persistent and creative (Ryan D Chen and Honomichl and Zhe, 2012).

- The class becomes lively, active and creates more learning motivation for students. Student learning outcomes are better in the learning process. (Suphi and Yaratana, 2016).
- Discovery learning helps teachers assess students' learning development. This teaching method is consistent with the current capacity development orientation. (Sobari and Husnussalam, 2019).

Disadvantages

- To apply this method, students must have the necessary knowledge and skills to perform exploratory tasks and find new knowledge. Average and weak students will have difficulty learning this method. (Trinh Duc Toan, 2016).
- The time of the process of discovering new knowledge takes up a lot of the entire learning process, so it depends on the content, the lesson objectives, and the way of teaching time distribution so that we can apply it. Therefore, discovery teaching cannot be applied to all lectures. (Emily Brown, 2006).
- The activity of discovering knowledge about geometry requires teachers to design many models, symbols, images, etc. Therefore, the facilities of teaching and learning must meet the desired results.
- The effort of teachers in discovery teaching is relatively large. Therefore, it is necessary to have special treatment for teachers for this teaching method compared to other teaching methods.
- This method is not effective to teach many students, because it will take a long time to help them find a theory or solve other problems. It is necessary for students in different regions to ask different discovery questions. (Ringgi Candraning Prawerti, 2014).
- Teachers spend a lot of effort and money in preparing tests and assessment questions for students by means of discovery teaching.

An example to illustrate the calculation of the side lengths of a right triangle using the Pythagorean theorem with the help of GeoGebra software

Tika Karlina Rachmawati (2016) wrote the article "An Analysis of Students' Difficulties in Solving Story Based Problems and Its Alternative Solutions" in the *Journal of Research and Advances in Mathematics Education*. The author analyzed the difficulties of students when solving problems and practical problems based on the Pythagoras theorem to calculate the length of the diagonal of the rectangle or the closest distance of the ship from the starting point. (Tika Karlina Rachmawati, 2016).

Vu Huu Binh and colleagues in the book "Improving Math 7" used the Pythagorean theorem to calculate the side lengths of a right triangle with exercises from simple to more complex. For example, the exercises show the lengths of any two sides of a right triangle and calculate the length of the other side; or practical related exercises using the Pythagorean theorem such as roofing a roof in the form of an isosceles triangle when knowing the height of the roof and the length of the ceiling. (Vu Huu Binh, 2014).

Fathiya Salsabila and colleagues (2017) presented "Students' Ability in Proving Pythagorean Theorem through Discovery Learning Model Using Geogebra Software" during The 6th International Conference on Multidisciplinary Research (ICMR) in conjunction with the International Conference on Electrical Engineering and Informatics (ICELTICs) 2017. The content is about training 8th graders the skills to use GeoGebra software to solve problems

discovering knowledge of the 8th-grade geometry program in general and content of Pythagorean theorem in particular. This has been tested in 2 classes; each class has 24 students, corresponding to one class with the traditional teaching method, the other class applying the discovery teaching method with the support of GeoGebra software. After two weeks of experimentation, the evaluation results show that the discovery-based class with the support of GeoGebra software has made great progress as shown in the test of math problems with content from basic to advanced. Students apply the Pythagorean theorem masterfully to solve real-life related problems in experiential activities quite well. (Fathiya Salsabila, 2017).

At the 3rd International Conference on Research of Educational Administration and Management (ICREAM 2019), Nguyen Ngoc Giang and Tran Trung mentioned the use of GeoGebra software in teaching geometry. For example, by teaching the Pythagoras theorem, we can use the dynamism of the software to help students distinguish static and dynamic elements in the lesson, thereby knowing how to construct and create moving elements by using the slide bar to develop, discover new problems with immediate results. (Nguyen Ngoc Giang and Tran Trung 2019).

From the above points of view, we think that to calculate the side lengths of a right triangle using the Pythagorean theorem with the help of GeoGebra software, first of all, students need to understand Pythagoras' theorem and the construction features as well as the elements of motion to discover many new and more complex problems.

The process of organizing the implementation of the situation

In order to form students' skills in solving problems applying the Pythagorean theorem and the ability to use GeoGebra dynamic geometry software in exercises to calculate the side lengths of a right triangle, we provide a procedure as follows:

Step 1. Learn contents to be explored with the help of GeoGebra

From the given data of the problem, it is necessary to analyze and determine the relationship between the initial data of the problem and the unknown quantity to be found. Learners discover for themselves which knowledge is reasonable to apply to solve the problem posed. After determining the data for the given problem, the teacher guides the students to bring the problem back to the actual triangle image. Ask students to tell where the triangle is right, which sides we already know the length of, and which theorem we use to find the length of the remaining side of the right triangle.

Step 2. Build shapes with the help of GeoGebra software

The teacher instructs students to build shapes using GeoGebra software for the data of the problem. Students need to discover relationships between unknown and known quantities, as well as detect the moving or fixed quantity of the problem. From there, students explore and discover how to build shapes with the help of this GeoGebra software.

Step 3. Solve the discovery task with the help of GeoGebra software

For the problem of applying the Pythagoras theorem, learners need to know the formula and the factor to be calculated here is to calculate the right angle or hypotenuse, thereby using the formula more accurately.

