

Comprehensive Project Risk Management Methodology

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ABSTRACT

The objective of this paper is to introduce and discuss the basics of a methodology called the Probabilistic Critical Path Method (PCPM) for managing the previously identified risks (uncertainty) of the three project management dimensions: time, cost and return (performance). An interactive Graphic User Interface (GUI) has been designed for visualizing the tradeoffs among these three dimensions as well as their uncertainties on a flat computer screen. The user can choose to visualize the probability of failure (exceeding some user given due date, budget or not exceeding a given Minimally Attractive Rate of Return – MARR) or the probability of success (not exceeding the due date and the budget and exceeding the MARR). PCPM allows for comprehensive project risk management and it constitutes a new integrative project risk management framework. This paper shows that it is possible to integrate all three project management dimensions (time, cost and return) and show their known risks as well as determining the optimal cost and the associated time and return for such optimal cost. Finally, it is possible to interactively show all this multidimensional information on a flat computer screen.

Keywords Risk management; project management; decision support systems.

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INTRODUCTION

One of the most important tasks in project management, particularly in technology and information-based organizations, is to successfully manage risk. Thus, project risk management becomes critical for successful project management (Buganová & Šimíčková, 2019; Khameneh, Taheri, & Ershadi, 2016; Odimabo & Oduoza, 2018; Serpella, Ferrada, Howard, & Rubio, 2014; Szymański, 2017).

The methodology introduced in this paper, called the Probabilistic Critical Path Method (PCPM) allows to manage the three project management dimensions: time, cost and performance (the latter being measured by the Internal Rate of Return, or simply return, of the project) and meaningfully link them to the identified risks of each dimension in the form of the probability of failure or, alternatively, the probability of success. It integrates these dimensions by allowing to visually see their tradeoffs as well as the probability of failure, that is, of exceeding the due date and the budget and the probability of not exceeding the Minimally Attractive Rate of Return (MARR), or

alternatively, to visualize the probability of succeeding by not exceeding the due date or the budget and exceeding the MARR. A specially designed and interactive Graphic User Interface (GUI) for successful project risk management decision making was created and even the optimal cost can be calculated given some specific indirect project cost slope (m). This methodology allows for comprehensive project risk management. This paper explains the basics of PCPM. Calculations have been automated by creating a Decision Support System (DSS) called *Schedule*¹, which can be downloaded for free from www.copertari.net/schedule.

Managing Risk And Uncertainty

A great deal of project management involves good risk management. Risk management in a project can be defined as *the consequence of the existence of significant uncertainty concerning the level of project achievement* (Chapman & Ward, 1997). Tight time, cost or performance goals increase time, cost and performance risks, respectively. A risky situation is often considered as the existence of potentially high and unacceptable costs due to events considered more or less likely to happen. This negative approach to risk leads to the idea that risk management essentially deals with reducing or removing the possibility of under-achievement. Risk analysis is not a 'throwing the dice' situation, but rather an area of study where a proactive, creative and intelligent prior planning approach must be used, instead of entrenching in a defensive position (Adams, 2001; Dey, 2001; McManus, 2001; Schimmoller, 2001; Walker, 2001).

Within this context, it is important to distinguish between risk and uncertainty. Risk is the possibility or probability of failure, whereas uncertainty is the variability of the relevant outcomes for a given risk or eventuality. Brealey and Myers (1991) define risk saying that more things can happen (at present time) that will happen (in the future). Uncertainty, on the other hand, is the degree in which an identified threat or risk (at present time, after previous analysis) will (presumably, based on experience, historical data or assumptions) vary. Uncertainty is an identified (and quantified) risk. Even then, the degree in which such identified risk will vary is unknown. Uncertainty constitutes the 'known' unknowns because although a specific risk has been identified, its exact impact is still unknown. Unidentified risks are 'unknown' unknowns because, generally speaking, a risk is unquantified uncertainty about something not yet being considered possible as a future eventuality. It will be assumed throughout this paper that risk identification has been successfully and completely carried out and will focus attention on the risks due to the uncertainty for the most relevant variables previously identified by decision makers. Risk sources are any factors that may affect the project dimensions (time, cost and performance). Setting a tight time target such as an optimistic due date increases time risk. Likewise, an irrationally low budget increases cost risk and setting a minimum Internal Rate of Return (IRR) increases performance risk. On the other hand, setting slack times, emergency budget allocations or a lower IRR reduces time, cost and performance risks, respectively (Dawson, 1998; Farrell, 1996; Lefley, 1997; Tavares, 1998).

¹ The heuristic specially designed for *probabilistic crashing* is proprietary software technology developed for *Schedule*.

Systemic Relationships Between The Project Dimensions: Time, Cost And Return (Performance)

Although the relations among the project management dimensions vary from time to time and from project to project, a systemic approach can be used to elucidate the nature of the underlying balances (Icmeli, 1996; Johnson & Schou, 1990; Sunde & Lichtenberg, 1995).

Figure 1a illustrates the systemic relationships between time and cost using influence diagrams. If the project is delayed (it takes longer) will cost more, so that there is a positive correlation between time and cost. But if in order to deliver the project on time, additional resources are used for critical activities, maintaining resources to a minimum for non-critical activities (which is called *crashing*) there is a negative correlation between cost and time (Winston, 1994). The existence of both a positive and a negative correlation between time and cost implies the existence of a balance point in which an optimal project completion can be achieved at a minimum cost. Figure 1b illustrates how the time/cost balancing is additionally influenced by performance. Improving the quality of the product requires investing more resources, which will increase cost and increase time if those resources are limited. But if more resources are invested and it takes longer to complete the project, it costs more, so that the Internal Rate of Return (IRR) of the project measuring its profitability is reduced. As a consequence, there must also be an optimal balance between time/cost achieving an optimal performance as measured according to the project's IRR.

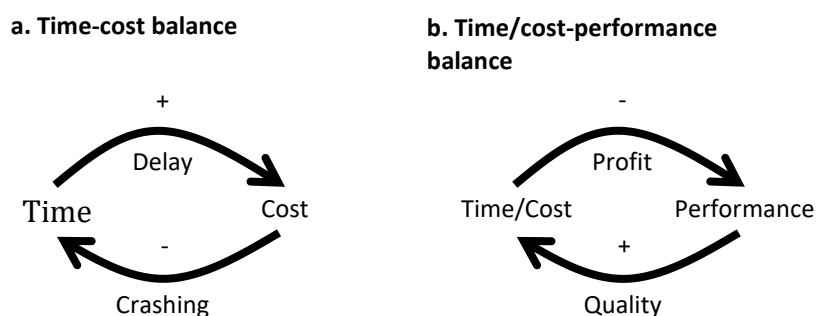


Figure 1. Balances among time, cost and performance.

Project Scope And Time, Cost And Performance Risks

The first and most important task in the management of a project in particular is determining its *scope*, that is, exactly what is intended to achieve with the project and exactly in what measure or proportion. From the scope determination, the different aspects of the project are planned. A procedure called *Work Breakdown Structure (WBS)* is used in order to identify the tasks and sub-tasks (or simply activities, being the total task the project itself) as well as activity precedence, that is, which activity should go before which other or if it has no precedence and it can begin at the start of the project. Two very important dimensions of the project are time and cost, which are closely linked by a procedure called *crashing*. An additional dimension is performance, although some speak about quality instead of performance. From the point of view of this paper, quality will be the degree of success in the performance of the project, and the performance will be measured using the Internal Rate of Return (IRR) of the project, for which the target is set based on the Minimally Attractive Rate of Return or MARR (Copertari, 2014).

