



Transformative Psychometric Research Trioengineering Modeling: Triangulating Digital Instrumentation, Implementation, and Data Analysis for Effective, Efficient, and Engaging Classroom Inquiry

James Edward Osler II
North Carolina Central University

ABSTRACT

This monograph provides an in-depth discourse on “Research Engineering” in the field of “Educational Science” first detailed in a 2012 Journal on Mathematics article. Research Engineering involves the methodology and the metrics used to conduct in-depth research investigations via the innovative Total Transformative Trichotomy-Squared [Tri-Squared] Test. The triangulation of this test aids in the comprehension of how the Tri-Squared calculation is a mixed methods research design based upon “Trichotomous Psychometrics”. Trichotomous Psychometrics involves the development, deployment, and analysis of “Trifold Assessments” for the holistic transformation of qualitative outcomes into quantitative data. This paper is a continuation of the published article entitled: The Psychometrics of Educational Science: Designing Trichotomous Inventive Investigative Instruments for Qualitative and Quantitative for Inquiry.

Keywords: Algorithmic Model, Cartesian Coordinates, Educational Science, Geometric Vectors, Models, Psychometrics, Research Engineering, Triangulation, Trichotomous Psychometrics. Trichotomy, Trioengineering, Trifold Assessment, and Tri-Squared.

INTRODUCTION

One of the most challenging areas of research in education involves the construction of specific instruments that are designed to measure qualitative outcomes and data. Although there are a great many measurement tools that analyze the cognitive and psychomotor domains, there remains a vacuum in the number of instruments especially designed to accurately measure the affective domain (the learning domain that contains attitudes, opinions, emotions, perception, and perspectives). This void is further expanded when the specific event under investigation is unique, specialized, has specific characteristics, serious legal constrictions, and issues regarding time. This often requires the research investigator to design an instrument that ideally measures the variables under investigation (Osler, 2013b).

The process of designing instruments for the purposes of assessment and evaluation is called “Psychometrics”. Psychometrics is broadly defined as the science of psychological assessment (Rust & Golombok, 1989). The Tri-Squared Test pioneered by the author in 2012, factors into the research design a unique event-based “Inventive Investigative Instrument”. This is the core of the

Trichotomous-Squared Test. The entire procedure is grounded in the qualitative outcomes that are inputted as Trichotomous Categorical Variables based on the Inventive Investigative Instrument. The specific assessment of the variables is completely dependent upon the outcomes determined by the researcher's instrument. The creation, production, and deployment of the trichotomous Inventive Investigative Instrument requires that the research investigator adopt the role of a "Trichotomous Psychometrician". A "Trichotomous Psychometrician" is an Educational Scientist that uses trichotomous-based psychometrics to develop a qualitative Inventive Investigative Instrument specifically designed capture qualitative responses during a specific event (Osler, 2013a). The design of Inventive Investigative Instruments is based on the mathematical "Law of Trichotomy" (Apostol, 1967). Trichotomy itself is derived from historic discussions surrounding higher cognition, general thought, and descriptions of intellect. Philosopher Immanuel Kant adapted the Thomistic acts of intellect in his "trichotomy of higher cognition" — (a) understanding, (b) judgment, (c) reason — which he correlated with his adaptation in the soul's capacities — (a) cognitive faculties, (b) feeling of pleasure or displeasure, and (c) faculty of desire (Kant, 2007). It is important to note that in mathematics, the law (or axiom) of trichotomy is most commonly the statement that for any (real) numbers x and y , exactly one of the following relations holds. Until the end of the 19th century the law of trichotomy was tacitly assumed true without having been thoroughly examined (Singh, 1997). A proof was sought by Logicians and the law was indeed proved to be true. Mathematically, Trichotomous relations are irreflexive and antisymmetric (Sensagent, 2012). A description of the entire Tri-Squared research process follows and is described in detail to provide the reader of the precise steps undertaken in the process of developing, designing, and ultimately implementing an Inventive Investigative Instrument (Osler, 2013a). "Trioengineering" is a novel term coined by the author and defined here as "the process involved in designing solutions by actively using the mathematical law of trichotomy to produce a measurable, scalable, and tangible results".

DEFINING THE FIELD OF EDUCATIONAL SCIENCE

The field of "Education Science" is also represented by the term "Eduscience" which is a portmanteau of the two terms "Education" and "Science". Similar to the field of "Bioscience", Eduscience is the study of education wherein applicable sciences (such as ergonomics, statistics, technology, etc.) are applied to enhance and improve learning. The primary purpose of the field of Eduscience is the study and application of solutions to improve and enhance the learning environment and learning in general. Eduscience is solution-driven and is actively concerned with the transfer and dissemination of knowledge. Education Science is a broad field and its professionals are directly involved in the field. Those who are actively involved in Eduscience can be referred to as "Education or Educational Scientists". Educational Scientists or "Eduscientists" are multifaceted professionals who have a variety of areas of expertise. They can assume multiple roles in the educational environment and can serve in a variety of offices and in a multitude of capacities. The primary positions that Eduscientists assume are in the following areas: Administration (as Leaders, Organizational Heads, and Organizational Management Professionals), Instruction (as Teachers, Professors, and Facilitators), Practice (as Practitioners in a variety Specified Areas and Arenas), and Technology (as Educational Technologists, Instructional Technologists, and Information Technologists). In these positions Eduscientists effectively use, analyze, study, and deploy novel instructional learning theories, methodologies, strategies, solutions, tools, and techniques in both traditional or virtual (pedagogical and andragogical) settings to bring about learning (Osler, 2012a).

