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Systematic Market Risk Adjusted for Liquidity Cost in the Banking Sector

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ABSTRACT

The emergence of one or more risks in the financial markets during a specific period causes a financial crisis. Financial crises impact financial stability, which is a key concern for all financial authorities, including central banks. One way to mitigate the risks any economy faces is to understand the origin of risk and how it spreads through the financial system. Liquidity risk goes hand-in-hand with market risk, as they affect each other during a crisis. After the 2007-2008 global financial crisis, banks showed that they need monitoring and efficient liquidity management during both stress and normal conditions; that is, they require better integration of bank liquidity and market risk management. In this study, we present a new methodology to forecast the systematic market risk adjusted for liquidity cost based on the Conditional Value at Risk (CoVaR) risk measure and asymmetric conditional copulas. We analyze a sample of international banks based on asset size in the US, EU, and Asia. Our hypothesis confirms that liquidity risk goes hand-in-hand with market risk, as they affect each other during a crisis. The results show dependence in the tails of the banks' returns and the market returns. These are very high generally, and even higher for the Covid-19 period than for other periods. The effect of both market risk and liquidity risk on banks during a crisis period is less than in non-crisis periods because banks are well informed institutions and can anticipate a financial crisis and mitigate risk, which explains our results.

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INTRODUCTION

The increased complexity of the financial market makes the measurement of market and liquidity risks a pressing topic. Regulators increased the level of control over banks to ensure that sufficient capital to hedge against market and liquidity risks. Although it is important to have a single risk measure to capture the risk a bank may face, it is also very important to have

a more granular level of detail that reflects performance and how to avoid risk. Risk measures and controls are used concurrently for internal control and by regulators to monitor banks' performance.

Market and liquidity risk management is one of the most challenging tasks in quantitative finance. The Basel Committee on Banking Supervision (BCBS) (2019) defines market risk as "the losses in on- and off-balance sheet risk positions arising from movements in market prices, the risks subject to market risk capital requirements include but are not limited to, default risk, interest rate risk, credit spread risk, equity risk, foreign exchange (FX) risk, and commodities risk for both trading and banking book instruments". While the BCBS (2008) defines liquidity risk as "the ability of a bank to fund increases in assets and meet obligations as they come due, without incurring unacceptable losses. The fundamental role of banks in the maturity transformation of short-term deposits into long-term loans makes banks inherently vulnerable to liquidity risk, both of an institution-specific nature and that which affects markets as a whole."

Banks were free to use their own market risk models to estimate the regulatory capital for their trading book positions. Such internal models were designed to calculate the Value at Risk (VaR) using a confidence level of 99%. According to Wimmerstedt (2015), VaR measures the "threshold loss over a time period that will not be exceeded with a given level of confidence." The VaR model is very well known because it is an easy concept to understand and apply. Moreover, VaR is very easy to back test because it relies on the historical number of losses, which can then be compared with the actual versus forecasted losses. Tavana et al. (2018) state that liquidity risk poses a devastating financial threat to banks if not well managed and can lead to irreversible consequences, such as the 2007-2008 Global Financial Crisis (GFC). Therefore, an accurate measurement to liquidity risk is needed. However, liquidity risk measurement is a complicated process, and must be well defined in order to provide a precise measurement. One of the problems of defining liquidity risk is to determine the factors that create an appropriate function that can predict and approximate its true value. Another challenge is that the definition of liquidity risk is vague and uncertain. It is vague because the term liquidity can refer to many dimensions simultaneously, especially when used with market risk or systemic liquidity risk. It is uncertain because it may have different meanings in different contexts.

Our study focuses on the integration of market risk and liquidity risk. We also measure liquidity risk by identifying the factors that capture liquidity risk accurately and determine the impact of the overall integration of market and liquidity risks.

The rest of this paper is organized as follows. Section 2 provides a literature review. Section 3 discusses the systematic liquidity risk measurement, which depends on the state of the market. Section 4 provides the model specification, in which we specify the marginal distributions and the Copula of returns. In Section 5, we apply our methodology to quantify the systematic liquidity risk conditional on the state of the market and report the results in Section 6. Finally, we offer our conclusions in Section 7.

LITERATURE REVIEW

The literature on market risk measurement and estimation is vast. The most common model for quantifying market risk is VaR, proposed by Jorion (1997) and Dowd (1998). Brien and

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Szerszeń (2017) introduced the more accurate generalized autoregressive conditional heteroskedasticity (GARCH) based VaR. Tripe and Malone (2017) measure systemic risk in banks using the z-score, building on prior works by Roy (1952); Boyd and Graham (1986); Hannan and Hanweck (1988); and Boyd, Graham, and Hewitt (1993). Mainik and Schaanning (2014) use the Conditional VaR (CoVaR) to capture market risk after Tobias and Brunnermeier (2016) introduced CoVaR to capture risk spillovers across financial institutions. However, Löffler and Raupach (2016) find that given the large weight of banks in a market, CoVaR can imply lower systemic risk in the market if banks increase their idiosyncratic risk. Teply and Kvapilikova (2017) derive a robust market-based measure of systemic risk during different financial cycles, called the wavelet CoVaR (WCoVaR). Karimalis and Nomikos (2018) propose a new methodology to estimate the CoVaR based on copulas and VaR under financial stress conditions.