The Probabilistic Critical Path Method (PCPM) considers the project and its dimensions from a probabilistic point of view. Thus, if the project is intended to be finished by a certain due date, the probability of success in time and the risk of not fulfilling the due date are illustrated in Figure 2a. Also, if the goal is to finish the project within a certain budget, project success in the cost dimension and cost risk are shown in Figure 2b. Finally, if the project goal is to reach a minimum MARR, performance success as measured by the project's IRR as well as the risk of not reaching the MARR are pictured in Figure 2c.

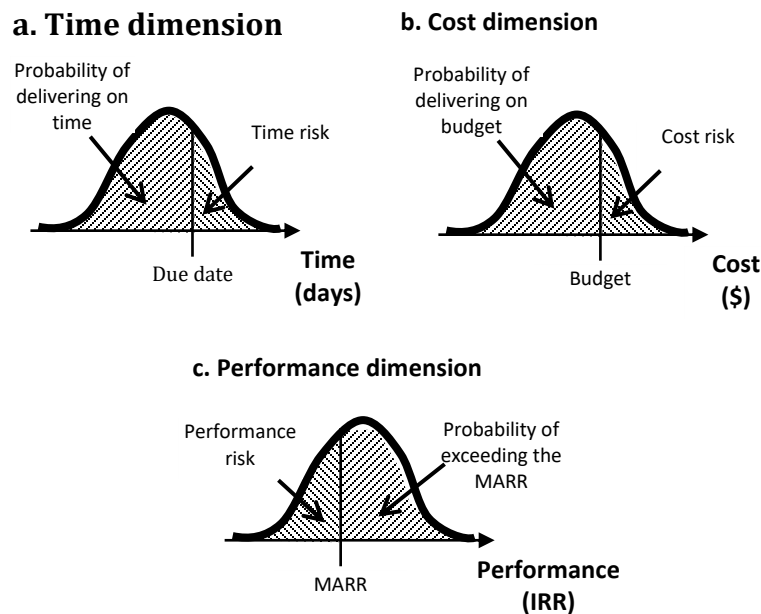


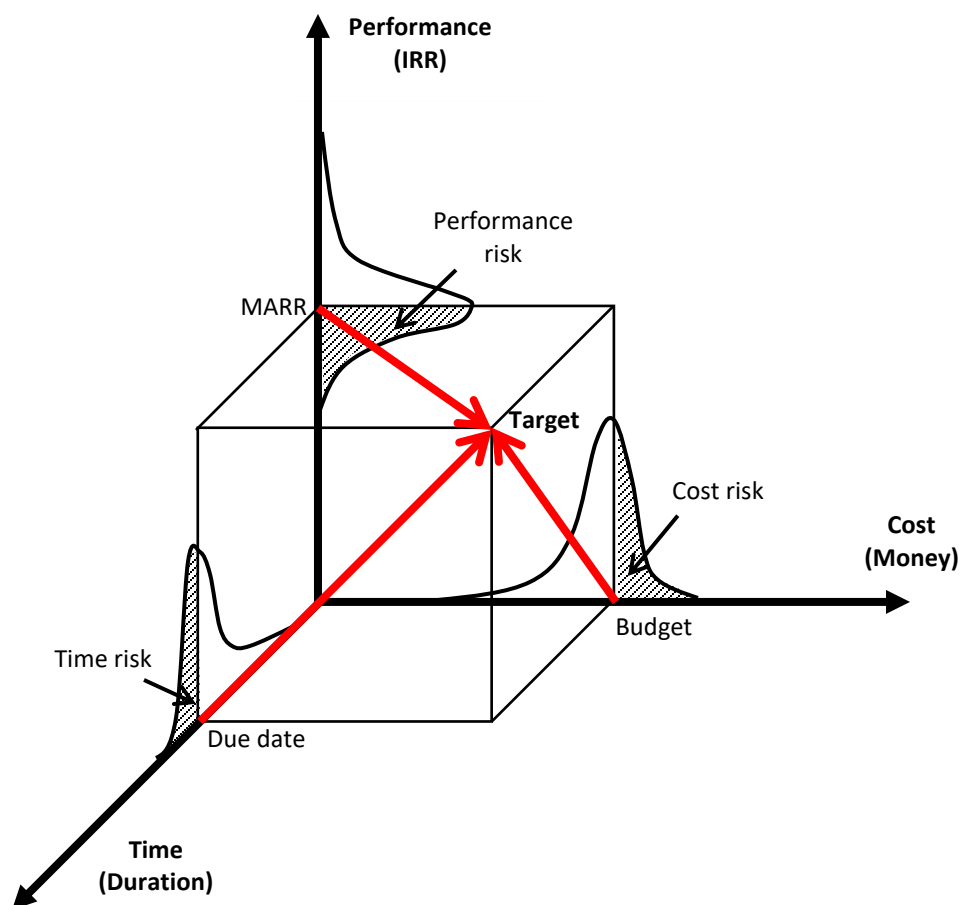
Figure 2. Project success and risk based on the time dimension (set by the due date), cost (set by the budget) and performance (set by the MARR and measuring performance according to the project's IRR).

The Targets Of Project Management

When we talk about project management, the most important aspects to consider in this area of study come up: project scope, time, cost and quality. Time and cost are clearly two of the most important dimensions of all projects and they are intimately connected. The quality of the project is something more difficult to measure. Generally speaking, it could be said that quality is satisfying the customer's expectations. However, who is the customer of a project? The project team members could be considered the customers, or the people receiving the project deliverables, or maybe the decision makers wishing the project to be successful so that they can innovate in their products or services or even the company stockholders wishing a successful project in order to receive a satisfactory return over their investment. It is precisely the latter point of view of quality measurement in a project, the one of the stockholders, being considered in order to define the way in which the project performance is measured. In this case, it is the Internal Rate of Return (IRR), or simply return, which is the rate of return over the investment the project yields, that is, that rate at which the sum of all cash flows is equal to zero, being those either positive (the final income to

obtain in a project) or negative (the different costs associated to each activity, being the activity critical or not and whether it is decided to crash it or not).

Figure 3. The dimensions of project management as an ideal balancing among the time, cost and performance dimensions.



In this way, for the purposes of this paper, there are three dimensions of the project to be measured: time, cost and performance. In PCPM time is measured probabilistically and the due date is used as a basis to indicate the project's time risk (see Figure 2a). The cost is also measured probabilistically and the budget is used as an evaluation point in order to know the probability of exceeding the budget, which is the cost risk. Finally, there is performance, measured as a dimension to exceed (while time and cost are dimensions not to exceed), having the IRR as indicator of performance and the MARR as the indicator of the threshold for success or failure. Figure 3 illustrates the three targets of project management and the way in which they can be visualized in a probabilistic three-dimensional chart.

THE SHAPE PARAMETERS EQUATION FOR THE BETA DISTRIBUTION

In general, we have that the mean (μ) and the variance (σ^2) can be calculated in practice according to equations (1) and (2), respectively, where the minimum, most like and maximum duration times are given as a , m , and b , respectively (Meredith & Mantel, 1995).

$$\mu = \frac{a+4m+b}{6} \quad (1)$$

$$\sigma^2 = \left(\frac{b-a}{6}\right)^2 \quad (2)$$

Also, the general formula for the mean and the variance of a beta probability density function with range (a and b) and shape (α and β) parameters (Hastings & Peacock, 1975) are given in equations (3) and (4), respectively.

$$\mu = \frac{a\beta+b\alpha}{\alpha+\beta} \quad (3)$$

$$\sigma^2 = (b-a)^2 \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \quad (4)$$

Since the values for μ and σ^2 are known from equations (1) and (2) based on PERT's three point estimates (a, m and b), equations (3) and (4) constitute a non-linear system of two equations with two unknowns (α and β), which can be solved. Equations (5) and (6) provide the solution to this system of non-linear equations, which yield the values for α and β , respectively.

$$\alpha = (\mu - a) \left(\frac{(\mu-a)(b-\mu)-\sigma^2}{(b-a)\sigma^2} \right), b > a \quad (5)$$

$$\beta = (b - \mu) \left(\frac{(\mu-a)(b-\mu)-\sigma^2}{(b-a)\sigma^2} \right), b > a \quad (6)$$

Equations (5) and (6) are novel and specific findings of the Probabilistic Critical Path Method (PCPM). What is the practical relevance of equations (5) and (6)? Now, it is possible to correctly calculate the shape parameters (α and β) of any activity or any variable described using a beta probability density function based on the minimum (a), maximum (b), mean (μ) and variance (σ^2) values. In turn, the mean (μ) and the variance (σ^2) values can be calculated using equation (1) and (2), respectively, which are based on three-point estimates for minimum (a), most likely (m) and maximum (b) values related to the variable being modelled using the beta probability density function.