After completing the shape construction steps, the teacher will guide students to go deep into applying the Pythagorean theorem to calculate the unknown side length, thereby discovering a way to solve the problem and calculating the required result of the problem.

Step 4. Deepen the discovery quest with the help of GeoGebra software

When using the support of GeoGebra software, learners will immediately verify whether their calculation is right or wrong, then correct it (if the calculation is wrong) to get the most accurate answer.

The teacher will provide accurate results and generalized overview comments about the task the students have discovered. After that, the advantages and limitations of students will be summarized when solving the discovery task with the help of GeoGebra software. In addition, based on the old problem, it is possible to exploit and develop the problem into a new problem. This approach helps learners develop thinking and creativity in the process of discovery.

Step 5. Use GeoGebra software to illustrate Pythagoras theorem on right triangles to create decorative works

Point out advantages and introduce outstanding features of GeoGebra software. In addition to drawing and verifying, GeoGebra can also illustrate images of Pythagoras' right triangles using the toolbar, especially the dynamic feature of the slider for students to refer, expand, and explore knowledge. From there, students can apply these features to decorate their graphic works.

Illustration

Example 1. A boy is holding a kite coil with his hand at a distance $AD = 1.6\text{m}$ from the ground and is standing and flying a kite at a distance $AB = 8\text{m}$ from a tree (as shown in the figure). He has released a kite string of length $DG = 17\text{m}$ and the kite is hovering above the tree. Calculate the distance GB of the kite from the ground.

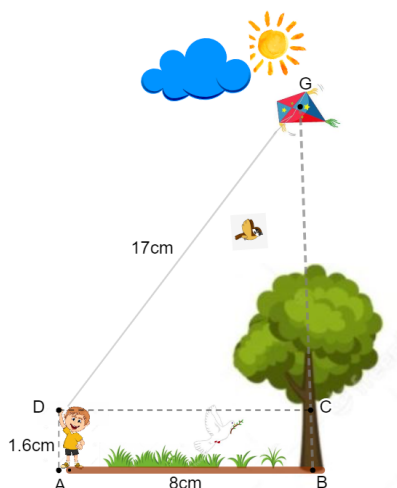


Figure 1. A boy flying a kite (Source: Freepik).

In the table of activities of teachers and students below, from step 1 to step 4, we use the operations of recognizing, applying, and discovering new problems. As for step 5, we use

manipulations of creative thinking such as generalization and analogy to represent elements of exploration, discovery, and shape building to create new decorative works with the help of tools and means of learning mathematics in the general education program of mathematics 2018.

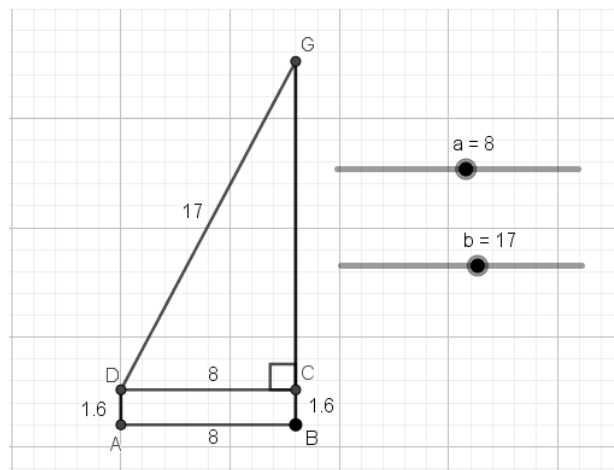


Figure 2. Building shapes from the problem of a boy flying a kite (Source: Personal Collection).

Activities of teachers and students:

Step 1. Learn contents to be explored with the help of GeoGebra

Teacher: Identify all the facts on the given problem.

Students:

- The distance from the boy's hand holding the kite coil relative to the ground is $AD = 1.6\text{m}$.
- The boy stands at a distance $AB = 8\text{m}$ from the tree.
- The length of the kite string is $DG = 17\text{m}$.

Teacher: Find a relationship between known quantities and the desired quantity. Describe how to calculate the distance of a kite from the ground.

Students: From the known quantities, the distance of the kite from the ground can be calculated as follows:

- + Calculate the GC distance using the Pythagorean theorem on the right triangle GDC at C.
- + Then, sum $GC + CB$, we get the desired result.

Step 2. Build shapes with the help of GeoGebra software

Teacher: Instruct students to transfer the picture to a specific triangle case.

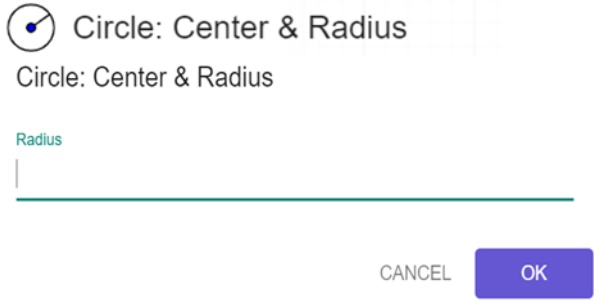
Use tools and math learning tools with the support of GeoGebra software to draw shapes according to the following steps:

Open the GeoGebra software.

For unnecessary objects, we can hide.

