

A Review of the Studies on the Development of Concepts of Decimals and Fractions in Japanese Elementary School Children

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ABSTRACT

This paper reviews the studies published in Japan on the development of the concept of decimals and fractions in Japanese children. The Google Scholar search using the keywords “concept of decimals” and “concept of fractions” hit 59 articles published during 1982–2020 in Japan. According to the *Course of Study*, the guideline for elementary school education in Japan set by the Ministry of Education, Culture, Sports, Science and Technology of Japan, decimals and fractions should be taught in arithmetic classes of elementary schools. However, only 17 studies explored the concepts of decimals and fractions and the development of those concepts of primary school children. Moreover, none of them were conducted utilizing a scientific research method with empirical data of Japanese elementary school children. It concluded that the research on the education of ratio, including decimals and fractions, in Japan was still in “the stone age” without utilizing genuine scientific methods. (150 words)

Keywords: studies published in Japan, elementary school children, arithmetic education, concept of decimals and fractions

INTRODUCTION

Uchida and Mori (2018) reported that there were 20% or more “fake math-dislikes” among Japanese junior high school students. Those students replied, “I dislike mathematics” in the questionnaire survey, while they performed positively toward “mathematics” assessed with a paper-based Implicit Association Test (Mori, Uchida, & Imada, 2008). Uchida and Mori (2018) also found that the number of “fake math-dislikes” increased with the year grades. There were more “fake math-dislikes” in the second and third graders than the first graders in Japanese junior high schools.

Since there were some proportion of students who showed the characteristics of the “fake math-dislikes” at the very beginning of the first year of junior high school, we should examine when students would turn into “fake math-dislikes” during elementary school days. As a math teacher at a junior high school, the present author has long experienced that many students lack appropriate knowledge and skills of decimal numbers and fractions that they should have learned in elementary school education. How do they learn the knowledge and skills of decimal numbers and fractions during elementary school?

Table 1: Objectives in Arithmetic Described in Course of Study for Elementary Schools (MEXT, 2017a)

Grade	Objectives
Grade 1	(1) Through activities using concrete objects and so on, to help pupils enrich their number sense. To help them understand the meaning and the representations of numbers, and to help them understand addition and subtraction, and explore ways of the calculations, and use the calculations. (2) Through activities using concrete objects and so on, to help pupils enrich their experiences that will form the foundation for understanding quantities and measurements, and enrich their sense of quantities. (3) Through activities using concrete objects and so on, to help pupils enrich their experiences that will form the foundation for understanding geometrical figures, and enrich their sense of geometrical figures. (4) Through activities using concrete objects and so on, to help pupils represent numbers and quantities as well as their relations by using words, numbers, algebraic expressions, figures and diagrams and interpret such representations.
Grade 2	(1) Through activities using concrete objects and so on, to help pupils enrich their number sense. To help them deepen their understanding of the meaning and the representations of numbers, as well as their understanding of addition and subtraction, and use the calculations. Furthermore, to help them understand the meaning of multiplication, explore ways of the calculation, and use the calculation. (2) Through activities using concrete objects and so on, to help pupils understand the units and measurements of length and volume and so on, and enrich their sense of quantities. (3) Through activities using concrete objects and so on, to help pupils understand geometrical figures such as triangles and quadrilaterals, and enrich their sense of geometrical figures. (4) Through activities using concrete objects and so on, to help pupils represent numbers and quantities as well as their relations by using words, numbers, algebraic expressions, figures, diagrams, tables, and graphs, and interpret such representations.
Grade 3	(1) To help pupils use addition and subtraction appropriately, to help them deepen their understanding of multiplication and use the calculation appropriately. To help them understand the meaning of division, explore ways of the calculation, and use the calculation. Furthermore, to help them understand the meaning and the representations of decimal numbers and fractions. (2) To help pupils understand the units and measurements of length, weight and time. (3) To help pupils understand geometrical figures such as isosceles triangles and equilateral triangles by paying attention to the elements that compose the geometrical figures. (4) To help pupils represent numbers and quantities as well as their relations by using words, numbers, algebraic expressions, figures, diagrams, tables, and graphs, and interpret such representations.
Grade 4	(1) To help pupils deepen their understanding of division and use the calculation appropriately. To help them deepen their understanding of the meaning and the representations of decimal numbers and fractions, understand the meaning of addition and subtraction of decimal numbers and fractions, explore ways of addition and subtraction, and use them. Moreover, to help them understand round numbers and use them according to their purposes. (2) To help pupils understand the units and measurements of area, determine the area of geometrical figures, and understand the unit and measurements of angle. (3) To help pupils understand plane figures, such as parallelograms and rhombuses, and solid figures, such as rectangular parallelepiped, by paying attention to the elements that compose the geometric figures and the relationships of those elements. (4) To help pupils represent numbers and quantities as well as their relations by using words, numbers, algebraic expressions, figures, diagrams, tables, and graphs, and investigate such representations.