THE STARTING OF ACTIVITIES IN A PROJECT MANAGEMENT NETWORK

Let k be any activity and N be the total number of activities in a project management network. In order to calculate the projects' IRR (or simply return) it is necessary to know when to apply activity costs. The problem with CPM is that there is no specific starting or finishing activity completion times if they have a slack greater than zero. Let d_k , ES_k , EF_k , LS_k , LF_k and S_k be a specific duration, early start, early finish, late start, late finish and slack for any activity k , where $k = 1, 2, \dots, N$. Also, let $C_k = \{1,0\}$ indicate with a 1 if the activity is critical and with a 0 if it is not. A critical activity delays the entire project, so they have a slack of zero. However, for non-critical activities, the slack is greater than zero. The problem is how to specify when non-critical activities start. Let $Start_k$ (or simply St_k) be the starting of activity k , where $k = 1, 2, \dots, N$. The user decides how optimistic ($0 \leq O \leq 1$) or pessimistic ($0 \leq P \leq 1$) wants to be. A completely optimistic user ($O = 1$) would start the activity at its early starting time (ES_k), whereas a completely pessimistic user ($P = 1$) would start the activity at its latest starting time (LS_k). Clearly, $O+P = 1$ and combinations of O and P different than one and zero are possible. Equation (7) indicate how to calculate the starting time ($Start_k$ or simply St_k). Notice that equation (8) is also satisfied.

$$Start_k = O \times ES_k + P \times LS_k \quad (7)$$

$$\text{Start}_k = \text{LF}_k - 0 \times S_k \quad (8)$$

EXAMPLE FOR TESTING THE METHODOLOGY

We are going to use a complex enough example of 9 activities in order to test the methodology. The example is shown in Table 1. It includes activity ID (k), activity name, activity precedence, minimum (a), most likely (m) and maximum (b) activity duration times, as well as the normal (CA) and crashed (CB) costs. This is data required by *Schedule* in order to automate computations. The elements of the Activity on Node (AON) notation used are illustrated in Figure 4.

ES _k	d _k	EF _k
k Name		C _k
LS _k	S _k	LF _k

Figure 4. Elements of the AON notation for project management network calculations.

Table 1. Complex enough PCPM example to test the methodology proposed.							
Activity		Precedence	Duration (days)			Costs	
ID (k)	Name		a	m	b	Normal (CA)	Crashed (CB)
1	A	-	0.5	2.125	3	\$100	\$200
2	B	-	1	3.25	4	\$50	\$250
3	C	1	2	8	8	\$300	\$400
4	D	1	0.5	3.375	4	\$150	\$350
5	E	1,2	0.5	4.625	5	\$200	\$500
6	F	1,2	3	9	9	\$50	\$100
7	G	4,5	2	8	8	\$120	\$180
8	H	3	1	7	7	\$200	\$300
9	I	6	0.5	5.875	6	\$75	\$150
						\$1,245	\$2,430

CALCULATING RANGE PARAMETERS (A AND B) AND MEAN ACTIVITY DURATION TIMES

Calculating the minimum (a), maximum (b) and mean (μ) activity duration times is very easy and can be done in the same way PERT does it. Figure 5, Figure 6 and Figure 7 show the project network minimum (a), maximum (b) and mean (μ) PCPM activity completion times calculations, respectively.

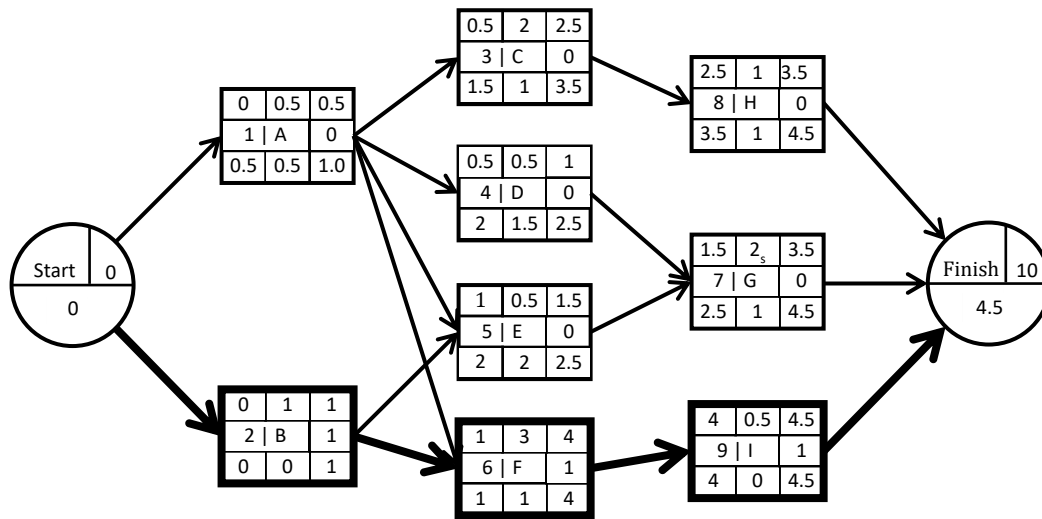


Figure 5. Minimum activity completion time calculations for the testing example.

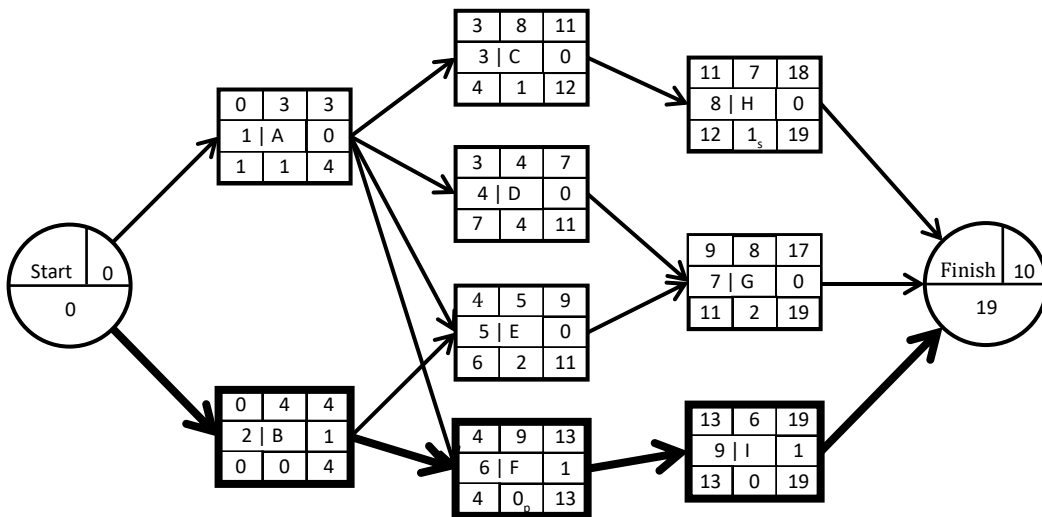


Figure 6. Maximum activity completion time calculations for the testing example.