Students: acquire and implement

Ordinal number	Shapes	Steps
1	 Line tool group.	Click on the line tool group
2	 Line	Select the symbol: The line passes through two points. Then draw a line x, keep point B and hide the other point.
3	 Other tool group	Click on the other tool group
4	 Slider	Select the icon: Slider Then fill in: Name: a = 8 Select: Number Minimum: 0 – Maximum: 20 (Depending on the value of the given topic, adjust accordingly) Then click OK, we get the slider as shown
5	 Circle and arc tool group	Click on the circle and arc tool group
6	 Circle: Center & Radius	Select the icon: Circle when the center and radius are known. Then select point B and enter radius as a and select OK.
7	 Point creation tool group	Click on the point creation tool group.
8	 Point	Select the icon: New point Then click to select the intersection of the circle with center B, radius a = 8cm with the line x. We get point A. (If you see the point with a different name, you can right-click => select rename.)
9	 Relationship tool group	Click on the group of relationship tools
10	 Perpendicular Line	Select the icon: Perpendicular line. Then, select line AB and point A.

		Do the same, select line segment AB and point B. We have 2 lines perpendicular to AB at points A and B.
11	 Circle and arc tool group	Click on the circle and arc tool group
12	 Circle: Center & Radius Circle: Center & Radius Radius CANCEL OK	Select the icon: Circle when the center and radius are known. Then select point A and enter radius as 1.6 and select OK. We have a circle with center A and a radius of 1.6cm.
13	 Point creation tool group	Click on the point creation tool group.
14	 Point	Select icon: New point Then click the mouse to select the intersection of the circle with center A, radius 1.6cm with the line perpendicular to AB at A. We get point D. (If you see the point with a different name, you can right-click => select rename.)
15	 Relationship tool group	Click on the group of relationship tools
16	 Parallel Line	Select icon: Parallel line. Then, choose point D and line AB, we get a line parallel to AB.
17	 Point creation tool group	Click on the point creation tool group.
18	 Point	Select the icon: New point Then click to select the intersection of the line parallel to AB and the line perpendicular to AB at B. We get point C. (If the point is shown with a different name, we can right-click => choose rename.)
19	 Line tool group	Click on the line tool group.
20	 Slider a=2	Select the icon: Slider Then fill in: Name: b = 17; Select: Number Minimum: 0 – Maximum: 30

		(Depending on the value of the given topic, adjust accordingly) Then click OK, we get the slider as shown
21	 Circle and arc tool group	Click on the circle and arc tool group
22	 Circle: Center & Radius	Select the icon: Circle when the center and radius are known. Then select point D and enter radius as b and select OK. We get a circle with center A and radius $b = 17\text{cm}$.
23	 Point creation tool group	Click on the point creation tool group.
24	 Point	Select the icon: New point Then click the mouse to select the intersection of the circle with center D, radius $b = 17\text{cm}$ with the line perpendicular to AB at B. We get the point G. (If the point is shown with a different name, we can right-click => select rename.
Hide unnecessary objects. Select the line perpendicular to AB at A, right-click and select: Show object. Do the same for the circle with center A and radius 1.6cm ; the circle with center B and radius $a = 8\text{cm}$; the circle with center D and radius $b = 17\text{cm}$; the line parallel to AB at D; the line perpendicular to AB at B		
25	 Line tool group	Click on the line tool group
26	 Segment	Select the icon: Line segment Then, connect 2 points A and B; A and D, D and C; B and C; D and G.
27	 Angle	Select the icon: Angle with a given magnitude Then choose the points G - C - D in turn, we get the angle GCD equal to 90°

Step 3: Solve the discovery task by finding the length of the remaining side of a right triangle based on the Pythagorean theorem

Teacher: After guiding students to find out the problem to be explored and build the shape, ask the students to proceed to solve the discovery problem with the help of GeoGebra software.

Teacher: Based on the picture, what formula do you use to calculate the GC edge?

Student: Based on the Pythagorean theorem

Teacher: Can you give a detailed solution to the problem?

Student: Consider triangle GCD right angled at C:

Applying the Pythagorean theorem, we have:

$$GC^2 + CD^2 = GD^2$$

$$\Rightarrow GC^2 = GD^2 - CD^2$$

$$\Rightarrow GC = \sqrt{GD^2 - CD^2}$$

$$\Rightarrow GC = \sqrt{17^2 - 8^2}$$

$$\Rightarrow GC = 15(\text{cm})$$

The distance of the kite from the ground is:

$$GB = GC + CB = 15 + 1,6 = 16,6 (\text{cm})$$

Step 4. Deepen the task of discovering, commenting by verifying the results with the help of GeoGebra software and mathematical creation (if any)

Teacher: Looking at the display area, it is easy to see how much the GC edge is?

Student: $GC = 15\text{cm}$

Teacher: Select the line icon and connect G and B (or use the Line segment command $\langle G, B \rangle$), look at the display area, how much is GB worth?

Student: $GB = 16.6\text{cm}$

The teacher then develops a new problem based on the old problem using GeoGebra software.

Teacher: Move the slider so that $a = 12$, $b = 20$, we get a new problem.