TRANSITIVE TRANSFORMATIVE KNOWLEDGE TRANSFER: THE ULTIMATE GOAL OF EDUCATION SCIENCE

Educational Scientists strive to make the process of knowledge transfer both transitive and transformational. A transitive and transformative knowledge transfer process is as seamless and as harmonious as possible in an effort to empower, enhance, and improve learning. Eduscientists are masters of teaching who also are also highly proficient practitioners who are able draw from personal and professional experiences to make the learning environment more viable (Accessible), usable (Ergonomic), teachable (Instructional), engaging (Relevant), approachable (Adaptive), exploration-based (Discovery), and inspirational (Transformative). The Total Transformative Trichotomy-Squared Test is a comprehensive multi-step research methodology that is employed by Eduscientists. It is especially designed to conduct qualitative and quantitative investigations in educational settings and the learning environment (Osler, 2013b).

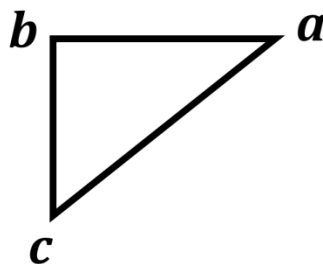
Education-Based Inquiry: Using the Tri-Squared Research Methodology as a Four Step Process to determine an appropriate research Effect Size, an ideal Sample Size, and an effective Alpha Level for Education-Based Experiments in the Learning Environment

The Tri-Squared research procedure (Osler, 2013c) consists of a four step approach designed to provide the researcher with a clear and precise set of data to conduct research, analyze data, and determine the level of significance required to either validate or reject the initial research hypothesis. The four Tri-Squared steps are as follows:

1. Design of an Inventive Investigative Instrument that has Trichotomous Categorical Variables and Trichotomous Outcome Variables.
2. Establish the Research Effect Size, Sample Size with associated Alpha Level.
3. Establish Mathematical Hypotheses.
4. Use the Tri-Squared Test as the Data Analysis Procedure following implementation.

The Upright Right Triangle Testing Model for the Tri-Squared Test

The Algorithmic Model of Triangulation (Osler, 2013c) is of the form:



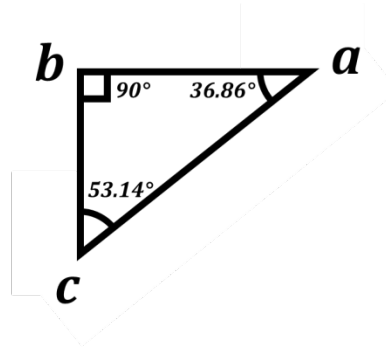
Where,

Vertex **a** = $\angle a$ = form "Vector [a→b]" as " $\overrightarrow{ab}$ " = "**authoring**" = a ratio of a measurement metric of 4 (used to convey information) = The Initial Tri-Squared Instrument Design;

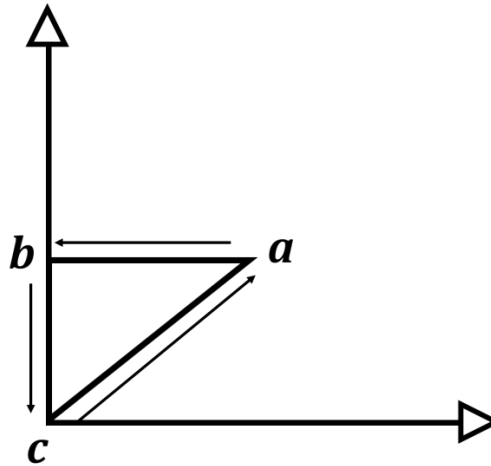
Vertex **b** = $\angle b$ = form "Vector [b→c]" as " $\overrightarrow{bc}$ " = "**building**" = a ratio of a measurement metric of 3 (as the Triostatistics: Tri-Squared Test) = The Tri-Squared Qualitative Instrument Responses

which will be analyzed via Tri-Squared Analysis; and Vertex $c = \angle c =$ form “Vector $[c \rightarrow a]$ ” as “ $[\overrightarrow{ca}]$ ” = “**conveying**” = The Final Tri-Squared Test Outcomes in a Quantitative Report.

Thus, the Triangulation Model is symbolized by a Right Triangle written as: “ ∇ ”. This symbol called “Trine” (meaning a group of three) is written mathematically as “ $\nabla = abc$ ” and is simplified into the mathematic geometric expression: ∇abc (meaning “Triangulation Model abc ” or more simply “Trine abc ”).



The angles have the following angular measurements in degrees: $\angle a = 36.86$, $\angle b = 90$, and $\angle c = 53.14$, that all add up to the standard 180° of any traditional triangle: $[36.86^\circ + 90^\circ + 53.14^\circ = 180^\circ]$. The connective lines between the Upright Right Triangle vertex points (i.e., the lines between points a , b , and c respectively) are geometric “vectors” (lines with both size [magnitude] and direction) making the model a systemic or cyclic process from the point of origin “ a ” back to the original point of origin “ a ”. This is illustrated in terms of Cartesian Coordinates as follows:



In terms of Vectors, the Upright Right Triangle as the Algorithmic Model of Triangulation now becomes the Trioengineering: Triangulation Model Right Triangle ∇abc is equal to three vectors that illustrate the movement in direction and magnitude from one completed task into another. The entire process is both cyclical and sequential with a “Trine Vector Equation” written as:

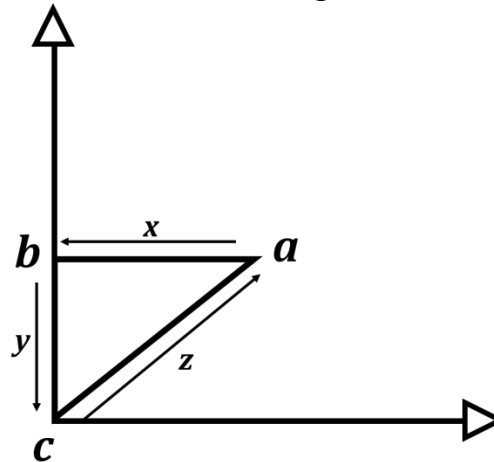
$$\nabla = [\leftarrow x] \rightarrow [\downarrow y] \rightarrow [\uparrow z], \text{ for } "[\overrightarrow{xyz}]"$$