Grade 5	(1) To help pupils deepen their understanding of properties of integers. To help them deepen their understanding of the meaning of multiplication and division of decimal numbers as well as the meaning of addition and subtraction of fractions, explore ways of the calculation, and use the calculations. (2) To help pupils determine the area of geometrical figures such as triangles and parallelograms and the volume of solid figures such as rectangular parallelepiped. To help them understand the average of measured value and the ratio of two quantities of different types. (3) To help pupils deepen their understanding of plane figures, and to help them understand solid figures such as prisms. (4) To help pupils investigate relationships between numbers/quantities. To help them investigate the features of data by using percentages and pie graphs.
Grade 6	(1) To help pupils deepen their understanding of the meaning of multiplication and division of fractions, explore ways of the calculations, and use the calculations. (2) To help pupils determine the area of circles and the volume of solid figures such as prisms. To help them understand speed and determine it. (3) To help pupils understand reduced figures, enlarged figures and symmetric figures, and deepen their understanding of geometrical figures. (4) To help pupils understand ratio and direct proportion, and use the idea of a function when exploring the relationships of numbers/quantities, and represent the relationships in algebraic expressions. To help them explore the distribution of data and investigate it statistically.

Note: The objectives related to decimals and fractions are in **bold fonts**.

The Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan prepared and publicly announced a new version of the *Course of Study for Elementary Schools* in 2017, starting in the academic year 2020–2021 (Ministry of Education, Culture, Sports, Science and Technology of Japan: MEXT, 2017a). The *Course of Study 2020* guides each elementary school to make its curriculum for 2020 onward. Therefore, we can determine how the decimal numbers and fractions are taught in elementary schools through the *Course of Study 2020*. For example, Table 1 shows the objectives of arithmetic education for each grade.

The *Course of Study 2020* sets the various goals of learning decimal numbers during grades 3–5 and fractions during grades 3–6 in elementary schools. It is noteworthy that there are no descriptions of decimal numbers and fractions in the *Course of Study for Junior High Schools* (MEXT, 2017b). In other words, learning of the concepts of decimals and fractions is completed in elementary education. With this fact in mind, we examine further details of the descriptions of the *Course of Study for Elementary Schools* for teaching the concepts of decimals and fractions (see Table 2). As Table 2 shows, the description “To get to know simple fractions such as $1/2$ and $1/3$,” appears as learning contents of grade 2, despite that the decimals and fractions first appear in the goals of grade 3. Except for this minor incongruence, the details of various aspects of decimals and fractions are ordered coherently in the *Course of Study*’s learning contents.

Table 2: Contents of Learning Decimals and Fractions Described in Course of Study for Elementary Schools (MEXT, 2017a)

Grade	Decimal Numbers	Fractions
Grade 1	(None)	(None)
Grade 2	(None)	To get to know simple fractions such as $1/2$ and $1/3$.