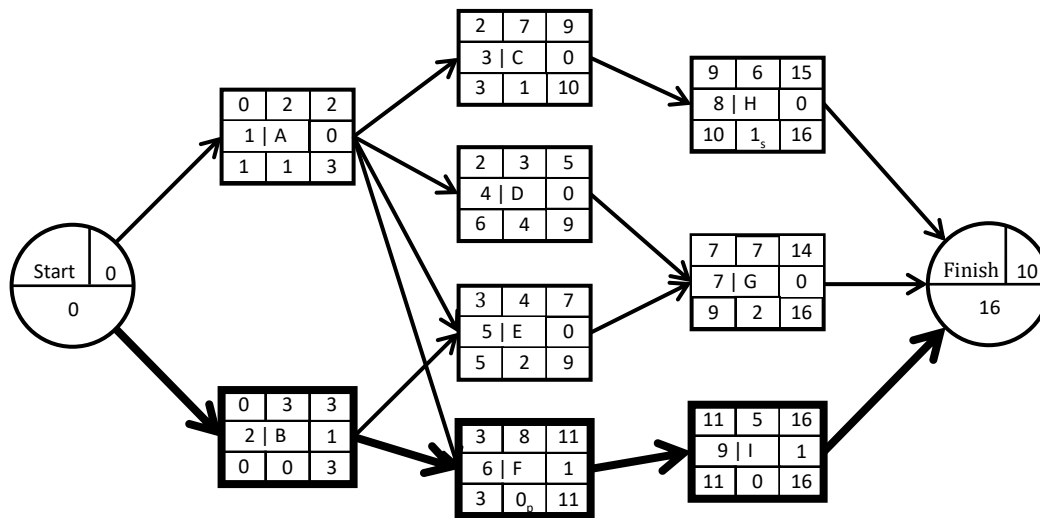


Figure 7. Mean activity completion time calculations for the testing example.

In Figures 5, 6 and 7, the critical path resulting in each case is highlighted using bold lines. The only thing missing in order to be able to calculate beta distributed completion times is the variance (σ^2) in order to be able to successfully apply equations (5) and (6). However, we are not going to use the PERT approach based on assuming the stochastic completion times of the activities to be represented using the mean activity duration time (μ) and CPM calculations. This is because we want more than just a probabilistic project completion PERT time, which assumes a normally distributed completion time that tends to underestimate the actual completion time, because we also want the probabilistic beta distributed completion times of all the activities.

Criticality and Project Completion Variance

Now we need a new concept, called *criticality*. Criticality in a PCPM network is the probability for each activity to be critical. Thus, C_k is no longer either zero or one, but a number between zero and one, since activities now have a given probability of being critical. For that, we are going to use simulation. Let s be the total number of simulation runs, $C_{k,i}$ be the criticality obtained for activity k ($k = 1, 2, \dots, N$) in iteration i ($i = 1, 2, \dots, s$), σ^2 the variance in the completion time for the whole project, σ_k^2 the variance of the completion time for activity k , path $\mathbf{p}$ be the set of all the activities (non-repetitive) of all the alternative project network paths in which activity k exists. Then, equations (9), (10) and (11) apply.

$$C_k = \frac{1}{s} \sum_{i=1}^s C_{k,i}, k = 1, 2, \dots, N, \text{ where } C_{k,i} = \begin{cases} 1, & \text{if the activity is critical} \\ 0, & \text{if the activity is not critical} \end{cases} \quad (9)$$

$$\sigma^2 = \sum_{k=1}^N C_k \sigma_k^2, 0 \leq C_k \leq 1 \quad (10)$$

$$\sigma_k^2 = \sum_{h \in \mathbf{p}} C_h \sigma_h^2, 0 \leq C_k \leq 1 \quad (11)$$

For the testing example shown in Table 1, a total of 1,000 runs and 10 re-runs were carried out. Table 2 shows the results. Notice that a slack is zero if it is between a zero value of ± 0.0001 .

Table 2. Criticality example using simulation (s = 10,000, r = 10) where the activity is critical if the slack is approximately zero.

k =	1	2	Block 1	3	4	5	6	Block 2	7	8	9	Block 3
r	C ₁	C ₂	C ₁ +C ₂	C ₃	C ₄	C ₅	C ₆	C ₃ +C ₄ + C ₅ +C ₆	C ₇	C ₈	C ₉	C ₇ +C ₈ + C ₉
1	32.5%	67.5%	100.0%	29.1%	0.0%	6.5%	64.5%	100.0%	6.5%	29.1%	64.5%	100.0%
2	31.9%	68.1%	100.0%	28.6%	0.1%	6.5%	64.9%	100.0%	6.5%	28.6%	64.9%	100.0%
3	32.7%	67.3%	100.0%	28.7%	0.1%	5.8%	65.4%	100.0%	5.9%	28.7%	65.4%	100.0%
4	32.4%	67.6%	100.0%	28.9%	0.0%	6.0%	65.1%	100.0%	6.0%	28.9%	65.1%	100.0%
5	32.9%	67.1%	100.0%	29.4%	0.0%	6.3%	64.3%	100.0%	6.3%	29.4%	64.3%	100.0%
6	32.3%	67.7%	100.0%	28.7%	0.0%	5.8%	65.4%	100.0%	5.9%	28.7%	65.4%	100.0%
7	32.4%	67.6%	100.0%	28.8%	0.0%	6.1%	65.1%	100.0%	6.1%	28.8%	65.1%	100.0%
8	32.4%	67.6%	100.0%	28.9%	0.0%	6.2%	64.9%	100.0%	6.3%	28.9%	64.9%	100.0%
9	32.8%	67.2%	100.0%	29.4%	0.0%	6.9%	63.6%	100.0%	7.0%	29.4%	63.6%	100.0%
10	31.7%	68.3%	100.0%	28.2%	0.0%	6.5%	65.3%	100.0%	6.5%	28.2%	65.3%	100.0%
C _k	32.4%	67.6%	100.0%	28.9%	0.0%	6.3%	64.9%	100.0%	6.3%	28.9%	64.9%	100.0%

The advantage of using criticality is that now we have beta distributed probabilistic completion times for selected activities marked as milestones in the project and not only a normally distributed probabilistic completion time.

THE OPTIMAL COST WITH UNLIMITED RESOURCES

Let the project completion time be denoted as T. The result of applying the probabilistic crashing heuristic is a series of points beginning with the maximum project completion (T = b) having the minimum overall cost (CA or U) and proceeding in a series of iterations until the minimum project completion (a) is reached having a maximum project cost (CB or V). For simplicity, crashing can be assumed to be an inversely proportional relationship between time and cost (Babu & Surech, 1996; Foldes & Soumis, 1993). Notice that crashing refers to a CPM technique, whereas probabilistic crashing refers to a PCPM heuristic technique specially designed for the PCPM methodology.

Generally speaking, however, it can be said that there are two kinds of costs: direct and indirect costs. Direct costs, which are inversely proportional to the project completion time (T) include the cost of material, equipment, and direct labor required to perform an activity, although in this case we will consider the entire project. Indirect costs include in addition to supervision and other overhead costs, interest charges on the cumulative project investment and overdue penalty costs (Auguston, 1993; Clark, 1992; Meredith & Mantel, 1995; Rivenbank, 2000; Tattersall, 1990).