Defined as Trine = “Concentration of Vector x into Concentration of Vector y into Concentration of Vector z ”, which is simplified into a more standardized Trine Vector Equation form written as:

$$\nabla = \tilde{x} \rightarrow \tilde{y} \rightarrow \tilde{z}, \text{ for } \overrightarrow{xyz}$$

Where,

vectors x, y, and z respectively are indicated on the “Algorithmic Triangulation Data Model” as:



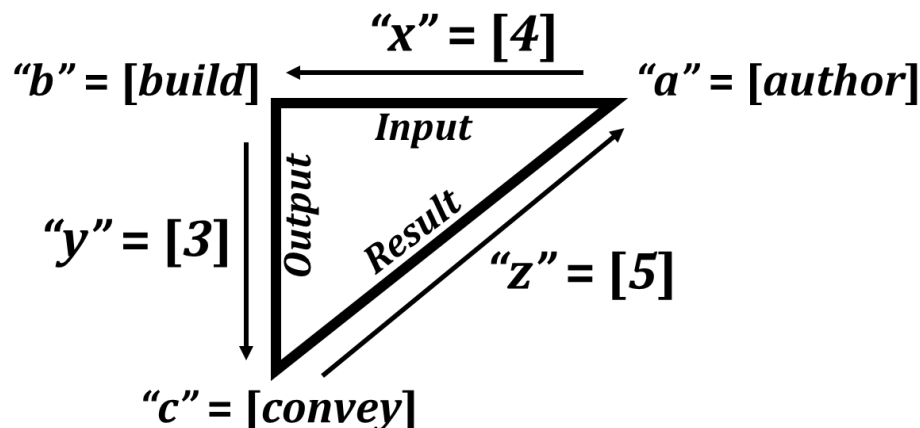
Indicating that the standardized form of the vectors x, y, and z are equivalent to the following geometric vectors that are the sequential Cartesian Coordinates relative to the size and magnitude of the research engineering phases that sequentially connect the respective angles $\angle a$, $\angle b$, and $\angle c$. This is written with the flow of the vector in opposite direction of its original presentation to indicate the more precise direction of arrows from right to left in a cyclical motion as follows:

$$\begin{aligned}\tilde{x} &= \overleftarrow{ab} \\ \tilde{y} &= \overleftarrow{bc} \\ \tilde{z} &= \overleftarrow{ac}\end{aligned}$$

THE COMPLETE TRI-SQUARED ANALYSIS ALGORITHMIC

Triangulation Model as the 4 to 3 to 5 Upright Right Triangle

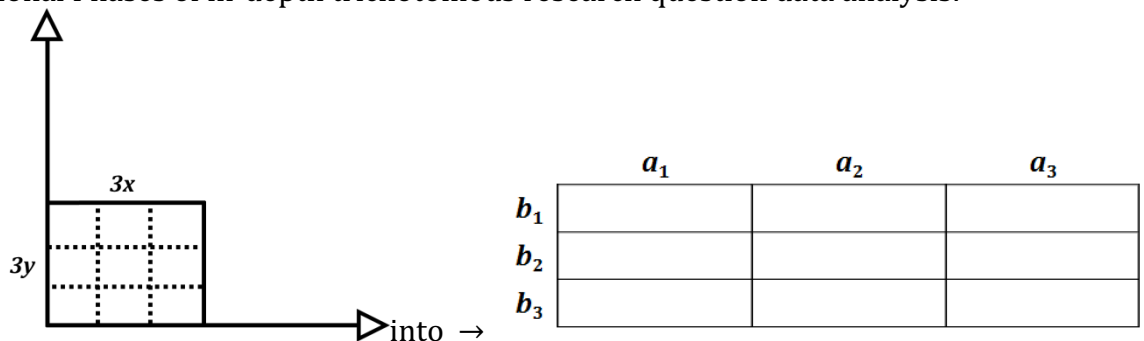
The three numeric Vector Operational Phases of the Triangulation Model that now expresses the full scope of the operational parameters of the Triostatistics Tri-Squared Test are completely defined in the following manner:



The entire Triangulation Model as a Research Engineering methodology begins with a breakdown of the Trine [↗] Operational Research Engineering Parameters and Geometric Vectors in the following manner:

The Tri-Squared Triangulation Model Research Engineering Process (highlighting the Operational Parameters and Phases of the Tri-Squared Test)

Geometric Vertex $a = \angle a = \text{"authoring"} = \text{The Initial Tri-Squared Instrument Design} = \text{Operational Parameter "a"} = \text{"author"} = \text{absolute value of } a = \text{"modulus a"} = |a| = \text{"Trine a"} = \sqrt{a} = \text{The creation of the Tri-Squared Inventive Investigative Instrument. This process can be seen in the following 3 by 3 cubic model which becomes the 3 by 3 rectangle table model of the Standard Tri-Squared Test Data Analysis Table (note: it becomes rectangular due to the top ratio author vector of "4" which represents the four initial sections of the Tri-Squared Test Triple-I instrument as the four Operational Phases of in-depth trichotomous research question data analysis:}$



This in turn, leads into [→] vector $x = \text{Geometric Vector } x = \text{"} \leftarrow x \text{"} = \overleftarrow{x} = \overleftarrow{ab} = \text{The Initial Tri-Squared Instrument Construction} = \text{Operational Phases "x"} = \text{absolute value of vector } x = \text{"norm x"} = \|x\| = \text{"Trine x"} = \sqrt{x} = \text{The creation of the Tri-Squared Inventive Investigative Instrument} = \text{The Pythagorean Triple of the Triangulation Model} = \text{"4"} = \text{The 4 Phases of Tri-Squared Inventive Investigative Instrument Construction as "Trichotomous Psychometrics"} which is composed of the following 4 Operational Phases:$