Measuring liquidity risk has been an area of focus since the 2007-2008 GFC. Many academics showed interest in modeling how to capture liquidity risk accurately. According to Tavana et al. (2018), the standard way to measure liquidity risk is by comparing the available funding sources against the expected cumulative cash shortfalls over a specific period. They conclude that artificial neural networks (ANNs) and Bayesian networks (BNs) can find the relevant critical risk factors that can measure liquidity risk accurately through functional and distributional estimations. DeYoung, Distinguin, and Tarazi (2018) study liquidity risk through the interrelationship between bank capital in the US and their liquidity prior to the Basel III regulations, which restricted liquidity positions. Bai, Krishnamurthy, and Weymuller (2018) develop a liquidity measurement system through the liquidity mismatch index (LMI). The LMI measures the mismatch between the funding liquidity of liabilities and the market liquidity of assets. Khan, Scheule, and Wu (2017) examine the effect of changing funding liquidity risk on bank risk taking capacity and the impact of bank size and capital buffers on bank risk capacity and funding liquidity risk through a panel regression. Their results support the idea that banks should move away from the short-term funding to reduce their riskiness and improve their assets quality. The findings of their study show that the bank size and capital buffer help curb the bank's risk-taking behavior in response to decrease the funding liquidity risk. This shows the importance of studying the interrelation between the market and liquidity risks. If banks size and capital buffer affect their liquidity risk profile it will impact their market risk as well.

Aniunas et al. (2017) created a model of liquidity risk management based on VaR and the best practices convenient for Lithuanian local commercial banks. Dionne, Pacurar, and Zhou (2015) propose the liquidity adjusted intraday VaR (LIVaR) as a measure of liquidity risk for high frequency data. It accounts for both the ex-ante liquidity risk of liquidating a position and market risk. Jobst (2014) developed the systemic risk-adjusted liquidity (SRL) model to measure systemic liquidity risk. Ruozi and Ferrari (2013) examine three well-known approaches to measuring liquidity risk: the cash flow matching approach, stock-based approach, and hybrid approach.

Many research efforts aimed to incorporate market liquidity risk into VaR models. Jarrow (1997) introduces a liquidity adjusted VaR that includes the volatility of the liquidity discount and the volatility of the liquidation time horizon considering both the execution lag and effect of trade size on the portfolio liquidation value. Almgren and Chriss (2001) estimate a simple linear cost model in which they construct an efficient frontier for a period that depends on the

liquidation strategy. Shamroukh (2000) demonstrates that to relate the holding period to the amount liquidated, the holding period must represent the liquidation amount. Dubil (2001) introduces a model that determines the optimal liquidation period for different assets.

However, few studies aim to integrate market risk and liquidity risk. Rodríguez and Estrada (n.d.) show that liquidity risk can become market risk and cause a financial crisis. According to Bangia et al. (2002), traditional market risk management focuses on the distribution of the portfolio value generated from the changes in the average bid and ask prices. The uncertainty of the spread between bid and ask prices creates liquidity risk, especially under unfavorable market conditions. Therefore, it is crucial to incorporate liquidity risk while modeling market risk.

Ioan and Adrian (2010) examine the integration of liquidity and market risks in the Romanian banking sector by comparing the liquidity at risk approach to the VaR approach (liquidity adjusted VaR). They conclude that ignoring the interaction between liquidity risk and market risk leads to an underestimation of risk and inadequate capital estimation. Meucci (2012) proposes a new methodology to combine market risk and liquidity risk that includes liquidity uncertainty in the market risk assessment for a future investment period. The liquidity uncertainty distribution depends on how much the bank will liquidate in the future, which depends on the market conditions (market risk). He uses both VaR and CoVaR in his estimations. Our model improved Meucci work, while he assumed normal distribution, we are not assuming normal distribution. Instead, we used we use Hansen's (1994) skewed t-distribution for the marginal distribution. In addition, Meucci did not use the copula for integration purposes, while we are used copula to integrate the effect of both market and liquidity risks.

SYSTEMIC LIQUIDITY RISK MEASURE

Adrian and Brunnermeier (2011) introduced CoVaR as a risk measure, where users generally apply CoVaR to measure the marginal contribution of an institution to overall systematic risk.

In this study, we use both VaR and CoVaR to forecast systematic liquidity risk in the banking system conditional on the state of the financial market. More precisely, we use the definition of CoVaR proposed by Adrian and Brunnermeier (2011) and extended by Girardi and Ergun (2013).

Let r_{it} be the return of bank i at time t and let Va$R_a(r_{it})$be the VaR of r_t at a significance level of $0 \leq \alpha \leq 1$, then the $VaR_a(r_{it})$ is simply the α-quantile of the distribution of r_t:

$$Pr\left[r_{it} \leq VaR_a(r_{it})\right] = \alpha. \qquad (1)$$

CoVaR is the VaR of bank i's return given that the market return is below its VaR. Adrian and Brunnermeier (2011) define the CoVaR risk measure, CoVaR$_{\alpha,\beta}(r_{it})$, as:

$$Pr\left[r_{it} \leq CoVaR_{\alpha,\beta}(r_{it}) \mid r_{mt} = VaR_\beta(r_{mt})\right] = \alpha, \qquad (2)$$

where α and β are any given significance levels and $r_{mt}$is the market return. The CoVaR defined by Adrian and Brunnermeier (2011) assumes the conditioning on r_{mt} to be exactly at its VaR level. One of the most important drawbacks of this measure is the absence of dependence

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consistency, as Mainik and Schaanning (2012) point out. To overcome this limitation Girardi and Ergun (2013) propose a modified CoVaR, defined as:

$$Pr\left[r_{it} \leq CoVaR_{a,\beta}(r_{it}) \mid r_{mt} \leq VaR_{\beta}(r_{mt})\right] = \alpha. \quad (3)$$

Here, the CoVaR $CoVaR_{\alpha,\beta}(r_{it})$ is the simply the VaR of r_{it} conditional on r_{mt} below its VaR. $CoVaR_{\alpha,\beta}(r_{it})$ is thus a measure of the conditional downside risk, including the effect of market distress on the bank's return. The systematic expected liquidity risk of bank i (i.e., $Liq_{\alpha,\beta}(r_{it})$) is therefore the difference between CoVaR and VaR. That is,
$Liq_{\alpha,\beta}(r_{it}) = CoVaR_{\alpha,\beta}(r_{it}) - VaR_{\alpha}(r_{it})$.