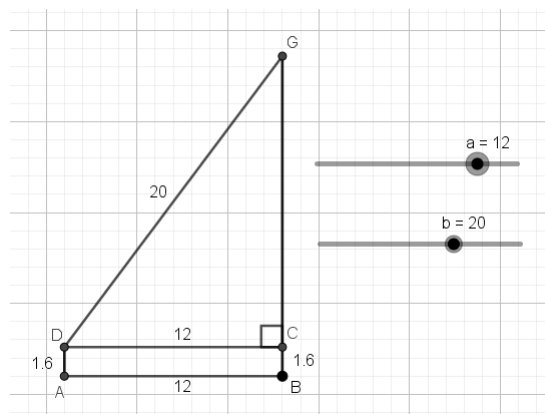


Figure 3. Construct a new problem shape (Source: Personal Collection).

Example 2. A boy 1.6 m tall was standing flying a kite 8 m from a tree. He dropped a rope 17 meters long and the kite was hovering above the tree. When it was windy, he wanted to go further and let the kite fly higher, so he went 4 meters away from the tree and dropped the rope 3 meters longer. Calculate the distance of the kite from the ground now.

Teacher: With the operation of using the slider as shown in Figure 3, through which we see that the drawing has changed. What is the distance of the kite from the ground at that time? (Based on the display area on GeoGebra software, we have absolutely accurate results without calculation)

Students: GB = 17,6cm

$r = \text{Segment}(G, B)$
 $\rightarrow 17.6$

Step 5. Use GeoGebra software to illustrate Pythagoras theorem on right triangles to create decorative works

In addition to using the Pythagoras theorem to calculate the side lengths of right triangles, GeoGebra software also supports users to create illustrations from equilateral triangles and polygons with toolbars. The work of verifying the results is known immediately without the need for repeated calculations.

Example 3. Given an isosceles right triangle ABC at B (Figure 4) with $AB = BC = 1$ cm. Using the toolbar in GeoGebra, we can illustrate consecutive right triangles with a fixed right angle side length of 1 cm, the other side formed by the hypotenuse of the previous right triangle.

- Calculate the area of the 4th and 10th right triangles in figure 5.
- Draw an illustration as shown in Figure 6.

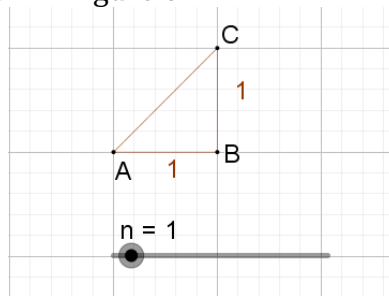


Figure 4. Constructing an isosceles right triangle ABC at B (Source: Personal Collection).

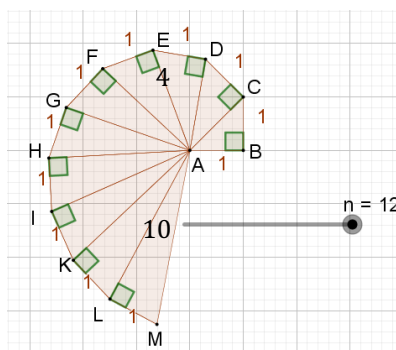


Figure 5. Constructing right triangles developed from Pythagoras theorem (Source: Personal Collection).

Teacher: To calculate the area of the 4th and 10th triangles. What do we need to know?

Students: The lengths of the 2 sides of the right angle of the 4th and 10th triangles.

Teacher: How do you calculate the lengths of the two sides of the right angle?

Students: With the given measures of sides AB and BC of triangle ABC, right-angled at B, the side AC can be calculated based on the Pythagorean theorem. Then, calculate the lengths of the hypotenuses of the 2nd, 3rd, 4th right triangle, respectively. Finally, calculate the area of the 4th triangle. For the 10th triangle, do the same.

Teacher: The above calculation is manual and takes a lot of time. Instruct students to use the software to get instant results without calculations.

Select the line icon and connect A and C,... (or use the command Line segment $\langle A,C \rangle, \dots$), do the same with the remaining lines, and then the lengths of the hypotenuses will appear on the display area. Thereby, it is possible to verify the results with the above calculation.

Students: Monitor, pay attention to follow the teacher's instructions on GeoGebra

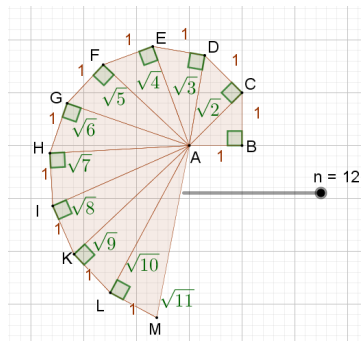
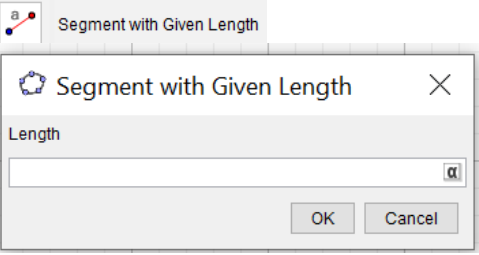
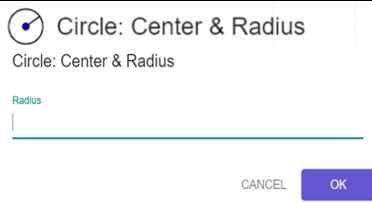
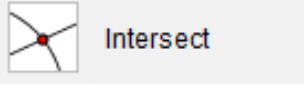


