

Markov-Switching Vector Autoregressive (MS-VAR) Modelling (Mean Adjusted): Application to Macroeconomic data

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ABSTRACT

Researchers very often model economic relationships using linear models. Even when non-linear models are used, they hardly consider the possibility of regimes, the transmission from one regime to another and also the duration of stay in a particular regime. These are not captured by linear models. To address this, we applied the Markov-Switching Mean Vector Autoregressive Model to model and estimate the interdependence between macroeconomic variables (International Trade and Macroeconomic Stability) within the context of the Nigerian economy. Specifically, the study analyzed the trends of total export, total import, exchange rate, and inflation rate in Nigeria within the study period, Estimated MSM-VAR models on the study variables and selected the best model using information criteria. The study empirically investigated the interdependence existing among the study variables; determined the probabilities of transition from one regime to another and the duration of stay in a regime; the study ascertained the relative significance of each random innovation in affecting the study variables. Monthly time series data of 246 months spanning through January 2000 to June 2020 were collected from the Central Bank of Nigeria Statistical Bulletin. The study used the Markov-Switching Mean Vector Autoregression in the Analysis. The results showed that all variables were stationary at first difference. The study chose 2 regimes and the model selection criteria selected [MS(2)-MVAR(2)]. The variables were largely self-explanatory and very strongly exogenous.

Keywords: Markov-Switching Variance Autoregressive Model (MS-VAR), MSM-VAR, International Trade, Macroeconomic Stability, Nigeria

INTRODUCTION

The dynamic interdependence existing between two or more macroeconomic variables is commonly modelled linearly without recourse to the existence of regimes. Consequently, the models are inappropriate with incomplete structural inference. Tuaneh and Essi (2021) reported that linear models ignore the unobservable state, regime switches and duration of stay

in a regime. The linear econometric methods often adopted by researchers suggest that the variables considered are always increasing or always decreasing. The periods of normalcy are not reflected. To bridge this gap in the literature, this study adopted a Multivariate Markov-Switching VAR (MS-VAR) model proposed by (Krolzig, 1997) to determine a regime-dependent dynamic interdependence between total export, total import, exchange rate and inflation rate. The MS-VAR is a flexible nonlinear multivariate framework that allows the parameters of the model to vary over time and according to different regimes determined by an often disregarded unobservable state variable.

There are basically two classes of the MS-VAR; the Markov-Switching Intercept VAR Model and the Markov-Switching Mean VAR Model. The main focus of the study was to apply the Markov-Switching Mean VAR Model, in modelling and estimating the interdependence between macroeconomic variables (International Trade and Macroeconomic Stability) within the context of the Nigerian economy. The study adopted total export and total import as proxies for International trade while exchange rate and inflation rate were used as proxies for Macroeconomic Stability. Specifically, it is the objective of the study to analyze the trends of total export, total import, exchange rate and inflation rate in Nigeria within the study period, Estimate the MSM-VAR models of total export, total import, exchange rate and inflation rate, and select the best model using information criteria, empirically investigate the interdependence existing between exchange rate and inflation in Nigeria and determine the probabilities of transition from one regime to another and the duration of stay in a regime. It is also our view to ascertain the relative significance of each random innovation in affecting the variables of the study.

MS-VAR models are used for prediction when there is state transmission (from an observable state to an unobservable state). The model describes a non-linear data generation process as piecewise linear through the restriction of the process to be linear in each state. The mechanism that produces the state is stochastic and is assumed differently. MS-VAR is a generalization of the basic finite VAR model of order p . the idea is that when the process is regime dependent, the stable VAR model might be inappropriate. The MS-VAR class of models has an underlying data generating process of an observed time series vector Y_t which depend upon an unobservable regime S_t . Observational equations do not completely describe the data-generating process. The models are formulated allowing deducing the evolution of regimes (Clement and Krolzig, 2002)

LITERATURE

Markov Switching Vector Autoregressive (MS-VAR) models have been applied to complex multivariate systems. In a series of papers, Krolzig (1997, 1998, 2000, and 2001) and Clement and Krolzig (2002) discuss the characterization and the tested business cycle asymmetries using the MS-VAR models. Series of authors, Krolzig (2001), Tilmann (2003) Chen and Shen 2007, Kiganda, Obange and Adhiambo 2017 used MS-VAR in macroeconomic research.

Tarsa and Bayat (2015) studied Inflation Targeting for Turkey, they analyzed the behaviour of the inflation rate in Turkey using monthly data spanning from January 2003 to August 2014. The study used the Markov Switching Intercept Autoregression (MSI-AR). The study identified two regimes and the results showed that the regime changes were slow in turkey.