To compute CoVaR parametrically, we use a copula to specify the joint distribution of r_{it} and r_{mt}. Assume that (r_{it},r_{mt}) has a copula C. Using Sklar's theorem (1959), the joint cumulative distribution function is:

$$Pr\left(r_{it} \leq r_{i,} r_{mt} \leq r_{m}\right) = C\left(u_{it,} u_{mt}\right),$$

where $u_{it} = F_{r_{it}}(r_i)$ and $u_{mt} = F_{r_{mt}}(r_m)$, and $F_{r_{it}}$ and $F_{r_{mt}}$ are the cumulative distribution functions for r_{it} and r_{mt}, respectively.

It follows that

$$Pr(r_{it} \leq r_i | r_{mt} \leq r_m) = \frac{C\left(u_{it,} u_{mt}\right)}{u_{mt}}$$

and Equation (3) is equivalent to

$$\frac{C\left(\beta, F_{r_{it}}\left[CoVaR_{\alpha,\beta}(r_{it})\right]\right)}{\beta} = \alpha. \quad (4)$$

From (4), we have
$C\left(\beta, F_{r_{it}}\left[CoVaR_{\alpha,\beta}(r_{it})\right]\right) = \alpha\beta$.

Moreover, assume that C is an Archimedean copula with a generator ψ; that is,

C $(u_{it,} u_{mt} - \psi^{-1}[\psi(u_{it}) + \psi_1(u_{mt})]$.

Therefore,

$$CoVaR_{\alpha,\beta}(u_{it}) = F_{r_{it}}^{-1}[\psi^{-1}(\psi(\alpha\beta) - \psi(\alpha))],$$

where $F_{r_{it}}^{-1}$ is the inverse cumulative distribution function of r_{it}.

MODEL SPECIFICATION

The Marginal Distribution

To model the marginal distribution, we use Hansen's (1994) skewed t-distribution, which has a general form of the standardized density function, as follows:

$$f(x) = \begin{cases} bc\left(1+\frac{1}{v-2}\left(\frac{bx+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}} & \text{if } x < -\frac{a}{b} \\ bc\left(1+\frac{1}{v-2}\left(\frac{bx+a}{1+\lambda}\right)^2\right)^{-\frac{v+1}{2}} & Otherwise \end{cases}, \quad (5)$$

where x is a real random variable, v is the shape parameter $(2 < v < \infty)$, the parameter λ governs the asymmetry of the distribution (-1< λ < 1), and

$$a = 4\lambda c\frac{v-2}{v-1},$$

$$b^2 = 1 + 3\lambda^2 + a^2$$

and

$$c = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi\,(v-2)\Gamma\left(\frac{v}{2}\right)}}.$$

The function $\Gamma(x)$ is the Gamma function given by

$$\Gamma(x) = \int_0^{+\infty} t^{x-1}exp[-t]\,dt.$$

The skewed t-distribution reduces to the t-distribution distribution when λ = 0[1].
Let $x = \frac{r_{t+1}-\mu_t}{\sigma_t}$; then, the scaled version of (5) is

$$f(r_{t+1}) = \begin{cases} \frac{bc}{\sigma_t}\left(1+\frac{1}{v-2}\left(\frac{b(r_{t+1}-\mu_t)+a\sigma_t}{(1-\lambda)\sigma_t}\right)^2\right)^{-\frac{v+1}{2}} & if\ r_{t+1} < \mu_t - \frac{a}{b}\sigma_t \\ \frac{bc}{\sigma_t}\left(1+\frac{1}{v-2}\left(\frac{b(r_{t+1}-\mu_t)+a\sigma_t}{(1+\lambda)\sigma_t}\right)^2\right)^{-\frac{v+1}{2}} & Otherwise \end{cases}, \quad (6)$$

where μ_t is the conditional mean of the return and σ_t^2 is its conditional variance.

Proposition 1 The CDF and the quantile of equation (6) are, respectively,

$$F(r_{t+1}) = \begin{cases} (1-\lambda)\mathrm{T}\left(\frac{b(r_{t+1}-\mu_t)+a\sigma_t}{(1-\lambda)\sigma_t}, v\right) & if\ r_{t+1} < \mu_t - \frac{a}{b}\sigma_t \\ \left((1+\lambda)\mathrm{T}\left(\frac{b(r_{t+1}-\mu_t)+a\sigma_t}{(1+\lambda)\sigma_t}, v\right) - \lambda\right) & Otherwise \end{cases}$$

[1] Oh and Patton (2018) apply the skewed t-distribution to financial econometrics.

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and

$$q_t = \begin{cases} \mu_t + \sigma_t \mathrm{T}^{inv}\left(\frac{u}{1-\lambda}, v\right) & if\ u < 1-\lambda \\ \mu_t + \sigma_t \mathrm{T}^{inv}\left(\frac{u+\lambda}{1+\lambda}, v\right) & Otherwise \end{cases},$$

where the function $T^{inu}(.,v)$ is the quantile of the standard Student t-distribution.
See the proof in Appendix A.