Grade 3	To get to know numbers that are 10 or 100 times as many or $\frac{1}{10}$ of another number, and the representations of the numbers. To use decimal numbers in expressing size of fractional part. To get to know the representations of decimal numbers to the tenths place. To understand the meaning of addition and subtraction of decimal numbers through the tenths place. To explore ways of the calculations and do the calculations through the tenths place.	To use fractions in expressing size of fractional parts or quantities obtained as a result of equal partitioning. To get to know the representations of fractions. To get to know that a fraction can be represented as a collection of unit fractions. To understand the meaning of addition and subtraction of fractions in simple cases, and to explore ways of the calculations.
Grade 4	To get to know the fact that decimal numbers are represented in the same manner as integers, and to deepen their understanding of the relative size of numbers. To explore ways of addition and subtraction of decimal numbers and to do the calculations. To explore ways of multiplication and division of decimal numbers in cases where multipliers and divisors are integers, and to do the calculations.	To pay attention to the fact that there are fractions that are the same in size in simple cases. To explore ways of addition and subtraction of fractions with the same denominators, and to do the calculations.
Grade 5	To make numbers that are 10 or 100 times as much/many, or $\frac{1}{10}$ or $\frac{1}{100}$ of another number, and to investigate their relationships. Based on the understanding of calculations in cases when the multiplier and the divisor are integers, to understand the meaning of multiplication and division in cases where the multiplier and the divisor are decimal numbers. To explore ways of multiplication and division of decimal numbers and to do the calculations. Also, to understand the meaning of the size of the remainder. To understand that the same relationships and rules can be applied to the multiplication and division of decimal numbers as in the case of integers.	To represent integers and decimals as fractions and to transform fractions into decimals. To understand that the results of division of integers can always be represented as a number when using fractions. To understand that a fraction obtained by multiplying or dividing the numerator and denominator of an existing fraction by the same number, has the same size as the existing fraction. To explore equivalence and size of fractions and summarize the comparison of sizes. To explore ways of addition and subtraction of fractions with different denominators, and to do the calculations. To understand the meaning of multiplication and division of fractions when multipliers and divisors are integers, explore ways of the calculations, and to do the calculations.
Grade 6	To help pupils consolidate their ability to calculate decimal numbers and fractions and extend their ability to use the calculations.	To understand the meaning of multiplication and division when multipliers and divisors are fractions, based on the concept of such calculations in cases where multipliers and divisors are integers. To explore ways of multiplication and division of fractions, and to do the calculations. With respect to multiplication and division of fractions, to understand that the same relationships and rules as integers can be applied.

However, in the National Assessment of Academic Ability (NAAA) administered every year by the National Institute for Educational Policy Research (NIEPR) for all ninth-grade students includes problems to assess the ability of calculation of decimals and fractions. The inclusion of these items shows the NIEPR and MEXT are concerned about accurately assessing the teaching of decimals and fractions in elementary school. The published reports of NAAA (National Institute for Educational Policy Research, 2014, 2016, 2017) revealed that their concerns were realistic. For example, the percentage of correct answers was 86.1% for the calculation of $3/4 \div 5/6$ in 2014, 67.8% for $2/5 \times 0.6$ in 2016, and 87.6% for $5/9 \times 2/3$ in 2017. There were no responses from 2.7%, 4.8%, and 1.2% of students, respectively. Surprisingly, there were even higher percentages—9.0%, 12.6%, and 8.2%, respectively—of students who gave “unexpected answers” that were not simple miscalculations. As NIEPR and MEXT worried, one out of seven ninth graders failed the multiplication and division of fractions, and a third of the students failed to calculate the multiplication of decimals and fractions.

Objectives and Methods of the Present Study

These findings, in concert with the first author’s own observations of Japanese students, motivated the present study, which aimed to review how Japanese researchers investigate Japanese children’s grasp of the concepts of decimals and fractions. We also aimed to examine the quality and quantities of research published in Japan concerning this issue from a perspective of developmental psychology.

We searched the literature using Google Scholar. First, we searched with the keywords in English, “concept of decimals” and “concept of fractions” with combinations of “elementary schools” and “Japan.” “Concept of decimals” hit 14 studies, and “concept of fractions” hit 82. However, most of them were studies on the international comparisons of academic achievement, textbooks, or the teaching methods for prospective teachers in the teaching license curriculum. None of them featured empirical data of Japanese elementary school children.

Consequently, we changed the searching strategy to use Japanese keywords in Google Scholar. We used “小数概念” (decimal concepts) or “小数の概念” (concepts of decimals) in conjunctions with “小学校” (elementary schools) or “小学生” (elementary school children). We also searched for studies with “分数概念” (fraction concepts) or “分数の概念” (concepts of fractions) with combinations of “小学校” (elementary schools) or “小学生” (elementary school children). There were fewer hits with the decimals than the fractions in general. Then, as for the studies concerning decimals, we included all the decimal studies irrespective of elementary schools (cf. Table 3).

From those studies, we excluded those without using Japanese elementary school children, unpublished ones, and unavailable ones. Eventually, there remained 59 papers for the present review. In the following sections, we describe the details of them under four different categories: research on the development of the concept of decimal numbers, studies on the development of the concept of fractions, series of studies conducted and led by Kuriyama and Yoshida, and other studies exploring various topics in arithmetic education including teaching the concepts of decimals and fractions.