Wai and Sek (2003) studied Intercept Adjusted Markov Switching Vector Autoregressive Model in Macro-economic Time Series Data. The study ascertained the asymmetry and state-switching behaviour of the data using the Markov Switching Vector Autoregressive models particularly the MSI-VAR model with adjusted Intercept. The study identified two regimes and the result showed a smooth transition of the stock index changes from recession state to growth state. The study showed that the gold price and oil price affected the stock exchange. It, however, concluded that MSI-VAR model provides significant, valid and reliable results.

Guidolin, (2012) applied Markov Switching Models in Empirical Finance. Tuaneh and Essi (2021) carried out a study on Markov-Switching Vector Autoregressive modelling (Intercept Adjusted); Application to International Trade and Macroeconomic Stability in Nigeria (2000–2019). The study identified two regimes and selected Markov-Switching Intercept (2 regimes) Vector Autoregressive (2 lags) Model [MSI-ARH (2) VAR (2)] Model as the best model

METHODOLOGY

In this section, we present the research design, sources and types of data, methods used in data collection, data analysis techniques and model specification.

Research Design

This quasi-experimental design was used. This is owing to the fact that the study sought to find out the causes or effects relationship among the variables of interest

Types and Sources of Data

We used monthly time series data on total export, total import, exchange rate, and inflation rate spanning from January 2001 to June 2020. The data were obtained from the Central Bank of Nigeria (CBN) Statistical Bulletin 2020.

Methods of Data Analysis

The Markov Switching Vector Autoregressive Model (MSVAR) principally the Markov Switching Mean Vector Autoregressive Model (MSM-VAR) which is a non-linear modelling method was used. However, the unit root tests were conducted to ascertain the nit root status of the study variables

Model Specification

The models used to evaluate the interdependence between variables that served as the dimension of international trade and macroeconomic stability indicators are specified in this section.

The general form of a standard VAR model is;

$$Y_t = \psi_0 + \sum_{i=1}^p \psi_i Y_{t-i} + \varepsilon_t \quad (1.0)$$

Following Krolzig 1997, the above is modified to allow for regime change so that Y_t follows a VAR procedure that is dependent on an unobservable discrete regime variable (s_t)

$$\text{MSI: } Y_t = \psi_0(s_t) + \sum_{i=1}^p \psi_i(s_t) Y_{t-i} + \varepsilon_t \text{ where: } \varepsilon_t \sim \text{iid}, N(0, \sum(s_t)) \quad (1.1)$$

$$\text{MSM: } Y_t - \mu_t(s_t) = \psi_1(s_t)(Y_{t-1} - \mu_t(s_{t-1})) + \dots + \psi_p(s_t)(Y_{t-p} - \mu_t(s_{t-p})) + \varepsilon_t \quad (1.2)$$

where; $m = 1, 2, \dots, m$ possible regimes and in period T when $S_t = m$ $\varepsilon_t \sim \text{iid}(0, \Sigma(s_t))$ and $\mu_t(s_t), \psi_1(s_t), \dots, \psi_p(s_t), \Sigma(s_t)$ are regime dependent parameters to be estimated.

In analytical applications, it is necessary to use a model in which certain parameters are restricted on the state of the Markov chain and the rest are regime invariant. The special cases of MS-VAR result from varying (state-dependent) the autoregressive parameters, the mean and intercepts parameter, and the heteroscedastic parameter. a regime-dependent covariance structure of the process might be considered as an additional feature.

The MS(m)-VAR(p) models, therefore, allows for a variety of specifications. Krolzig (1997) established a common notation expressing the models where the parameters are subject to regime dependence

Note: μ = mean, ψ_0 = intercept term, Σ = Variance and ψ_i = matrix of autoregressive parameter.

The eight (8) basic classes Markov Switching Mean Vector Autoregressive (MSM-VAR) models noted earlier are tabulated below. An overview of Markov switching vector autoregressive models is presented in table 1.