Location and scale parameters

We use an ARMA-GARCH model to allow for time-varying conditional means and variances of returns. We use the model ARMA(1,1) for μ_t

$r_t = \mu_t + \varepsilon_t$,

where $r_t$is the return at time t

$$\mu_t = \mu + \emptyset r_{t-1} + \varphi\epsilon_{t-1}, \qquad (7)$$

We model the conditional variance σ_t^2 as an asymmetric GARCH (1,1), that is, AGARCH (1,1), as in Engle and Ng (1993):

$$\sigma_{i,t}^2 = \sigma_i + \alpha_i(\varepsilon_{t-1} - \delta_i)^2 + \beta_i\sigma_{i,t-1}^2 (i = 1,2). \qquad (8)$$

The parameters α_i, δ_i, and β_i capture the effect of news, volatility persistence, and leverage effect, respectively. For instance, if $\delta_i > 0$, then bad news has a higher effect on volatility than good news. We can see this clearly by rewriting the variance equation as

$$\sigma_{i,t}^2 = \sigma_i^* + \alpha_i\varepsilon_{t-1}^2 - \alpha_i\delta_i\varepsilon_{t-1} + \beta_i\sigma_{i,t-1}^2 (i = 1,2),$$

where σ_i^*=σ_i + $\alpha_i\delta_i^2$ and δ_i^* = $\alpha_i\delta_i$. Since δ_i^* is positive, bad news ($\varepsilon_{t-1} < 0$) has a higher effect on volatility than good news ($\varepsilon_{t-1} > 0$). Note that because the distribution is asymmetric, $\mu_t$is the conditional mode rather than conditional mean.

Dependence specification

In our applications, we use the Clayton copula to model the dependence between the returns. The copula density is given by $\left(u^{-\theta} + v^{-\theta} - 1\right)^{-\frac{1}{\theta}}$. The Clayton copula is a member of the Archimedean copula family. It has dependence parameter $\theta \in (0, +\infty)$ and generator function $\psi(u) = \frac{u^{-\theta}-1}{\theta}$, where perfect dependence is approached at $\theta \to +\infty$ while independence is attained at $\theta = 0$. Recall that the CoVaR is:

$$CoVaR_{\alpha,\beta,t}(Y_t) = [\psi^{-1}(\psi(\alpha\beta) - \psi(\alpha))] \qquad (9)$$

and the quantile is

$$q_t = \begin{cases} \mu_t + \sigma_t \mathrm{T}^{inv}\left(\frac{u}{1-\lambda}, v\right) & if\ \ u < 1-\lambda \\ \mu_t + \sigma_t \mathrm{T}^{inv}\left(\frac{u+\lambda}{1+\lambda}, v\right) & Otherwise \end{cases}.$$

It follows that VaR and CoVaR are, respectively,

$$VaR_{\alpha}(r_{it}) = \begin{cases} \mu_t + \sigma_t \mathrm{T}^{inv}\left(\frac{\alpha}{1-\lambda}, v\right) & if\ \alpha < 1-\lambda \\ \mu_t + \sigma_t \mathrm{T}^{inv}\left(\frac{\alpha+\lambda}{1+\lambda}, v\right) & Otherwise \end{cases}$$

and

$$CoVaR_{\alpha,\beta}(r_{it}) = \begin{cases} \mu_t + \sigma_t \mathrm{T}^{inv}\left(\frac{\left((\alpha\beta)^{-\theta}-\alpha^{-\theta}+1\right)^{-\frac{1}{\theta}}}{1-\lambda}, v\right) & if\ \left((\alpha\beta)^{-\theta}-\alpha^{-\theta}+1\right)^{-\frac{1}{\theta}} < 1-\lambda \\ \mu_t + \sigma_t \mathrm{T}^{inv}\left(\frac{\left((\alpha\beta)^{-\theta}-\alpha^{-\theta}+1\right)^{-\frac{1}{\theta}}+\lambda}{1+\lambda}, v\right) & Otherwise \end{cases} \quad (10)$$

The systematic liquidity risk is the difference between CoVaR and VaR:

$$Liq_{\alpha,\beta}(r_{it}) = CoVaR_{\alpha,\beta}(r_{it}) - VaR_{\alpha}(r_{it}).$$

LIQUIDITY COST

The bid–ask quote is by far the most demonstrable indicator of overall liquidity, but closing prices often deviate from the quotes as trades are consummated at different prices from, or even outside, the quotes. In addition, quotes are not always available in all markets and for all periods (Lesmond, 2005). Fleming and Remolona (1999) use the relative spread. Relative spread is the most studied liquidity measure because it is easy to calculate and allows a comparison of the spreads of different shares. Some authors call this measure the "inside spread." If the last trade occurred long before the absolute spread was measured, then prices for the trade and the relative spread may be irrelevant to the actual market situation (Wyss, 2004). Amihud et al. (2005) infer that relative spread is not good measure of liquidity because it does not cover all aspects of liquidity.