Table 3: Keywords Used in the Google Search

Keywords	Number of Hits
Decimal Concepts, Elementary Schools (小数概念, 小学校)	20
Concepts of Decimals, Elementary Schools (小数の概念, 小学校)	20
Decimal Concepts, Elementary School Children (小数概念, 小学生)	9
Concepts of Decimals, Elementary School Children (小数の概念, 小学生)	6
Decimal Concepts (小数概念)	27
Concepts of Decimals (小数の概念)	33
Fraction Concepts, Elementary Schools (分数概念, 小学校)	128
Concepts of Fractions, Elementary Schools (分数の概念, 小学校)	128
Fraction Concepts, Elementary School Children (分数概念, 小学生)	47
Concepts of Fractions, Elementary School Children (分数の概念, 小学生)	41

Note: The actual Japanese phrases used in the search were shown in parentheses.

Research on Development of the Concept of Decimal Numbers in Japanese Children

Our Google Scholar search uncovered only the following single study on the development of the concept of decimal numbers. Kuriyama and Yoshida (2000) conducted a two-year longitudinal study on the development of the understanding of decimals in Japanese third, fourth, and fifth graders. They administered two different arithmetic tasks to the children, one requiring the procedural knowledge and the other requiring the conceptual knowledge, and found that the acquisition of the concept of decimal sizes was more difficult than the acquisition of the procedural knowledge of addition and subtraction of decimal numbers. They analyzed the children's mistakes and found that the existing knowledge of integers hindered decimal concepts. Kuriyama and Yoshida proposed a three-stage acquisition model of decimal concepts through their examinations. In the first stage, children understand the concepts of decimals adequately based on their daily knowledge. However, in the second stage, they tend to misinterpret the decimal concepts with their arithmetic knowledge, such as integers. Finally, they acquire the correct concepts of decimals based on the correct knowledge of decimals.

Although not explicitly mentioning the development of the concept of decimals, Sakamoto (1998) studied the factors involved in solving the decimal ratio problems among fifth-grade children. Sakamoto (1998) administered two experiments with fifth graders for investigating what factors were crucial for solving the decimal ratio problems. Through these two experiments, Sakamoto found that the achievement levels of the decimal ratio problems were firmly related to the acquisition of the following three factors: mastering the three types of ratio calculation, understanding the concept of decimals, and manipulating the terms in a formula. The three types of ratio calculation are as follows:

Type 1 Ratio = Amount-to-be-compared $\div$ Amount-to-base-on

Type 2 Amount-to-be-compared = Amount-to-base-on $\times$ Ratio

Type 3 Amount-to-base-on = Amount-to-be-compared $\div$ Ratio

Studies on the Development of the Concept of Fractions in Japanese Children

There were nine studies exploring the development of the fraction concept. Yoshida and Kuriyama (1991) conducted a longitudinal study in which they assessed the knowledge and skills of third-grade children in fractions six times during their three-year research. They administered two types of assessments: comparisons of fraction sizes and calculations of fractions. By analyzing the incorrect answers, they proposed a four-stage model of the acquisition of the fraction concepts. The first stage is the pre-learning stage before entering primary education. However, children in this stage may have acquired correct knowledge of fractions through their knowledge of numbers, such as integers, even before learning at elementary schools. In the second stage, children first learn fractions at school. Children in the third grade learn fractions as a formal style for describing quantities smaller than one. However, three years of learning of fractions may not be sufficient for fully integrating the concept of fractions with children's existing knowledge of integers. They tend to interpret fractions through their knowledge of integers. At stage 3, children have integrated the fraction concept only partially into their existing knowledge. They understand that fractions are different from the integers, but the knowledge of integers still cause misinterpretations of fractions. Only at the final, the fourth stage, children have appropriately integrated the pieces of knowledge of fractions and integers. Children at this stage can distinguish the two systems and treat them appropriately.

Nevertheless, Yoshida and Kuriyama (1991) stated that it was not clear whether all children follow these four stages to obtain the concept of the correct fraction. Their data, obtained from experimental studies and classroom observations, showed about 10% of children seemed to have attained directly, from the first stage to the fourth, through learning at school. Meanwhile, about 20% of children remained in the second stage even after learning fractions for three years.