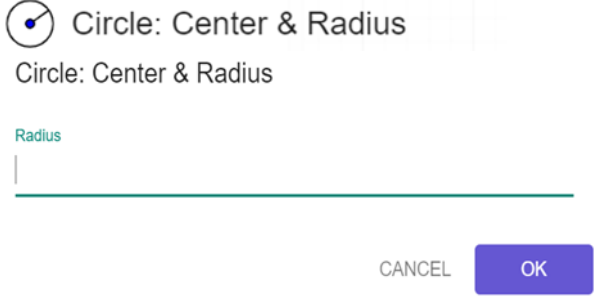
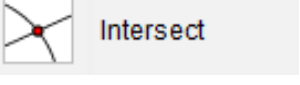
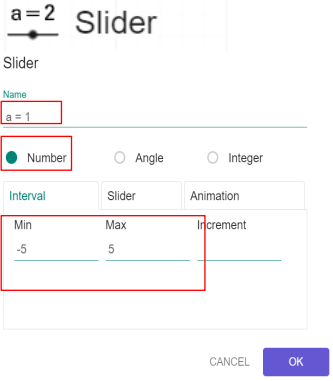
Figure 6. Constructing shapes and calculating the measures of the hypotenuses of right triangles evolved from the Pythagorean theorem (Source: Personal Collection).

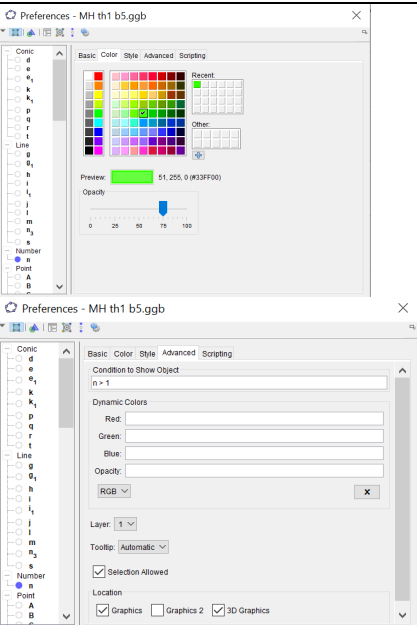
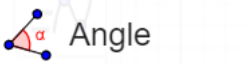
Teacher:

- In addition to using GeoGebra to draw shapes and verify results, GeoGebra software can also illustrate Pythagorean theorems to create decorative works.
- Instruct students to draw Figure 6, decorate with colors like Figure 7 on GeoGebra
- With Figure 7, it is possible to draw a lot more triangles, in the picture the teacher only draws 10 triangles as a representative. Ask students to do the same to draw 5 more triangles in Figure 7.

Students: receive and follow the teacher's instructions to draw Figure 7.

Ordinal number	Shapes	Steps
1	 Line tool group.	Click on the line tool group
2		Select the icon: Line with a given length Then, select any point, enter the length: 1. We get $AB = 1\text{cm}$
3	 Relationship tool group	Click on the group of relationship tools
4	 Perpendicular Line	Select the icon: Perpendicular line. Then select line AB and point B.
5	 Circle and arc tool group	Click on the circle and arc tool group
6		Select the icon: Circle when the center and radius are known. Then select point B and enter radius as 1 and select OK. We have a circle with center B and a radius of 1cm.
7	 Point creation tool group	Click on the point creation tool group.
8	 Intersect	Select the icon: The intersection of 2 objects. Then choose a line perpendicular to AB and a circle with center B with a radius of 1cm. We get the intersection.
9	 Point	Select the icon: New point Then click to select the intersection of the line perpendicular to AB and the circle with center B and radius 1cm. We get point C. Select a line perpendicular to AB, a circle with center B and radius 1cm, right-click and select show object.
10	 Relationship tool group	Click on the group of relationship tools
11	 Perpendicular Line	Select the icon: Perpendicular line. Then select the line AC and point C.

12	 Circle and arc tool group	Click on the circle and arc tool group
13	 Circle: Center & Radius Circle: Center & Radius Radius CANCEL OK	Select the icon: Circle when the center and radius are known. Then select point C and enter radius as 1 and select OK. We have a circle with center C and a radius of 1cm.
14	 Point creation tool group	Click on the point creation tool group.
15	 Intersect	Select the icon: The intersection of 2 objects. Then choose a line perpendicular to AC and a circle with center C and radius 1cm. We get the intersection.
16	 Point	Select the icon: New point Then click to select the intersection of the line perpendicular to AC and the circle with center C and radius 1cm. We get point D. Select the line perpendicular to AC, a circle with center C and radius 1cm, right-click and select display object.
Performing the same commands as steps from step 10 to step 16 9 times continuously, we will get 10 triangles as shown in Figure 5.		
17	 Other tool group	Click on the other tool group
18	 Slider Slider Name a = 1 Number Angle Integer Interval Slider Animation Min Max Increment -5 5 CANCEL OK	Select the icon: Slider Then fill in: Name: n Select: Number Minimum: 0 – Maximum: 20 – Increment: 1 (Depending on the value of the given topic, adjust accordingly) Then click OK, we get the slider as shown
19	 Triangle t1 = 0.5	Select t1 = 0.5 in the display area, right-click and select properties => Color => Customize according to your needs. Then, select the advanced item, the line condition to show object: enter