Glosten and Milgrom (1985) model the cost of trading as the bid–ask spread. Although the bid–ask spread is the most common measure, spread is not always available for all bonds or for all periods. We use Bangia et al.'s (2002) measure of the liquidity spread, given by:

$$spread_t = 2\frac{P_t^{Ask} - P_t^{Bid}}{P_t^{Ask} + P_t^{Bid}},$$

where P_t^{Ask} is the ask price and P_t^{Bid} is the bid price. The induced liquidity cost is:

$$L_cost_t = \frac{1}{2}(\mu_{t-1}^{s} + \kappa\,\sigma_{t-1}^{s}),$$

where κ is a scaling factor that depends on the underlying distribution, μ_{t-1}^{s} is the next period's expected spread, and σ_{t-1}^{s} is its conditional standard deviation. The stylized fact about the spread is that its distribution has positive skewness and fat tails. Therefore, as in the case of returns, we use the distribution to model the spread distribution with the same dynamic as its moments, such as returns. It follows that the scaling factor κ is:

$$\kappa = \frac{\left(1+\lambda\right)\mathrm{T}^{inv}\left(\frac{\alpha+\lambda}{1+\lambda}, v\right) - a}{b}.$$

If $\lambda = 0$, then we get the quantile of the Student t-distribution

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$$\kappa = \mathrm{T}^{inv}(\alpha, v).$$

In addition, at the limit when $v \to \infty$, we obtain the quantile of the normal distribution

$$\kappa = \Phi^{inv}(\alpha),$$

where Φ^{inv} is the inverse of the CDF of the normal distribution.

RESULTS

We use daily observations from January 2000 to June 2020 retrieved from the Bloomberg Database for 39 banks from the US, Canada, the UK, France, Italy, Germany, Switzerland, Sweden, the Netherlands, Malaysia, Japan, Singapore, and China. The data consist of the stock closing price and the bid and ask prices. For bank-specific data, we use annual observations collected from Reuters and DataStream for 1980 to 2019. The bank-specific data include the debt-to-equity ratio, return on equity ratio, and loan to assets ratio.

From Tables 2 and 3, the summary statistics of the returns and bid–ask spreads of the 40 banks suggest skew and leptokurtotic distributions. The distribution of returns has moderate skewness and kurtosis compared to the distribution of the bid–ask spread. The distribution of the spread is more skewed to the right, with extremely fat tails for most banks. The normality and symmetry of the distributions are rejected for all series.

Table 1: Descriptive Statistics: Returns

Mean		**Minimum**	**Maximum**	Standard Deviation	**Skewness**	**Kurtosis**
JP MORGAN CHASE&CO.	0.020	-23.228	22.392	2.542	0.248	18.888
GOLDMANSACHSGP.	0.001	-21.022	23.482	2.379	0.227	18.512
SUNTRUSTBANKSDEAD	-0.005	-31.705	26.669	2.936	-0.403	22.916
CITIGROUP -0.067		-49.470	45.632	3.515	-0.533	37.214
USBANCORP 0.002		-20.047	20.572	2.288	-0.095	19.580
PNCFINL.SVS.GP. 0.011		-53.436	31.547	2.632	-1.477	65.751
CHARLESSCHWAB 0.018		-16.448	17.940	2.395	0.105	10.111
STATE STREET -0.001		-89.250	27.266	3.112	-6.510	206.205
BANK OF NEWYORKMELLON -0.002		-31.687	22.159	2.451	-0.230	24.274
TORONTO DOM.BK.(NYS)	0.012	-16.531	14.898	1.664	-0.291	17.089
FIFTHTHIRDBANCORP	-0.019	-57.317	47.229	3.765	-0.392	47.820
WELLS FARGO&CO -0.008		-27.210	28.341	2.756	0.691	24.943
CAPITALONEFINL. -0.003		-28.824	23.452	3.032	-0.482	19.162
NEW YORK COMMUNITY BANCORP -0.012		-14.856	13.060	2.014	-0.253	12.659
AMERICAN EXPRESS0.014		-19.352	19.789	2.385	0.099	16.472
BANKOFAMERICA -0.022		-34.206	30.210	3.323	-0.314	25.012
MORGANSTANLEY -0.010		-29.966	62.585	3.263	1.521	56.303
ROYAL BANKOFCANADA	0.010	-18.799	15.021	1.696	-0.140	18.525

STANDARDCHARTERED	-0.042	-19.471	30.718	2.680	0.205	15.591
BARCLAYS -0.062		-27.691	56.181	3.392	0.785	34.535
ROYAL BANKOFSCTL.GP.	-0.121	-112.020	31.763	3.934	-7.088	201.178
BNPPARIBAS -0.027		-21.208	20.461	2.735	0.110	11.537
SOCIETEGENERALE -0.062		-25.125	22.910	3.085	-0.290	10.995
CREDITAGRICOLE -0.039		-19.772	23.578	2.920	0.052	9.990
UNICREDIT -0.090		-29.257	20.491	3.243	-0.270	9.619
INTESASANPAOLO -0.037		-28.150	19.447	2.843	-0.578	12.310
COMMERZBANK -0.111		-29.437	20.048	3.201	-0.205	10.476
DEUTSCHEBANK -0.068		-19.131	24.829	2.856	0.218	10.864
CREDITSUISSEGROUP	-0.051	-19.093	23.842	2.520	0.118	12.926
NORDEABANK(OME)	-0.014	-17.222	19.427	2.398	0.251	10.863
INGGROEP -0.044		-31.796	27.128	3.240	-0.107	16.837
CIMBGROUPHOLDINGS	-0.007	-15.824	15.094	1.718	-0.049	11.497
MALAYANBANKING -0.008		-8.326	9.779	1.336	-0.326	9.215
HSBCHOLDINGS -0.036		-27.632	18.228	1.712	-1.442	40.342
MITSUBISHIUFJFINL.GP.	-0.031	-15.939	14.458	2.069	0.255	7.407
SUMITOMOMITSUIFINL.GP.	-0.035	-16.450	13.967	2.167	0.194	9.845
MIZUHOFINL.GP. -0.049		-16.049	16.887	2.121	0.227	11.088
DBSGROUP HOLDINGS	0.005	-10.048	10.391	1.588	0.002	8.528
OVERSEA•CHINESEBKG.	0.008	-11.761	9.953	1.455	-0.150	8.557