Let D denote the direct costs of the whole project. Then, direct cost is inversely proportional to time (T), as indicated in equation (12).

$$D \propto \frac{1}{T} \quad (12)$$

In order to transform such proportionality relationship into an equality, proportionality constants are added, as indicated in equation (13).

$$D = \frac{\chi}{T} + \phi \quad (13)$$

In this case, there are for practical purposes, two direct costs: V, which is associated to time $T = a$, and U, which is associated with time $T = b$. Thus, equations (14) and (15) are created.

$$V = \frac{\chi}{a} + \phi \quad (14)$$

$$U = \frac{\chi}{b} + \phi \quad (15)$$

These system of two equations have four known values (after probabilistic crashing and time calculations): V, U, a and b and two unknowns: χ and ϕ . Thus, it can easily be solved. The solution for the unknown variables (χ and ϕ) are indicated in equations (16) and (17), respectively.

$$\chi = \frac{ab(V-U)}{b-a}, b > a \quad (16)$$

$$\phi = \frac{Ub-Va}{b-a}, b > a \quad (17)$$

Thus, substituting from equations (16) and (17) into equation (13) results in the direct cost expression, shown in equation (18).

$$D = \frac{ab(V-U)}{T(b-a)} + \frac{Ub-Va}{b-a} \quad (18)$$

The project indirect cost, I, is directly proportional to the project completion time (T), as indicated in equation (19).

$$I \propto T \quad (19)$$

The proportionality expression in equation (19) can be transformed into an equality adding proportionality constants. Let O denote the minimum overhead (indirect) cost at time $T = 0$, and m the increments (slope) of the indirect cost as a function of time. Then, equation (20) indicates the relationship between time and indirect cost.

$$I = O + m T \quad (20)$$

The total cost (C) is the result of adding the indirect (I) and the direct (D) costs, as indicated in equation (21).

$$C = I + D \quad (21)$$

Substituting I from equation (20) and D from equation (18) into equation (21) yields equation (22), which is the sum of the indirect and direct costs, or total cost.

$$C = (O + m T) + \left(\frac{ab(V-U)}{T(b-a)} + \frac{Ub-Va}{b-a} \right) \quad (22)$$

Taking the derivative of the total cost (C) with respect to time (T) from equation (22) and equating that to zero yields the optimal project completion time (T^*). The derivative is shown in equation (23).

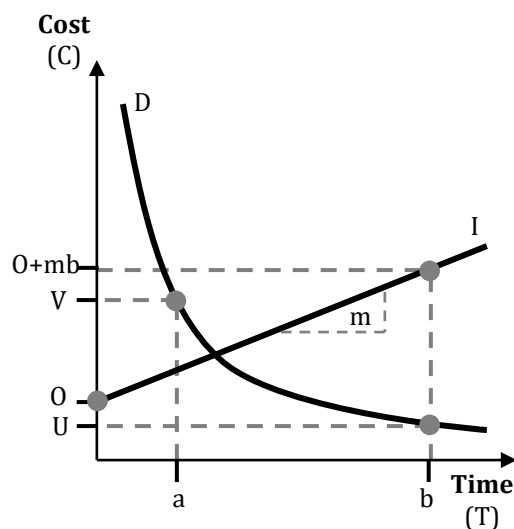
$$\frac{\delta C}{\delta T} = m - \frac{ab(V-U)}{T^2(b-a)} = 0 \quad (23)$$

Solving for T from equation (23) yields the optimal completion time (T^*) as indicated in equation (24).

$$T^* = \sqrt{\frac{ab(V-U)}{m(b-a)}} \quad (24)$$

Figure 8a illustrates the direct (D) and indirect (I) costs, whereas Figure 8b shows the total cost and it highlights the fact that the derivative, when equating to zero, would result in the optimal project completion time (T^*) for the optimal minimal total cost (C^*).

a. Direct and indirect project costs.



b. Optimal project cost.

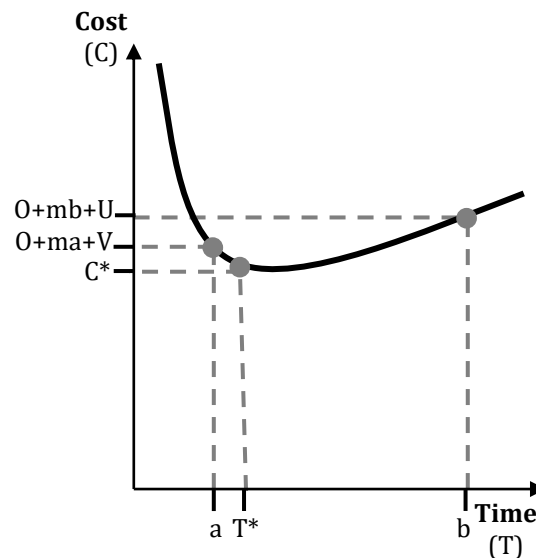


Figure 8. Time-cost tradeoffs.

TIME-COST TRADEOFF

When considering the total cost of the project, a concept that is central not only to CPM but also to PCPM is the time-cost tradeoff. When considering there are unlimited resources, it is possible in CPM as well as in PCPM to calculate a tradeoff between time and cost.

Generally speaking, the longer it takes to complete the project because fewer resources are used, the lower the cost of such doing is. If we use additional resources, it should be possible to reduce the time it takes to complete the project at the expense of incurring in additional costs. By following an optimal algorithm, a curve similar to the one corresponding to the direct costs in Figure 8a is generated. However, because there are too many activities in realistic projects, it is not possible to consider all alternatives of activity crashing. A project with 10 activities would have $2^{10} = 1,024$ crashing combinations to consider. A project with 20 activities would have $2^{20} = 1,048,576$ crashing combinations. *Schedule* allows to have a maximum of 254 activities. Considering 2^{254} crashing combinations is out of any practical algorithm that could be devised. Even a relatively realistic

project with 50 activities (2^{50} combinations) would mean disaster for a combinatorial algorithm to find the optimal time-cost tradeoff.

In any case, the theory holds that if such optimal time-cost crashing calculations could be performed, the result should look like the direct cost curve in Figure 8a. Let iteration $i = 0$ correspond to the case where there is no activity crashing at all having the maximum project completion time (b), and the maximum number of iterations (I) be the case where all activities are crashed. Clearly, iteration $i = I$ is reached only after the minimum project completion time (a) has been obtained in the previous iteration (that is, iteration $i = I-1$).

Let CA_k be the normal activity duration cost (corresponding to the maximum activity duration time b_k) and CB_k be the crashed activity duration cost (corresponding to the minimum activity duration time a_k) for activities $k = 1, 2, \dots, N$. Also, let time t_i be the completion time of the project for iteration i and cost c_i be the corresponding cost to such project completion time.

Figure 9 shows the ideal (optimal) time-cost tradeoff sequence of I iterations. For this case, the minimum project cost (CA) is calculated according to equation (25) and the maximum project cost (CB) is given according to equation (26).