		$n > 1$. (second triangle: $n > 2$..., 10th triangle: $n > 10$) Do the same with the remaining triangles. Then select the right angle of the first triangle in the display area, right-click, select properties. Then, select the advanced item, the line condition to show object: enter $n > 1$. (right angle of 2nd triangle: $n > 2$..., 10th triangle: $n > 10$) Do the same with the remaining right angles.
20		Select the icon: Angle with a given magnitude Then choose the points C - B - A in turn, we get the angle CBA equal to 90°. Do the same with the remaining triangles.
21	In the display area, select all objects, right-click and select show objects to hide all.	
22		Right click on the slider, select Animation On, we get the triangles as shown in illustration 2.7

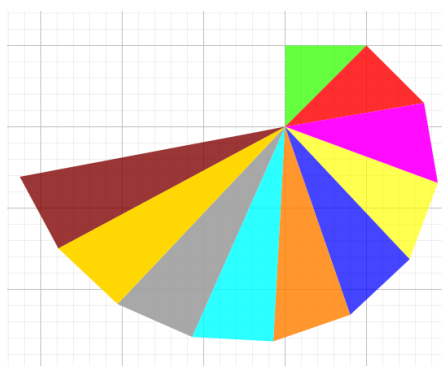


Figure 7. Decorative construction of right triangles developed from Pythagoras' theorem (Source: Personal Collection).



Figure 8. Illustration of Pythagoras theorem on right triangles in a spiral shape to create decorative patterns on canvas (Source: Freepik).

DISCUSSION

To test the feasibility and evaluate the effectiveness of the teaching method, we asked the teacher to give the students of the experimental class and the control class a 45-minute test. Quantitative analysis is based on test results.

We conducted a study of the two classes before the experiment through information from the homeroom teacher and the math teacher of the two classes. The result is that two classes have the same strength in math.

We used SPSS statistical software to conduct analysis during the experimental process.

We experimented in the school year 2020-2021 at Dong Nai Pedagogical Practical High School (Bien Hoa City, Dong Nai Province, Vietnam) following the basic curriculum in Vietnam.

- Experimental class 8A1 includes 26 students. Math teacher: Nguyen Thuy Hong Anh.
- The control class 8A2 consists of 25 students. Math teacher: Bui Thi Giang.

We obtain the frequency distribution table of experimental math scores of the experimental class and the control class as follows:

Table 1. Frequency distribution table of math test scores of two classes.

		Test scores												Total
		4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	
Classes	Experimental	0	0	1	0	2	3	5	6	4	2	2	1	26
	Control	1	1	2	3	2	4	3	3	2	2	1	1	25
Total		1	1	3	3	4	7	8	9	6	4	3	2	51

We test the normal distribution and find that the experimental math score distribution is normally distributed.

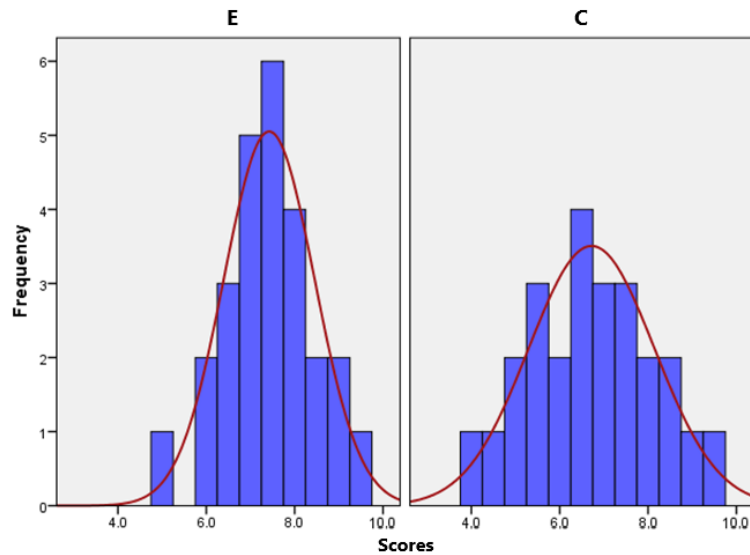


Chart 1. Distribution of math test scores of two classes.