Table 1: Special Cases of Markov-Switching Mean Variance Autoregressive (MSMVAR) Models

S/n	Markov Switching Mean (MSM) VAR Model	M	AR _i	Σ
1	MSM(m)-VAR(p)	✓	×	×
2	MSMAR(m)-VAR(p)	✓	✓	×
3	MSMH(m)-VAR(p)	✓	×	✓
4	MSMARH(m)-VAR(p)	✓	✓	✓
5	MSAR(m)-MVAR(p)	×	✓	×
6	MSH(m)-MVAR(p)	×	×	✓
7	MSARH(m)-MVAR(p)	×	✓	✓
8	LINEAR-(m)-MVAR(p)	×	×	×

Where: (m) = Number of regime, (p) = number of lags, AR = autoregressive parameter, H(Σ) = Variance (Heteroscedastic parameter), M = Mean

VAR Lag length Selection Criteria chose

Too few lags and too many lags may result in auto-correlated errors and overfitting respectively. The information criteria are used to minimize errors. This study used the Akaike Information Criteria (AIC), and Schwartz Information Criteria (SC). The lag length selection result indicated that a lag length of 2 was chosen by these information criteria. The models were consequently specified using lag length 2 and subsequent estimation was done using 2 lags

Specification of Markov Switching Variance Autoregressive Model with 2 Regimes and 2 Lags

Specification of Markov Switching Variance Autoregressive Model with 2 Regimes and 2 Lags

$$\begin{aligned}
 Y_t = \left\{ \begin{aligned}
 &st(1|s_{t-1}) \begin{pmatrix} \Delta \text{TEX} \\ \Delta \text{TIM} \\ \Delta \text{EXR} \\ \Delta \text{INR} \end{pmatrix} = \begin{pmatrix} \omega_{1,111} & \omega_{1,112} & \omega_{1,113} & \omega_{1,114} \\ \omega_{2,111} & \omega_{2,112} & \omega_{2,113} & \omega_{2,114} \\ \omega_{3,111} & \omega_{3,112} & \omega_{3,113} & \omega_{3,114} \\ \omega_{4,111} & \omega_{4,112} & \omega_{4,113} & \omega_{4,114} \end{pmatrix} \begin{pmatrix} \Delta \text{TEX}_{t-1} \\ \Delta \text{TIM}_{t-1} \\ \Delta \text{EXR}_{t-1} \\ \Delta \text{INR}_{t-1} \end{pmatrix} + \begin{pmatrix} \omega_{1,121} & \omega_{1,122} & \omega_{1,123} & \omega_{1,124} \\ \omega_{2,121} & \omega_{2,122} & \omega_{2,123} & \omega_{2,124} \\ \omega_{3,121} & \omega_{3,122} & \omega_{3,123} & \omega_{3,124} \\ \omega_{4,121} & \omega_{4,122} & \omega_{4,123} & \omega_{4,124} \end{pmatrix} \\
 &\begin{pmatrix} \Delta \text{TEX}_{t-2} \\ \Delta \text{TIM}_{t-2} \\ \Delta \text{EXR}_{t-2} \\ \Delta \text{INR}_{t-2} \end{pmatrix} + St \begin{pmatrix} \omega_{11} \\ \omega_{21} \\ \omega_{31} \\ \omega_{41} \end{pmatrix} + st(\varepsilon_t \varepsilon_t') \\
 \\
 &st(2|s_{t-1}) \begin{pmatrix} \Delta \text{TEX} \\ \Delta \text{TIM} \\ \Delta \text{EXR} \\ \Delta \text{INR} \end{pmatrix} = \begin{pmatrix} \omega_{1,211} & \omega_{1,212} & \omega_{1,213} & \omega_{1,214} \\ \omega_{2,211} & \omega_{2,212} & \omega_{2,213} & \omega_{2,214} \\ \omega_{3,211} & \omega_{3,212} & \omega_{3,213} & \omega_{3,214} \\ \omega_{4,211} & \omega_{4,212} & \omega_{4,213} & \omega_{4,214} \end{pmatrix} \begin{pmatrix} \Delta \text{TEX}_{t-1} \\ \Delta \text{TIM}_{t-1} \\ \Delta \text{EXR}_{t-1} \\ \Delta \text{INR}_{t-1} \end{pmatrix} + \begin{pmatrix} \omega_{1,221} & \omega_{1,222} & \omega_{1,223} & \omega_{1,224} \\ \omega_{2,221} & \omega_{2,222} & \omega_{2,223} & \omega_{2,224} \\ \omega_{3,221} & \omega_{3,222} & \omega_{3,223} & \omega_{3,224} \\ \omega_{4,221} & \omega_{4,222} & \omega_{4,223} & \omega_{4,224} \end{pmatrix} \\
 &\begin{pmatrix} \Delta \text{TEX}_{t-2} \\ \Delta \text{TIM}_{t-2} \\ \Delta \text{EXR}_{t-2} \\ \Delta \text{INR}_{t-2} \end{pmatrix} + (s_t) \begin{pmatrix} \omega_{12} \\ \omega_{22} \\ \omega_{32} \\ \omega_{42} \end{pmatrix} + st(\varepsilon_t \varepsilon_t')
 \end{aligned} \right.
 \end{aligned}
 \tag{1.3}$$