Sakamoto (1995) conducted an assessment study on the acquisition of the concept of fractions in sixth-graders. The assessment included the problems on the concept of fractions, the three types of ratio equations, and the procedural knowledge of multiplications and divisions of fractions. The error analyses revealed that even those students who had acquired the knowledge of the three types of rational equations and had mastered how to solve word problems with integers often failed to solve word problems involving fractions by making an appropriate formula. The ability to make an adequate formula did not relate to other knowledge and skills, such as the concept of fraction sizes, or the definitions of improper and mixed fractions.

Based on the theory of Vygotsky, Yoshida (2002) hypothesized that children would acquire the concept of fractions through their daily activities and experiences of equal division. Then, Yoshida

(2002) surveyed daily activities and experiences of second and third graders, who had not learned fractions at school yet. The results supported the hypothesis in general. However, it also showed that daily experiences often caused a misunderstanding of fractions.

Kuriyama and Son (2003) investigated the development of the concept of division in children of preschool ages, first grade, and second grade. They reported that the children seemed to have obtained an informal knowledge of dividing a whole into parts.

Harada (2013) administered an arithmetic test on the division of fractions to sixth graders, who had already learned it in class. The results revealed that a considerable proportion of the students tended to mistake a divisor for a divided. With this finding, Harada (2013) pointed out the deficit of classroom instructions for fraction division procedures.

Kuriyama and Murase (2015) surveyed assessing the achievement levels of sixth and seventh graders on ratio problems. The contents of the survey comprised problems on the three types of ratio equations (see above), and the graphical representations and the tabulations of equal-whole. By analyzing the errors and mistakes, they found that even the seventh graders had difficulties understanding the concept of equal-whole when the reference variables were different.

Ogawa and Makino (2012; 2013) explored the origins of the misconception of sizes of fractions in fifth and sixth graders. They assessed five times the answering patterns on the size comparison problems of fractions and analyzed the characteristics of errors. Their analyses revealed that the errors stemmed from the misconception of fractions, treating numerators and denominators in the same way as integers.

Uragami and Oka (2017) was a case report of cognitive counseling (Ichikawa, 1993) of a fifth-grade boy with difficulties understanding decimals and fractions. He had trouble with the calculation of decimals and fractions. Through cognitive counseling emphasizing inductive instruction, they reported, the boy gradually made fewer calculation errors.

Series of Studies Conducted and Led by Kuriyama and Yoshida

Kuriyama, Yoshida, and their collaborators have rigorously examined the children's concepts of decimal numbers and fractions for 30 years. Of their studies, those of Kuriyama and Murase (2015), Kuriyama and Son (2003), Kuriyama and Yoshida (2000), and Yoshida and Kuriyama (1991) appear elsewhere in this review. In this section, we describe the rest of their studies in chronological order. Sawano and Yoshida (1997) administered an assessment of the ability of equal dividing and ratio judgment to fifth graders. The results reproduced those of Yoshida and Kuriyama (1991). The knowledge of equal dividing in fifth graders was at a sufficient level, considering that they had not learned it at school yet.

Kuriyama (2005) conducted a paper test on fifth graders to assess the achievement of various ratio calculations. The examination comprised two problems to convert decimals into percentages (D-to-P), two to convert reversely (P-to-D), and one each for the three types of ratio calculations we listed above. The percentages of correct answers for each type of problem were 72%–89% for the D-to-P problems, 84%–91% for the P-to-D problems, 48% for Type 1, 60% for Type 2, and 38% for Type 3. Based on these results, Kuriyama (2005) identified two sources of difficulties for children in

understanding ratio: the biases from the knowledge children already acquired and the confusion of the amount-to-be-compared and the amount-to-base-on.

Kuriyama (2006) examined the strategies necessary for solving the ratio problems at the sixth-grade level. The researcher interviewed sixth graders individually after they had learned about ratios four months before the interview. During the interview, he asked each sixth grader the same questions used in Kuriyama (2005), namely, the D-to-P and P-to-D conversions, and the three types of ratio equations. The percentages of correct answers were 62%–82% for the D-to-P conversions, 76%–79% for the reverse conversions, and 27%, 57%, and 30% for each of the three types of ratio equations, respectively. The strategy analyses revealed that the children utilized a variety of strategies that were different from the formal ones taught at school. Accordingly, Kuriyama argued the necessity of investigating the children's strategies and recommended taking the "children's daily knowledge" into account for school lessons.