1. a_0 = The Instrument Name (Asset Security Optional);
2. a_1 = Section One of the Research Instrument. Constructed from the first series of instrument items (a. through c.) derived from the research investigation questions as the Qualitative Trichotomous Categorical Variables (as the Initial Investigation Input Variables), evaluated via the Qualitative Trichotomous Outcomes (as the Resulting Outcome Output Variables = b_1 , b_2 , and b_3 respectively);
3. a_2 = Section Two of the Research Instrument. Constructed from the second series of instrument items (d. through f.) derived from the research investigation questions as the Qualitative Trichotomous Categorical Variables (as the secondary Investigation Input Variables), evaluated via the Qualitative Trichotomous Outcomes (as the Resulting Outcome Output Variables = b_1 , b_2 , and b_3 respectively); and
4. a_3 = Section Three of the Research Instrument. Constructed from the third series of instrument items (g. through i.) derived from the research investigation questions as the Qualitative Trichotomous Categorical Variables (as the tertiary Investigation Input Variables), evaluated via the Qualitative Trichotomous Outcomes (as the Resulting Outcome Output Variables = b_1 , b_2 , and b_3 respectively);

The following table provides the metrics for the construction of the Inventive Investigative Instrument following the parameters indicated in phases 1. through 4. of the first vector [$\vec{x} = \overline{ab}$ = 4] of the Triangulation Model (Figure 1.):

$a_0 =$	Inventive Investigative Instrument —Name— [Optional Asset Security]				
$a_1 =$	Section 1. Research Question One. The First Series of Questions from the Qualitative Trichotomous Categorical Variables are listed below.				
	Responses: [Select only one from the list.] ►		b_1	b_2	b_3
	a.	Item One	☐	☐	☐
	b.	Item Two	☐	☐	☐
	c.	Item Three	☐	☐	☐
$a_2 =$	Section 2. Research Question Two. The Second Series of Questions from the Qualitative Trichotomous Categorical Variables are listed below.				
	Responses: [Select only one from the list.] ►		b_1	b_2	b_3
	d.	Item Four	☐	☐	☐
	e.	Item Five	☐	☐	☐
	f.	Item Six	☐	☐	☐
$a_3 =$	Section 3. Research Question Three. The Third and Final Series of Questions from the Qualitative Trichotomous Categorical Variables are listed below.				
	Responses: [Select only one from the list.] ►		b_1	b_2	b_3
	g.	Item Seven	☐	☐	☐
	h.	Item Eight	☐	☐	☐
	i.	Item Nine	☐	☐	☐

Figure 1.

This Tabular Triangulation Model can be fully exemplified in the aforementioned example provided by Figure 2.

Thus, Figure 2. is provided again (in an enlarged form) to illustrate a sample Inventive Investigative Instrument (Osler and Waden, 2012b):

The Osler-Waden 9th Grade Academies, Centers, and Center Models Assessment Instrument ©

A. Has the 9th Grade Academy, Center, or Center Model been:

	Yes	No	Missing
1. Successful?	☐	☐	☐
2. Made a Difference?	☐	☐	☐
3. Aided in Retention?	☐	☐	☐

How as the Academy/Center been successful, made a difference, or aided in retention, if at all?

B. Did the 9th Grade Academy, Center, or Center Model Result in the following:

	Yes	No	Missing
4. Positive Impact?	☐	☐	☐
5. Active Participation?	☐	☐	☐
6. Decline in Dropout Rate?	☐	☐	☐

How as the Academy/Center been positive, aided in participation, or decreased the dropout rate, if at all?

C. How did the 9th Grade Academy, Center, or Center Model have an impact on the following:

	Yes	No	Missing
7. Positively Effect Standardized Testing?	☐	☐	☐
8. Increase Graduation Rate?	☐	☐	☐
9. Increase Attendance?	☐	☐	☐

How as the Academy/Center positively affected testing, graduation rates, and attendance, if at all?

How long has the model/program (freshman/Ninth Grade Academy been operation in your school?

How long has the interviewee (yourself) been (working) there and what is the level of his or her involvement in the program such as Assistant Principal, Principal, teacher or other staff member?

What is the role of those interviewed during the interviews, and their knowledge of the program, whether their knowledge was medium knowledge low level of knowledge etc.

Figure 2.

This defined in the first Operational Geometric Vector as (Figure 3.):

$a_0 =$	The Osler-Waden 9th Grade Academies, Centers, and Center Models Assessment Instrument © [with Asset Security = ©]				
$a_1 =$	Has the 9th Grade Academy, Center, or Center Model been:				
	Responses: [Select only one from the list.] ►		$b_1 =$ Yes	$b_2 =$ No	$b_3 =$ Missing
	a. = 1.	Successful?	☐	☐	☐
	b. = 2.	Made a Difference?	☐	☐	☐
	c. = 3.	Aided in Retention?	☐	☐	☐
$a_2 =$	Did the 9th Grade Academy, Center, or Center Model Result in the following:				
	Responses: [Select only one from the list.] ►		$b_1 =$ Yes	$b_2 =$ No	$b_3 =$ Missing
	d. = 4.	Positive Impact?	☐	☐	☐
	e. = 5.	Active Participation?	☐	☐	☐
	f. = 6.	Decline in Drop Out Rate?	☐	☐	☐
$a_3 =$	How did the 9th Grade Academy, Center, or Center Model have an impact on the following:				
	Responses: [Select only one from the list.] ►		$b_1 =$ Yes	$b_2 =$ No	$b_3 =$ Missing
	g. = 7.	Positively Effect Standardized Testing?	☐	☐	☐
	h. = 8.	Increase Graduation Rate?	☐	☐	☐
	i. = 9.	Increase Attendance?	☐	☐	☐

Figure 3.