Where: $Y_t = \begin{pmatrix} \Delta \text{TEX}_t \\ \Delta \text{TIM}_t \\ \Delta \text{EXR}_t \\ \Delta \text{INR}_t \end{pmatrix}$ and $\begin{pmatrix} \omega_{11} \\ \omega_{21} \\ \omega_{31} \\ \omega_{41} \end{pmatrix} = \begin{pmatrix} \omega_{12} \\ \omega_{22} \\ \omega_{32} \\ \omega_{42} \end{pmatrix}$ regime 1 and 2 have a common intercept term.

Where: $Y_t = \begin{pmatrix} \Delta \text{TEX}_t \\ \Delta \text{TIM}_t \\ \Delta \text{EXR}_t \\ \Delta \text{INR}_t \end{pmatrix}$ and $\begin{pmatrix} \omega_{11} \\ \omega_{21} \\ \omega_{31} \\ \omega_{41} \end{pmatrix} = \begin{pmatrix} \omega_{12} \\ \omega_{22} \\ \omega_{32} \\ \omega_{42} \end{pmatrix}$ regime 1 and 2 have a common intercept term.

The order of numbering is; model, regime, lag, and variable for the coefficients and Model, regime for the intercept term.

MSARH(2)-MVAR(2) model requires the estimation of a 4x4 matrix for each regime which gives 64 autoregressive parameters, a column of 4 intercept terms common for the 2 regimes, a matrix of 4 variances and 6 co-variances for each regime, and 2 independent transition probabilities, given a total of 90 parameters It is often assumed that the state variable is governed by the Markov chain:

$$\begin{aligned}
 P(s_t = 1|s_{t-1} = 1) &= P_{11} \\
 P(s_t = 2|s_{t-1} = 1) &= P_{12} \\
 P(s_t = 1|s_{t-1} = 2) &= P_{21}
 \end{aligned}$$

$$P(s_t = 1 | s_{t-1} = 2 = P_{22})$$

Which is often presented as:

$$P_{ij} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

These transition probabilities are restricted so that $P_{11} + P_{12} = 1$ and $P_{21} + P_{22} = 1$

RESULTS AND DISCUSSION

Number of Unobservable State/Regime

Table 2: Testing for the Number of Unobservable State/Regimes

Since $S_t = 1, 2, 3, \dots, N, N \geq 2$

Regime	AIC	SC
2	8.1427	8.2677
3	9.1765	9.2345
4	10.0819	10.1335

Table 2 showed that the study identified 2 regimes, regime 1 (expansion) and regime 2 (contraction). The best regime is selected based on minimum information criteria. The commonly used information criteria are the Akaike information criterion and Schwarz information criterion (SIC). Regime 2 has the minimum information criteria with an AIC of 8.1427 and SC of 8.2677. Tuaneh and Essi (2021) in their study on Markov-Switching Vector Autoregressive Modelling (Intercept Adjusted); Application to International Trade and Macroeconomic Stability in Nigeria (2000–2019) identified 2 regimes.

Model Selection

Eight models of the Markov switching mean model were estimated. The information criteria results are summarized in Table 3 below.

Table 3: Markov Switching Vector Autoregressive Models Selection

S/n	Estimated Models	Log-likelihood	Akaike info criterion	Schwarz criterion	Number of coefficients
1	MSM(2)-VAR(2)	739.6634	-5.954	-5.179	52
2	MSMAR(2)-VAR(2)	843.5392	-6.576	-5.324	84
3	MSMH(2)-VAR(2)	1013.128	-8.235	-6.311	62
4	MSMARH(2)-VAR(2)	1005.387	-7.891	-6.490	94
5	MSAR(2)-MVAR(2)	772.3266	-5.994	-4.802	80
6	MSH(2)-MVAR(2)	932.0147	-7.567	-6.703	58
7	MSARH(2)-MVAR(2)	1046.364	-8.280	-6.939	90
8	LINEAR(2)-MVAR(2)	738.514	-5.953	-5.192	51

Source: Researchers' Computation with E-views 11.0

The Markov-Switching Mean Vector Autoregressive Model permits a variety of specifications including means switching, autoregressive parameter switching and heteroscedastic parameter switching. Consequently, eight (8) Markov switching Mean Vector Autoregressive

Model were estimated and summarized in Table 3. The Markov switching autoregressive heteroskedastic (2) Mean Vector Autoregression model (2) c model of the Markov switching mean model had the highest log likelihood (1046.364) and the least information criteria (Akaike Information Criteria = -8.28, Schwarz Information Criteria = -6.939), consequently, the MSARH(2)-MVAR(2) with 90 parameters was selected.