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Table 2: Descriptive Statistics: Spread

	Mean	Minimum	Maximum	Standard Deviation	Skewness	Kurtosis
JP MORGAN CHASE &CO.	0.001	-0.014	0.091	0.003	19.016	514.854
GOLDMAN SACHS GP.	0.001	-0.005	0.068	0.002	13.825	276.611
SUNTRUST BANKS DEAD	0.001	-0.003	0.103	0.005	17.512	339.069
CITIGROUP	0.001	-0.021	0.070	0.003	8.929	136.725
US BANCORP	0.001	-0.002	0.109	0.004	20.767	476.457
PNC FINL.SVS.GP.	0.001	-0.001	0.152	0.005	23.142	654.345
CHARLES SCHWAB	0.001	-0.010	0.608	0.015	37.334	1460.180
STATE STREET	0.001	-0.001	0.120	0.004	20.085	496.555
BANK OF NEW YORK MELLON	0.001	-0.003	0.290	0.007	39.258	1626.628
TORONTO DOM.BK. (**NYS**)	0.001	0.000	0.331	0.009	27.857	906.025
FIFTH THIRD BANCORP	0.001	-0.005	0.028	0.001	9.310	160.388
WELLS FARGO &**CO**	0.001	-0.004	0.096	0.003	17.075	408.070
CAPITAL ONE FINL.	0.001	-0.001	0.210	0.004	42.582	2144.743
NEW YORK COMMUNITY BANCORP	0.002	0.000	0.199	0.009	16.682	314.864
AMERICAN EXPRESS	0.001	-0.001	0.061	0.002	19.025	488.459
BANK OF AMERICA	0.001	-0.017	0.230	0.005	38.208	1863.985
MORGAN STANLEY	0.001	-0.002	0.045	0.002	10.560	146.124
ROYAL BANK OF CANADA	0.001	0.000	0.027	0.001	5.909	74.231
STANDARD CHARTERED	0.001	0.000	0.025	0.001	8.403	126.498
BARCLAYS	0.001	0.000	0.202	0.004	49.599	2761.470
ROYAL BANK OF SCTL.GP.	0.001	-0.054	0.026	0.002	-5.443	279.776
BNP PARIBAS	0.000	-0.039	0.023	0.001	-7.074	450.608
SOCIETE GENERALE	0.000	-0.021	0.046	0.001	14.953	515.147
CREDIT AGRICOLE	0.001	-0.019	0.014	0.001	-0.470	106.414
UNICREDIT	0.002	-0.006	0.053	0.004	3.692	22.809
INTESA SANPAOLO	0.003	-0.155	0.162	0.007	-0.808	211.292
COMMERZBANK	0.004	-0.002	0.087	0.002	11.704	363.250
DEUTSCHE BANK	0.003	0.000	0.032	0.001	3.823	60.641
CREDIT SUISSE GROUP	0.001	-0.028	0.006	0.001	-30.677	1516.026
NORDEA BANK (**OME**)	0.001	0.000	0.032	0.001	12.132	230.765
ING GROEP	0.001	-0.076	0.025	0.002	-20.209	724.599
CIMB GROUP HOLDINGS	0.004	0.001	0.385	0.009	32.404	1271.895
MALAYAN BANKING	0.003	0.001	0.288	0.006	32.286	1534.081
HSBC HOLDINGS	0.001	0.000	0.011	0.000	8.609	134.537
MITSUBISHI UFJ FINL.GP.	0.002	-0.085	0.056	0.003	-5.624	210.838
SUMITOMO MITSUI FINL.GP.	0.002	-0.197	0.020	0.004	-30.655	1432.864
MIZUHO FINL.GP.	0.004	-0.096	0.085	0.005	-4.210	131.904
DBS GROUP HOLDINGS	0.001	-0.034	0.017	0.001	-3.000	117.909
OVERSEA•CHINESE BKG.	0.002	-0.004	0.016	0.002	2.705	12.922

We follow Bouaddi and Moutanabbir (2020) and use the following ratio:

$$R^{L}_{\alpha,\beta} = \frac{CoVaR^{L}_{\alpha,\beta}(Y_t) - VaR^{L}_{\beta}(Y_t)}{VaR^{L}_{\beta}(Y_t)}, \qquad (11)$$

where $CoVaR^{L}_{\alpha,\beta}(Y_t)$ is the β quantile of a given bank given that the US market is below its αquantile. $VaR^{L}_{\beta}(Y_t)$ is given by:

$$Pr\left[Y_t \leq VaR^{L}_{\beta}(Y_t)\right] = \beta,$$

which is the lower quantile computed from the marginal distribution of the market. This ratio captures the effect of the US market on the banks' risk when it is in an extreme bad state. We highlight that α and β are very small probabilities (e.g., equal to 1% or 5%). Thus, $R^{L}_{\alpha,\beta}$ is a measure of the potential expected systematic extreme loss percentage change. In our analysis, we set α and β to 5%.