Where, the Operational Geometric Vector phases are defined respectively as:

a_0 = The Osler–Waden 9th Grade Academies, Centers, and Center Models Assessment Instrument ©;

a_1 = Has the 9th Grade Academy, Center, or Center Model been;

a_2 = Did the 9th Grade Academy, Center, or Center Model Result in the following;

a_3 = How did the 9th Grade Academy, Center, or Center Model have an impact on the following;

b_1 = Yes;

b_2 = No; and

b_3 = Missing

This is followed by:

Geometric Vertex $b = \mathbf{L}b = \text{“build”}$ = The Tri-Squared Qualitative Instrument Responses = Operational Parameter “ b ” = “build” = absolute value of b = “modulus b ” = $|b|$ = “Trine b ” = ∇b = The effective deployment of the Tri-Squared Inventive Investigative Instrument to elicit responses and aggregate the response data. This process can be seen in the following model (Figure 4.):

	a_1	a_2	a_3
b_1	a_1b_1	a_2b_1	a_3b_1
b_2	a_1b_2	a_2b_2	a_3b_2
b_3	a_1b_3	a_2b_3	a_3b_3

Figure 4.

A Standard 3×3 Tri-Squared Table of Comprehensive Inputted Qualitative Research Responses would resemble the following model represented by Figure 5. Where, T_n = The Total Number of Responses (based upon a one to one ratio of each Trichotomous Testing Input Variables contrasted directly with each Trichotomous Results Output Variable) in each of the individual cells of the Standard 3×3 Tri-Squared Table.

	a_1	a_2	a_3
b_1	$T_{a_1b_1}$	$T_{a_2b_1}$	$T_{a_3b_1}$
b_2	$T_{a_1b_2}$	$T_{a_2b_2}$	$T_{a_3b_2}$
b_3	$T_{a_1b_3}$	$T_{a_2b_3}$	$T_{a_3b_3}$

Figure 5.

Where Figure 5. is defined as,

$T_{a_1b_1}$ = Total Number of Responses for Cell One in the Standard 3×3 Tri-Squared Table;
 $T_{a_2b_1}$ = Total Number of Responses for Cell Two in the Standard 3×3 Tri-Squared Table;
 $T_{a_3b_1}$ = Total Number of Responses for Cell Three in the Standard 3×3 Tri-Squared Table;
 $T_{a_1b_2}$ = Total Number of Responses for Cell Four in the Standard 3×3 Tri-Squared Table;
 $T_{a_2b_2}$ = Total Number of Responses for Cell Five in the Standard 3×3 Tri-Squared Table;
 $T_{a_3b_2}$ = Total Number of Responses for Cell Six in the Standard 3×3 Tri-Squared Table;
 $T_{a_1b_3}$ = Total Number of Responses for Cell Seven in the Standard 3×3 Tri-Squared Table;
 $T_{a_2b_3}$ = Total Number of Responses for Cell Eight in the Standard 3×3 Tri-Squared Table; and
 $T_{a_3b_3}$ = Total Number of Responses for Cell Nine in the Standard 3×3 Tri-Squared Table;

This leads into $[\rightarrow]$ vector y = Geometric Vector $y = \downarrow y = \bar{y} = \overline{bc} =$ The Initial Tri-Squared Instrument Construction = Operational Phases “ y ” = absolute value of vector $y = \text{“norm } y\text{”} = \|y\| = \text{“Trine } y\text{”} = \nabla y =$ The creation of the Tri-Squared Inventive Investigative Instrument = The Pythagorean Triple of the Triangulation Model = “3” = The 3 Phases of Tri-Squared Inventive Investigative Instrument Deployment which is composed of the following 3 Operational Phases:

1. Inventive Investigative Instrument Deployment;
2. Inventive Investigative Instrument Completion; and lastly
3. Inventive Investigative Instrument Data Aggregation;

AN EXAMPLE OF TRIANGULATION—TRIFOLD ASSESSMENT: THE INVENTIVE INVESTIGATIVE INSTRUMENT EXTRACTED FROM THE INITIAL TRICHOTOMOUS RESEARCH QUESTIONS AND ASSOCIATED CATEGORICAL AND OUTCOME VARIABLES USED IN THE RESEARCH STUDY

In this sample research study the researcher Inventive Investigative Instrument items were derived from the aforementioned research questions and outcome variables. The instrument was used to obtain data from administrators as responses to research questions relating to the impact of Ninth Grade Centers, Academies or similar models on minority ninth grade students. The purpose of this instrument was to provide data from 9th Grade Academies, Centers, and Center Models for non-parametric Tri-Squared analysis. Data that was not responded to was reported as “Missing” (a separate Categorical Variable designed to report all research results). The Inventive Investigative Instrument was a qualitative method of data collection designed to accurately provide responses to carefully answer research questions. Ultimately, the research data analysis methodology (Tri-Squared) qualitatively and quantitatively determined the academic success of ninth grade students

who participated in ninth grade centers or similar models at four year high schools in North Carolina based upon the research hypothesis. A sample Inventive Investigative Instrument containing all three outcome variables is provided to illustrate how the 3 × 3 Trichotomous Table is constructed from the Inventive Investigative Instrument (Osler 2013b).

The Waden 9th Grade Academies, Centers, and Center Models Assessment Instrument ©

A. Has the 9th Grade Academy, Center, or Center Model been:

	Yes	No	Missing
1. Successful?	☐	☐	☐
2. Made a Difference?	☐	☐	☐
3. Aided in Retention?	☐	☐	☐

How as the Academy/Center been successful, made a difference, or aided in retention, if at all?_____

B. Did the 9th Grade Academy, Center, or Center Model Result in the following:

	Yes	No	Missing
4. Positive Impact?	☐	☐	☐
5. Active Participation?	☐	☐	☐
6. Decline in Dropout Rate?	☐	☐	☐

How as the Academy/Center been positive, aided in participation, or decreased the dropout rate, if at all?_____

C. How did the 9th Grade Academy, Center, or Center Model have an impact on the following:

	Yes	No	Missing
7. Positively Effect Standardized Testing?	☐	☐	☐
8. Increase Graduation Rate?	☐	☐	☐
9. Increase Attendance?	☐	☐	☐

How as the Academy/Center positively affected testing, graduation rates, and attendance, if at all?_____

How long has the model/program (freshman/Ninth Grade Academy been operation in your school?