Kuriyama (2007) continued his research using the interview method with another group of sixth graders who had learned a ratio four months before. In this study, only the three types of ratio equations were used for the assessment. The results were similar to previous studies.

In the Kuriyama (2010) study, he gave sixth-grade children the ratio problems using fractions in the type 2 equation and the part-whole ratio calculation. The percentages of correctness were 11–45%. Through the error analyses, he concluded that the acquisition of the part-whole concept was indispensable for solving the ratio problems with fractions.

Most recently, Kuriyama and Yoshida (2014) repeated their previous study with another sample of sixth graders who had learned ratio at school ten months before. In the assessment, the three-type-ratio-equation problems and the part-whole problems were used. The results showed a similar pattern to the previous studies, but the percentages of correctness were higher, in general.

Other Studies Exploring the Concepts of Decimals and Fractions in Japanese Children

The only 17 studies described above empirically examined the development of the concepts of decimals and fractions with research data obtained from elementary school children. There were other studies utilizing classroom observational methods. Shiraishi (2006) and Kawamura and Kurosaki (2007) reported their classroom observations teaching decimal numbers. There were several more classroom observational studies of teaching fractions at school (Baba, 2004; Hasegawa, 1997; Isaka, 2008; Kitabori & Tsuji, 2019; Kurosaki & Marui, 2011; Naito, Enpukuji, Watanabe, Murayama, Ikeda, Hama, Hosaka, & Higuchi, 1990; Nakajima, 2011; Nakanishi, 2011; Okubo, Yamamoto, Abe, Shoji, Shimanuki, Morii, Suehara, Hirano, Murakami, Sakamaki, Sekine, & Nagatani, 2003; Shimomura, 2016; Utagawa, & Tsuruta, 2019; Yamaguchi, 2007; Yoshida, 2001). There were some literature studies on decimal education (Nakajima, 2011; Ota, 2015; Uno, 2011). Hayashi (1987) and several other studies were based on the literature review on teaching fractions (Fujimura, 2011; Hashimoto, 2014; Kobayashi, 2007; Koyama, 1998; Mitsuda, 1996; 2006; Ota, Inoue & Kim, 2018; Tajika & Hotta, 2020; Yoshie, 2007).

Kobayashi and Takahashi (1987) published an annual review of studies on learning and instructions, including arithmetic teaching in educational psychology in Japan. There were several more reviews on various aspects of Japanese education that included decimal and fraction teaching

(Amano, 1995; Deguchi, 2000; Goto & Morishita, 1993; Ochiai, 1988; Sakamoto, 2000; Takagaki, 2011).

There were some international comparative studies, such as Yamaguchi (2010). Yamaguchi (2010) made comparisons among the courses of study in the United Kingdom, Finland, and the United States of America. Numata (2000) stated drawbacks of the Japanese fraction teaching through comparisons with that of the US. Kumagai (1992) conducted a comparative study of the school lessons in Japan and the US. The school curricula of Japan and China were studied and compared by Li (1996). Itoi, Shibata, Saito, and Goto (2007) was another comparative study between the elementary mathematics textbooks used in Japan and Korea. Although we found only a few studies, there also were historical studies of the fraction education (Ito, 1982) and the achievement levels of decimals and fractions in modern Japan after World War II (Kubo, Makino, & Mori, 2005).

Conclusions

In the present review, we thoroughly searched and found 59 articles published in Japan that studied the development of the concept of decimals and fractions in Japanese children. Our careful scrutiny of these articles leads us to the following conclusions. First, of these 59 papers, only 17 were empirical ones utilizing an actual sample of elementary school children. Secondly, not many of them used scientific methods, such as psychological experiments, among those empirical studies. Finally, Kuriyama and Yoshida and their collaborators conducted most of the data-based studies. The school education of the ratio in Japan has heavily relied on their research.

In contrast with the trend of evidence-based education (Davies, 1999), the decimal and fraction education in Japan has hardly been evidence-based. There were no experimental studies with randomized control treatments (RCTs). No meta-analysis studies, either, with no RCT studies published. The research on the education of ratio, including decimals and fractions, in Japan, is still in the stone age, or at least in the 20th century, without utilizing genuine scientific methods.

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