Table 3: Potential Expected Systematic Extreme Loss percentage change, $R^{L}_{\alpha,\beta}$

	Non-**Crisis Periods**	**Crisis Periods**	**Covid19 Crisis Period**	**Debt-to-Equity Ratio**	**Loans to Assets**	**Return On Equity**
JP MORGAN CHASE &CO.	-28.653	-38.370	-32.537	9.998	33.130	293.911
GOLDMAN SACHS GP.	-19.291	-15.620	-46.262	11.685	6.980	663.708
SUNTRUST BANKS DEAD	-10.251	-3.549	-30.204	5.653	69.266	113.105
CITIGROUP	-4.204	16.519	-16.602	1.562	35.561	350.403
US BANCORP	4.904	4.372	-18.205	14.726	63.372	195.368
PNC FINL.SVS.GP.	10.700	4.967	-3.592	10.052	58.118	152.004
CHARLES SCHWAB	3.257	-0.637	-7.078	18.284	7.244	33.795
STATE STREET	-16.658	-6.600	-40.248	9.063	7.702	144.305
BANK OF NEW YORK MELLON	-25.772	-33.968	-34.178	7.385	16.502	96.000
TORONTO DOM.BK. (NYS)	6.443	6.146	-10.594	14.268	51.340	231.727
FIFTH THIRD BANCORP	-10.719	-4.606	-17.751	8.018	67.661	134.225
WELLS FARGO &CO	6.242	-0.875	-19.373	11.751	55.939	214.647
CAPITAL ONE FINL.	-11.590	-6.972	-57.580	7.891	65.632	207.779
NEWYORKCOMMUNITYBANCORP	15.910	4.807	-40.779	6.736	73.054	253.424
AMERICAN EXPRESS	-20.067	-16.002	-50.115	25.797	53.453	367.868
BANK OF AMERICA	-17.658	-13.386	-16.221	4.520	43.425	272.367
MORGAN STANLEY	-14.200	-13.348	-51.553	6.193	9.534	600.425
ROYAL BANK OF CANADA	-4.112	-5.815	-18.905	17.383	43.678	260.808
STANDARD CHARTERED	-21.784	-26.698	-27.456	8.034	52.492	288.291
BARCLAYS	-22.592	-16.812	-36.446	7.532	31.127	714.046
ROYAL BANK OF SCTL.GP.	-12.347	-10.352	-23.891	NA	NA	NA
BNP PARIBAS	-20.803	-15.980	-34.870	7.666	36.543	314.720
SOCIETE GENERALE	-5.080	-5.328	-18.413	4.421	36.025	348.028
CREDIT AGRICOLE	-10.778	-5.748	-16.196	3.282	43.127	727.924
UNICREDIT	-20.773	-37.325	-6.819	9.143	63.326	538.691
INTESA SANPAOLO	4.916	7.074	4.791	4.360	61.362	469.743
COMMERZBANK	-7.323	-7.846	-3.023	0.978	51.528	921.288
DEUTSCHE BANK	-13.443	-10.655	-45.203	1.164	25.440	687.852
CREDIT SUISSE GROUP	-5.417	-5.189	-14.735	3.145	28.473	849.988
NORDEA BANK (OME)	-9.573	2.915	-17.637	11.444	56.611	775.038
ING GROEP	-11.597	-11.707	-21.274	10.739	76.820	709.873

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CIMB GROUP HOLDINGS	25.665	28.990	15.987	12.474	68.199	142.037
MALAYAN BANKING	23.201	29.950	-0.329	12.713	58.692	172.196
HSBC HOLDINGS	-8.068	-9.654	-10.303	7.452	45.446	263.770
MITSUBISHI UFJ FINL.GP.	-4.133	2.957	-24.386	18.609	57.594	155.673
SUMITOMO MITSUI FINL.GP.	3.414	10.087	-7.382	2.810	2.408	119.443
MIZUHO FINL.GP.	14.350	23.178	-12.449	6.982	41.294	946.668
DBS GROUP HOLDINGS	-16.582	-14.062	-38.672	10.721	67.068	135.528
OVERSEA-CHINESE BKG.	-10.901	-10.331	-19.212	12.408	56.382	129.370
AVERAGE	**-6.80**	**-5.27**	**-22.30**	**9.13**	**45.30**	**368.32**

Note: The average potential expected systematic extreme loss percentage change ($R^{L}_{\alpha,\beta}$) from equation (11)

Table 4 presents the results for $R^{L}_{\alpha,\beta}$ for the 39 banks. We find that the dependence in the tails of the banks' returns and the market returns are very high, and higher during the Covid-19 period than for other periods. Conditional on the US market being extremely down, the potential expected systematic extreme loss percentage change is very high during the Covid-19 period, with an average absolute change of around 24% compared to around 13% in other periods, as the last row of Table 4 reports.

Results in row 41 of Table 4 reflect the fact that the banking system sees its risk decreasing on average during crisis periods compared to non-crisis periods because the proportion (the percentage increase) of market risk, including liquidity risk (not the total amount) affecting banks during a crisis is less than under normal conditions. The potential expected systematic extreme loss decreases by 22.26% on average during the Covid-19 period compared with 6.66% and 5.14% during the non-crisis periods and the 2007- 2008 GFC, respectively. However, not all banks experienced a reduction in the extreme potential loss, as we see in columns 1-3 of the table. This supports our hypothesis that banks must account for the dependence between liquidity and market risks. Although some banks showed that the percentage increase of the expected systematic extreme loss decrease during crisis period than non-crisis period, other banks showed an increase. In both cases, ignoring the impact of the dependence of market and liquidity risks will underestimate the riskiness of the bank profile, which can lead to the underestimation of the required capital that can affect the banks financial viability.