How long has the interviewee (yourself) been (working) there and what is the level of his or her involvement in the program such as Assistant Principal, Principal, teacher or other staff member? _____

What is the role of those interviewed during the interviews, and their knowledge of the program, whether their knowledge was medium knowledge low level of knowledge etc.

The Tri-Squared Research Design Using the Inventive Investigative Instrument

Step One

Design of an Inventive Investigative Instrument that has Trichotomous Categorical Variables and Trichotomous Outcome Variables.

To effectively use Tri-Squared in a research investigation the researcher must first develop a series of trichotomous categorical variables based on associated trichotomous outcome variables. This is the first initial and crucial step to using Tri-Squared as a valid, reliable, and objective means of analyzing data. Second, an “Inventive Investigative Instrument” must be created and implemented in a manner compliant with the initial trichotomous categorical variables and outcomes stated at the outset of the research investigation. This insures that the research investigation is consistent throughout the study and that the later Tri-Squared computations are validly reporting what actually took place in the research environment during the time period in which the actual study was conducted. As previously stated the “Inventive Investigative Instrument” can be psychometrically delivered as a test or qualitatively delivered in the form of a research questionnaire, survey, interview or another type of metric. As long as the trichotomous categorical variables are measured according to the established associated trichotomous outcome variables then the research has merit within the confines and strict requirements of the Tri-Squared Test.

Step Two

Establish the Research Effect Size, Sample Size with associated Alpha Level.

The Tri-Squared Effect Size Formula

$$Tri_{Eff}^2 = \left[T_{CR} - (T_C T_R : n_{Tri^2}) \right] : C_S R_S (n_{Tri^2} - 1)$$

RESULTS

The Effect Size Table for the calculated Tri-Squared Effect Size Formula for the study was determined based off of the Standard 3×3 Tri-Squared Table calculated to be 0.125 (or small in overall effect size). The total number of participants that were identified in the study from the outset was $n_{Tri} = 17$. Thus, an alpha level of 0.975 is best for this small sample size from a corresponding small effect size according to the calculated effect size for the Standard 3×3 Tri-Squared Table (specifically identified as 17 to 33 participants for $\alpha = 0.975$). This is further illustrated in the following Tri-Squared Distribution Tables (Figure 6.):

Tri-Squared Effect Size										Tri_{eff}^2
<i>Small</i>										<i>0.10</i>
<i>Medium</i>										<i>0.30</i>
<i>Large</i>										<i>0.50</i>

Tri-Squared Probability Distribution Table											
Number of Research Participants Placed in Intervals Based off of Tri-Squared Effect Size Magnitude: [Small, Medium, or Large] is Based off of the Tri-Squared Mean = [d.f.] = 4											
Magnitude	Small Unit Intervals: Multiple of 1 = 4[4] = 4 · 4 = 16 Therefore, Interval has Increments of 16					Medium Unit Intervals: Multiple of 2 = 4[16] = 4[4 · 4] = 64 Therefore, Interval has Increments of 64			Large Unit Intervals: Multiple of 3 4[4 · 4 · 4] = 4[64] = 256		
Number of Participants	1–16	17–33	34–40	41–57	58–74	75–139	140–204	205–269	270–526	527–783	784–1040+
Probability P(x)	0.995	0.975	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.002	0.001

Tri-Squared Distribution Table Displaying Primary Alpha Levels with Associated Critical Values for Hypothesis Tests											
α Level	0.995	0.975	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.002	0.001
$Tri_{[i]}^2 =$ $d.f. = 4$	0.207	0.484	5.989	7.779	9.488	11.143	11.668	13.277	14.860	16.924	18.467
$Tri_{eff}^2 =$ Effect Size	<i>Small</i> 4[4]	<i>Small</i> 4[4]	<i>Small</i> 4[4]	<i>Small</i> 4[4]	<i>Small</i> 4[4]	<i>Medium</i> 4[16]	<i>Medium</i> 4[16]	<i>Medium</i> 4[16]	<i>Large</i> 4[64]	<i>Large</i> 4[64]	<i>Large</i> 4[64]+
$Tri_{sm}^2 =$ Sample Size [Intervals]	1–16	17–33	34–40	41–57	58–74	75–139	140–204	205–269	270–526	527–783	784–1040+

Figure 6.

Step Three

Establish Mathematical Hypotheses from Research Hypotheses.

Sample Research Hypotheses

The following were the research hypotheses for the example study:

H₀: There are significant differences in the perception of the success of Ninth Grade Academy Models in terms of graduation rates, dropout rates, high stakes testing, retention, and attendance by high school administrators.

H₁: There are no significant differences in the perception of the success of Ninth Grade Academy Models in terms of graduation rates, dropout rates, high stakes testing, retention, and attendance by high school administrators.

The aforementioned hypotheses yielded the following mathematical hypotheses using the Tri-Squared Test as the data analysis procedure:

An Example of Two-Tailed Mathematical Hypotheses

$$H_0: Tri^2 = 0$$

$$H_1: Tri^2 \neq 0$$

Step Four

Use the Tri-Squared Test as the Data Analysis Procedure following implementation.

Report the Results of the Research Data Analysis: Sample Historic Research Table, Qualitative, and Quantitative 3 × 3 Tri-Squared Tables

Quantitative Research Data Analysis Procedure: Results of the Tri-Squared Test

The Tri-Squared Test statistical analysis procedure was used to analyze data in the study. An alpha-level of 0.10 was considered in light of the research context that was evidence-based in the prescribed schools that had restricted and controlled learning environments that allowed for very few chance factors to affect the outcomes of the research investigation ($n_{Tri} = 17$ with $17 \times 3 = 51$ items per the outcomes of the Trichotomous Variables on the Inventive Investigative Instrument). This was coupled with the data gathered from administrators that allowed for minimal chance factors to affect research outcomes. The 0.10 estimate was reasonable for this particular study based on the calculated Tri-Squared Effect Size. In addition, due to the exploratory context and nature of the research investigation (in an area where little previous research has been done regarding 9th Grade Academies, Centers, and Center Models) a less stringent level of significance of 0.10 best fit the research study. The study yielded the following final results using the Tri-Squared analysis procedure in tabular format presented on the pages that follow (Osler, 2013).