From the data on table 1, we have chart 2 to compare the scores of the test results of the two classes:

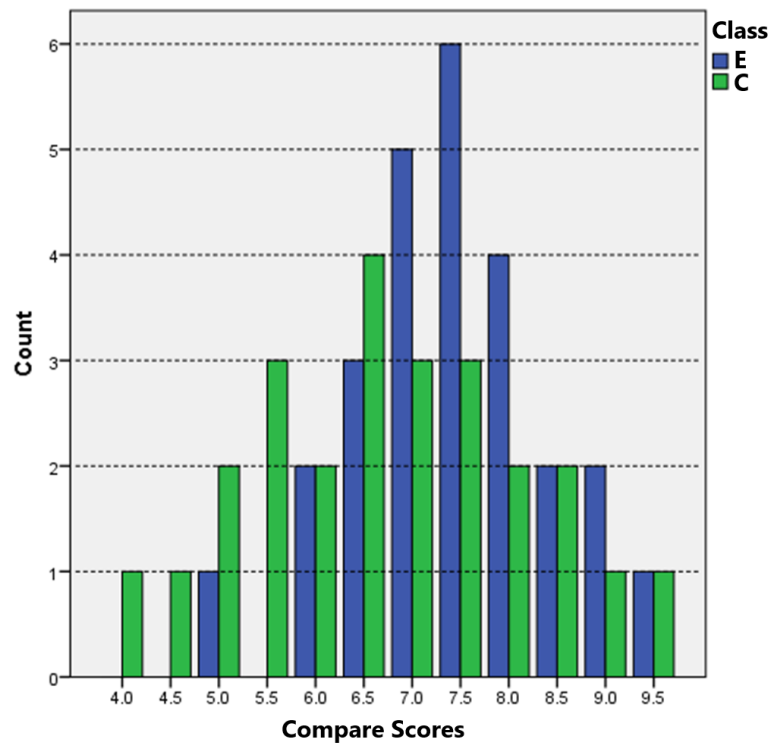


Chart 2. Column chart comparing test scores of two classes.

Looking at chart 2, we can see that the height of the score columns and the score distribution of the two classes have been different. The scores of the experimental class are from 5-9.5 points or more and most are 6.5-8 points. The score distribution of the control class is from 4-9.5 points and the majority is from 5.5-7.5.

In addition, we also obtain the characteristic parameters of statistics as in Table 2.

Table 2. Table of statistical characteristic parameters of test scores of two classes.

Class	Mean	Variance	Std. Deviation
Experimental	7.42	1.05	1.03
Control	6.72	2.02	1.42

Reading the above test results, we have the average score of the two classes:

$$\overline{X}_{TN} = 7.42; \overline{X}_{DC} = 6.72. \text{ Variance: } S_{TN}^2 = 1.05; S_{DC}^2 = 2.02.$$

Comment: there is a big difference between the mean scores and the variance of the two classes. The average score of the experimental class is higher; The standard deviation and variance of the experimental class are lower, so the concentration around the mean is higher than that of the control class. We can say that the score of the experimental class is higher than that of the control class.

In order to accurately assess the difference (or the big and small) between the mean scores of the two experimental classes, we test the mean between the two classes, with the significance level $\alpha = 0,05$ through Table 3 with the following two assumptions:

Assumption H_0 : "The mean scores of experimental and control classes are similar".

H_1 : "The average score of the experimental class is higher than that of the control class".

Table 3. Table of average T-test of test scores of two classes.

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Experimental test score	Equal variances assumed	3.244	.078	2.030	49	.048	.7031	.3463	.0072	1.3990
	Equal variances not assumed			2.018	43.573	.050	.7031	.3485	.0006	1.4056

- Levene test has the value $Sig. = 0.078 > \alpha = 0,05$, so the variances of the two classes are almost equal, using the results of the Independent-samples T-test corresponding to the case where the equal variances of the two samples are assumed.

- Independent-samples T-test, we have $Sig.(2-tailed) = 0.048 < \alpha = 0.05$, so we reject hypothesis H_0 , accept hypothesis H_1 . Therefore, the mean score of the experimental class is higher than that of the control class at the 5% level of significance.

Thus, by the test method between classes with equivalent academic ability, the results show that the experimental class after being taught according to the experimental lesson plan has better results. Its average score is higher than that of the control class. It can be seen that the teaching method used for the experimental class is completely feasible and effective in teaching.

CONCLUSION

Discovery teaching about calculating the length of a right triangle's side using the Pythagorean theorem with the help of GeoGebra software is a way of active teaching and promoting the qualities and capacities of learners. When learners participate in the learning process, they will create knowledge for themselves. Therefore, this teaching method will help students remember longer than the conventional traditional teaching method. Learners through each learning task, with their own dose, will reveal all their advantages as well as overcome their own limitations. In discovery teaching, teachers are companions, guides, and facilitators for students, not the one-way transmission of knowledge to students as before. Teachers through appropriate leading questions will lead students to the knowledge they need to achieve. Compared with other conventional teaching methods, discovery teaching has outstanding advantages. However, discovery teaching also has disadvantages. Teachers will spend more effort than traditional teaching. Because the teacher must have each question, the teaching method meets each student's individuality, so the time spent is very large. For a class with a large number of students, it is difficult to accommodate all students in a short amount of time. In the article, we have come up with a discovery teaching method about calculating the side lengths of right triangles using the Pythagoras theorem with the help of GeoGebra software. By experiments, we found that the effectiveness of teaching is much higher than the teaching method that teachers dictate to students. Through data processing with SPSS software, we find that the teaching method is feasible.