Estimation of Model Parameters**Table 4: Estimation of Model Parameters of the selected Model (MS-ARH(2)-VAR(2))**

	Regime 1				Regime 2			
	DTEX	DTIM	DEXR	DINR	DTEX	DTIM	DEXR	DINR
Regime Dependent Intercept								
C	0.003	0.01 ⁵	0.00003	-0.011	0.003	0.015	0.00003	-0.011
	[0.365]	[1.669]	[0.107]	[-1.054]	[0.365]	[1.669]	[0.107]	[-1.054]
Regime Dependent Autoregressive Parameter								
DTEX(-1)	-0.289*	0.118	-0.001	0.023	-0.584*	0.172	-0.033	-0.579
	[-3.726]	[0.997]	[-0.788]	[0.304]	[-4.864]	[0.813]	[-0.686]	[-1.331]
DTEX(-2)	-0.159*	-0.025	-0.002	-0.013	0.011	0.148	-0.076	-0.692
	[-2.125]	[-0.213]	[-1.532]	[-0.196]	[0.097]	[0.723]	[-1.609]	[-1.614]
DTIM(-1)	0.073	-0.640*	0.001	0.006	0.001	-0.279*	0.017	0.108
	[1.463]	[-8.583]	[0.784]	[0.134]	[0.015]	[-1.974]	[0.512]	[0.366]
DTIM(-2)	0.043	-0.236*	0.00005	-0.019	-0.039	-0.099	0.005	-0.616*
	[0.854]	[-3.131]	[0.068]	[-0.420]	[-0.472]	[-0.693]	[0.151]	[-2.092]
DEXR(-1)	4.917*	-2.244	0.346*	-4.241*	0.306	0.1297	0.321*	0.443
	[2.394]	[-0.763]	[9.615]	[-2.481]	[0.835]	[0.205]	[2.227]	[0.338]
DEXR(-2)	1.061*	-0.484	-0.016*	-0.150	0.308	0.005	-0.015	-0.122
	[2.262]	[-0.674]	[-2.158]	[-0.362]	[0.590]	[0.005]	[-0.074]	[-0.066]
DINR(-1)	0.089	-0.017	0.002*	0.166*	-0.020	-0.114	0.002	-0.927*
	[1.769]	[-0.223]	[2.173]	[3.572]	[-0.464]	[-1.575]	[0.099]	[-6.074]
DINR(-2)	0.066	0.086	0.001	-0.102*	-0.082*	0.057	0.012	0.428*
	[1.263]	[1.078]	[1.239]	[-2.042]	[-2.188]	[0.878]	[0.802]	[3.154]
Regime Dependent Heteroschedastic								
SIGMA-DTEX	0.021*	0.006*	0.00001	-0.003*	0.020*	0.006	0.001	0.016
	[9.271]	[2.528]	[0.572]	[-1.987]	[5.085]	[1.315]	[0.822]	[1.553]
SIGMA-DTIM	0.006*	0.0493*	0.00005	0.002	0.006	0.059*	0.000	-0.028
	[2.528]	[9.317]	[1.244]	[0.901]	[1.315]	[5.053]	[-0.049]	[-1.634]
SIGMA-DEXR	0.00001	0.00005	0.000005*	0.00002	0.001	0.000	0.003*	-0.001
	[0.572]	[1.244]	[8.024]	[1.153]	[0.822]	[-0.049]	[5.068]	[-0.183]
SIGMA-DINR	-0.003*	0.002	0.00003	0.016*	0.016	-0.028	-0.001	0.260*
	[-1.987]	[0.901]	[1.153]	[7.450]	[1.553]	[-1.634]	[-0.183]	[5.113]