CONCLUSION

We confirmed our hypothesis that liquidity risk goes hand-in-hand with market risk, as they affect each other during a crisis. We present a new methodology to forecast the systematic market risk adjusted for liquidity cost based on the CoVaR risk measure and asymmetric conditional copulas. Our results show dependence in the tails of the banks' returns and the market returns. These are very high generally, and even higher for the Covid-19 period than for other periods. The effect of both market risk and liquidity risk on banks during a crisis period is less than in non-crisis periods because banks are well informed institutions and can anticipate a financial crisis and mitigate risk, which explains our results. However, this result does not mean that the total amount of risk (both market and liquidity risk) that affects banks stemming from the market is less during a crisis. In other words, the percentage increase of the risk during crisis period is less than the percentage increase of the risk during non-crisis period, since banks behavior during crisis period changes and reflects different measures than when in operating in non-crisis conditions.

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APPENDIX (A): PROOF OF PROPOSITION 1

In the following proof, we derive the CDF and the quantile functions of equation (5). The CDF is:

$$F(x)=\begin{cases}\displaystyle\int_{-\infty}^{\mathrm{x}} bc\left(1+\frac{1}{v-2}\left(\frac{bt+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}}dt & if\quad t<-\frac{a}{b}\\ \displaystyle\int_{-\infty}^{-\frac{a}{b}} bc\left(1+\frac{1}{v-2}\left(\frac{bt+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}}dt & \\ \displaystyle+\int_{-\frac{a}{b}}^{\mathrm{x}}\frac{bc}{\sigma_t}\left(1+\frac{1}{v-2}\left(\frac{bt+a}{1+\lambda}\right)^2\right)^{-\frac{v+1}{2}}dt & Otherwise\end{cases}.$$

Let $z=\frac{bt+a}{(1-\lambda)}$; then,

$$\int_{-\infty}^{x} bc\left(1+\frac{1}{v-2}\left(\frac{bt+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}}dt=(1-\lambda)\int_{-\infty}^{\frac{bx+a}{1-\lambda}} c\left(1+\frac{1}{v-2}z^2\right)^{-\frac{v+1}{2}}dz$$

$$=(1-\lambda)\mathrm{T}\left(\frac{bx+a}{1-\lambda},v\right),\qquad(12)$$

where $T\ (s,v)$ is the CDF of the standard Student t-distribution of s with degree of freedom v. We deduce from (12) that:

$$\int_{-\infty}^{-\frac{a}{b}} bc\left(1+\frac{1}{v-2}\left(\frac{bx+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}}dt=\frac{1-\lambda}{2}.$$

Similarly, let $z=\frac{bt+a}{(1+\lambda)}$; then,

$$\int_{-\frac{a}{b}}^{x} bc\left(1+\frac{1}{v-2}\left(\frac{bt+a}{(1+\lambda)}\right)^2\right)^{-\frac{v+1}{2}}dt=(1+\lambda)\int_{0}^{\frac{bx+a}{1+\lambda}} c\left(1+\frac{1}{v-2}z^2\right)^{-\frac{v+1}{2}}d_z$$

$$=(1+\lambda)\int_{-\infty}^{\frac{bx+a}{1+\lambda}} c\left(1+\frac{1}{v-2}z^2\right)^{-\frac{v+1}{2}}d_z$$

$$-(1+\lambda)\int_{-\infty}^{0} c\left(1+\frac{1}{v-2}z^2\right)^{-\frac{v+1}{2}}d_z.$$

Thus, $-(1+\lambda)\mathrm{T}\left(\frac{bx+a}{1+\lambda},v\right)-\frac{1+\lambda}{2}$,

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$$F(x) = \begin{cases} (1-\lambda)\mathrm{T}\left(\dfrac{bx+a}{1-\lambda}, \mathrm{v}\right) & if \quad x < -\dfrac{a}{b} \\ \dfrac{1-\lambda}{2} + (1+\lambda)\mathrm{T}\left(\dfrac{bx+a}{1+\lambda}, \mathrm{v}\right) & Othetwise \end{cases} .$$

It follows that

$$F(x) = \begin{cases} (1-\lambda)\mathrm{T}\left(\frac{bx+a}{1-\lambda}, v\right) & if \;\; x < -\frac{a}{b} \\ (1+\lambda)\mathrm{T}\left(\frac{bx+a}{1+\lambda}, v\right) - \lambda & Otherwise \end{cases} .$$

The resulting quantile function is

$$F^{inv}(x) = \begin{cases} T^{inv}\left(\frac{u}{1-\lambda}, v\right) & if \quad u < \frac{1}{2} \\ T^{inv}\left(\frac{u+\lambda}{1+\lambda}, v\right) & Otherwise \end{cases}, \qquad (13)$$

where u is a uniform random variable (i.e., $u \in [0,1]$). For the rescaled version with mean μ_t and variance σ_t^2, we have:

$$F(r_{t+1}) = \begin{cases} (1-\lambda)T\left(\dfrac{b\left(r_{t+1-\mu_t}\right) + a\sigma_t}{(1-\lambda)\sigma_t}, v\right) & if \;\; r_{t+1} < \mu_t - \dfrac{a}{b}\sigma_t, \\ \left((1+\lambda)\, T\left(\dfrac{b(r_{t+1} - \mu_t)a\sigma_t}{(1+\lambda)\sigma_t}, v\right) - \lambda\right) & Otherwise \end{cases},$$

and the rescaled quantile is

$$q_t = \begin{cases} \mu_t + \sigma_t \dfrac{(1-\lambda)T^{inv}\left(\frac{u}{1-\lambda}, v\right) - a}{b} & if \;\; u < 1-\lambda \\ \mu_t + \sigma_t \dfrac{(1+\lambda)T^{inv}\left(\frac{u+\lambda}{1+\lambda}, v\right) - a}{b} & Otherwise \end{cases} . \qquad (14)$$