Table One: Example Psychometric Qualitative Outcomes of the Tri-Squared Test

Data Analyzed Using the Trichotomous-Squared Three by Three Table designed to analyze the research questions from an Inventive Investigative Instrument with the following Trichotomous Categorical Variables: a_1 = Successful via: Impact, and Positive Testing [the summation of the outcomes of the assessment instrument item 1]; a_2 = Made a Difference via: Participation, and Graduation Rate [the summation of the outcomes of the assessment instrument item 2]; and a_3 = Aided in Retention via: Drop Out Rate, and Attendance [the summation of the outcomes of the assessment instrument item 3]. The 3 × 3 Table has the following Trichotomous Outcome Variables: b_1 = Yes; b_2 = No; and b_3 = Missing. The Inputted Qualitative Outcomes are reported as follows:					
$n_{Tri} = 17$	TRICHOTOMOUS CATEGORICAL VARIABLES				
$\alpha = 0.975$					
TRICHOTOMOUS OUTCOME VARIABLES		a_1	a_2	a_3	
	b_1	48	43	38	
	b_2	2	4	5	
	b_3	1	4	8	
$Tri^2 d.f. = [C - 1][R - 1] = [3 - 1][3 - 1] = 4 = Tri^2_{[8]}$					

The Tri-Square Test Formula for the Transformation of Trichotomous Qualitative Outcomes into Trichotomous Quantitative Outcomes to Determine the Validity of the Research Hypothesis:

$$Tri^2 = T_{Sum} \left[\left(Tri_x - Tri_y \right)^2 : Tri_y \right]$$

Tri² Critical Value Table = 8.131 (with *d.f.* = 4 at $\alpha = 0.975$). For *d.f.* = 4, the Critical Value for $p > 0.975$ is 0.484. The calculated Tri-Square value is 8.131, thus, the null hypothesis (H_0) is rejected by virtue of the hypothesis test which yields the following: Tri-Squared Critical Value of $0.484 < 8.131$ the Calculated Tri-Squared Value.

Table One illustrates the qualitative mathematical application of the Trichotomous-Squared (“Trichotomy-Squared”, “Tri-Squared” or “Tri-Square”) statistical analysis procedure. “Tri-Squared” is the mathematical transformation of qualitative data into quantitative data for the purpose of validating a research hypothesis. The Table One 3×3 Table reports the qualitative outcomes based on the Inventive Investigative Instrument Trichotomous Categorical Variables according to participant responses as the Trichotomous Outcome Variables. Table One shows that participants primarily and overwhelmingly selected the “Yes” Categorical Variable ($a_1b_1 = 48$, $a_2b_1 = 43$, and $a_3b_1 = 38$) rather than the alternative Categorical Variables of either “No” or “Missing” (the “Missing” C. V. indicated unselected or inapplicable responses to an item). The mathematical formula for the Tri-Squared is reported illustrating the final outcome of the research hypothesis test: the null hypothesis (H_0) is rejected at $p > 0.975$ is 0.484. Table Two follows and provides the outputted quantitative outcomes of the Tri-Squared Test.

Table Two: Example Psychometric Quantitative Outcomes of the Tri-Squared Test

Data Analyzed Using the Trichotomous-Square Three by Three Table designed to analyze the research questions from an Inventive Investigative Instrument with the following Trichotomous Categorical Variables: a_1 = Successful via: Impact, and Positive Testing [the summation of the outcomes of the assessment instrument item 1]; a_2 = Made a Difference via: Participation, and Graduation Rate [the summation of the outcomes of the assessment instrument item 2]; and a_3 = Aided in Retention via: Drop Out Rate, and Attendance [the summation of the outcomes of the assessment instrument item 3]. The 3×3 Table has the following Trichotomous Outcome Variables: b_1 = Yes; b_2 = Negative No; and b_3 = Missing. The Outputted Quantitative Outcomes are reported as follows:					
$n_{Tri} = 17$	TRICHOTOMOUS CATEGORICAL VARIABLES				
$\alpha = 0.975$					
TRICHOTOMOUS OUTCOME VARIABLES		a_1	a_2	a_3	
	b_1	43	43	43	
	b_2	3.67	3.67	3.67	
	b_3	4.33	4.33	4.33	
$Tri^2 d.f. = [C - 1][R - 1] = [3 - 1][3 - 1] = 4 = Tri^2_{[8]}$					

The Tri-Square Test Formula for the Transformation of Trichotomous Qualitative Outcomes into Trichotomous Quantitative Outcomes to Determine the Validity of the Research Hypothesis:

$$Tri^2 = T_{Sum} \left[\left(Tri_x - Tri_y \right)^2 : Tri_y \right]$$

Tri² Calculated Tri-Squared = $[0.581] + [0] + [0.581] + [0.760] + [0.030] + [0.482] + [2.561] + [0.025] + [3.111] = 8.131$ (with *d.f.* = 4 at $\alpha = 0.975$). For *d.f.* = 4, the Critical Value for $p > 0.975$ is 0.484. Thus, we can reject the null hypothesis (H_0) by virtue of the hypothesis test: Tri-Squared Critical Value of $0.484 < 8.131$ the Calculated Tri-Squared Value.