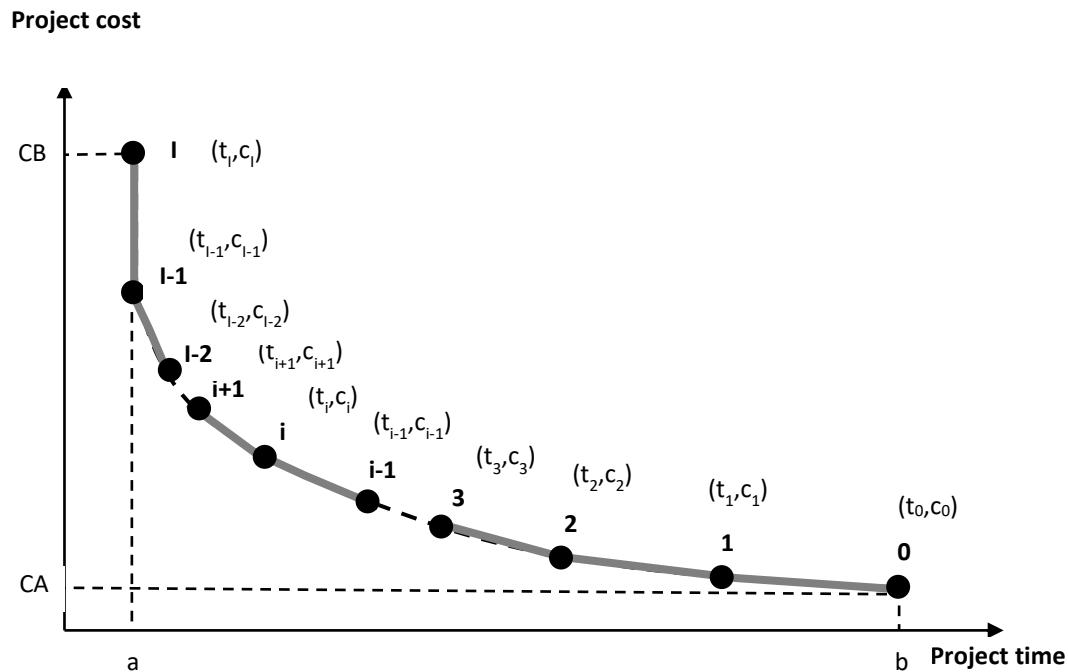


Figure 9. Ideal (optimal) time-cost tradeoff crashing sequence of I iterations.

$$CA = \sum_{k=1}^N CA_k, k = 1, 2, \dots, N \quad (25)$$

$$CB = \sum_{k=1}^N CB_k, k = 1, 2, \dots, N \quad (26)$$

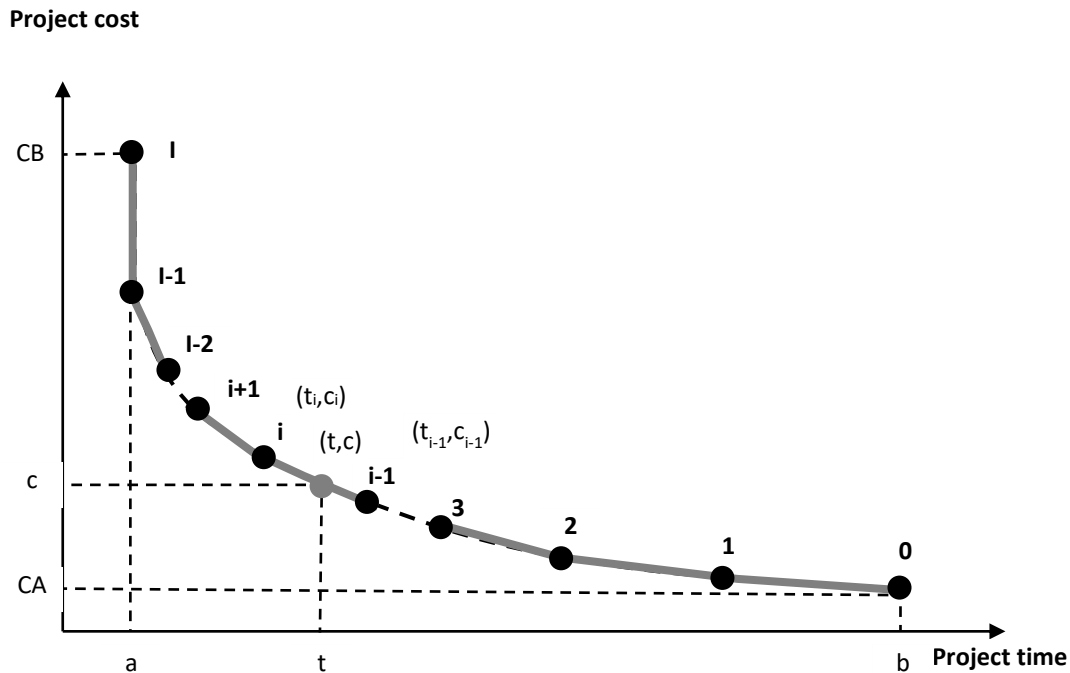
The variance in the cost of activity k ($C\sigma_k^2$) is given according to equation (27).

$$C\sigma_k^2 = \left(\frac{CB_k - CA_k}{6} \right)^2, k = 1, 2, \dots, N \quad (27)$$

The only information we need to be able to apply the beta distribution shape parameters equations is the mean cost. For that, it is necessary to interpolate the mean cost ($C\mu$) corresponding to the mean project completion time (μ).

Now, we need to consider the case where there is a time t corresponding to a cost c between iterations i and $i-1$. Equation (28) calculate the interpolated cost c corresponding to an interpolation time t , whereas equation (29) yield the interpolated time t corresponding to an interpolation cost c . Figure 10 illustrates the situation.

Figure 10. Ideal (optimal) time-cost tradeoff crashing sequence of I iterations for a given interpolation time and cost (t, c).



$$c = c_i - \frac{(t_{i-1} - t)(c_i - c_{i-1})}{t_i - t_{i-1}}, c_i \geq c > c_{i-1} \text{ and } t_i \leq t < t_{i-1} \quad (28)$$

$$t = t_i + \frac{(c_i - c)(t_{i-1} - t_i)}{c_i - c_{i-1}}, t_i \leq t < t_{i-1} \text{ and } c_i \geq c > c_{i-1} \quad (29)$$

Thus, having a mean interpolation time μ , it is possible to calculate the interpolated cost $C\mu$ according to equation (30).

$$C\mu = c_i - \frac{(t_{i-1} - \mu)(c_i - c_{i-1})}{t_i - t_{i-1}}, c_i \geq C\mu > c_{i-1} \text{ and } t_i \leq \mu < t_{i-1} \quad (30)$$

Now, having the minimum project cost (CA), maximum project cost (CB), the mean project cost (C_μ) and the variance project cost ($C\sigma^2$), we can calculate the shape parameters ($C\alpha$ and $C\beta$) for the corresponding beta distributed cost probability density function as given by equations (31) and (32).

$$C\alpha = (C_\mu - CA) \left(\frac{(C_\mu - CA)(CA - C_\mu) - C\sigma^2}{(CB - CA)C\sigma^2} \right), CB > CA \quad (31)$$

$$C\beta = (CB - C_\mu) \left(\frac{(C_\mu - CA)(CB - C_\mu) - C\sigma^2}{(CB - CA)C\sigma^2} \right), CB > CA \quad (32)$$

THE RETURN (PERFORMANCE) DIMENSION

The Net Present Value (NPV) and the Internal Rate of Return (IRR) or simply return

Let T be the project duration for certain iteration i ($i = 0, 1, 2, \dots, I$) and the Start_k (St_k as a shortcut) the beginnings of each activity for $k = 1, 2, \dots, N$. It is assumed a certain cost k (c_k) occurs at the beginning of each activity (at time St_k). Also, suppose there is an income, which is assumed to be collected at the end of the project (at time T for each time-cost tradeoff iteration). The costs of each activity are the ones collected for such iteration. Finally, such income (Income) is fixed and should in principle (but not necessarily) be greater than the maximum project cost (CB). It is initially assumed that the Internal Rate of Return (IRR) is, as a consequence, positive. In order to simplify calculations, let X denote the value for $1 + \text{IRR}$, as indicated in equation (33).

$$X = 1 + \text{IRR} \quad (33)$$

Then, the IRR is the rate at which the Net Present Value (NPV) equals zero. The equation that explains the NPV calculation when it is zero is shown in equation (34).

$$\text{NPV} = 0 = \text{Income} - \sum_{k=1}^N c_k X^{T - \text{St}_k} \quad (34)$$

Equation (34) obtains the value of X when the NPV = 0. From that X, based on equation (33), we can calculate the value of the IRR. It is assumed there is one IRR for the project, regardless of the duration units (days, weeks, months or years). This is because calculating the annualized rate of return brings additional complications. Thus, we have a function of X, $f(X)$, indicated in equation (35).