Source: Researcher's computation with Eviews 11.0, detail in appendix 9, Table 4.67

$$\begin{aligned}
 DTEX &= -0.289*DTEX_{(t-1)} - 0.159*DTEX_{(t-2)} + 0.074*DTIM_{(t-1)} + 0.043*DTIM_{(t-2)} + 4.917*DEXR_{(t-1)} \\
 &+ 1.061*DEXR_{(t-2)} + 0.089*DINR_{(t-1)} + 0.066*DINR_{(t-2)} + 0.003 \\
 DTIM &= 0.118*DTEX_{(t-1)} - 0.025*DTEX_{(t-2)} - 0.640*DTIM_{(t-1)} - 0.236*DTIM_{(t-2)} - 2.244*DEXR_{(t-1)} - \\
 &0.484*DEXR_{(t-2)} - 0.017*DINR_{(t-1)} + 0.086*DINR_{(t-2)} + 0.015 \\
 DEXR &= -0.001*DTEX_{(t-1)} - 0.002*DTEX_{(t-2)} + 0.001*DTIM_{(t-1)} + 0.00005*DTIM_{(t-2)} + 0.346*DEXR_{(t-1)} \\
 &- 0.016*DEXR_{(t-2)} + 0.002*DINR_{(t-1)} + 0.001*DINR_{(t-2)} + 0.00003 \\
 DINR &= 0.023*DTEX_{(t-1)} + -0.013*DTEX_{(t-2)} + 0.006*DTIM_{(t-1)} - 0.019*DTIM_{(t-2)} - 4.241*DEXR_{(t-1)} - \\
 &0.150*DEXR_{(t-2)} + 0.166*DINR_{(t-1)} - 0.102*DINR_{(t-2)} - 0.011
 \end{aligned}$$

Regime 1 Variance Covariance Matrix				
SIGMA	DTEX	DTIM	DEXR	DINR
DTEX	0.021*	0.006*	0.00001	-0.003*
DTIM	0.006*	0.0493*	0.00005	0.002
DEXR	0.00001	0.00005	0.000005*	0.00002
DINR	-0.003*	0.002	0.00003	0.016*

Regime 1: The Markov Switching Mean Autoregressive Heteroschedastic Vector Autoregressive Model in regime 1 also presented in Table 4 shows that; the first and the second lag of total export, and the first and the second lag of exchange rate significantly affected total export. However, only the lags 1 and 2 of total import had significant effects on total import. Exchange rate Lag 1 and lag 2, and lag 1 of exchange rate had significant effects on exchange rate while the first leg of the exchange rate and lags 1 and 2 of inflation rate significantly affected inflation rate. This agrees with the findings of Tuaneh and Essi 2021.

$$\begin{aligned}
 DTEX &= -0.584*DTEX_{(t-1)} + 0.011*DTEX_{(t-2)} + 0.001*DTIM_{(t-1)} - 0.039*DTIM_{(t-2)} + 0.306*DEXR_{(t-1)} \\
 &+ 0.308*DEXR_{(t-2)} - 0.0120*DINR_{(t-1)} - 0.082*DINR_{(t-2)} + 0.003 \\
 DTIM &= 0.172*DTEX_{(t-1)} + 0.148*DTEX_{(t-2)} - 0.279*DTIM_{(t-1)} - 0.099*DTIM_{(t-2)} + 0.1288*DEXR_{(t-1)} \\
 &+ 0.005*DEXR_{(t-2)} - 0.114*DINR_{(t-1)} + 0.057*DINR_{(t-2)} + 0.015 \\
 DEXR &= -0.033*DTEX_{(t-1)} - 0.076*DTEX_{(t-2)} + 0.017*DTIM_{(t-1)} + 0.005*DTIM_{(t-2)} + 0.321*DEXR_{(t-1)} \\
 &- 0.015*DEXR_{(t-2)} + 0.002*DINR_{(t-1)} + 0.012*DINR_{(t-2)} + 0.00003 \\
 DINR &= -0.579*DTEX_{(t-1)} - 0.692*DTEX_{(t-2)} + 0.108*DTIM_{(t-1)} - 0.616*DTIM_{(t-2)} + 0.443*DEXR_{(t-1)} \\
 &- 0.122*DEXR_{(t-2)} - 0.927*DINR_{(t-1)} + 0.428*DINR_{(t-2)} - 0.011
 \end{aligned}$$

Regime 2 Variance Covariance Matrix				
SIGMA	(DTEX)	(DTIM)	(DEXR)	(DINR)
(DTEX)	0.020*	0.006	0.001	0.016
(DTIM)	0.006	0.059*	0.000	-0.028
(DEXR)	0.001	0.000	0.003*	-0.001
(DINR)	0.016	-0.028	-0.001	0.260*

Regime 2: The Markov Switching Mean Autoregressive Heteroschedastic Vector Autoregressive Model in regime 2 also presented in Table 4 shows that; the first lag of total export and the second lag of inflation rate significantly affected total export. Similarly, only lags 1 of total import had significant effects on total import. More so, only Lag 1 of the exchange rate had significant effects on the exchange rate while lag 1 of total import and lags 1 and 2 of the

inflation rate significantly affected inflation rate. This result agrees with the findings of Tuaneh and Essi 2021.