Table Two illustrates the quantitative mathematical application of the Trichotomous-Squared ("Trichotomy-Squared", "Tri-Squared" or "Tri-Square") statistical analysis procedure. "Tri-Squared" is the mathematical transformation of qualitative data into quantitative data for the purpose of validating a research hypothesis (clearly illustrated in this Table). Table Two illustrates and validates the process of transforming qualitative data into quantitative data as a means of in-depth mixed methods for the purposes of discrete data analysis. The Table Two 3×3 Table reports the transformed quantitative outcomes based on the Inventive Investigative Instrument Trichotomous Categorical Variables according to participant responses as the Trichotomous Outcome Variables. Table Two data displays that participants primarily and overwhelmingly selected the "Yes" Categorical Variable ($a_1b_1 = 43$, $a_2b_1 = 43$, and $a_3b_1 = 43$) rather than the alternative Categorical Variables of either "No" or "Missing" (the "Missing" C. V. indicated unselected or inapplicable responses to an item). The mathematical formula for the Tri-Squared is reported illustrating the final outcome of the research hypothesis test: the null hypothesis (H_0) is rejected at $p > 0.975$ is 0.484 because the Tri-Squared Test Critical Value of $0.484 < 8.131$ the Calculated Tri-Squared Test Value. Qualitative Interview Data follows reported in visual format with associated percentages.

SUMMARY

The purpose of this paper is to provide further support for the design of specific psychometric instruments in educational research (specifically the novel field of "Education Science"). The novel field of "Educational Science" with the associated Tri-Squared Test was provided as the ideal environment that supports and validates the use of "Trichotomous Psychometrics" as a means of producing event-specific trichotomous instrumentation. Further support was provided through a research study that used a trichotomous instrument to determine the impact of Ninth Grade Academies, Freshman Academies or similar models upon ninth grade minority student achievement. The study provided an ideal example of just how effective trichotomous psychometric instruments are and how they can be used as qualitative data gathering instruments in a research study. Ultimately, the example study determined that the research participants for the most part agreed that 9th Grade Academies, Centers, and Center Models were effective in their respective schools. This outcome is supported by the final results of the Tri-Square analysis which yielded the following: A Critical Tri^2 value of $0.484 < 8.131$ the Calculated Tri^2 value. Thus, the research null hypothesis was rejected and it can therefore be stated that 9th Grade Academies, Centers, and Center Models do have an effect on the academic success, make a positive difference, and aid in the retention of students (Osler, 2012b).

The Tri-Squared Test Algorithmic Model and associated Statistical Methodology were provided to illustrate and display how this method can be put to use both practically and rapidly. The Total Transformative Trichotomy-Squared Research Design Methodology is a universally applicable in-depth investigative procedure that is an ideal way of examining the unique, diverse, and many times specialized procedures inherent to a variety of research models and investigative inquiry. Qualitative outcomes can be transformed into measurable quantitative outcomes to determine the outcome of research hypotheses. Both qualitative and quantitative methods are seamlessly incorporated into a unified data analysis methodology that delivers a reporting procedure that specifically aligns with the initial research objectives, hypotheses, and variables of the investigator. This method of statistical analysis allows the researcher to use an integration of qualitative and quantitative research methodologies by seamlessly combining the two opposing branches of

research investigation for maximized statistical power. This provides “Eduscientists” with a discrete and comprehensively stringent data analysis tool that is purely objective, extremely reliable, and very valid. Thus, investigative inquiry will grow, the body of knowledge will continue to expand, thereby empowering present and future generations of researchers.

The implementation of “Trioengineered Trichotomous Psychometrics” as Tri-Squared Inventive Investigative Instruments empowers 21st Century educational researchers as “Educational Scientists” who are embarking on novel approaches to answer age old questions about education, pedagogy, and the process of learning. These “Eduscientists” are seeking instruments that satisfy specific investigative needs, address specific research questions, and validate unique research situations. This type of psychometrics also enables instruments to be especially designed to address the specific needs of educational institutions, specialized learning environments, and distinctive pedagogical situations. It illustrates the value of analyzing small groups, sample sizes, and individuals without sacrificing statistical power or the value of research due to the uniqueness of the environment under examination. As stated in the by the author in an article in the Journal on Educational Psychology, “In this manner, psychometrics expands, new and novel research instruments are developed, and the field of Educational Science is validated” (Osler, 2013b).

References

- Apostol, T. M. (1967). Calculus, second edition, Volume one: One-variable calculus, with an introduction to linear algebra. Waltham, MA: Blaisdell.
- Kant, I. (2007). Critique of pure reason (based on Max Müller’s translation). P. Classics a Division of Pearson PLC. New York, NY.
- Osler, J. E. (2013a). The Psychometrics of Educational Science: Designing Trichotomous Inventive Investigative Instruments for Qualitative and Quantitative for Inquiry. December–February Journal on Educational Psychology, 8 (3), pp. 15–22.
- Osler, J. E. (2013b). The Psychological Efficacy of Education as a Science through Personal, Professional, and Contextual Inquiry of the Affective Learning Domain. February–April Journal on Educational Psychology, 6 (4), pp. 36–41.
- Osler, J. E. (2013c). Algorithmic Triangulation Metrics for Innovative Data Transformation: Defining the Application Process of the Tri-Squared Test. April–June Journal on Mathematics, 2 (2), pp. 10–16.
- Osler, J. E. (2012a). Trichotomy–Squared – A novel mixed methods test and research procedure designed to analyze, transform, and compare qualitative and quantitative data for education scientists who are administrators, practitioners, teachers, and technologists. July–September Journal on Mathematics, 1 (3), pp. 23–31.
- Osler, J. E. & Waden, C. (2012b). Using Innovative Technical Solutions as an Intervention for At Risk Students: A Meta–Cognitive Statistical Analysis to Determine the Impact of Ninth Grade Freshman Academies, Centers, and Center Models upon Minority Student Retention and Achievement. September–November Journal on School Educational Technology, 7 (3), pp. 11–23.
- Rust, J. & Golombok, S. (1989). Modern psychometrics: The science of psychological assessment (2nd ed.). Florence, KY, US: Taylor & Frances/Routledge.
- Sensagent: Retrieved, May 9, 2012: [http://dictionary.sensagent.com/trichotomy+\(mathematics\)/en-en/](http://dictionary.sensagent.com/trichotomy+(mathematics)/en-en/)
- Singh, S. (1997). Fermat’s enigma: The epic quest to solve the world’s greatest mathematical problem. A. Books a Division of Random House. New York, NY.