References

- Vu Thi Lan Anh (2020). Discovery teaching of the topic of inequalities and systems of first-order inequalities in class 10. Master thesis in Mathematics pedagogy, Vietnam National University, Hanoi.
- Le Vo Binh (2007). Teaching geometry in the last grades of lower secondary school in the direction of a discovery teaching method. Doctoral Thesis in Educational Sciences, Vinh University.
- Vu Huu Binh (2014). *Fostering Mathematics 7*. Vietnam Education Publishing House.
- Emily Brown (2006). Discovery Learning in the Classroom. *Science and Skepticism*.
- Jerome S Bruner (1960). *The Process of Education*. Harvard University
- Joyce Castonova (2002). Discovery Learning for the 21st Century: What Is It and How Does It Compare to Traditional Learning in Effectiveness in the 21st Century. *Action Research Exchange* 1(1):1-12.
- Ryan D Chen, Honomichl and Zhe (2012). The Role of Guidance in Children's Discovery Learning. *Focus Article* 3:615-22.
- Multi Efrini (2016). Discovery Learning Method Based on Scientific Approach in Curriculum 2013 in Teaching English at Senior High School. in *Proceeding International Seminar on Education 2016 Faculty of Tarbiyah and Teacher Training*.
- Fathiya Salsabila, Rahmah Johar and Susanti Panca Wahyuni (2017). Students' Ability in Proving Pythagorean Theorem through Discovery Learning Model Using Geogebra Software in *Proceedings of The 7th Annual International Conference (AIC) Syiah Kuala University and The 6th International Conference on Multidisciplinary Research (ICMR) in conjunction with the International Conference on Electrical Engineering and Informatics (ICELT)*.

- Tran Ba Hoanh (2004). Teaching Through Guided Discovery Activities. *Educational Science Information* (102).
- Markus Hohenwarter and Judith Hohenwarter (2008). Introduzione a GeoGebra. <http://www.geogebra.org/book/intro-it.zip>
- Nguyen Thi Huong and Le Tuan Anh (2018). Exploiting GeoGebra Software in Some Teaching Cases of Grade 9 Math. *Education Magazine*, June 2018 Special Issue 163–67; 114.
- Tram Huy Khoi (2019). Teaching the topic of spatial geometry with the help of GeoGebra software. Master Thesis of Science in Education, Dong Thap University.
- Le Viet Minh Triet (2018). Using Geogebra According to the Case Theory Approach in Teaching Flat Geometry in High Schools. *Education Magazine*, Issue 425(March 1, 2018)) 44–46.
- Nguyen Phu Loc (2010). Teaching Exploring Math Concepts. *Journal of Science*, 14–16.
- Dariusz Majerek (2014). Application Of Geogebra For Teaching Mathematics. *Advances in Science and Technology Research Journal* 8:51–54.
- Richard E Mayer (2004). Should There Be a Three-Strikes Rule Against Pure Discovery Learning? *American Psychologist* 59:14–19.
- Nguyen Danh Nam (2012). GeoGebra with an Interactive Help System Generates Abductive Argumentation during Proving Process. *North American GeoGebra Journal* 1. <https://www.researchgate.net/publication/262839935>
- Nguyen Ngọc Giang and Tran Trung. 2019. Teaching Geometry According to the Discovery Method with GeoGebra Software. *Advances in Social Science, Education and Humanities Research* 400.
- Pho Duc Hoa and Ngo Duy Son (2008). *Application of Information Technology in Active Teaching*. Education publisher.
- Ringgi Candraning Prawerti (2014). The Effectiveness of Using Discovery Learning Method in Teaching Writing Skill Viewed from the Students' Creativity. English education department graduate school, Sebelas Maret University.
- Judith Preiner (2008). Introducing Dynamic Mathematics Software to Mathematics Teachers: The Case of GeoGebra. Dissertation in Mathematics Education Faculty of Natural Sciences, University of Salzburg.
- Tika Karlina Rachmawati (2016). An Analysis of Students' Difficulties in Solving Story Based Problems and Its Alternative Solutions. *Journal of Research and Advances in Mathematics Education* 1:140–53.
- Novriana Rahma Siagian (2018). The Effect of Applying Discovery Learning Method on the Students' Ability in Writing Descriptive Text at Second Grade of Smk Al-Washliyah 3 Medan. Department of english education of tarbiyah and teachers training of state islamic, University of North Sumatera Medan.
- Mochamad Roni Sobari and Hendra Husnussalam (2019). The Use of Discovery Learning Model to Improve The Students' Descriptive Writing. *Professional Journal of English Education* 2:2019.
- Soheila Belgheis and Rosemaliza Kamalludeen (2018). The Intention to Use GeoGebra in the Teaching of Mathematics Among Malaysian Teachers. *Malaysian Online Journal of Educational Technology* 6(1).
- Nilgün Suphi and Hüseyin Yaratın (2016). Effects Of Discovery Learning And Student Assessment On Academic Success. *He Turkish Online Journal of Educational Technology* 829–35.
- Trinh Duc Toan (2016). Exploiting GeoGebra Software to Support Discovery-Driven Grade 7 Geometry Teaching. Master of Mathematics Pedagogy, Northwestern University.
- Tracy Bicknell-Holmes and Paul S. Hoffman (2000). "Engage, Elicit, Experience, Explore: Applying Discovery Learning to Library Instruction. https://digitalcommons.unl.edu/library_talks/31
- Morgan K Williams (2017). John Dewey in the 21st Century. *Journal of Inquiry & Action in Education. Journal of Inquiry & Action in Education*, 9(1)