Transition probabilities

The constant Markov transition probabilities for sample 2000M1-2019M6 is presented below.

Note that $P(i, k) = P(s(t) = k | s(t_1) = i)$

Where row = i and column = k

$$P_{ij} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 0.894 & 0.106 \\ 0.346 & 0.657 \end{bmatrix}$$

Hence $P_{11} = 0.894$, and $P_{22} = 0.657$. Where; $P_{11} + P_{12} = 1$, $P_{21} + P_{22} = 1$

The result implied that; given that the current state is in expansion, the probability of transitioning to expansion in the next period is 0.894 ie, and the probability of transitioning to a contraction in the next period is 0.106. Also, given that the current state is in contraction, the probability of transitioning to expansion in the next period is 0.346 and the probability of transitioning to a contraction in the next period is 0.657.

Expected Duration of Stay

The expected duration derived by $\left(\frac{1}{1-P_{ij}}\right)$ is 9.43 in regime 1 and 2.92 in regime 2 as also shown in the expected duration. The result implied that there is 89% probability of staying in regime 1 for a duration of 9 months 13 days. Also, there is a 66% probability of staying in regime 2 for a duration of 2 months 27 days.

Smoothened and Filtered Probability of MSIH(2)-VAR(2)

The smoothed probabilities and filtered probabilities are shown below provide inference on s_t conditional on all available sample information. In regime one $pr(s_t = 1)$, we find out that the smoothing process is high and remain high till 2014 and fluctuated up and down at the end of the regime. Regime two $pr(s_t = 2)$, contraction smoothing process is in opposite direction similar to regime one smoothing start from down and remained down till 2013 and fluctuate up and down till the end. These indicate an unstable price of oil in international which may lead to the financial crisis. The filtered probability has a similar pattern with the smoothed probability.

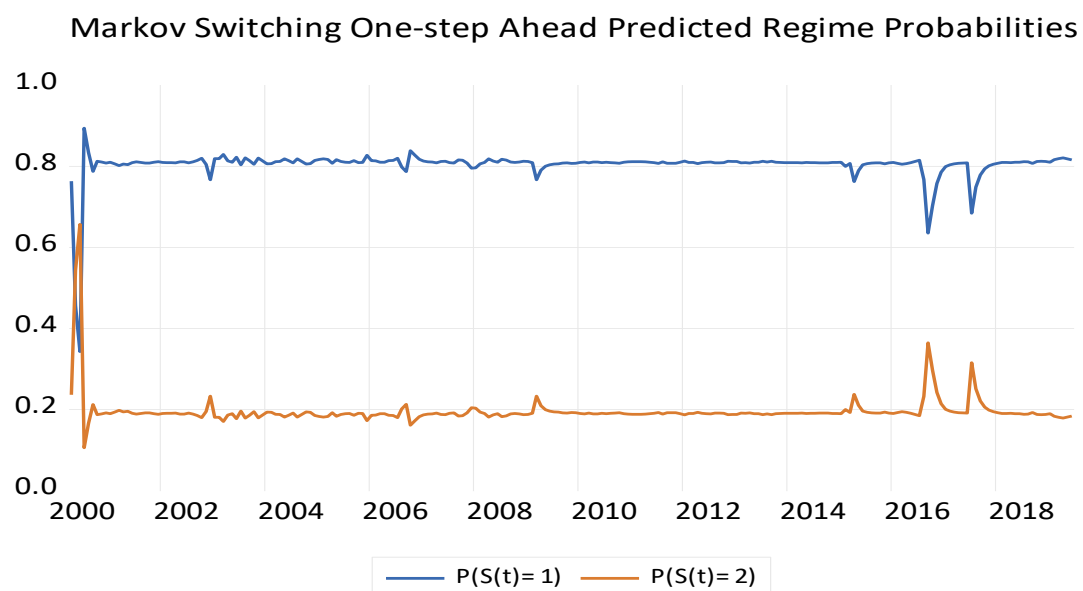


Table 5: Variance Decomposition Table (MSARH(2)-MVAR(2))