$$f(X) = \text{Income} - \sum_{k=1}^N c_k X^{T - \text{St}_k} \quad (35)$$

We need to find the value (and there should be only one because there is one sign change only in the cash flows) for which $f(X) = 0$. For that, we rely on the Newton-Raphson method of assured convergence (Burden & Faires, 1985). Beginning with any given value for X (such as 1.5, for example), we proceed from such initial value ($X_0 = 1.5$ for $j = 0$) and continue to the next iteration. The process for each new iteration j is indicated in equation (36).

$$X_{j+1} = X_j - \frac{f(X_j)}{f'(X_j)} \quad (36)$$

We can see that equation (36) requires us to calculate the derivative of the function $f(X_j)$, that is $f'(X_j)$. That can easily be done by derivation of equation (35), since X is the unknown variable, which is shown in equation (37).

$$f'(X) = - \sum_{k=1}^N c_k (T - St_k) X^{T-St_k-1} \quad (37)$$

The rate of return obtained (IRR) does not necessarily increases as the time-cost tradeoff iterations proceed, because the result obtained according to *Schedule's* heuristic is not necessarily convex. Thus, we take the minimum such rate of return and call it RA as well as the maximum rate of return and call it RB . Let RR be the rate of return corresponding to some given time completion value t . Thus, we can relate the rate of return RR to a given time t and calculate a given time t for a given rate of return RR , as indicated in equations (38) and (39), where a is the minimum (all crashed) project completion time and b the maximum (normal) project completion time.

$$RR = RA + \frac{(t-a)(RB-RA)}{b-a}, b > a \quad (38)$$

$$t = a + \frac{(RR-RA)(b-a)}{RB-RA}, RB > RA \quad (39)$$

The mean rate of return ($R\mu$) is assumed to be the rate of return at which the mean project completion time (μ) occurs. Thus, we have equation (40).

$$R\mu = RA + \frac{(\mu-a)(RB-RA)}{b-a}, b > a \quad (40)$$

The return variance ($R\sigma^2$) is calculated according to the original PERT equation, which is shown in equation (41).

$$R\sigma^2 = \left(\frac{RB-RA}{6} \right)^2 \quad (41)$$

Since we have the minimum (RA), maximum (RB), mean ($R\mu$) and variance ($R\sigma^2$) of the IRR or simply return, we can calculate the beta shape parameters ($R\alpha$ and $R\beta$) according to equations (42) and (43).

$$R\alpha = (R\mu - RA) \left(\frac{(R\mu-RA)(RA-R\mu)-R\sigma^2}{(RB-RA)R\sigma^2} \right), RB > RA \quad (42)$$

$$R\beta = (RB - R\mu) \left(\frac{(R\mu-RA)(RB-R\mu)-R\sigma^2}{(RB-RA)R\sigma^2} \right), RB > RA \quad (43)$$

Testing The Theory With The Example From Table 1.

Table 3 shows the time (t_i), cost (c_i) and return (r_i) for each iteration (where $I = 15+1$) calculated for the example from Table 1 as given by *Schedule*.

Table 3. Time, cost and return calculations from *Schedule* for the realistic enough example shown in Table 1 where Income = \$2,500.

I	Time (t_i)	Cost (c_i)	Return (r_i)
0	19	\$ 1,245.0000	5.36516914%
1	18	\$ 1,253.3333	5.52892210%
2	17	\$ 1,278.3333	5.64622149%
3	15	\$ 1,348.3333	6.03133936%
4	13	\$ 1,418.3333	6.59978418%
5	12	\$ 1,458.6364	6.99933285%
6	11	\$ 1,498.9394	7.67486409%
7	10	\$ 1,692.8788	6.50907407%
8	9	\$ 1,732.8788	6.18065098%
9	8	\$ 1,839.5455	5.23776127%
10	7.5	\$ 1,892.8788	4.91881442%
11	7	\$ 1,926.2121	4.87586633%
12	6.5	\$ 1,959.5455	4.81307255%
13	6	\$ 2,029.7835	4.47847154%
14	4.5	\$ 2,260.9524	3.03750096%
15	4.5	\$ 2,430.0000	0.94333776%

Figure 11 shows the time-cost tradeoffs and Figure 12 shows the return for the time/cost-return calculations from Table 3.

Notice in Figure 11 that the time-cost tradeoff obtained according to *Schedule's* proprietary probabilistic crashing heuristic is not a convex function as shown in Figure 8a, Figure 9 and Figure 10. However, as the completion time decreases, the cost always increases.

Also, notice that in Figure 12 the returns obtained are not always increasing as the completion time increases. This is because of the complexity of the functions in equations (35) and (37).

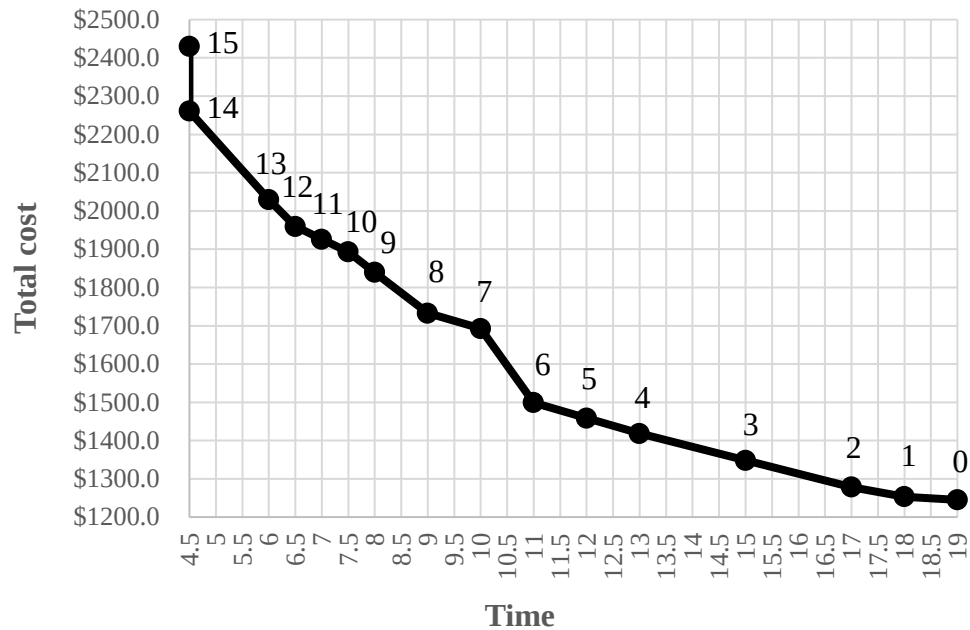


Figure 11. Time-cost tradeoff for the realistic enough example from Table 1.

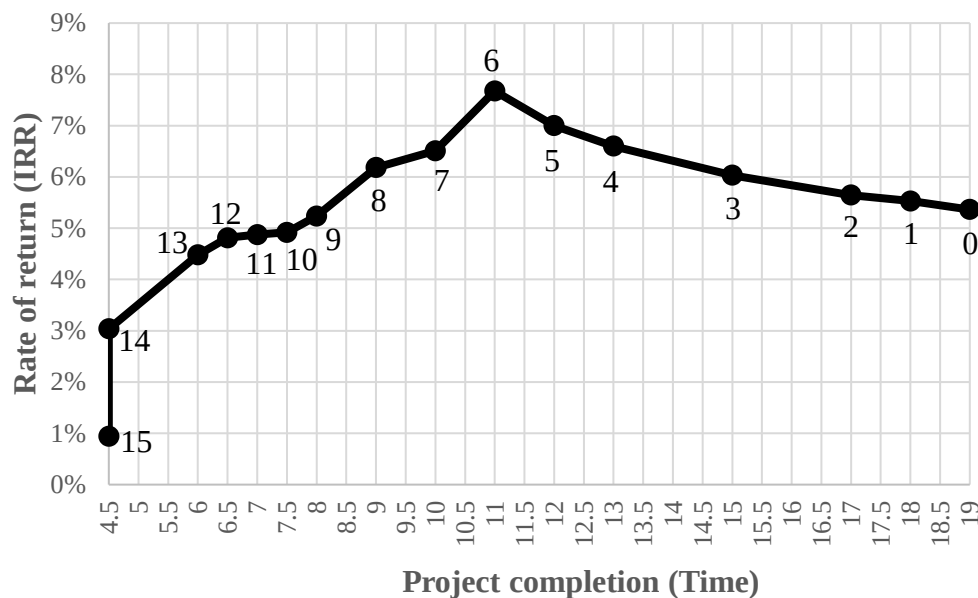


Figure 12. Time-return calculations for the realistic enough example from Table 1.

THE INTERACTIVE GRAPHIC USER INTERFACE (GUI) FOR *SCHEDULE*

As we have previously seen, it is possible to calculate beta distributed time, cost and return probability density functions, which give us the “risk” of failure (and alternatively success) of each dimension. Furthermore, these dimensions are related through the time-cost tradeoff curve. Figure 13 shows a scheme of this interactive user interface.

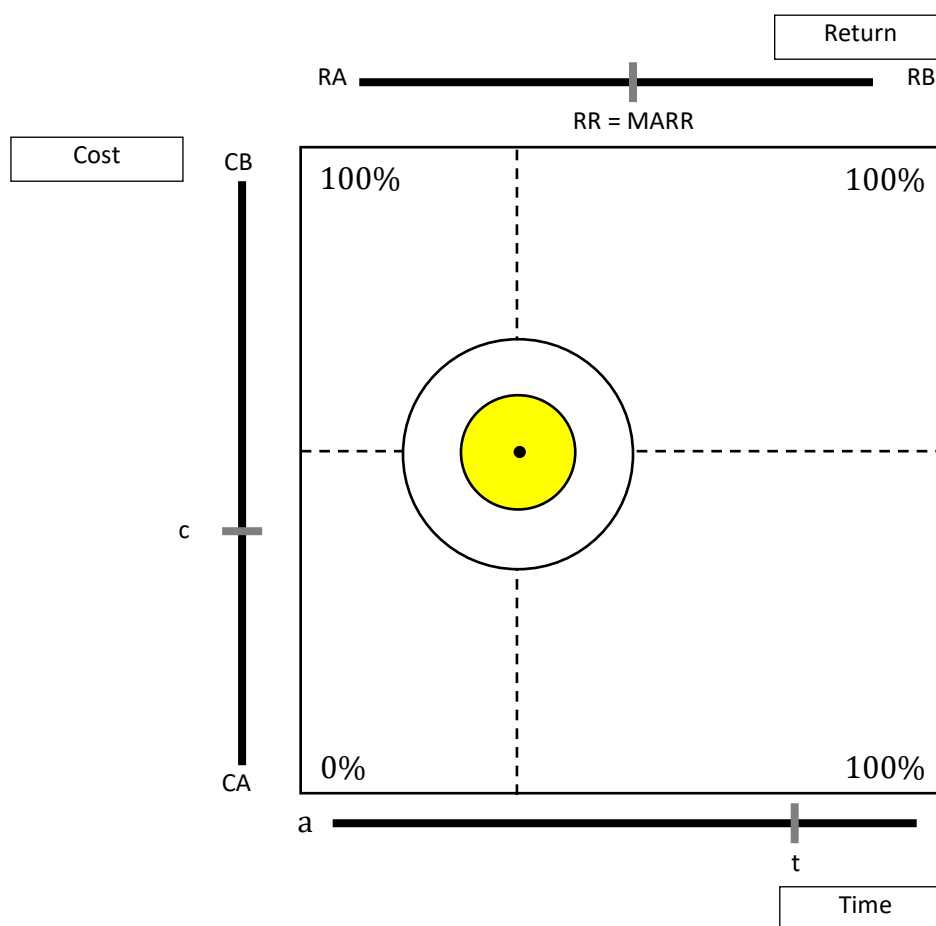


Figure 13. Interactive GUI for *Schedule*.

These dimensions are related according to equations (28), (29) and (38). If we change the due date (t), it immediately changes the budget (c) and the return (RR) and if we change the budget (c) it changes the due date (t) and the return (RR). To avoid a cyclical user interaction, we can change the MARR at will and see the effect on the bubble.

DISCUSSION AND CONCLUSION

CPM presents a considerable constraint: activity durations must be deterministic. Besides, there is the inherent problem with the CPM that for the non-critical activities, they can begin at any time between the Early Start (ES) of the activity and such ES plus the slack (S). Also, activities can finish at any time between the Early Finish (EF) of the activity and such EF plus the slack (S). In all these cases, when do the actual beginning and end of the non-critical activities actually occurs?

We require, then, a modification to CPM. Using the optimistic percentage (O) and its corresponding pessimistic one ($P = 1 - O$), it is possible to specify values given the starting and finishing times of the activities to know when it is they occur in each iteration of the simulation required to calculate the criticality of each activity.

The CPM has, nevertheless, something very important, which is the crashing method, which can be extended to PCPM (calling it probabilistic crashing using the proprietary heuristic for the computations) to obtain a time and cost relationship and also a time-cost and return relationship, which allows to link all variables among them, even probabilistically, so that the risk element for each dimension is added.

PERT, on the other hand, offers the idea of minimum, most likely and maximum times for the duration of each activity, which is used in PCPM to calculate the mean and the variance of each activity according to the PERT textbook formula. However, PERT tends to lead to optimistic estimates of the duration of the activities by assuming a normal distribution (and not a beta distribution, as with PCPM) representing the project duration. In principle, this is theoretically wrong, because there is an absolute minimum project duration (called *a* in PCPM) and an absolute maximum project duration (called *b* in PCPM). The normal distribution (unlike the beta distribution) does not have minimum nor maximum duration, but rather a duration time that goes from $-\infty$ to $+\infty$, despite the fact that the probabilities of having these extreme durations are infinitesimal.

There is, nonetheless, an additional limitation to PERT: what is the project duration variance when there is more than one critical path? We could assume that in this case the sum of the variances of the activities in each and every critical path are used and choose whichever is the highest of all possible critical paths. However, PERT does not provide such consideration for assuming it unlikely to happen, which constitutes a serious limitation of PERT. On the other hand, PCPM considers the variance of the project based on the criticalities and it does not assume a normal distribution for such duration, but rather a beta distribution, so that it does not lead to optimistic (nor pessimistic) assumptions concerning the project duration. Instead, it leads to a precise probabilistic assessment. Furthermore, it allows to calculate the beta distributed completion times of all the activities considered to be milestones.

Notice that in this case the return calculation (IRR) was consistent with the theory, that is, as time increases, the related cost is reduced and the return increases. It is possible that the return calculations are not always following this precise trend. It is possible to have fluctuations such that with time, return not always increases. In any case, the lowest project duration (*a*) is associated with the minimum return (RA) and the largest project duration (*b*) is associated with the highest return obtained (RB). However, the actual results can be consulted to find out the correct return for any given time and cost, even for the optimal time and the optimal cost.

Finally, PCPM is the only method leading to a project duration, cost and return described by a beta distribution, so that it is possible to estimate the time, cost and return risks for given due dates, budgets or MARR, respectively, and consequently, define probabilistic tradeoffs among these dimensions. In conclusion, PCPM not only emphasizes the risk element to the project, but it also offers a comprehensive approach to Project Management (Project Risk Management to be more specific) by considering the three dimensions and their inherent tradeoffs: time, cost and return (the latter one measuring project performance).

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