Regime 1						Regime 2				
Period	S.E.	DTEX	DTIM	DEXR	DINR	S.E.	DTEX	DTIM	DEXR	DINR
Variance Decomposition of DTEX:										
1	0.144	100.00	0.000	0.000	0.000	0.140	100.00	0.000	0.000	0.000
2	0.152	97.411	1.431	0.628	0.530	0.164	98.495	0.025	1.155	0.325
3	0.152	97.096	1.474	0.650	0.780	0.172	95.894	0.034	1.690	2.382
.	.	.							.	.
.	.	.							.	.
12	0.153	96.90246	1.646	0.660	0.792	0.555	11.513	22.92	0.816	64.749
Variance Decomposition of DTIM:										
1	0.222	3.909	96.09	0.000	0.000	0.242	3.424	96.576	0.000	0.000
2	0.263	2.964	96.99	0.039	0.006	0.253	3.125	92.102	0.173	4.600
3	0.267	3.125	96.59	0.051	0.225	0.273	3.794	80.933	0.183	15.090
.	.	.							.	.
.	.	.							.	.
12	0.268	3.297	96.375	0.058	0.270	0.997	2.836	30.530	0.651	65.984
Variance Decomposition of DEXR:										
1	0.002	0.210	0.799	98.991	0.000	0.056	1.307	0.081	98.612	0.000
2	0.002	0.395	1.572	97.268	0.765	0.059	1.257	0.365	98.360	0.018
3	0.002	1.861	1.523	95.074	1.542	0.060	2.725	0.356	96.302	0.617
.	.	.							.	.
.	.	.							.	.
12	0.002	1.877	1.533	94.992	1.599	0.069	2.698	6.402	72.961	17.939
Variance Decomposition of DINR:										
1	0.127	2.649	1.128	0.887	95.335	0.510	4.780	7.612	0.339	87.268
2	0.129	2.572	1.138	1.195	95.095	0.717	8.519	8.567	0.703	82.211
3	0.130	2.554	1.378	1.390	94.678	1.019	5.916	16.455	0.806	76.822
.	.	.							.	.
.	.	.							.	.
12	0.130	2.572	1.385	1.397	94.646	5.756	2.624	26.202	0.683	70.492
Cholesky Ordering: DTEX DTIM DEXR DINR										

Source: Researcher's computation with Eviews 11.0.

Variance Decomposition of the Selected Markov Switching Mean VAR Model Result **Variance Decomposition of Total Export [MSARH (2) -MVAR (2)]**

The percentage of the forecast error variance in regime 1 as shown in Table 5 showed that in the short run, 100% forecast variance in total export was self-explained. Total import, exchange rate, and inflation rate, however, were strongly exogenous. Moving forward, total export

decreases while total import increases. c of total export was 96.9% in the long run while the percentage forecast variance of total import, in the long run, was 1.63%. The percentage of the forecast error variance as also shown in Table 5 showed that in the short run, 100% forecast variance in total export was self-explained. The other variables were strongly exogenous. Progressing onward, total export decreases while total import increased. In the long run, the percentage forecast variance of total export was 11.5% while that of total import was 22.9%

Variance Decomposition of Total Import [MSARH (2) -MVAR (2)]

96.09% of forecast error variance of total import was self-explained other variables were strongly exogenous. In regime 2, the percentage of the forecast error variance in the short run was 96.57% self-explained. Other variables were strongly exogenous in predicting total imports. However, the forecast error variance of total import decreased while that of the other increased at a very slow rate. In the long run, the percentage forecast variance of total import was 80.9% while total export was 3.79%

Variance Decomposition of Exchange Rate [MSIARH (2) -VAR (2)]

In regime 1, the percentage of the forecast error variance of the exchange rate as shown in Table 5 was 98.99% self-explained. Total export, total import, and inflation rate, very strong exogeneity. Moving into the future, the exchange rate decreases slowly while the other variables also increased slowly. In the long run, the percentage forecast variance of the exchange rate was 0.94% while the percentage forecast variance of total export and total import, in the long run, was 1.86 and 1.52% respectively.

CONCLUSION AND RECOMMENDATIONS

The VAR model selected lag length 2, the MS-MVAR analysis identified two regimes (High and low). The information criteria selected the Markov-Switching Autoregressive Heteroschedastic 2 Vector Auto-regression 2 Model [MSIARH (2) - VAR (2)] as the best model with the least information criteria. The Variance Decomposition of the endogenous variables showed that the variables were self-explanatory in the short run. In the long run, the influence decreased but at a very slow pace. The strong exogeneity of the other variables also increased at a slow pace.

The right-hand side variables (supposed exogenous variable) should be tested for endogeneity before the decision on a single equation or system equation estimation. The possibility of regimes should be determined. All the classes of the Markov-Switching Vector Autoregressive Models should be estimated and the best selected using the information criteria.

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